Slab magnetised non-relativistic low-beta electron-positron plasmas: collisionless heating, linear waves and reconnecting instabilities

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The properties of a non-relativistic magnetised low beta electron-positron plasma in slab geometry are investigated. The two species are taken to be drift-kinetic while we retain Larmor radius effects in quasi-neutrality, and inertia in Ohm's law. A linear analysis shows that, for small magnetic perturbations, Alfvénic perturbations travel at the electron Alfvén speed, which is based on the electron mass. We discuss the role of the displacement current when Larmor scales and Debye scales effects are both retained. We predict the existence of a kinetic electron Alfvén wave which connects to the K-modes of Mishchenko et al. (2017) in the electrostatic limit. It is found that linear drift waves are not supported by the system if the two species have the same temperature. Tearing modes can be driven unstable by equilibrium current density gradients. Also in this case, the characteristic time is based on the electron Alfvén speed. Nonlinear hybrid fluid-kinetic equations are also derived. It is shown that each species is described, to leading order, by the Kinetic Reduced Electron Heating Model (KREHM) kinetic equation of Zocco and Schekochihin [Physics of Plasmas 18, 102309 (2011)]. The model is extended to retain first order Larmor radius effects. It supports collisionless dispersive waves, which can greately impact nonlinear magnetic reconnection. Diamagnetic effects enter the nonlinear equations via the first order magnetic compressibility. A minimal nonlinear model for $2D$ low-frequency isothermal pair plasmas is derived.

1. Introduction

Electron positron plasmas have played a crucial role in the theory of magnetic reconnection. By exploiting the similarities of a simple fluid model [Chacón *et al.* (2008)] and electron magnetohydrodynamics (EMHD) with electron inertia [(Chacón *et al.* 2007; Zocco *et al.* 2008, 2009)], Chacón *et al.* (2008) have shown that dispersive waves are not the cause of fast magnetic reconnection. This result came as a confirmation of ealier particle-in-cell simulation results [(Bessho & Bhattacharjee 2005; Daughton & Karimabadi 2007)]. Non-relativistic electron positron plasmas, however, are not merely models which are useful to settle controversies among theoreticians. There is now great excitement about the creation of a laboratory electron positron plasma [(Pedersen *et al.* 2012; Saitoh *et al.* 2014)] which, by itself, justifies new investigations in this field.

In this article we revisit some fluid equations similar to those of Chacón *et al.* (2008), but in the framework of gyrokinetics for magnetic reconnection [(Zocco & Schekochihin 2011; Loureiro *et al.* 2013; Zocco *et al.* 2015; Zocco 2015; Loureiro *et al.* 2016)]. The new aspect here introduced is in the quasineutrality equation. The smallest kinetic scale that enters the quasineutrality equation is taken to be the Larmor scale, $\rho_e = v_{the}/\Omega_c$, which is assumed to be much smaller than the intertial scale $d_e = \rho_e/\sqrt{\beta}$, where β is the ratio of kinetic to plasma pressure, and $\Omega_c = eB/(mc)$ is the cyclotron frequency. The inclusion of the Debye length is avoided [(Helander 2014; Helander & Connor 2016)], as this would require a covariant treatment. However, the possible effects of the displacement current are discussed. Some considerations on linear waves and reconnecting instabilities are in Sec. (2). In Section (3), we propose an adaptation of the Kinetic Reduced Electron Heating Model (KREHM) equations [(Zocco & Schekochihin 2011)] to low-beta pair plasmas. Particular attention is devoted to its isothermal limit. Conclusions are in Section (4).

2. Linear analysis

In our system, density fluctuations are calculated by taking the zeroth moment of perturbed distribution function $\delta f_{e\mp} = -e_{\pm}\varphi F_0/T_0 + h_{e\mp}$, where the non-adiabatic part of δf satisfies the electromagnetic gyrokinetic equation of Frieman & Chen (1982)

$$\begin{aligned}\frac{dh_{e\mp}}{dt} + v_{\|}\hat{\mathbf{b}}\cdot\nabla h_{e\mp} &= \frac{e_{\mp}F_0}{T_0}J_0\left(k_\perp\rho_e\hat{v}_\perp\right)\frac{\partial}{\partial t}\left(\varphi - \frac{v_\|}{c}A_\|\right)\\ &-\frac{c}{B_0}\mathbf{e}_z\cdot\nabla\left(\varphi-\frac{v_\|}{c}A_\|\right)\times\nabla F_0 + \left(\frac{dh_{e\mp}}{dt}\right)_{coll},\end{aligned} \tag{2.1}$$

where $d/dt = \partial_t + B_0^{-1}\{J_0\varphi,\}$, $\hat{\mathbf{b}}\cdot\nabla = \partial_z - B_0^{-1}\{J_0A_\|,\}$, $\{A,B\} = \partial_x A\partial_y B - \partial_y A\partial_x B$, $v_\|$ is the particles velocity in the $z-$direction, parallel to the guide field of modulus B_0, F_0 is the Maxwellian equilibrium, x and y are Cartesian space co-ordinates orthogonal to z, and J_0 is the Bessel function. The result is the familiar continuity equation

$$\frac{d}{dt}\frac{\delta n_{e\mp}}{n_0} = -\hat{\mathbf{b}}\cdot\nabla u_{\|e\mp} - i\omega_{*e\mp}\frac{e^{\mp}\varphi}{T_0}, \tag{2.2}$$

where we are using the local approximation

$$\mathbf{v}_E\cdot\frac{\nabla n_0}{n_0} = -i\omega_{*e\mp}\frac{e^{\mp}\phi}{T_0} \tag{2.3}$$

for the background density gradient, which introduces effects associate with the diamagnetic frequency $\omega_{*e\mp} = \mp i v_{the}/(2L_n)\rho_e\partial_y$, where $\nabla n_0/n_0 \approx -L_n^{-1}$, and $(\partial h/\partial t)_{coll}$ is the collisional operator, and $k_\perp\rho_e \ll 1$.

