Sum-frequency ionic Raman scattering

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In a recent report sum-frequency excitation of a Raman-active phonon was experimentally demonstrated for the first time. This mechanism is the sibling of impulsive stimulated Raman scattering, in which difference-frequency components of a light field excite a Raman-active mode. Here we propose that ionic Raman scattering analogously has a sum-frequency counterpart. We compare the four Raman mechanisms, photonic and ionic difference- and sum-frequency excitation, for three different example materials using a generalized oscillator model for which we calculate the parameters with density functional theory. Sum-frequency ionic Raman scattering completes the toolkit for controlling materials properties by means of selective excitation of lattice vibrations.

I. INTRODUCTION

Ultrashort electromagnetic pulses are an established tool to control the electronic and structural phases of matter. Intense laser pulses in the terahertz spectral range provide direct access to the excitation of optical phonons and have become practical only during the past decade [1][4]. Highly excited optical phonons govern a variety of physical phenomena, such as phase transitions [5], induced or enhanced superconductivity [6][8], and control of magnetic order [9][10]. It is therefore necessary to understand the fundamental mechanisms that underly the excitation of coherent optical phonons with light. Infrared-active phonons carry an electric dipole moment and can therefore be excited directly by coupling to the electric field component of electromagnetic radiation. For Raman-active phonons, which do not possess an electric dipole moment, another indirect way has to be taken.

An established route is to disturb the electronic system with an ultrashort light pulse, which then mediates energy to Raman-active phonons via electron-phonon interaction [11][12]. For coherent, nonresonant excitation below the band gap [13], the primary mechanism involved is impulsive stimulated Raman scattering (ISRS), in which a virtual electronic state serves as intermediate energy level for the Raman scattering of the incident light by the phonon, see figure 1(a) [14][16]. In this case, the difference frequency of two photons from the light pulse is resonant with a vibrational transition of the phonon mode. We will in the following refer to this as a "photonic" Raman mechanism.

A second route to exciting Raman-active phonons is via scattering with infrared-active phonons. This was proposed nearly half a century ago as ionic Raman scattering (IRS), and has only been demonstrated within this decade due to the advancement of intense terahertz sources [17][18]. In ionic Raman scattering, a highly excited infrared-active phonon serves as the intermediate state for Raman scattering, see figure 1(b). This effect is mediated through anharmonic phonon-phonon coupling rather than electron-phonon interaction, and it is less dissipative than its photonic counterpart due to the lower energy of the excitation [19][20].

In a recent experiment by Maehrlein et al. [21], a third route has been demonstrated, in which the 40 THz Raman-active phonon of diamond is excited by a terahertz pulse in a two-photon absorption process, see figure 1(c). This is the sum-frequency excitation (SFE) counterpart to ISRS, which combines the possibility to excite phonons in compounds that do not possess infrared-active phonons with the advantage of low-energy excitation by terahertz radiation.

The purpose of this theoretical study is two fold: First, we complete the map of photonic and ionic difference- and sum-frequency Raman mechanisms with the “missing” sum-frequency part of ionic Raman scattering (SF-IRS), see figure 1(d). Second, we compare the four mechanisms for three different example materials: Diamond, erbium ferrite (ErFeO₃), and bismuth ferrite (BiFeO₃). Assuming realistic experimental conditions, we show that sum-frequency excitation, both photonic and ionic, are able to coherently control Raman-active phonons in the electronic ground state in a way that is complementary to previous nonlinear phononics studies.

