Reply

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1. Introduction

In his comments Pierson has raised a number of interesting questions. To place the discussion in proper perspective, we should perhaps first point out that the purpose of our paper was not to experiment with various parametrical representations of the wave spectrum—as interesting as these are—but to apply recent results on the energy balance of the wave spectrum to the practical task of wave prediction. These results, summarized in Hasselmann *et al.* (1973, hereafter called J), have changed the structure of the wave prediction problem.

On the one hand, the problem has apparently be-

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come more complicated through the dominant role of the nonlinear resonant energy transfer mechanism in determining the form of the spectrum and the rate of shift of the spectral peak in a growing wind sea. With present day (and foreseeable) computers the rigorous Boltzmann integral expression for the energy transfer cannot be integrated numerically, even approximately, for an arbitrary wave spectrum within the framework of a numerical wave prediction scheme. In another respect, however, the problem has in fact become simpler, because the nonlinear energy transfer continually readjusts the energy distribution within the spectrum to a selfstabilizing, quasi-universal form in which the individual geometric signatures of the generating wind fields are largely lost. Both factors point to the need for simplified parametrical representations in which the usual discretization of the wave spectrum in terms of several hundred degrees of freedom is replaced by far simpler representations containing only a few free parameters.

Although parametrical representations clearly have their specific problems, as Pierson points out, we feel this is not the principal difficulty confronting the development of wave-prediction models. For this reason, many of the valid points raised by Pierson were not discussed in detail in the paper. For example, it is clear that since the wave spectrum is a statistical quantity, the parameters describing the wave spectrum will also be statistical variables. Thus, whenever presenting a spectral parameter one should also indicate its statistical sampling variability in the same way that plots of power spectra are normally presented with error bars denoting their confidence limits. This was not done in our paper, as it is obvious from a cursory inspection of our data that the natural geophysical variability of the parameters shown by far exceeds the sampling variability associated with the statistical uncertainty of the spectra to which the parametrical forms were fitted (see also J, § 2.4). The understanding of this large natural variability, which is apparent even in the highly selected JONSWAP data, is one of the important problems which must be faced in assessing the ultimate limitations of numerical wave prediction models.

After these general comments we turn to the specific points raised by Pierson, following his subdivision into four groups of questions.

2. The parameters α and γ

It is not claimed that our method of fitting parameters to fetch-limited and fully developed spectra is unique. After experimenting with a number of different functions, we chose the five-parameter function (H 2.1) as a manageable but sufficiently flexible form simply because 1) it gave a good fit to essentially all the spectra we encountered, and 2) it was conceptually rather simple, being derived from the well-known Pierson-Moskowitz spectral shape by multiplying with a narrow band "peak-enhancement" function, effective only near the peak, to model the observed variations in the width and amplitude of the peak. We agree with Pierson that Salfi's (1974) method of representing fetch-limited spectra in which the fully developed PM spectrum is replaced by a low-frequency cutoff function G below some fetch-dependent peak frequency also yields a spectrum rather similar in shape to the mean JONSWAP spectrum, with an equivalent peak enhancement factor γ near 3 for small fetches. However, it should be noted that Salfi's form contains only one free parameter, the peak frequency, which then determines the peak enhancement factor and the equivalent left- and right-sided peak widths. Thus Salfi's form is not general enough to investigate the observed variability of spectral shapes, which was the principal reason we introduced a parametrical representation of the spectrum containing three free-shape parameters, in addition to the peak frequency and energy-scale parameter α. However, when considering only the *mean* evolution of a growing wind sea as a function of fetch, where we found γ, σ_a and σ_b in our representation to be essentially constant until one comes very close to the fully developed state, any one of the different empirical formulas quoted in J and H are probably acceptable within the scatter of the observations. Our only criticism of Salfi's representation is that it fails to reproduce the decrease of α with fetch found in J and by a number of other workers (cf. J, Fig. 2.7). (The minor differences—less than 20%—in computing α according to Salfi's or our procedure are negligible in this context.)

As pointed out in J, however, the reason for introducing a parametrical representation of our fetch-limited data was not to present yet another empirical formula for the growth of wind waves for the ideal case of a uniform wind blowing orthogonally off a straight shore, but rather to clarify the physics responsible for the observed wave growth in order to develop a wave prediction model applicable for *arbitrary* wind fields and boundaries. As a result of our dynamical analysis, we believe that wave prediction models which ignore the nonlinear energy transfer and are based solely on the combination of a Miles-Phillips generation mechanism and a limiting equilibrium range characterized by a universal Phillips constant α are physically incorrect. While these models can be tuned to give similar results to our model for the ideal fetch-limited situation (except for the fetch-dependence of α), we believe they will produce incorrect predictions when applied to other wind fields. These conclusions are based on rigorous calculations of the nonlinear energy transfer for observed spectra, rather than actual comparisons of the performance of different models for complex wind fields. Such experiments are clearly needed.