Poisson's equation for the electrostatic potential, φ, is derived from the quasineutrality condition

$$\int d^3\mathbf{v}\delta f_{e^-} = \int d^3\mathbf{v}\delta f_{e^+}. \tag{2.4}$$

We consider $h_e = \left(e^{\mp}\varphi/T_0 + \delta n_{e\mp}/n_0 + 2v_\|u_{\|e\mp}/v_{the}^2\right)F_{0e\mp} + g_{e\mp}$, where $\int d^3\mathbf{v}g_{e\mp} = \int d^3\mathbf{v}v_\|g_{e\mp} \equiv 0$. Then

$$2\frac{T_0}{e}\left(\frac{\delta n_{e^+}}{n_0} - \frac{\delta n_{e^-}}{n_0}\right) = -\rho_e^2\nabla^2\varphi, \tag{2.5}$$

where we see that

$$\frac{\delta n_{e^+}}{n_0} - \frac{\delta n_{e^-}}{n_0} \sim \beta\frac{e\varphi}{T_0} \ll \frac{e\varphi}{T_0}, \tag{2.6}$$

if $k_\perp d_e \sim 1$, with $d_e = c/\omega_{pl}$, $\omega_{pl}^2 = 4\pi n_0 e^2/(cm_e)$, $\beta = 8\pi n_0 T_0/B^2 \ll 1$, and

$$d_e \gg \rho_e \gg \lambda_D, \tag{2.7}$$

guarantees balance in Eq. (2.5). Equation (2.5) is valid, stricly speaking, only if perpendicular temperature fluctuations, associated with finite Larmor radius effects, are neglected. We consider this regime for the moment, and leave a more rigorous treatment of temperature fluctuations for the following Section which is concern with nonlinear physics. Before proceeding, however, we notice that Eq. (2.5) is similar to the quasineutrality condition used in Ref. Helander (2014) where the Debye length is replaced by the Larmor scale. This is a fundamental requirement to avoid the otherwise awkward condition $d_e \sim \lambda_D$, which would imply $v_{the} \sim c$.

We calculate the $v_\parallel-$moment of Eq. (2.1), to obtain

$$\begin{aligned}\frac{d}{dt}\left(A_\parallel + \frac{mc}{e^\mp}u_{\parallel e^\mp}\right) &= -c\frac{\partial\varphi}{\partial z} - \frac{T_0 c}{e^\mp}\hat{\mathbf{b}}\cdot\nabla\left(\frac{\delta n_{e^\mp}}{n_0} + \frac{\delta T_{\parallel e^\mp}}{T_0}\right)\\ &+ i\omega_{*e^\mp}\left(1+\eta_e\right)A_\parallel + \frac{mc}{e}\nu\left(u_{\parallel e^-} - u_{\parallel e^+}\right),\end{aligned} \tag{2.8}$$

where $\eta_e = n_0\nabla T_0/(T_0\nabla n_0)$, ν a constant collision frequency, $\delta T_{\parallel e^\mp} = n_0^{-1}\int dv_\parallel 2\hat{v}_\parallel^2 h_{e^\mp}$ and a simple Lenard-Bernstein collisional model operator has been used [Lenard & Bernstein (1958); Zocco & Schekochihin (2011)]. Parallel Ampère's law gives

$$\frac{e}{mc}d_e^2\nabla^2 A_\parallel = u_{\parallel e^-} - u_{\parallel e^+}. \tag{2.9}$$

Equation (2.2), if we take into account of Eq. (2.5), implies that

$$\omega \sim k_\parallel v_{A,e} \;\text{ and }\; \frac{v_{the}}{c}A_\parallel \sim \sqrt{\beta}\varphi, \tag{2.10}$$

where $v_{A,e} = B/\sqrt{4\pi m_e n_0}$ is the Alfvén speed based on the electron mass. The system is closed with an equation for the parallel temperature fluctuations, derived using a highly collisional fluid closure, $\nu \gg \omega$, for the flux of energy [Zocco & Schekochihin (2011); Zocco *et al.* (2015)]

$$\begin{aligned}\frac{d}{dt}\frac{\delta T_{\parallel e^\mp}}{T_0} &= \frac{v_{the}^2}{2\nu}\left(\hat{\mathbf{b}}\cdot\nabla\right)^2\frac{\delta T_{\parallel e^\mp}}{T_0} - i\frac{v_{the}^2}{2\nu}\hat{\mathbf{b}}\cdot\nabla\eta_{e^\mp}\omega_{*e^\mp}\frac{e^\mp A_\parallel}{T_0}\\ &- i\eta_{e^\mp}\omega_{*e^\mp}\frac{e^\mp\varphi}{T_0} - 2\hat{\mathbf{b}}\cdot\nabla u_{\parallel e^\mp}.\end{aligned} \tag{2.11}$$

This is just a choice that falicitates the forthcoming discussion. More precisely, we are considering the semi-collisional limit [Drake & Lee (1977)]

$$1 \ll \sqrt{\frac{\nu}{\omega}} \sim \frac{k_\parallel v_{the}}{\omega} \ll \frac{\nu}{\omega}, \tag{2.12}$$

with(i) $\omega \sim \omega_{*e}$. Electron thermal conduction effects are negligible for

$$1 \ll \frac{k_\parallel v_{the}}{\omega} \ll \sqrt{\frac{\nu}{\omega}}, \tag{2.13}$$

then Eq. (2.11) becomes

$$\frac{d}{dt}\frac{\delta T_{\parallel e^\mp}}{T_0} \approx -i\omega_{*e^\mp}\frac{e^\mp\varphi}{T_0} - 2\hat{\mathbf{b}}\cdot\nabla u_{\parallel e^\mp}. \tag{2.14}$$

The system could easily be left completely kinetic, then Eq. (2.11) would couple to higher order moments. However, each of these moments would follow a universal equation when projected on the basis of Hermite polynomials which allow for an efficient treatment of the non-isothermal case $\delta T_\parallel \neq 0$ [Zocco & Schekochihin (2011); Loureiro *et al.* (2013);

(i) The numerical factor does not coincide with the one evaluated by Braginskii (1965), since we are using a collision operator model.

Zocco *et al.* (2015); Zocco (2015); Schekochihin *et al.* (2016)]. In the truly collisionless case the hierarchy of Hermite moments generates a plasma response which was proven to be equivalent to the collisionless response evaluated via Landau contour integration [Zocco (2015)]. The isothermal approximation instead, $\delta T_\parallel \equiv 0$, would be described by the electron response of the nonlinear model of Schep *et al.* (1994). In the context of linear magnetic reconnection, the presence of temperature fluctuations is a technicality that has an impact on the transition from collisional to collisionless regimes, but it is irrelevant when one wants to estimate reconnection rates for very small but finite collisionality. The inclusion of the resonant electron response (i.e. Landau resonance) is not necessary to obtain corret reconnection rates in the collisionless limit, since the relevant condition to transition into a collisionless reconnection regime is that the inertial scale exceeds the resistive one, and this can occur even when collisions are finite (ii). For this reasons, we are justified to use Eq. (2.11) and yet consider a collisionless limit for linear magnetic reconnection. Nonlinearly, the role of high order moments that couple to the equation for temperature fluctuations is very important, as it was showed by Loureiro *et al.* (2013).

