

**Conformal standard model, leptogenesis, and dark matter**Adrian Lewandowski,<sup>1,\*</sup> Krzysztof A. Meissner,<sup>2</sup> and Hermann Nicolai<sup>1</sup><sup>1</sup>*Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut) Mühlenberg 1, D-14476 Potsdam, Germany*<sup>2</sup>*Faculty of Physics, University of Warsaw, ul. Pasteura 5, 02-093 Warsaw, Poland*

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The conformal standard model is a minimal extension of the Standard Model (SM) of particle physics based on the assumed absence of large intermediate scales between the TeV scale and the Planck scale, which incorporates only right-chiral neutrinos and a new complex scalar in addition to the usual SM degrees of freedom, but no other features such as supersymmetric partners. In this paper, we present a comprehensive quantitative analysis of this model, and show that all outstanding issues of particle physics proper can in principle be solved “in one go” within this framework. This includes in particular the stabilization of the electroweak scale, “minimal” leptogenesis and the explanation of dark matter, with a small mass and very weakly interacting Majoron as the dark matter candidate (for which we propose to use the name “minoron”). The main testable prediction of the model is a new and almost sterile scalar boson that would manifest itself as a narrow resonance in the TeV region. We give a representative range of parameter values consistent with our assumptions and with observation.

DOI: [10.1103/PhysRevD.97.035024](https://doi.org/10.1103/PhysRevD.97.035024)**I. INTRODUCTION**

The conspicuous absence of any hints of “new physics” at LHC, and, more pertinently, of supersymmetric partners and exotics [1,2] has prompted a search for alternative scenarios beyond the Standard Model (SM) based on the hypothesis that the SM could survive essentially *as is* all the way to the Planck scale, modulo “minor” modifications of the type discussed here, see [3–22] for a (very incomplete) list of references. In this paper we follow up on a specific proposal along these lines which is based on our earlier work [6], and demonstrate that this proposal in principle allows for a comprehensive treatment of all outstanding problems of particle physics proper. This list includes perturbativity and stability of the model up to the Planck scale and an explanation of leptogenesis and the nature of dark matter,<sup>1</sup> in a way which is in complete accord with the

fact that LHC has so far seen nothing, and furthermore appears to be fully consistent as a relativistic QFT all the way up to the Planck scale  $M_{\text{PL}}$  (for which we use the reduced value  $M_{\text{PL}} \approx 2.4 \times 10^{18}$  GeV). The consistency up to that scale, but not necessarily beyond, is in accord with our essential assumption that, at the Planck scale, an as yet unknown UV complete theory of quantum gravity and quantum space-time takes over that transcends space-time based relativistic QFT. Importantly, the present approach is essentially “agnostic” about what this theory is.

Added motivation for the present investigation comes from very recent LHC results which indicate that the low-energy supersymmetry (SUSY) paradigm which has dominated much of particle physics over the past three decades is close to failure, unless one resorts to the more exotic possibility that “low-energy” ( $N = 1$ ) supersymmetry is broken at a very high scale. In our opinion, however, the latter option would defeat the original purpose of solving the hierarchy problem, and thus lack the plausibility of the original minimal supersymmetric standard model (MSSM). One crucial question is therefore how the “naturalness” of the electroweak scale can be explained without appealing to supersymmetric cancellations. In this paper we offer one possible such alternative explanation based on [25]; another possibility which bears some resemblance to the present scheme as far as physics up to  $M_{\text{PL}}$  is concerned (but not beyond) is to invoke asymptotic safety, see e.g. [13,26,27].

In its original form the model proposed in [6] tried to exploit the fact that, with the exception of the scalar mass

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<sup>1</sup>We thus exclude dark energy and the origin of inflation from this list; these are problems that, in our view, will likely require a proper theory of quantum gravity for their complete resolution. We note, however, that Higgs inflation [23] can be easily incorporated into the present model [24].