II. THEORETICAL MODEL

A. General equation of motion for the excitation of phonons

To investigate the time evolution of a phonon mode we numerically solve its equation of motion:

\[ \ddot{Q} + \kappa \dot{Q} + \frac{\partial}{\partial Q} V(Q) = \sum_i Z_i E_i + \varepsilon_0 \sum_{ij} R_{ij} E_i E_j, \]

where \( Q \) is the normal mode coordinate (or amplitude) of the phonon at the Brillouin-zone center in units of \( \sqrt{n} \), with \( n \) being the atomic mass unit. \( \kappa \) is the phonon linewidth, and \( V(Q) \) is the (an)harmonic lattice potential of the phonon. \( E_i \) is the electric field component of the
Exciting electromagnetic pulse, and the indices \( i \) and \( j \) run over the spatial coordinates. \( Z_i = \left( \sum_n Z^*_n u_n \right)_i \) is the mode effective charge of the phonon with \( Z^*_n \) being the Born effective charge tensor and \( u_n \) the displacement vector of atom \( n \), and the sum runs over all atoms in the unit cell \([22]\). \( R_{ij} = V_i \partial Q \chi_{ij} \) is the Raman tensor, which is given by the linear electric susceptibility, \( \chi_{ij} \), and the volume of the unit cell, \( V_i \). Phonon linewidths lie in the range of \( \kappa \approx \Omega / 10 \) to \( \Omega / 20 \) for the materials that we consider here, where \( \Omega \) is the eigenfrequency of the phonon mode \([9, 21, 23]\). For a detailed derivation, see for example reference \([24]\). We model the electric field component of a light or terahertz pulse as \( E(t) = E_0 \exp(-t/t_0)^2/(2(\tau/2\sqrt{2}\ln2)^2) \cos(\omega_0 t + \varphi_{\text{CEP}}) \), where \( E_0 \) is the peak electric field, \( \omega_0 \) is the center frequency, \( \varphi_{\text{CEP}} \) is the carrier-envelope phase, and \( \tau \) is the full width at half maximum duration of the pulse.

**B. Equations of motion for photonic and ionic Raman scattering**

For photonic Raman scattering we assume a harmonic lattice potential, \( V(Q) = \Omega^2 R^2 Q^2 / 2 \), as anharmonicities are not important for the process. Furthermore in centrosymmetric crystals \( Z_i = 0 \) for Raman-active phonons, and for a linearly polarized laser pulse, we can write equation (1) as

\[
\dot{Q}_R + \kappa_R \dot{Q}_R + \Omega^2 Q_R = \varepsilon_0 R E^2(t). \tag{2}
\]

In contrast, ionic Raman scattering is described by a quadratic-linear coupling of an infrared-active with a Raman-active phonon. The anharmonic lattice potential in its simplest form can be expressed as \( V(Q_R, Q_{IR}) = \Omega_{IR}^2 Q_{IR}^2 + \Omega_R^2 Q_R^2 + c Q_{IR}^2 Q_R \), where \( c \) is the quadratic-linear coupling coefficient given in meV/\((\sqrt{\text{Å}})^3\) \([19]\). We have to solve the equations of motion for both phonons, and equation (1) can be written respectively as

\[
\dot{Q}_{IR} + \kappa_{IR} \dot{Q}_{IR} + (\Omega_{IR}^2 + 2c Q_R) Q_{IR} = Z_{IR} E(t), \tag{3}
\]

\[
\dot{Q}_R + \kappa_R \dot{Q}_R + \Omega_R^2 Q_R = c Q_{IR}^2(t). \tag{4}
\]

The driving force of the Raman-active phonon in photonic Raman scattering is the square of the electric field, \( E^2(t) \), see equation (2) while in ionic Raman scattering it is the square of the “phonon field” of the infrared-active phonon, \( Q_{IR}^2(t) \), see equation (3). In addition, the Raman-active mode feedback affects the initially excited infrared-active mode by dynamically renormalizing its frequency as \( \Omega_{IR}^2 \rightarrow \Omega_{IR}^2 + 2c Q_{IR}^2 \), see equation (3).