2.1. *Waves*

2.2. *High thermal conductivity*

We consider the approximation of high thermal conductivity, therefore

$$\frac{d}{dt}\frac{\delta T_{\parallel e^\mp}}{T_0} \approx -i\omega_{*e^\mp}\frac{e^\mp\varphi}{T_0} - 2\hat{\mathbf{b}}\cdot\nabla u_{\parallel e^\mp}. \tag{2.15}$$

We use Eq. (2.15) in Eqs. (2.8), we then add the parallel moment equations (2.8) of the two species, to notice that diamagnetic effects cancel exactly. Thus, we obtain

$$A_\parallel - \frac{k_\parallel c}{\omega}\varphi = \frac{\nu}{i\omega}\left(1 - i\frac{\omega}{2\nu} - \frac{3}{4}\frac{k_\parallel^2 v_{the}^2}{i\omega\nu}\right)k_\perp^2 d_e^2 A_\parallel. \tag{2.16}$$

On the other hand, Poisson's equation (2.5), after using the continuity equations and Ampère's law (2.9), becomes

$$\varphi = \frac{1}{2\beta_e}\frac{k_\parallel v_{the}}{\omega}\frac{v_{the}}{c}A_\parallel. \tag{2.17}$$

By combining Eq. (2.16) and (2.17), we obtain

$$\omega^2 = \frac{1}{2}\frac{k_\parallel^2 v_{A,e}^2}{1 + \left(1 + i\frac{2\nu}{\omega}\right)k_\perp^2 d_e^2/2}, \tag{2.18}$$

where we are taking the limit

$$k_\perp^2\rho_e^2 \sim \beta \ll 1. \tag{2.19}$$

Thus, we find no drift wave, a result also obtained by Helander (2014). In the "collisionless" regime ($\omega \gg \nu$) we find the dispersive waves

$$\omega^2 = \frac{1}{2}\frac{k_\parallel^2 v_{A,e}^2}{1 + k_\perp^2 d_e^2/2}, \tag{2.20}$$

(ii) See, for istance, [Zocco *et al.* (2015)] where the truly collisoinless electron conductivity, evaluated via Landau contour integration, is reproduced vey well by a truncated continued fraction solution generated by a Hermite expansion of the electron distribution function.

which, at long wavelengths, becomes a shear Alfvén wave,

$$\omega^2 \approx \frac{k_\parallel^2 v_{A,e}^2}{2}, \tag{2.21}$$

where the factor of 2 in the denominator stems from from unconventional definition of v_A after Eq. (2.10), which only involved half the density.

In the presence of collisions, electron thermal conduction induces a damping at short wavelengths

$$\omega \approx \frac{-i}{2k_\perp^2 d_e^2} \frac{k_\parallel^2 v_{A,e}^2}{\nu}. \tag{2.22}$$

Perhaps not surprisingly, Eq. (2.22) defines the semicollisional scale introduced by Drake & Lee (1977).

Had we retained the Debye length instead of the Larmor radius in Eq. (2.5) ($\rho_e^2 \to k_\perp^2$), we would have found two waves travelling at the speed of light, which we prefer not to allow for, because the displacement current has been neglected in Ampère's law (2.9). This would have been true also in the isothermal limit ($\delta T_\parallel = 0$). Then, Eq. (2.18) would have been

$$\omega = \pm \frac{k_\parallel c}{\sqrt{2}}, \tag{2.23}$$

which, again, cannot be accepted. Had one retained the whole hierarchy of moments coupled to Eq. (2.11), valid for arbitrary collisionality, they would still have entered the dispersion relation via the $k_\perp^2 \lambda_D^2$ term and yielded a wave travelling at the speed of light. We conclude that the collisionless electromagnetic limit must be at least Lorentz-Poincaré invariant. As already anticipated, the reason is more apparent if one ponders the consequences of allowing the electrostatic potential to vary on the Debye scale, while letting the current varying on the inertial scale, d_e. This implies

$$\lambda_D \sim d_e \to v_{the} \sim c, \tag{2.24}$$

which demands a relativistic description. An electromagnetic gyrokinetic theory that retains Larmor radius effects seems to suffer from a similar problem, since in this case

$$\lambda_D \sim \rho_e \to v_{A,e} \sim c, \tag{2.25}$$

where ρ_e is the Larmor radius and $v_{A,e}$ the Alfvèn speed. However, while Eq. (2.24) is a condition on the kinetic energy of particles, which can be met in extreme conditions, Eq. (2.25) is simply stating that Alfvén waves must be allowed to travel at the speed of light. This is a perfectly acceptable physical requirement provided Maxwell's equations are kept consistent with a covariant description, therefore including the displacement current in Ampère's law. Some of these aspects have also been pointed out by a recent work of Stenson *et al.* (2017).

2.3. *Alfvén waves*

We could insist on keeping a finite Debye length in our electromagnetic equations. In this case, the displacement current in Ampère's law should be retained, since it plays a crucial role in establishing charge neutrality and generating Langmuir waves. In this case our model equations will indeed suitable for a covariant formulation. Thus, we consider a modified version of Eqs. (2.3b) of Helander & Connor (2016) where the diplacement current has been added

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) A_\parallel - \frac{1}{c}\frac{\partial}{\partial t}\nabla_\parallel \varphi = \frac{4\pi n_0 e}{c}\left(u_{\parallel e^-} - u_{\parallel e^+}\right). \tag{2.26}$$

Poisson's law, since we are using the radiation (Coulomb) gauge, $\nabla \cdot \mathbf{A} = 0$, does not change. Let us first show that the collisionless isothermal response derived using our Eqs. (2.2) and (2.8) gives the same result of Eq. (3.7) of Helander & Connor (2016) for $d_e^{-1} \ll k_\perp \ll \lambda_D^{-1}$, which is the regimes of interest of the future APEX experiment Pedersen *et al.* (2012); Saitoh *et al.* (2014) . Now quasineutrality Eq. (2.5)(with $\rho_e \to \sqrt{2}\lambda_D$) and Ohm's law Eq. (2.8) (with $\nu \to 0$) become

$$\frac{1}{4}\frac{k_\parallel c}{\omega}\left[\left(k_\perp^2 - \frac{\omega^2}{c^2}\right) A_\parallel + \frac{\omega}{c} k_\parallel \varphi\right] = k_\perp^2 \varphi, \tag{2.27}$$

and

$$\frac{\omega}{k_\parallel c}\left\{A_\parallel + \frac{d_e^2}{2}\left[\left(k_\perp^2 - \frac{\omega^2}{c^2}\right) A_\parallel + \frac{\omega}{c} k_\parallel \varphi\right]\right\} = \left(1 + k_\perp^2 \lambda_D^2\right)\varphi. \tag{2.28}$$

For $\lambda_D^{-1} \gg k_\perp \gg d_e^{-1}$, when $\lambda_D \ll d_e$, we obtain a dispersive Langmuir wave

$$\omega^2 \approx \frac{k_\parallel^2}{2k_\perp^2}\omega_{pl}^2, \tag{2.29}$$

where ω_{pl} is the plasma frequency. Equation (2.29) indeed coincides with the high frequency limit of the electrostatic wave that solves Eq. (3.7) of Helander & Connor (2016). For the range of wavelengths of interest, the displacement current does not seem to have an effect. A full kinetic treatment would damp this wave [See Mishchenko et al. (2017)]