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term that triggers spontaneous breaking of electroweak symmetry, the SM Lagrangian is classically conformally invariant. For this it relied on the Coleman-Weinberg (CW) mechanism [28] to break electroweak symmetry and to argue that mass scales can be generated purely by the quantum-mechanical breaking of classical conformal invariance. In this paper a modified version of this model is presented which has explicit mass terms but which is still conformal in the sense that it postulates the absence of any intermediate scales between 1 TeV and the Planck scale  $M_{\text{PL}}$ —hence the name, *conformal standard model* (CSM). The model nevertheless achieves a stabilization of the electroweak hierarchy thanks to an alternative proposal for the cancellation of quadratic divergences presented in [25], and it is in this sense that we speak of *softly broken conformal symmetry* (SBCS). This term is meant to comprise three main assumptions, namely: (i) the avoidance of quadratic divergences, (ii) the smallness (with respect to the Planck scale) of all dimensionful quantities, and (iii) the smallness of all dimensionless couplings up to  $M_{\text{PL}}$ . With these assumptions the model is indeed “almost conformal,” and the quantum-mechanical breaking of conformal invariance (as embodied in the CW correction to the effective potential) remains a small correction over this whole range of energies. Here, we extend our previous considerations towards a more complete picture, in an attempt to arrive at a minimal comprehensive solution to the outstanding problems of particle physics, in a way that remains compatible with all available LHC results. More specifically, we here focus on the question whether the model can offer a viable explanation for leptogenesis and the origin of dark matter. The main message of this paper, then, is that indeed these problems can be solved at least in principle within this minimal SBCS scheme. However, as we said, no attempt will be made towards a solution of cosmological constant problem, nor inflation or dark energy, as these probably require quantum gravity.

The present paper thus puts together different ideas most of which have already appeared in different forms in the literature (in particular, conformal symmetry, extra sterile scalars and “Higgs portals,” low mass heavy neutrinos, resonant leptogenesis, and quantum gravity induced violations of the Goldstone theorem), though, to the best of our knowledge, never in the combination proposed and elaborated here. Let us therefore summarize the distinguishing special features and assumptions underlying the present work:

- (i) There are no large intermediate scales between the TeV scale and the Planck mass; in particular there is no grand unification nor grand unified theory (GUT) scale physics.
- (ii) There is no low-energy supersymmetry; instead the electroweak hierarchy is stabilized by the alternative

mechanism for the cancellation of quadratic divergences proposed in [25].<sup>2</sup>

- (iii) The consistency of the model up to the Planck scale is ensured by demanding absence of Landau poles and of instabilities or metastabilities up to that scale. Possible pathologies that might appear if the model is extrapolated *beyond* that scale are assumed to be taken care of by quantum gravity, hence are not relevant for the present analysis.
- (iv) The model naturally incorporates resonant leptogenesis [29–32] with low mass heavy neutrinos, where we show that a range of parameters exists which meets all requirements. Furthermore, the predictions of the model do not in any way affect the SM tests that have so far confirmed the SM *as is*.
- (v) The Majoron, i.e. the Goldstone boson of spontaneously broken lepton-number symmetry, is assumed to acquire a small mass  $\sim 10^{-3}$  eV due to a (still conjectural) folklore theorem according to which there cannot exist unbroken continuous global symmetries in quantum gravity, as a consequence of which it becomes a possible dark matter candidate (whose abundance comes out with the right order of magnitude subject to our assumptions). The ensuing violation of the Goldstone theorem entails calculable couplings to SM particles from radiative corrections, which are naturally very small.
- (vi) The main testable prediction of the model is a new scalar resonance at  $\mathcal{O}(1 \text{ TeV})$  or even below that is accompanied by a (in principle measurable) reduction of the decay width of the SM-like Higgs boson. The couplings of the new scalar to SM particles are strongly suppressed in comparison with those of the SM Higgs boson by a factor  $\sin \beta$ , where the angle  $\beta$  parametrizes the mixing between the SM Higgs boson and the new scalar. The only new *fermionic* degrees of freedom are three right-chiral neutrinos.
- (vii) Because our model contains no new scalars that carry charges under SM gauge symmetries it can be easily discriminated against many other models with an enlarged scalar sector, such as two doublet models.