The two photonic processes, ISRS and THz-SFE, can be described by the same equation of motion \([2]\). The two mechanisms are only distinguished by the duration of the pulse and its center frequency, \( \omega_0 \), which is higher than the phonon frequency in ISRS, \( \omega_0 > \Omega_R \), and ideally half the phonon frequency in THz-SFE, \( \omega_0 = \Omega_R / 2 \). We can draw an analogy for ionic Raman scattering here, which has so far been always connected to the coupling of a high-frequency infrared-active phonon with a low-frequency Raman-active phonon, \( \Omega_{IR} > \Omega_R \), in which difference-frequency components of the phonon field \( Q_{IR}^2(t) \) are responsible for the excitation of the Raman-active phonon \([18, 19, 25\text{–}32]\). Here, we will show that this mechanism can be extended to a sum-frequency counterpart that fulfills \( \Omega_{IR} = \Omega_R / 2 \) just analog to the photonic Raman processes \([21]\). A summary of the discussion in this section is given in table \( I \) in which also the results for phase sensitivity and impulsiveness from the following sections are shown.

**C. Computational details**

We calculated the phonon eigenfrequencies, eigenvectors, and the Raman tensors from first-principles using the density functional theory formalism as implemented in the Vienna ab-initio simulation package (VASP) \([33, 34]\), and the frozen-phonon method as implemented in the phonopy package \([35]\). To calculate the frequency-dependent Raman tensor we followed the scheme of reference \([36]\). We used the default VASP PAW pseudopotentials for every considered atom and converged the Hellmann-Feynman forces to \(10^{-5} \text{ eV/Å} \) using...
a plane-wave energy cut-off of 950 eV and a 9×9×9 k-point Monkhorst-Pack mesh [37] to sample the Brillouin zone for diamond and 850 eV, 6×6×6 for BiFeO₃. For the exchange-correlation functional, we chose the PBEsol form of the generalized gradient approximation (GGA) [38]. For BiFeO₃ we found that an on-site Coulomb interaction of 4 eV and a Hund’s exchange of 1 eV optimally reproduce both the G-type antiferromagnetic ordering and lattice dynamical properties [39, 40]. Our fully relaxed structures with lattice constants 3.55 Å for diamond and 3.94 Å with pseudocubic angle 90.44° for BiFeO₃ fit reasonably well to common experimental values [41, 42], as do our calculated phonon frequencies. Our calculated phonon eigenfrequency for the F₂g mode in diamond is 39.2 THz, though for simplicity we keep referring to it as the “40 THz mode”. For the details on ErFeO₃, we refer the reader to the computational details of reference [23].

III. RESULTS

A. THz-SFE versus ISRS in diamond

We begin by reproducing the experiments of references [21] and [43], in which the 40 THz F₂g Raman-active phonon of diamond was excited via THz-SFE and ISRS, respectively. Both mechanisms can be described by equation (2), for which we use the experimental excitation pulses with a center frequency of \( \omega_0 / 2\pi = 20 \) THz and pulse duration of \( \tau = 0.2 \) ps for the terahertz pulse for THz-SFE, and \( \omega_0 / 2\pi = 375 \) THz (395 nm) and \( \tau = 10 \) fs for the visible light pulse for ISRS. The electric field is oriented along the Raman-active [110] direction with a peak electric field of \( E_0 = 8 \text{ MV/cm} \) in both cases. The calculated parameters for the equation of motion are given in table I, and the eigenvector of the F₂g mode is illustrated in figure 2(a). We show the response of the F₂g mode to each optical excitation as described by equation (2) in figure 2(b).

The response for THz-SFE shows a gradual increase of the phonon amplitude with the onset of the terahertz pulse, which illustrates that the mechanism is nonimpulsive. The maximum phonon amplitude reaches \( Q = 0.28 \times 10^{-2} \sqrt{\mu} \text{A} \), and the phase of the oscillation is sensitive to the carrier-envelope phase of the terahertz pulse, \( \varphi_{\text{CEP}} \). The response for ISRS shows an abrupt onset of the phonon amplitude at \( t = 0 \), which is characteristic for the impulsive nature of the mechanism. The maximum phonon amplitude reaches \( Q = 0.04 \times 10^{-2} \sqrt{\mu} \text{A} \), which agrees well with the results of recent time-dependent density functional theory studies for ISRS [44, 45]. Here the phase of the oscillation is independent of \( \varphi_{\text{CEP}} \).