We now turn our attention to Alfvénic perturbations, which should connect to the wave just found in the electrostatic limit, $\lambda_D \gg \rho_e$. We rewrite Poisson's law and Ampère's law for the collisionless drift-kinetic case [Eq. (3.3) and (3.4) of Helander & Connor (2016)]

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} \varphi \\ \frac{v_{the}}{c} A_\parallel \end{pmatrix} = 0. \tag{2.30}$$

where the coefficients were evaluated by Helander & Connor (2016): $a_{11} = 1 + \lambda_D^2 k_\perp^2 + xZ(x)$, $a_{22} = -x^2\left[1 + xZ(x)\right] + k_\perp^2 d_e^2$, $a_{21} = -a_{12} = x\left[1 + xZ(x)\right]$, with $x = \omega/(k_\parallel v_{the})$, and Z is the plasma dispersion function Fried *et al.* (1968). When Eq. (2.26) is used instead of Eq. 2.9, we have that

$$k_\perp^2 d_e^2 \to k_\perp^2 d_e^2 \left(k_\perp^2 - \frac{\omega^2}{c^2}\right), \tag{2.31}$$

and

$$a_{21} \to x\left[1 + xZ(x)\right] + x k_\parallel^2 \lambda_D d_e. \tag{2.32}$$

For alfvenic perturbations we must expand the plasma dispersion function for large arguments(i), since

$$x = \frac{\omega}{k_\parallel v_{the}} = \frac{\omega}{\sqrt{\beta} k_\parallel v_{A,e}} \sim \frac{1}{\sqrt{\beta}} \gg 1. \tag{2.33}$$

Setting to zero the determinant of the matrix of Eq. (2.30), for $k_\perp \lambda_D \sim k_\perp \rho_e \sim \sqrt{\beta} \ll 1$,

(i) Notice that Helander & Connor (2016) keep $x \sim 1$.

then gives

$$\frac{\omega^2}{k_\parallel^2 v_{A,e}^2} = \beta \frac{d_e^2\left(k_\perp^2 - \frac{\omega^2}{c^2}\right)}{(k_\perp^2\rho_e^2 + k_\perp^2\lambda_D^2)\left[1 + 2d_e^2\left(k_\perp^2 - \frac{\omega^2}{c^2}\right)\right] + k_\parallel^2\lambda_D d_e}, \tag{2.34}$$

which is a kinetically modified Alfvén wave. Since

$$\frac{\omega}{k_\perp c} = \frac{\omega}{k_\parallel v_{A,e}}\frac{v_{A,e}}{c}\epsilon_{GK} \ll 1, \tag{2.35}$$

we have

$$\frac{\omega^2}{k_\parallel^2 v_A^2} = \beta \frac{d_e^2 k_\perp^2}{(k_\perp^2\rho_e^2 + k_\perp^2\lambda_D^2)\left[1 + 2d_e^2 k_\perp^2\right] + k_\parallel^2\lambda_D d_e}. \tag{2.36}$$

When $(k_\parallel^2/k_\perp^2)v_{A,e}/c \equiv \epsilon_{GK}^2 v_{A,e}/c \gg \sqrt{\beta}$, for $\rho_e \sim \lambda_D$, the $k_\parallel^2\lambda_D d_e$ in the denominator is dominant, and we have

$$\omega^2 \approx k_\perp^2\rho_e^2\frac{v_{A,e}^2}{\lambda_D d_e}. \tag{2.37}$$

When $\epsilon_{GK}^2 v_{A,e}/c \ll \sqrt{\beta}$, the same term is negligible, and we have

$$\frac{\omega^2}{k_\parallel^2 v_A^2} = \beta \frac{d_e^2 k_\perp^2}{(k_\perp^2\rho_e^2 + k_\perp^2\lambda_D^2)\left[1 + 2d_e^2 k_\perp^2\right]}, \tag{2.38}$$

which, in the subsidiary $d_e^2 k_\perp^2 \to \infty$ limit, for $\lambda_D \gg \rho_e$ gives

$$\omega^2 \approx \frac{k_\parallel^2}{2k_\perp^2}\omega_{pl}^2. \tag{2.39}$$

This result agrees with (2.29) and with the high frequency solution of Eq. (3.7) of Helander & Connor (2016). Our results suggest that, in a low-beta gyrokinetic theory, the displacement current can be neglected only if

$$\epsilon_{GK}^2\frac{v_{A,e}}{c} \sim \epsilon_{GK}^2\frac{\lambda_D}{\rho_e} \ll \sqrt{\beta}. \tag{2.40}$$

2.4. *Tearing instability*

When considering a sheared slab, in the neighbourhood of a resonant surface, we have

$$k_\parallel \approx k_y\frac{x}{L_s}, \tag{2.41}$$

where L_s is the shear length. Poisson's law and Ohm's law become, respectively

$$\rho_e^2\frac{\partial^2\varphi}{\partial x^2} = -\frac{k_y v_{the}}{2\omega}\frac{v_{the}}{c}\frac{x}{L_s}d_e^2\frac{\partial^2 A_\parallel}{\partial x^2}, \tag{2.42}$$

and

$$A_\parallel - \frac{k_y c}{\omega}\frac{x}{L_s}\varphi = \left(i\frac{\nu}{\omega} + \frac{1}{2} - \frac{3}{4}\frac{k_y^2 v_{the}^2}{\omega^2}\frac{x^2}{L_s^2}\right)d_e^2\frac{\partial^2 A_\parallel}{\partial x^2}, \tag{2.43}$$

which can easily be cast in the form presented in Ref. [(Zocco & Schekochihin 2011)]. Now, we have

$$-\frac{x}{\delta}\left(A_\parallel - \frac{x}{\delta}\tilde{\varphi}\right)\sigma\left(\frac{x}{\delta}\right) = 2\rho_e^2\frac{\partial^2\tilde{\varphi}}{\partial x^2}, \tag{2.44}$$

and

$$-\frac{x}{\delta}d_e^2\frac{\partial^2 A_\parallel}{\partial x^2} = 2\rho_e^2\frac{\partial^2\tilde{\varphi}}{\partial x^2}, \tag{2.45}$$

where $\delta = L_s\omega/(k_y v_{the})$, $\tilde{\varphi} = (c/v_{the})\varphi$, and

$$\sigma\left(\frac{x}{\delta}\right) = \frac{1}{i\frac{\nu}{\omega} + \frac{1}{2} - \frac{3}{4}\frac{x^2}{\delta^2}}. \tag{2.46}$$