We note that a comprehensive “global” and quantitative analysis of the type performed here would be rather more cumbersome, or even impossible, for more extensive scenarios beyond the SM with more degrees of freedom and more free parameters. For instance, even with a very restricted minimal set of new degrees of freedom and parameters as in the present setup, closer analysis shows

<sup>2</sup>However, this assumption by no means excludes the possibility that (extended) supersymmetry does play an essential role at the Planck scale to ensure finiteness (UV completeness) of a unified theory of quantum gravity.

that in order to arrive at the desired physical effects such as resonant leptogenesis with the right order of magnitude for the lepton asymmetry a very careful scan over parameter space is required, as the physical results can depend very sensitively on *all* parameters of the model, so some degree of fine-tuning may be unavoidable.

The structure of this paper is as follows. In Sec. II we describe the basic properties of the model, and explain how to maintain perturbativity and stability up to the Planck scale. Section III is devoted to a detailed discussion of leptogenesis in the CSM, and shows that a viable range of parameters exists for which resonant leptogenesis can work. In Sec. IV we discuss  $(B - L)$  breaking and the possible role and properties of the associated pseudo-Goldstone boson (“minoron”) as a dark matter candidate. Although we present a representative range of parameters consistent with all our assumptions and with observations, we should emphasize that our numerical estimates are still quite preliminary. Of course, these estimates could be much improved if the new scalar were actually found and its mass value measured. For the reader’s convenience we have included an appendix explaining basic properties of neutrino field operators in Weyl spinor formalism.

## II. THE CSM

The CSM is a minimal extension of the Standard Model that incorporates right-chiral neutrinos and an additional complex scalar field, which is charged under SM lepton-number, like the right-chiral neutrinos, and generates a Majorana mass term for the right-chiral neutrinos after spontaneous breaking of lepton-number symmetry. In keeping with our basic SBCS hypothesis of softly broken conformal symmetry, that is, the absence of large intermediate scales between the TeV scale and the Planck scale  $M_{\text{PL}}$ , this mass is here assumed to be of  $\mathcal{O}(1 \text{ TeV})$ . To ensure the stability of the electroweak scale it makes use of a novel mechanism to cancel quadratic divergences [25], relying on the assumed existence of a Planck scale finite theory of quantum gravity, as a consequence of which the cutoff is a physical scale that is not taken to infinity. The phase of the new scalar is a Goldstone boson that within the framework of ordinary quantum field theory remains massless to all orders due to the vanishing  $(B - L)$  anomaly, but will be assumed to acquire a tiny mass by a quantum gravity induced mechanism, as a result of which it acquires also small and calculable nonderivative couplings to SM matter. The viability of the model up to the Planck scale will be ensured by imposing the consistency requirements listed above. In particular, the extra degrees of freedom that the CSM contains beyond the SM are essential for stability: without these extra degrees of freedom the SM does suffer from an instability (or rather, metastability) because the running scalar self-coupling becomes negative around  $10^{10} \text{ GeV}$  [33].

The field content of the model is thus almost the same as for the SM (see e.g. [34–36] for further details, and [37] for a more recent update). For the fermions we will mostly use  $\text{SL}(2, \mathbb{C})$  Weyl spinors  $\chi_\alpha$  in this paper, together with their complex conjugates  $\bar{\chi}_{\dot{\alpha}}$ , see e.g. [38] for an introduction. The quark and lepton  $SU(2)_L$  doublets are thus each composed of two  $\text{SL}(2, \mathbb{C})$  spinors

$$Q^i \equiv \begin{pmatrix} u_\alpha^i \\ d_\alpha^i \end{pmatrix}, \quad L^i \equiv \begin{pmatrix} \nu_\alpha^i \\ e_\alpha^i \end{pmatrix},$$

where indices  $i, j, \dots = 1, 2, 3$  label the three families. In addition we have their  $SU(2)_L$ -singlet partners  $U_\alpha^i, D_\alpha^i$  and  $E_\alpha^i$ . The new fermions in addition to the ones present in the SM are made up of a family triplet  $N_\alpha^i$  of gauge singlet neutrinos. The scalar sector of the model consists of the usual electroweak scalar doublet  $H \equiv (H_1, H_2)^\top$  and a new gauge-sterile complex singlet scalar  $\phi(x)$ , which carries lepton number. This field couples only to the sterile neutrinos and, via the Higgs portal, to the electroweak doublet  $H$ .