3. Sampling variability and bias

Since the observed spectra to which our parametrical form (2.1) is fitted is a statistical estimate of the "true" spectrum, the fitted parameters f_m, α, γ, σ_a and σ_b also represent statistical variables. If the parameters depend linearly on the spectrum, their statistical properties can readily be evaluated from the standard Tukey statistics of spectral estimates. Unfortunately, it is a characteristic of most parametrical fits, including ours, that the fitting algorithm represents a rather complex nonlinear functional dependence of the parameters on the spectrum. Thus it is possible to give only rather crude estimates of the statistical variability of the parameters. For a typical JONSWAP spectrum computed with 36 degrees of freedom, the standard deviations of the individual spectral estimates are approximately 24% of the spectral values. We estimate the standard deviations or our parameters, expressed as percentages of the variables themselves, to be of the following orders: f_m: 5–10%; α: 4–6%; γ: 20–40%; σ_a, σ_b: 20–50%. These variations are relatively insignificant compared with the much larger natural variability of the parameters found even for the highly selected, "ideal" JONSWAP cases.

Pierson has further suggested that our γ estimates are biased toward too high values through our rejection of multiple-peaked spectra. Multiple-peaked spectra were rejected only for the Moskowitz (1963) set of fully-developed spectra. The rejections were limited to cases where the deviation of the spectrum from a single-peaked Pierson-Moskowitz form were considerably greater than could be explained by the variability of spectral estimates. Multiple-peaked spectra which were consistent with the estimated statistical variability were retained. Similarly, all multiple-peak spectra of the other data sets were retained. We believe our rejection of these multiple-peaked spectra to be fully justified and certainly as rigorous a procedure as that of Pierson and Moskowitz when they rejected the averaged 18.01 m s^{-1} spectrum in the development of their model because it "seems to be distorted in shape compared to the other four" (Pierson and Moskowitz, 1964, p. 5183).

However, this point need hardly be argued. We reanalyzed Moskowitz' fully developed spectra, because it was clear that Moskowitz' method of averaging all spectra within a 5 kt wind band would necessarily yield a flatter average spectrum than the technique we used, in which the spectral shape parameters were first fitted to each spectrum separately and then the shape parameters averaged afterward. We suspected that this may be the reason the JONSWAP data indicated no systematic decrease of γ relative to its mean value of 3.3 with increasing fetch, although the fully developed Pierson-Moskowitz spectrum corresponds to a value of $\gamma=1$. Contrary to our suspicions, however, our reanalysis of Moskowitz' data *did* indicate a significantly lower value of $\gamma=1.4$ for fully developed spectra, and our conclusion was that the Pierson-Moskowitz spectrum was indeed a fair description of a fully developed sea, despite the ambiguities in defining an average spectrum. In view of the standard deviation of 62% in the γ-values of the Moskowitz set of fully developed spectra (cf. H, Table 1) it appears a rather fine point to debate whether the residual difference between our value of $\gamma=1.4$ and the Pierson-Moskowitz value $\gamma=1$ is due to the genuine difference in averaging techniques or the rejection of multiple-peaked spectra which were considered—correctly or incorrectly—to be swell contaminated.

4. The use of winds at 10 m

We have used 10 m winds consistently throughout our analysis except at one point, where we have mistakenly substituted the 10 m wind instead of the 19.5 m wind into the expression for the peak frequency of the fully developed PM spectrum. We are grateful to Pierson for drawing attention to this error. It can be corrected, as he points out, by setting $\nu=0.13$ instead of $\nu=0.14$ as the transition point from a fetch-limited to a fully developed sea. Fortunately, the error is such that it led only to the exclusion of some marginal cases of almost fully developed seas from our fetch-limited data sets, rather than the incorrect inclusion of fully-developed cases. Thus our results are not materially affected.

5. The equilibrium range

Throughout J and H, the f^{-5} "equilibrium" range referred for field data always to frequencies in the gravity-wave region between the peak frequency and the natural cutoff of most of the instruments used, around 0.5 Hz. Within this range, the data could be scaled quite well using Kitaigorodskii's scaling relations.

The α-$\tilde{x}$ relation (2.44) in J, from which the α-ν relation (6.1) in H was derived, was based not only on field data but also on laboratory results. These were included because they were still within the gravity-wave range and scaled quite well with the field data in accordance with Kitaigorodskii's relations. However, it was pointed out in J that the JONSWAP data alone would have produced a steeper α-$\tilde{x}$ relation (as evidence in the steeper α-ν relation in H, Fig. 9a), so that Kitaigorodskii's relations apparently do not apply exactly.

The relation (6.1) was nevertheless retained in H as it was found to give good agreement again with the composite data set (cf. H, Fig. 9j). We agree with Pierson that one must expect Kitaigorodskii's scaling laws to break down in the capillary-wave range, but large systematic discrepancies were not evident in the field data for gravity-wave frequencies below 0.5 Hz.

REFERENCES

Hasselmann, K., T. P. Barnett, E. Bouws, H. Carlson, E. E. Cartwright, K. Enke, J. A. Ewing, H. Gienapp, D. E. Hasselmann, P. Kruseman, A. Meerburg, P. Müller, D. J. Olbers, K. Richter, W. Sell, and H. Walden, 1973: Measurements of wind-wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP). *Deut. Hydrogr. Z.*, A8, No. 12.