Since we are always assuming $\rho_e \ll d_e \sim \delta$, we are effectively in a one-fluid limit, the ultralow-beta discussed in Ref. [(Zocco & Schekochihin 2011)]. We report on the collisionless case, the results apply to the collisional case in a straightforward manner. The analysis is known but we reproduce some key steps for the sake of clarity. One can introduce the function $\chi(\xi) = \xi A'_\parallel - A_\parallel$, where $\xi = x/\delta_{in}$, and $\delta_{in} = \sqrt{4\delta\rho_e^2}$, to obtain one equation for $\tilde{\chi} = -1 + \chi/\chi_0$,

$$\xi^2\frac{d}{d\xi}\left[\frac{1}{\xi^2} + \alpha^2 G\right]\tilde{\chi}' - \left(\xi^2 + \lambda^2\right)\tilde{\chi} = \lambda^2, \tag{2.47}$$

where $\lambda^2 = 4\delta\rho_e/d_e^2$, $\alpha = \sqrt{2\rho/\delta}$, $G = (\delta^2/x^2)(\sigma^{-1} - 2)$, and χ_0 is a constant of integration. The dispersion relation for the rescaled eigenvalue λ^2 is then

$$\int_0^\infty d\xi\frac{\tilde{\chi}'}{\xi} = -\frac{\Delta'\delta_{in}}{2}, \tag{2.48}$$

where Δ' is the parameter that measures the discontinuity of the derivative of $A_\parallel^{MHD}$ across the reconnection layer, and $A_\parallel^{MHD}$ is the stable solution found in the ideal MHD region, $x \to \infty$ s.t. $E_\parallel \to 0$.[(Furth *et al.* 1963)]. As already pointed out in Ref. [(Zocco & Schekochihin 2011)], there is no need to solve Eq. (2.47) to derive scaling laws for reconnection rates. We can apply to our case Eq. (B47) that the authors suggest, and obtain

$$\frac{\gamma}{k_y v_{the}} \sim \left(\Delta' d_e\right)^2\frac{d_e^2}{\rho_e L_s}. \tag{2.49}$$

This is the equivalent of the collisionless result found for electron-ion plasmas by Drake & Lee (1977), where the Alfvén speed is based on the electron mass. The collisional counterpart is recovered by replacing $d_e \to \sqrt{\nu d_e^2/\gamma}$, to obtain the traditional result of Furth *et al.* (1963) (but based on the electron Alfvèn speed). When $\Delta'\delta_{in} \gg 1$, the current is limited by the scale δ_{in}, so that $\partial_x^2 A_\parallel \sim A_\parallel/\delta_{in}^2$. Then the dispersion relation becomes $\lambda^2 \sim 1$, which yields [Basu & Coppi (1981)]

$$\frac{\gamma}{k_y v_{the}} \sim \frac{d_e^2}{\rho_e L_s}, \tag{2.50}$$

which gives the scaling $\gamma \sim (\nu d_e^2)^{1/3}$ of Coppi *et al.* (1976) in the collisional limit.

3. Improved nonlinear model

The inclusion of the Larmor scale in Eq. (2.5), instead of the Debye length, allowed us to avoid a covariant treatment. The use of the drift-kinetic model of Zocco & Schekochihin (2011) helped us, but we did not exploit its full nonlinear potential yet. For this, fields amplitudes must be ordered more carefully. Equation (2.1), in fact, is nothing more than a drift-kinetic equation that one could have cosidered regardless of the results of Zocco & Schekochihin (2011). If one wants to consider the nonlinear $\mathbf{E}\times\mathbf{B}$ frequency $\omega \sim (c/B_0)k^2\varphi$ and the streaming term, after using Eq (2.5), we find that

$$\frac{\delta n}{n_0} \sim \sqrt{\beta}\epsilon_{GK}. \tag{3.1}$$

So, a nonlinear electromagnetic gyrokinetic theory that retains inertial effects must be developed to first order in a small Larmor radius expansion. This also implies that magnetic compressibility now must be retained to first order, since

$$\frac{\delta B_\parallel}{B_0} \sim \beta \frac{e\varphi}{T_0} \sim \sqrt{\beta}\epsilon_{GK} \sim \frac{\delta n}{n_0}. \tag{3.2}$$

Perpendicular magnetic fluctuations are ordered by balancing the electrostatic and the vector potential amplitudes of the gyrokinetic potential $\chi = \varphi - (v_\parallel/c)A_\parallel$, then

$$\frac{\delta B_\perp}{B_0} \sim \frac{u_\perp}{v_{A,e}} \frac{1}{\sqrt{\beta}}, \tag{3.3}$$

where $u_\perp \sim ck_\perp\varphi/B_0$. In many relevant situations, the spatial variation of all quantities along the exact magnetic field is required, then $k_\perp \delta B_\perp \sim k_\parallel B_0$, which implies

$$\frac{u_\perp}{v_{A,e}} \sim \sqrt{\beta}\epsilon \to k_\perp \rho_e \sim \sqrt{\beta}, \tag{3.4}$$

and is naturally consistent with our fundamental ordering $k_\perp d_e \sim 1$. We therefore use

$$\frac{\delta B_\perp}{B_0} \sim \epsilon_{GK}, \tag{3.5}$$

which is different from what the continuity equation (2.2) would have implied

$$\frac{\delta B_\perp}{B_0} \sim \beta \epsilon_{GK}. \tag{3.6}$$

Having completed the amplitude orderings, in order to obtain fluid-like equations, one can separate the first two moments of the perturbed distribution function, but considering first order terms

$$h_{e\mp} = \left[\frac{e^{\mp}\left(\varphi^{(0)} + \varphi^{(1)}\right)}{T_0} + \left(\frac{\delta n^{(0)}_{e\mp}}{n_0} + \frac{\delta n^{(1)}_{e\mp}}{n_0} \right) + 2\frac{v_\parallel \left(u^{(0)}_{\parallel e\mp} + u^{(1)}_{\parallel e\mp} \right)}{v^2_{the}} \right] F_0 \tag{3.7}$$
$$+ g^{(0)} + g^{(1)} + \mathcal{O}(k_\perp^4 \rho_e^4),$$

where $\int d^3\mathbf{v}g \equiv \int d^3\mathbf{v}v_\parallel g \equiv 0$ to all orders in $k_\perp^2\rho_e^2 \sim \beta \ll 1$. When magnetic compressibility is retained, the gyrokinetic potential on the RHS of Eq. (2.1) becomes

$$\varphi - \frac{v_\parallel}{c}A_\parallel \to \varphi - \frac{v_\parallel}{c}A_\parallel + \frac{T_0}{e}\hat{v}_\perp^2 \frac{\delta B_\parallel}{B_0} \equiv \chi, \tag{3.8}$$

where

$$\frac{\delta B_\parallel}{B_0} = -\beta \sum_{\mp} \int d^3\mathbf{v}\hat{v}_\perp^2 h_{e\mp}. \tag{3.9}$$