### A. Lagrangian

Apart from the ghosts and gauge fixing terms [34], the CSM Lagrangian takes the form

$$\mathcal{L}_{\text{CSM}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_Y - V, \quad (1)$$

with gauge-invariant kinetic terms

$$\begin{aligned} \mathcal{L}_{\text{kin}} = & \mathcal{L}_{\text{kin}}^{\text{SM}} + (D_\mu H)^\dagger \mathcal{D}^\mu H \\ & + \partial_\mu \phi^* \partial^\mu \phi + i \bar{N}_\alpha^j \bar{\sigma}^{\mu\dot{\alpha}\beta} \partial_\mu N_\beta^j, \end{aligned} \quad (2)$$

where we only display the kinetic term of the Higgs doublet and the kinetic terms of the new fields, while  $\mathcal{L}_{\text{kin}}^{\text{SM}}$  takes the standard form that can be found in any textbook, see [34–36]. The scalar potential reads

$$\begin{aligned} V = & -m_1^2 H^\dagger H - m_2^2 \phi^* \phi \\ & + \lambda_1 (H^\dagger H)^2 + 2\lambda_3 H^\dagger H \phi^* \phi + \lambda_2 (\phi^* \phi)^2, \end{aligned} \quad (3)$$

with  $m_1^2, m_2^2 > 0$ . Exploiting the symmetries of the action we assume that the vacuum expectation values take the form<sup>3</sup>

$$\sqrt{2}\langle H_i \rangle = v_H \delta_{i2}, \quad \sqrt{2}\langle \phi \rangle = v_\phi, \quad (4)$$

with non-negative  $v_H$  and  $v_\phi$ . Clearly, we are interested in a situation in which both the electroweak symmetry and lepton-number symmetry are broken, and therefore we

<sup>3</sup>For the vacuum expectation values we adopt the normalization conventions of [25].

TABLE I. Exemplary Values.

$M_\varphi$ [GeV]	$s_\beta$	$M_N$ [GeV]	$v_\phi$ [GeV]	$\mathcal{Y}_{B0}$	$\Gamma_h$ [MeV]	$\Gamma_\varphi$ [GeV]	$\text{Br}(\varphi \rightarrow [\text{SM}])$	$\text{Br}(\varphi \rightarrow hh)$
1030	-0.067	1604	17 090	$7.9 \times 10^{-11}$	4.19	4.02	0.78	0.2
893	-0.076	1238	11 331	$1.2 \times 10^{-10}$	4.186	3.3	0.76	0.2
839	-0.082	1181	11 056	$1.2 \times 10^{-10}$	4.182	3.08	0.76	0.2
738	-0.093	1052	10 082	$1.1 \times 10^{-10}$	4.174	2.66	0.76	0.22
642	-0.11	1303	19 467	$1.7 \times 10^{-10}$	4.16	2.34	0.76	0.22
531	-0.13	949	12 358	$1.5 \times 10^{-10}$	4.138	1.92	0.74	0.22
393	-0.18	801	12 591	$1.0 \times 10^{-10}$	4.07	1.28	0.72	0.26
362	-0.20	815	14 534	$1.6 \times 10^{-10}$	4.04	1.06	0.68	0.3
350	-0.21	738	12 302	$7.4 \times 10^{-11}$	4.028	0.96	0.66	0.32
320	-0.23	751	14 437	$1.3 \times 10^{-10}$	3.984	0.86	0.66	0.32
279	-0.28	683	14 334	$9.6 \times 10^{-11}$	3.896	0.68	0.7	0.28
258	-0.31	675	15 752	$1.3 \times 10^{-10}$	3.824	0.54	0.78	0.20

assume that  $v_H$  and  $v_\phi$  are nonzero. The values (4) correspond to the stationary point of (3), provided that the mass parameters are chosen as follows:

$$m_1^2 = \lambda_3 v_\phi^2 + \lambda_1 v_H^2, \quad m_2^2 = \lambda_3 v_H^2 + \lambda_2 v_\phi^2.$$