Despite the smaller value of the Raman tensor \( R^{(3)}(\text{THz}) \) compared to \( R^{(3)}(\text{VIS}) \), see table I, THz-SFE is roughly one order of magnitude stronger than ISRS for a similar electric field strength, \( E_0 \). This is because the 40 THz frequency component of the driving force, \( E^2(t) \), resulting from the sum frequency of the 20 THz pulse is roughly a factor of ten higher than that resulting from the difference frequency of the 395 nm pulse. This order-of-magnitude difference in excitation strength persists for pulses throughout the visible spectrum [44].

To take into account the total pulse energy, we show the dependence of the coherent phonon amplitude on the duration of the pump pulse in figure 2(c) for two distinct cases: (i) constant peak field, \( E_0 \), and (ii) constant pulse energy. In ISRS there is an optimal value of pulse duration for constant \( E_0 \) that corresponds to a bandwidth of the pulse, for which the difference-frequency components at 40 THz are maximal. When the energy of the pulse is fixed, a shorter pulse will trade off for a higher \( E_0 \) and therefore increase the effect until the pulse gets too short and approaches the single cycle regime. For THz-SFE the situation is different: Due to its nonimpulsive

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Diamond</th>
<th>ErFeO₃</th>
<th>BiFeO₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega_R ) (THz)</td>
<td>39.2</td>
<td>16.5</td>
<td>3.2</td>
</tr>
<tr>
<td>( Z_R ) (e/( \sqrt{\mu} ))</td>
<td>0.67</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td>( R(\text{VIS}) ) (Å²/( \sqrt{\mu} ))</td>
<td>70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R(\text{THz}) ) (Å²/( \sqrt{\mu} ))</td>
<td>50</td>
<td>-9</td>
<td>-41</td>
</tr>
<tr>
<td>( c ) (meV/(Å²( \sqrt{\mu} )))</td>
<td>7.8</td>
<td>8.0</td>
<td></td>
</tr>
</tbody>
</table>
FIG. 2. (a) Eigenvector of the 40 THz $F_{2g}$ mode of diamond. (b) Evolution of the $F_{2g}$ mode following THz-SFE and ISRS by a terahertz and visible light pulse, respectively. The envelopes of the excitation pulses are shown schematically. (c) Dependence of the normalized phonon amplitude, $Q/Q_0$, on the duration of the terahertz and visible light pulse, $\tau_{\text{THz-SFE}}$ and $\tau_{\text{ISRS}}$. $Q_0$ is the maximum phonon amplitude at $\tau_{\text{THz-SFE}} = 0$ ps and $\tau_{\text{ISRS}} = 10$ fs. We show two cases, one in which $E_0$ is kept constant while changing $\tau$ (circles), and one in which the total pulse energy is kept constant while changing $\tau$ and $E_0$ accordingly (triangles).

nature, a longer duration of the pulse will continuously increase the coherent phonon amplitude, when $E_0$ is kept constant. In this case, the amplitude will build up until damping, $\kappa$, and excitation force are balanced. In contrast, keeping the total pulse energy constant, a longer pulse will trade off for a lower $E_0$ and the effect decreases.