We notice that, due to its parity in velocity space, the new $\delta B_\parallel$ term enters in the equation for density fluctuations. Let us evaluate the density moment of Eqs. (2.1) after using Eq. (3.8), and subtract the two results obtained, one for each species. To leading order we have

$$\hat{\mathbf{b}} \cdot \nabla d_e^2 \nabla^2 A^{(0)}_\parallel = 0. \tag{3.10}$$

To next order, Eq. (2.4) gives

$$\left(\frac{\delta n^{(1)}_{e^+}}{n_0} - \frac{\delta n^{(1)}_{e^-}}{n_0} \right) = -\frac{\rho_e^2 \nabla^2}{2} \frac{e\varphi^{(0)}}{T_0} - \frac{\rho_e^2 \nabla^2}{4} \left(\frac{\delta T^{(0)}_{\perp e^+}}{T_0} - \frac{\delta T^{(0)}_{\perp e^-}}{T_0} \right), \tag{3.11}$$

which now proves our previous statement on the validity of Eq. (2.5), since here

$$\frac{\delta T_{\perp e^{\pm}}}{T_0} = \frac{1}{n_0}\int d^3\mathbf{v}\hat{v}_\perp^2 g_{e^{\pm}} \tag{3.12}$$

are the perpendicular temperature fluctuations. As usual, they drive parallel magnetic compressional perturbations

$$\frac{\delta B_\parallel^{(1)}}{B_0} = -\frac{\beta_e}{2}\left(\frac{\delta n_{e^+}^{(0)}}{n_0} + \frac{\delta n_{e^-}^{(0)}}{n_0} + \frac{\delta T_{\perp,e^-}^{(0)}}{T_0} + \frac{\delta T_{\perp,e^+}^{(0)}}{T_0}\right), \tag{3.13}$$

which have to be taken into account in the first order continuity equation. The electrostatic potential is determined by the difference of the first order density fluctuations of the two species. We then subtract the two first order continuity equations, and obtain the generalised vorticity equation

$$\begin{aligned}
&\frac{d}{dt}\left[\rho_e^2\nabla^2\frac{e\varphi^{(0)}}{T_0} - \frac{\rho_e^2\nabla^2}{4}\left(\frac{\delta T_{\perp e^-}^{(0)}}{T_0} - \frac{\delta T_{\perp e^+}^{(0)}}{T_0}\right) - \beta_e\left(\frac{\delta T_{\perp e^-}^{(0)}}{T_0} + \frac{\delta T_{\perp e^+}^{(0)}}{T_0}\right)\right] = \\
&-\hat{\mathbf{b}}\cdot\nabla\left[\frac{e}{mc}d_e^2\nabla^2 A_\parallel^{(1)} + \beta_e\left(u_{\parallel e^-}^{(0)} + u_{\parallel e^+}^{(0)}\right)\right] \\
&+\frac{1}{B_0}\left\{A_\parallel^{(1)} + \frac{\rho_e^2\nabla^2}{4}A_\parallel^{(0)}, \frac{e}{mc}d_e^2\nabla^2 A_\parallel^{(0)}\right\} \\
&-\frac{cT_0}{B_0 e}\left\{\frac{\rho_e^2\nabla^2}{4}\frac{e\varphi^{(0)}}{T_0} + \frac{\delta B_\parallel^{(1)}}{B_0}, \frac{\delta T_{\perp e^-}^{(0)}}{T_0} - \frac{\delta T_{\perp e^+}^{(0)}}{T_0}\right\} \\
&+\frac{v_{the}}{B_0}\left\{\frac{\rho_e^2\nabla^2}{4}A_\parallel^{(0)}, \frac{1}{n_0}\int d^3\mathbf{v}\hat{v}_\parallel\hat{v}_\perp^2\left(g_{e^-}^{(0)} - g_{e^+}^{(0)}\right)\right\},
\end{aligned} \tag{3.14}$$

where the terms multiplying an explicit β_e come from (3.13). We now evaluate all the terms of the RHS. To zeroth order, Eq. (2.8) is valid, and we have

$$\begin{aligned}
&\frac{d}{dt}\left(u_{\parallel e^+}^{(0)} + u_{e^-}^{(0)}\right) = -\frac{v_{the}^2}{2}\hat{\mathbf{b}}\cdot\nabla\left(\frac{\delta n_{e^+}^{(0)}}{n_0} + \frac{\delta n_{e^-}^{(0)}}{n_0} + \frac{\delta T_{\perp,e^-}^{(0)}}{T_0} + \frac{\delta T_{\perp,e^+}^{(0)}}{T_0}\right) \\
&+\frac{v_{the}^2}{L_n}(1+\eta_e)\frac{\partial_y A_\parallel^{(0)}}{B_0},
\end{aligned} \tag{3.15}$$

where we see that diamagnetic effects do not cancel. The sum $\delta n_{e^+}^{(0)} + \delta n_{e^-}^{(0)}$ can be evaluated by using the zeroth order continuity equations

$$\frac{d}{dt}\left(\frac{\delta n_{e^+}^{(0)}}{n_0} + \frac{\delta n_{e^-}^{(0)}}{n_0}\right) = -\hat{\mathbf{b}}\cdot\nabla\left(u_{\parallel e^+}^{(0)} + u_{e^-}^{(0)}\right) - \frac{v_{the}}{L_n}\rho_e\partial_y\frac{e\varphi^{(0)}}{T_0}. \tag{3.16}$$

To first order, only the difference $u_{\parallel e^-}^{(1)} - u_{\parallel e^+}^{(1)} = (e/mc)d_e^2\nabla^2 A_\parallel^{(1)}$ enters the vorticity equation, thus we obtain