The tree-level potential (3) is bounded from below provided that the quartic couplings obey

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \text{and} \quad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}. \quad (5)$$

If, in addition to (5),  $\lambda_3 < \sqrt{\lambda_1 \lambda_2}$ , then (4) is the global minimum of  $V$ . The physical spin-zero particles are then two  $CP$ -even scalars  $h$  and  $\varphi$ , and one  $CP$ -odd scalar  $a = \sqrt{2}\text{Im}(\phi)$ .<sup>4</sup> The latter is the Goldstone boson, which—as we will argue later—acquires a small mass due to quantum gravity effects (see Sec. IV A).

The two heavy scalar bosons are thus described as mixtures of the two real scalar fields with nonvanishing vacuum expectation values ( $s_\beta \equiv \sin \beta$ ,  $c_\beta \equiv \cos \beta$ ),

$$\begin{pmatrix} h \\ \varphi \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \sqrt{2}\text{Re}(H_2 - \langle H_2 \rangle) \\ \sqrt{2}\text{Re}(\phi - \langle \phi \rangle) \end{pmatrix}, \quad (6)$$

with masses  $M_h$  and  $M_\varphi$ . The angle  $\beta$  thus measures the mixing between the SM Higgs boson and the new scalar. In order not to be in conflict with existing data the angle  $\beta$  must obviously be chosen small, and furthermore such that  $h$  can be identified with the observed SM-like Higgs boson with  $M_h = (125.6 \pm 0.4)$  GeV [39]. Introducing the tree-level SM quartic coupling

<sup>4</sup>Here we employ a linear parametrization of the scalar fields, i.e.  $\phi = \text{Re}\phi + i\text{Im}\phi$ , because this is the most convenient one for loop calculations. Later, however, we will switch to an exponential parametrization, see Eq. (73) below, which is more convenient to study properties of the Goldstone boson, but where the renormalizability of the model is no longer manifest.

$$\lambda_0 \equiv \frac{1}{2} \frac{M_h^2}{v_H^2} \approx 0.13, \quad (7)$$

one can conveniently parametrize the tree-level values of unknown parameters  $v_\phi$ ,  $M_\varphi$  and  $\beta$  in terms of the five parameters ( $v_H, \lambda_0, \lambda_1, \lambda_2, \lambda_3$ ) as follows:

$$v_\phi = v_H \sqrt{\frac{\lambda_0(\lambda_1 - \lambda_0)}{\lambda_2(\lambda_1 - \lambda_0) - \lambda_3^2}}, \quad M_\varphi^2 = 2 \frac{\lambda_1 \lambda_2 - \lambda_3^2}{\lambda_0} v_\phi^2 \quad (8)$$

and

$$\tan \beta = \frac{\lambda_0 - \lambda_1}{\lambda_3} \frac{v_H}{v_\phi}, \quad s_\beta \equiv + \frac{\tan \beta}{\sqrt{1 + \tan^2 \beta}}. \quad (9)$$

As a consequence, the model predicts the appearance of a “heavy brother” of the usual Higgs boson, which would manifest itself as a *narrow* resonance in or below the TeV region (see Table I below; the narrowness of the resonance is due to the small mixing  $\sin^2 \beta$  and the relatively large  $v_\phi$  scale). The SM-like Higgs boson  $h$  can, in principle, decay into a pair of pseudo-Goldstone bosons. The corresponding branching ratios are, however, very small (for all exemplary points in Table I they do not exceed 0.2%). Thus, the decay width of  $h$  is decreased with respect to the SM value by a factor  $\cos^2 \beta$ . In the numerical analysis we assume that  $|\beta| \leq 0.3$ ; we note that available LHC data leave enough room for such a modification of SM physics [40,41].

