“Advanced”-\(\delta f\) simulations of the NBI heating in Wendelstein 7-X

Martin Schmidt, Andreas Werner
Max-Planck-Institut für Plasmaphysik, EURATOM Association, 17491 Greifswald, Germany

Introduction
The \(\delta f\) Monte Carlo technique is a powerful tool for kinetic calculations in tokamak and stellarator plasmas [1-3]. The idea of the commonly used \(\delta f\) approach depends on the determination of the deviation \(\delta f\) from a Maxwellian. Starting from the drift kinetic equation, linearization yields a marker equation for the weights of the test particles. In the case of NBI heating, one wants to simulate the slowing down behavior of suprathermal particles, which are far away from a Maxwellian. The application of the commonly used \(\delta f\) Monte Carlo technique in such a case offers no advantage over the “full-\(f\)” method. The “full-\(f\)” method in this case is a very time consuming task, because one has to calculate several slowing down times of fast particles. In contrast to tokamaks, the radial drift of ripple-trapped particles results in a significantly enhanced radial transport in classical stellarators. In the quasi-isodynamic W7-X configurations, fast particles remain near their “birth” flux surface for a long time. With the assumption of slowing-down on the flux surfaces, a Fokker-Planck solver delivers a rather good estimate being very fast and accurate. A new approach for the NBI heating starts with the \textit{ansatz} for the distribution function \(f = f_M + f_1 + \delta f\), where \(f_M\) represents the Maxwellian. The second part, \(f_1\), describes the flux-surface-averaged part of the deviation from \(f\). These two parts, \(f_M\) and \(f_1\), are used in the marker equation with the appropriate sources and sinks. A \(\delta f\) scheme formulated in this way solves the problem in 2nd order to determine a \(\delta f\) which contains the desired information on drift effects. Therefore the “advanced” \(\delta f\) Monte Carlo method offers improved statistical behavior compared with full-\(f\) and usual \(\delta f\) methods.

The “advanced” \(\delta f\) method
The new “advanced” \(\delta f\) method is an extension of the \(\delta f\) approach which additionally to the Maxwellian \(f_M\) takes a non-equilibrium part \(f_1\) out of the distribution function and concentrates the simulation on the remaining part \(\delta f = f - (f_M + f_1)\). \(f_1\) is a solution of the flux-surface-averaged equation which delivers a good estimate, so that \(\delta f\) should be small compared to \(f_1\).

Let’s consider the drift kinetic equation. Take the distribution function \(f(x, p, v)\) with \(p = v_\parallel /v\). Then

\[
V(f) - C^p(f) - C^v(f) = S_{NBI} - S_l
\]
where
\[ V = \left( \frac{u_p}{B} B + v_{\nabla B} + v_{E \times B} \right) \cdot \nabla + \dot{p} \frac{\partial}{\partial p} + \dot{v} \frac{\partial}{\partial v} \]  
represents the Vlasov operator where \( u_p B / B \), \( v_{E \times B} \) and \( v_{\nabla B} \) are the drift velocities of the guiding center and the terms \( \dot{p}, \dot{v} \) are
\[ \dot{p} = -\frac{1-p^2}{2B^2} \left( v B \cdot \nabla B + \frac{p}{B} (B \times \nabla B) \cdot \nabla \Phi \right), \dot{v} = -v \frac{1+p^2}{2B^3} (B \times \nabla B) \cdot \nabla \Phi. \]

\( C^p(f) \) represents a pitch angle and \( C^v(f) \) an energy collision operator. The term \( S_{NB} \) describes a particle source due to the NBI and \( S_l \) describes the particle losses.