$$\begin{aligned}
&\frac{d}{dt}\left(A_\parallel^{(1)} - \frac{d_e^2\nabla^2}{2}A_\parallel^{(1)} + \frac{\rho_e^2\nabla^2}{4}A_\parallel^{(0)}\right) = -c\hat{\mathbf{b}}\cdot\nabla\varphi^{(1)} \\
&\frac{cT_0}{2e}\hat{\mathbf{b}}\cdot\nabla\left[\frac{\delta n_{e^-}^{(0)}}{n_0} - \frac{\delta n_{e^+}^{(0)}}{n_0} + \frac{\delta T_{\parallel e^-}^{(1)}}{T_0} - \frac{\delta T_{\parallel e^+}^{(1)}}{T_0}\right] \\
&+\frac{c}{B_0}\left\{\varphi^{(1)} + \frac{\rho_e^2\nabla^2}{4}\varphi^{(0)} + \frac{T_0}{e}\frac{\delta B_\parallel^{(1)}}{B_0}, \frac{d_e^2\nabla^2}{2}A_\parallel^{(0)}\right\} \\
&-\frac{cT_0}{eB_0}\left\{A_\parallel^{(1)}, \frac{\delta T_{\parallel e^-}^{(0)}}{T_0} - \frac{\delta T_{\parallel e^+}^{(0)}}{T_0}\right\} \\
&-\frac{cT_0}{eB_0}\left\{\frac{\rho_e^2\nabla^2}{4}A_\parallel^{(0)}, \frac{1}{n_0}\int d^2\mathbf{v}\hat{v}_\parallel^2\hat{v}_\perp^2\left(g_{e^-}^{(0)} - g_{e^+}^{(0)}\right)\right\} \\
&+\frac{c}{B_0}\left\{\frac{\rho_e^2\nabla^2}{4}\varphi^{(0)} + \frac{T_0}{e}\frac{\delta B_\parallel^{(1)}}{B_0}, \frac{mc}{2e}\frac{1}{n_0}\int d^3\mathbf{v}v_\parallel\hat{v}_\perp^2\left(g_{e^-}^{(0)} - g_{e^+}^{(0)}\right)\right\} \\
&-\frac{cm}{2en_0}\int d^3\mathbf{v}v_\parallel\sum_s\left(\frac{\partial h_s}{\partial t}\right)_{coll}.
\end{aligned} \tag{3.17}$$

An equation for $g^{(0)}$ is derived by subtracting Eqs. (2.2) and (2.8) from the gyrokinetic equation, and using Eq. (3.7). The result is

$$\begin{aligned}
&\frac{d}{dt}g_{e\mp}^{(0)} + v_\parallel\hat{\mathbf{b}}\cdot\nabla\left(g_{e\mp}^{(0)} - \frac{\delta T_{\parallel e\mp}^{(0)}}{T_0}F_0\right) - C[g_{e\mp}^{(0)}] = \left(1 - 2\frac{v_\parallel^2}{v_{the}^2}\right)\hat{\mathbf{b}}\cdot\nabla u_{\parallel e\mp}^{(0)}F_0 \\
&-\frac{cT_0}{eB_0}\frac{\partial_y}{L_T}\left[\left(\hat{v}^2 - \frac{3}{2}\right)\frac{e\varphi^{(0)}}{T_0} - \left(\hat{v}^2 - \frac{5}{2}\right)\frac{v_\parallel}{c}\frac{eA_\parallel^{(0)}}{T_0}\right]F_0,
\end{aligned} \tag{3.18}$$

where

$$C[g_{e\mp}^{(0)}] = \left(\frac{\partial h_{e\mp}^{(0)}}{\partial t}\right)_{coll} - 2\frac{v_\parallel F_0}{v_{the}^2 n_0}\int d^3\mathbf{v}\left(\frac{\partial h_{e\mp}^{(0)}}{\partial t}\right)_{coll}.$$

Equation (3.18) is the extension of the Kinetic Reduced Electron Heating Model equation [(Zocco & Schekochihin 2011)] to the case of finite density and temperature gradients [(Zocco *et al.* 2015; Loureiro *et al.* 2016)]. For the evolution of the fields, again, the important quantities are the difference and the sum the electron and positron equations. From the difference we obtain

$$\frac{d}{dt}\left(g_{e^-}^{(0)} - g_{e^+}^{(0)}\right) + v_\parallel\hat{\mathbf{b}}\cdot\nabla\left[g_{e^-}^{(0)} - g_{e^+}^{(0)} - \left(\frac{\delta T_{\parallel e^-}^{(0)}}{T_0} - \frac{\delta T_{\parallel e^+}^{(0)}}{T_0}\right)F_0\right] = C[g_{e\mp}^{(0)} - g_{e^+}^{(0)}], \tag{3.19}$$

where we used the fact that $\hat{\mathbf{b}}\cdot\nabla\nabla^2 A_\parallel^{(0)} = 0$, and the diamagnetic cancellation. On the other hand, the sum gives

$$
\begin{aligned}
&\frac{d}{dt}\left(g_{e^-}^{(0)}+g_{e^+}^{(0)}\right)+v_\|\hat{\mathbf{b}}\cdot\nabla\left[g_{e^-}^{(0)}+g_{e^+}^{(0)}-\left(\frac{\delta T_{\|e^-}^{(0)}}{T_0}+\frac{\delta T_{\|e^+}^{(0)}}{T_0}\right)F_0\right]=C[g_{e^+}^{(0)}+g_{e^-}^{(0)}]\\
&\left(1-2\frac{v_\|^2}{v_{the}^2}\right)\hat{\mathbf{b}}\cdot\nabla\left(u_{\|e^-}^{(0)}+u_{\|e^+}^{(0)}\right)F_0\\
&-2\frac{cT_0}{eB_0}\frac{\partial_y}{L_T}\left[\left(\hat{v}^2-\frac{3}{2}\right)\frac{e\varphi^{(0)}}{T_0}-\left(\hat{v}^2-\frac{5}{2}\right)\frac{v_\|}{c}\frac{eA_\|^{(0)}}{T_0}\right]F_0
\end{aligned}
\tag{3.20}
$$

and the diamagnetic contribution does not cancel. It remains to specify the collisional operator. We notice that, for sufficiently large collisionality, temperature fluctuations must isotropize, that is $\delta T_\perp=\delta T_\|$. This is achieved with the simple collision operator model

$$
\left(\frac{\partial h_{e\mp}}{\partial t}\right)_{coll}=\nu\left\{\frac{1}{2}\frac{\partial}{\partial\hat{v}_\|}\left(\frac{\partial}{\partial\hat{v}_\|}+\hat{v}_\|\right)h_{e\mp}+2\frac{v_\|u_{\|,e\pm}}{v_{the}^2}+\left(1-2\hat{v}_\|^2\right)\frac{\delta T_{\perp,e\mp}}{T_0}F_0\right\}. \tag{3.21}
$$