The Yukawa couplings are given by

$$\begin{aligned} \mathcal{L}_Y = & \left\{ -Y_{ji}^D D^{j\alpha} H^\dagger Q_\alpha^i + Y_{ji}^U U^{j\alpha} H^\dagger \epsilon Q_\alpha^i \right. \\ & - Y_{ji}^E E^{j\alpha} H^\dagger L_\alpha^i + Y_{ji}^V N^{j\alpha} H^\dagger \epsilon L_\alpha^i \\ & \left. - \frac{1}{2} Y_{ji}^M \phi N^{j\alpha} N_\alpha^i \right\} + \text{H.c.}, \quad (10) \end{aligned}$$

with the usual Yukawa matrices  $Y^E$ ,  $Y^D$  and  $Y^U$  of the SM, where  $\epsilon$  is the antisymmetric  $SU(2)_L$  metric. The matrix  $Y^\nu$  mediates the coupling of the SM fields to the sterile neutrino components, while the complex symmetric matrix  $Y^M$  describes the interactions of the latter with the new scalar  $\phi$ . By fermionic field redefinitions that preserve Eq. (2), one can assume that  $Y^M$ ,  $Y^E$  and  $Y^U$  are diagonal and non-negative, and that  $Y^D$  differs from a positive and diagonal  $\tilde{Y}^D$  by the (inverse of) unitary CKM matrix  $Y^D = \tilde{Y}^D V_{\text{CKM}}^\dagger$ . In more customary notation the fermionic fields are described by 4-component Dirac spinors of charged leptons and up-type quarks

$$\Psi_E^i = \begin{bmatrix} e_\alpha^i \\ \bar{E}^{i\alpha} \end{bmatrix}, \quad \Psi_U^i = \begin{bmatrix} u_\alpha^i \\ \bar{U}^{i\alpha} \end{bmatrix}, \quad (11)$$

together with the analogous 4-spinor field  $\Psi_D^i$

$$\Psi_D^i = \begin{bmatrix} d_\alpha^i \\ \bar{D}^{i\alpha} \end{bmatrix} \quad (12)$$

for the down quarks, with a  $V_{\text{CKM}}$ -induced rotation  $d_\alpha^i \rightarrow d_\alpha^{\prime i}$  of the upper components [34].<sup>5</sup>

After spontaneous symmetry breaking the neutrino mass terms are

$$-\mathcal{L} \supset m_{Dij} N^{ai} \nu_\alpha^j + m_{Dij}^* \bar{N}_\alpha^i \bar{\nu}^{aj} + \frac{1}{2} M_{Nij} N^{i\alpha} N_\alpha^j + \frac{1}{2} M_{Nij}^* \bar{N}_\alpha^i \bar{N}^{aj} \quad (13)$$

with

$$M_N \equiv Y^M v_\phi / \sqrt{2}, \quad m_D \equiv Y^\nu v_H / \sqrt{2}. \quad (14)$$

The masses for light neutrinos are thus obtained via the seesaw mechanism [42–44] and follow easily by diagonalizing the symmetric tree-level mass matrix (see also [45])

$$\mathcal{M} = \begin{bmatrix} 0 & m_D^\top \\ m_D & M_N \end{bmatrix}. \quad (15)$$

Introducing unitary  $3 \times 3$  matrices  $U_0$  and  $V_0$  and the  $6 \times 6$  block matrix

$$\mathbb{V} = \begin{bmatrix} \mathbb{X}_1 & \mathbb{X}_2 \\ \mathbb{X}_3 & \mathbb{X}_4 \end{bmatrix}, \quad (16)$$

with the  $3 \times 3$  submatrices

<sup>5</sup>The *chiral* Dirac fields usually employed are thus  $e_L \equiv P_L \Psi_E = (e, 0)$  and  $e_R \equiv P_R \Psi_E = (0, \bar{E})$ , etc.

$$\begin{aligned} \mathbb{X}_1 &= i \left\{ \mathbb{1} - \frac{1}{2} m_D^\dagger M_N^{-1\dagger} M_N^{-1} m_D \right\} U_0, \\ \mathbb{X}_2 &= m_D^\dagger M_N^{-1\dagger} V_0, \\ \mathbb{X}_3 &= -i M_N^{-1} m_D U_0, \\ \mathbb{X}_4 &= \left\{ \mathbb{1} - \frac{1}{2} M_N^{-1} m_D m_D^\dagger M_N^{-1\dagger} \right\} V_0, \end{aligned}$$