B. THz-SFE versus ionic Raman Scattering in ErFeO$_3$

We will now compare THz-SFE to the conventional, difference-frequency type of ionic Raman scattering (DF-IRS) at the example of ErFeO$_3$. For orthorhombic ErFeO$_3$ with space group $Pnma$, experimental and theoretical studies are available that show the coupling of the Raman-active 3.2 THz $A_g$ mode, see figure 3(a), with the infrared-active 16.5 THz $B_{3u}$ mode fulfilling the condition $\Omega_{\text{IR}} > \Omega_{\text{R}}$ [9,23]. Therefore, we model two different terahertz pulses, one with $\omega_0/2\pi = 1.6$ THz and pulse duration of $\tau = 1$ ps; the electric field for DF-IRS is oriented along the infrared-active $a$ direction with $\omega_0/2\pi = 16.5$ THz and $\tau = 0.2$ ps. We assume a peak electric field of $E_0 = 8$ MV/cm in both cases. The calculated parameters for the equations of motion are given in table II and the eigenvector of the $A_g$ mode is illustrated in figure 3(a). We show the response of the $A_g$ mode to each optical excitation as described by equations 2–4 in figure 3(b).

As in the case of diamond, the response for THz-SFE shows a continuous increase of the phonon amplitude with the onset of the pulse, reaching a maximum of $Q = 2.6 \times 10^{-12} \sqrt{\mu}$A. The response for DF-IRS shows an impulsive onset of the phonon amplitude at $t = 0$ that is not sensitive to the carrier-envelope phase, $\varphi_{\text{CEP}}$ [43], as well as the typical displacive feature of nonlinear phononics [18,19,23]. In this case, the maximum phonon amplitude reaches $Q = 0.9 \times 10^{-12} \sqrt{\mu}$A.

The amplitude of the $A_g$ mode induced by THz-SFE is higher than the amplitude induced by DF-IRS by a
At frequencies of 15.3 THz and 7.4 THz. Thus by exciting the system with a single pulse with a center frequency of $\omega_0/2\pi = 7.6$ THz we expect both THz-SFE and SF-IRS to occur at the same time: The pulse directly excites the $A_1(7)$ mode via infrared absorption, which then mediates energy to the $A_1(15)$ mode via SF-IRS. Simultaneously, the pulse excites the $A_1(15)$ mode via THz-SFE (but not via infrared absorption as the $A_1(15)$ phonon lies well outside the 2.9 THz bandwidth of the pulse). To make this process clearer, we show a schematic of the excitations in figure 3(a). Note that one could also drive the 15.3 THz mode directly via infrared absorption. For consistency to the previous sections, we label the 7.4 THz mode as “IR” and the 15.3 THz mode as “R”, and both criteria, $\omega_0 \approx \Omega_R/2$ and $\Omega_{IR} \approx \Omega_R/2$ are fulfilled. We model the terahertz pulse with a center frequency of $\omega_0/2\pi = 7.6$ THz and a duration of $\tau = 0.3$ ps. The electric field is oriented along the Raman- and infrared-active [111] direction with a peak of $E_0 = 8$ MV/cm. The calculated parameters for the equations of motion are given in table I and the eigenvectors of the $A_1$ modes are illustrated in figure 3(b). We show the responses of both $A_1$ modes to the optical excitation as described by equations 2 - 4 in figures 3(c),(d).

The response for the simultaneous excitation via THz-SFE and SF-IRS in figure 3(c) shows a fundamentally different behavior from the other three mechanisms: A beat signal arises and the phonon amplitude reaches by far the highest value of all three examples, $Q = 0.37 \sqrt{\mu}$. We separate the effects of THz-SFE and SF-IRS in figure 3(d). The response for SF-IRS entirely captures the new feature, while the THz-SFE contribution leads only to a negligible increase of the phonon amplitude and phase shift. The beat signal is caused by a mutual exchange of energy between the infrared-active and Raman-active phonon, and the beat frequency is determined by the strength of the anharmonic phonon coupling, $c$, and through $Q_R$ by the strength of the terahertz pulse, see equation 6. A node of the beat signal of the $A_1(15)$ mode corresponds to a maximum of the beat signal of the $A_1(7)$ mode, see black arrows in figure 3(c) - the maximum is swallowed by the damping, however. Naturally, for sum-frequency excitation, the phase of the response is sensitive to the carrier-envelope phase of the terahertz pulse, $\varphi_{CEP}$. Higher-order anharmonicities in the potential $V(Q_R, Q_{IR})$ affect the amplitude, beat-frequency and even introduce new beats, however to a much smaller degree than the quadratic-linear coupling, $Q_R^2 Q_{IR}$.