Finally, with the equation for $g_{e\mp}^{(1)}$, we can close our system. This is obtained from

$$
\begin{aligned}
&\frac{\partial h_\mp^{(1)}}{\partial t}+v_\|\frac{\partial h_\mp^{(1)}}{\partial z}+\frac{c}{B_0}\left\{\chi^{(0)},h_\mp^{(1)}\right\}+\frac{c}{B_0}\left\{\chi^{(1)}+\hat{v}_\perp^2\frac{\rho_e^2\nabla^2}{4}\chi^{(0)},h_\mp^{(0)}\right\}=\\
&\frac{e^\mp F_0}{T_0}\frac{\partial}{\partial t}\left(\chi^{(1)}+\hat{v}_\perp^2\frac{\rho_e^2\nabla^2}{4}\chi^{(0)}\right)\\
&-\frac{cn_0}{B_0L_n}\partial_y\left(\chi^{(1)}+\hat{v}_\perp^2\frac{\rho_e^2\nabla^2}{4}\chi^{(0)}\right)\left[1+\eta_{e\mp}\left(\hat{v}^2-\frac{3}{2}\right)\right]F_0\\
&+\left(\frac{\partial h_\mp^{(1)}}{\partial t}\right)_{coll.},
\end{aligned}
\tag{3.22}
$$

where the equations for all the fields have already been derived. We leave this expression implicit, and proceed considering the limiting case of collisionless isothermal species. This is the limit in which the KREHM equations [(Zocco & Schekochihin 2011)] reduce to the Schep Pegoraro and Kuvshinov two-fluid model [(Schep *et al.* 1994)]. However, now we expect a new result. Indeed, even if our quasineutrality equation, Eq. (3.11), looks similar to a long-wavelength electrostatic ion response

$$
\delta n_{e^-}-\delta n_{e^+}\rightarrow\delta n_e=\delta n_i\propto\nabla^2\rho_i^2\varphi, \tag{3.23}
$$

its physical content is rather different. This is due to the fact that positrons, which must provide charge neutrality, possess a compressional dynamics that will manifest and generate a qualitatively different set of equations from the equivalent electron-ion ones. Let us derive this new set of nonlinear equations for collisionless magnetic reconnection and turbulence in magnetised isothermal pair plasmas.

We introduce the field-line-following co-ordinate, l, and integrate Eq. (3.10) along the perturbed field lines. Since, for $l\rightarrow\pm\infty$, the solution must decay to zero, we set to zero the resulting constant of integration. The electric field, therefore, to zeroth order satisfies the ideal MHD equation

$$
\frac{\partial}{\partial t}A_\|^{(0)}+c\hat{\mathbf{b}}\cdot\nabla\varphi^{(0)}=0, \tag{3.24}
$$

with

$$\nabla^2 A_\parallel^{(0)} = 0. \tag{3.25}$$

We are considering the isothermal response, thus, $g_{e\mp} \equiv 0$. By inspecting Eqs. (3.18) and (3.22), we observe that they are homogeneous, and therefore consistent with these solutions, only if $L_n \to \infty$. We consider this limit.

Then, the generalised Ohm's law reduces to

$$\frac{d}{dt}\left(A_\parallel^{(1)} - \frac{d_e^2\nabla^2}{2}A_\parallel^{(1)}\right) = -c\hat{\mathbf{b}}\cdot\nabla\varphi^{(1)} + \frac{cT_0}{2e}\hat{\mathbf{b}}\cdot\nabla\frac{\rho_e^2\nabla^2}{2}\varphi^{(0)}, \tag{3.26}$$

while the vorticity equation gives

$$\frac{d}{dt}\rho_e^2\nabla^2\frac{e\varphi^{(0)}}{T_0} = -\hat{\mathbf{b}}\cdot\nabla\left[\frac{e}{mc}d_e^2\nabla^2 A_\parallel^{(1)} + \beta_e V_\parallel^{(0)}\right]. \tag{3.27}$$

Due to finite magnetic compressibility, this is coupled to the equations for the total density $2\delta n^{(0)} = \delta n_{e^-}^{(0)} + \delta n_{e^+}^{(0)}$ and the velocity $V_\parallel^{(0)} = u_{\parallel e^-}^{(0)} + u_{\parallel e^+}^{(0)}$

$$\frac{d}{dt}V_\parallel^{(0)} = -v_{the}^2\hat{\mathbf{b}}\cdot\nabla\frac{\delta n^{(0)}}{n_0}, \tag{3.28}$$

$$2\frac{d}{dt}\frac{\delta n^{(0)}}{n_0} = -\hat{\mathbf{b}}\cdot\nabla V_\parallel^{(0)}. \tag{3.29}$$

We found a set of six equations for the six unknown $A_\parallel^{(0)}$, $\varphi^{(0)}$, $A_\parallel^{(1)}$, $\varphi^{(1)}$, $V_\parallel^{(0)}$, and $\delta n^{(0)}$. We see that compressible effects enter the vorticity equation (3.27) through β_e because quasineutrality is valid to first order in the small Larmor radius expansion. The new term, proportional to $V_\parallel = u_{\parallel e^-}^{(0)} + u_{\parallel e^+}^{(0)}$, couples the electromagnetic system of equations for $\varphi^{(1)}$ and $A_\parallel^{(1)}$ to the fluid equations (3.28) and (3.29). It is easy to show that these equations support the compressional sound wave $\omega = v_{the}k_\parallel/\sqrt{2}$, which is linearly strongly damped in the electrostatic limit [Helander & Connor (2016); Mishchenko *et al.* (2017)]. In the low-frequency limit $\omega \ll k_y v_{the}(\partial_x A_\parallel^{(0)}/B_0) \sim k_z v_{the}$, such damping is neglible. Thus, Eqs. (3.24)-(3.29) are valid for sub-Alfvénic instabilities, $\omega \sim k_y v_{A,e}\lambda$, with $\lambda \ll \sqrt{\beta_e} \ll 1$, provided non-linear collisionless heating is negligible ($g_{e^\pm} \equiv 0$). This is definitely true for the reconnecting instabilities presented in Section 2.4.

4. Conclusion

We presented a simple study of non-relativistic electron-positron plasmas in a magnetised sheared slab. The two species were described by using the drift-kinetic model. Linear results are presented for nearly electrostatic perturbations, $\delta B_\perp/B_0 \sim \beta\epsilon_{GK} \ll e\varphi/T_0 \sim \epsilon_{GK}$, where $\epsilon_{GK} = k_\parallel/k_\perp$ is the small expansion parameter of gyrokinetic theory, $k_\parallel$ and $k_\perp$ are the wave vectors for perturbations parallel and perpendicular to the equilibrium magnetic field, B_0, and β is the ratio of kinetic to magnetic plasma pressure. We emphazised the role of the electron Alfvén wave. This is supported by the system and connects, in the electrostatic limit, to high frequency K-modes. Current driven instabilities are described by a formalism equivalent to that of an electron-ion plasma, but with the Alfvén speed based on the electron mass. A non-linear electromagnetic model is also presented. Here magnetic fluctuations are allowed to be of the same order of electrostatic ones, $\delta B_\perp/B_0 \sim \epsilon_{GK}$. The resulting hybrid fluid-kinetic model is an extension of the Kinetic Reduced Electron Heating Model (KREHM) of Zocco & Schekochihin (2011), where finite Larmor radius effects are retained. The specific case of isothermal species is derived in detail, and its limitations are discussed. This constitutes a new set of equations for nonlinear electromagnetic phenomena in magnetised pair plasmas.

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