The new linearization ansatz for the NBI heating of \( f \) yields \( f = f_M + f_1 + \delta f \) where \( f_M \) represents the Maxwellian as 0th order distribution function and the second part \( f_1 \) describes the flux-surface-averaged part of the deviation of \( f \) and is obtained as the solution of the equation
\[ -C^p(f_1) - C^v(f_1) = \langle S_{NB} \rangle - \langle S_l \rangle, \]
where \( \langle ... \rangle \) is the flux-surface-averaging operator and the term \( S_l \) is used, e.g. \( \dot{n} \) and \( \dot{T} \) modeling, for the particle and power balance to obtain a stationary solution. \( f_1(r, p, v) \) can be obtained by a Fokker-Planck solver which is very fast. Ignoring the Maxwellian \( f_M \) for the neoclassical transport leads to an inhomogeneous equation for \( \delta f \)
\[ V(\delta f) - C^p(\delta f) - C^v(\delta f) = -v_{\nabla B} \left| \frac{\partial f_1}{\partial r} \right|_r - \dot{p} \frac{\partial f_1}{\partial p} - \dot{v} \frac{\partial f_1}{\partial v}, \]
where \( S_{NB} - \langle S_{NB} \rangle \) and \( S_l - \langle S_l \rangle \) are neglected. The right hand side leads to the marker equation
\[ w = -\int \left( v_{\nabla B} \left| \frac{\partial f_1}{\partial r} \right|_r + \dot{p} \frac{\partial f_1}{\partial p} + \dot{v} \frac{\partial f_1}{\partial v} \right) dt. \]
In order to get the part \( \delta f \) one assumes that the distribution function is highly localized, so one can represent \( \delta f \) as the sum of delta functions in the form
\[ \delta f = \sum_i w_i \delta(x - \bar{x}_i) \delta(p - p_i) \frac{1}{v^2} \delta(v - v_i) \]
and the appropriate convolution yield to the desired quantities.

**Simulation setup**

As a proof of principle an example for the planned stellarator project W7-X, a \( H^0 \) in \( H^+ \) injection with four nearly perpendicular (pitch= 0.19) beam sources of 1.75 MW each and injection energy of 55 kV was chosen. The density and temperature profiles are shown in figure 1 and the power deposition profiles of fast particles are shown in figure 2; the average magnetic field is 2.5 T.
In the first step, the distribution function $f_1$ and its derivatives are calculated by the Fokker-Planck solver. To initialize the simulation particles, their toroidal angle $\phi$ and the poloidal angle $\theta$ are distributed randomly. The radial position $r$, the velocity coordinates $p$, $v$ and the weight $w$ of each particle are chosen with a random shooting process according to the distribution function $f_1$. $v$ is in every case chosen suprathermal. If a particle gets lost or thermalized it is replaced by a new one and so the number of particles is kept constant.

The simulation then starts by advancing the particle by an appropriate time step. After each integration step the collision operators are called. The simulation runs until the total weight of all particles converges.

During the simulation the time average over the weights of the particles at their radial positions is calculated in order to get the radial profile of the distribution function $\delta f$. For every call of the energy collision operator the separate power transfer from simulation particles to background ions and electrons is stored to derive the power deposition profiles. The ion-slowing down current can then be directly determined as a moment of the distribution function.

**Simulation results**

In this section the results obtained by the $\delta f$ method are benchmarked with the results of the equivalent full-$f$ method and compared with the results of the flux-surface-averaged equation. Both Monte Carlo methods show, as expected, nearly the same results. The only difference between the advanced $\delta f$ and the full-$f$ method is the simulation time. In the case of the full-$f$ method the simulation time depends on the slowing down time of highly energetic ions, which is in this case about 20 milliseconds. For the same integration time step, the simulation
time of the advanced $\delta f$ method was 2 milliseconds and so one obtained a speed-up of one order of magnitude. In the simulation 1024 particles and an integration time step $\Delta t = 10^{-8}$ s were used.

First, the resulting power deposition profiles to electrons and ions are plotted in figure 3. The flux-surface-averaged solution belonging to $f_1$ shows an overestimation at inner radii and an underestimation at outer radii in the energy transfer to background ions. The power deposition profiles to electrons obtained by both Monte Carlo methods show a good agreement with the result of the flux-surface-averaged solution (figure 3). Only at inner radii do they exhibit the same behaviour as the ion deposition profile. This overestimation in the Fokker-Planck solution is caused by the decrease of the “birth” profiles (figure 2) at this position and is affected by radial diffusion in the Monte Carlo simulations.

In figure 4 the ion-slowing down current densities are compared. Here, $f_1$ shows an underestimation in the region from $r = 10$ to 40 cm compared to the $\delta f$ method.

Figure 5 shows the “power deposition pattern” of escaping ions at the last closed magnetic surface. If this is extrapolated to the inner vessel one gets an estimation of the expected wall
Conclusion
The “advanced” $\delta f$ method is a more sophisticated and faster approach compared with the “full-$f$” method. It determines a correction to a good numerical approximation of the dominating part $f_1$ of the distribution function whereas the “full-$f$” approach solves the problem for the entire distribution function. In the latter case one must run the simulation the slowing down time of maximum energy whereas in the $\delta f$ method one reaches convergence on the radial diffusion timescale.

References