IV. DISCUSSION

We completed the map of photonic and ionic Raman scattering for the excitation of Raman-active phonons in insulators with the missing sum-frequency part of ionic Raman scattering, see figure 3(d). The difference-frequency mechanisms are impulsive in nature and not sensitive to the carrier-envelope phase of the driving field.
FIG. 4. (a) Schematic of the excitation by the terahertz pulse. (b) Eigenvectors of the coupled 15.3 THz (blue) and 7.4 THz (green) $A_1$ modes from a view along the [111] direction of rhombohedral BiFeO$_3$. The motion of bismuth ions in these modes is negligible. (b) Full evolution of the $A_1$ modes following the excitation by a single 7.6 THz pulse. The $A_1(15)$ mode is excited via infrared absorption. The $A_1(15)$ mode is excited simultaneously via THz-SFE and SF-IRS. Black arrows point to the beat nodes of the $A_1(15)$ mode that correspond to beat maxima of the $A_1(7)$ mode. (c) Evolution of the $A_1(15)$ mode as in (b), but separating the effects of the two mechanisms. The marginal THz-SFE contribution is magnified by a factor of ten for better visibility. The envelope of the excitation pulse is shown schematically.

$\varphi_{\text{CEP}}$, whereas the sum-frequency mechanisms are non-impulsive and therefore sensitive to $\varphi_{\text{CEP}}$. A summary of the properties is shown in table I.

Among the investigated phonon excitations, the up-conversion of frequency components of the driving force is more efficient than the down-conversion. An increase of the total pulse energy will only enhance difference-frequency excitation if it is due to a higher peak electric field, $E_0$, but not due to a longer pulse duration, $\tau$. In contrast, an increase of either $E_0$ or $\tau$ leads to a stronger sum-frequency excitation. This property is particularly relevant for narrowband excitation pulses, for example generated by accelerator-based mid-infrared and terahertz sources. The resulting frequency components are weighted by the Raman tensor and anharmonic phonon coupling, which both depend on the material properties.

For homonuclear materials that do not possess infrared-active phonons, such as diamond, only photonic difference- and sum-frequency excitation is possible. Here the more efficient conversion of sum-frequency components also leads to a higher selectivity for THz-SFE compared to ISRS. Generally, the selectivity depends on the symmetries and frequencies of the phonon modes in the material. In the photonic Raman mechanisms the electric field has to be oriented along the Raman-active direction of the target $Q_R$ mode, while in the ionic Raman mechanisms it has to be oriented along the infrared-active direction of the coupling $Q_{IR}$ mode. Consequently, the selectivity depends on whether “unwanted” phonon modes lie within the bandwidth and polarization direction of the driving force $E(t)$ (infrared-active) or $E^2(t)$ (Raman-active) in addition to our target $Q_{IR}$ and $Q_R$ modes. For lattice driven phenomena in the electronic ground state, all three terahertz excitation mechanisms, THz-SFE, DF- and SF-IRS, are favorable over commonly used visible-light or near-infrared ISRS in order to avoid parasitic electronic excitations. The
sum-frequency processes provide an additional route to excite optical phonons within the “new terahertz gap” of 5-15 THz, for which powerful sources are yet missing [3].

With the increased availability of strong terahertz and mid-infrared sources, we anticipate that the presented map of photonic and ionic Raman mechanisms will serve as guide for the selective excitation of crystal lattice vibrations in future. Specifically, we expect that strong excitation of Raman-active phonons will complement the effects arising from infrared-active phonons in the context of spin-phonon and electron-phonon coupled phenomena.

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