one has  $\mathbb{V}^\dagger \mathbb{V} = \mathbb{1} + \mathcal{O}(\|m_D\|^3)$ , and

$$\mathcal{M}_{\text{ph}} \equiv \mathbb{V}^\top \mathcal{M} \mathbb{V} = \begin{bmatrix} U_0^\top \mathcal{M}_\nu U_0 & 0 \\ 0 & V_0^\top \mathcal{M}_N V_0 \end{bmatrix} + \mathcal{O}(\|m_D\|^3), \quad (17)$$

with complex symmetric matrices

$$\mathcal{M}_\nu = m_D^\top M_N^{-1} m_D, \quad (18)$$

$$\mathcal{M}_N = M_N + \frac{1}{2} M_N^{-1*} m_D^* m_D^\top + \frac{1}{2} m_D m_D^\dagger M_N^{-1\dagger}. \quad (19)$$

Observe that up to  $\mathcal{O}(m_D^3)$  the matrix  $\mathbb{V}$  achieves the diagonalization of the  $6 \times 6$  matrix  $\mathcal{M}$  in (15) into the two blocks of  $3 \times 3$  matrices exhibited above, but that the latter are not necessarily in diagonal form yet. Employing the Casas-Ibarra parametrization [46] of the Dirac mass matrix  $m_D$  (or equivalently the Yukawa matrix  $Y^\nu$ ; as explained above,  $M_N$  can be assumed positive diagonal)

$$m_D = M_N^{1/2} R_{\text{CI}}^\top [\text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})^{1/2}] U_{\text{MNS}}^\dagger, \quad (20)$$

with the unitary Maki-Nakagawa-Sakata  $U_{\text{MNS}}$  matrix (see [46] and references therein) and a complex orthogonal Casas-Ibarra matrix  $R_{\text{CI}}$  ( $R_{\text{CI}}^\top R_{\text{CI}} = \mathbb{1}$ ) one has

$$\mathcal{M}_\nu = U_{\text{MNS}}^* [\text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})] U_{\text{MNS}}^\dagger, \quad (21)$$

which shows that  $m_{\nu i}$  are light neutrino masses at the tree level. The main advantage of the Casas-Ibarra parametrization, and the reason we use it here, is that it provides a clear separation of the parameters of  $Y^\nu$  into the ones that are relevant for neutrino oscillations, namely  $m_{\nu i}$  and the CKM-like unitary matrix  $U_{\text{MNS}}$ , and the ones describing heavy neutrinos and their properties ( $M_N$ ,  $R_{\text{CI}}$ ).

The matrix  $V_0$  is now chosen such as to make  $V_0^\top \mathcal{M}_N V_0$  a positive diagonal matrix [note that  $\mathcal{M}_N$  differs from  $M_N$ , cf. (19)]. The matrix  $\mathcal{M}_{\text{ph}}$  in Eq. (17) is then diagonal provided that  $U_0 = U_{\text{MNS}}$ ; however, we will be mainly interested in light neutrino states that participate in specific fast interactions during the leptogenesis, i.e. that are approximate eigenstates of weak interactions. Therefore, we take  $U_0 = \mathbb{1}$ , and change the basis in the field space so that  $\mathcal{M}_{\text{ph}}$  in Eq. (17) is a new mass matrix (in other words, we are using interaction eigenstates rather than mass

eigenstates for the light neutrinos). Henceforth,  $\nu_\alpha^i$  and  $N_\alpha^j$  denote neutrino fields in this new basis, unless stated otherwise, and are referred to as light and heavy neutrinos. It is sometimes convenient to assemble these 2-component Weyl spinors into Majorana 4-spinors

$$\psi_N^i = \begin{bmatrix} N_\alpha^i \\ \bar{N}^{i\dot{\alpha}} \end{bmatrix}, \quad \psi_\nu^i = \begin{bmatrix} \nu_\alpha^i \\ \bar{\nu}^{i\dot{\alpha}} \end{bmatrix}. \quad (22)$$

Note that, as a result of the rotation with  $\mathbb{V}$ , the (new)  $N$ 's do couple to the massive gauge bosons already at the tree level

$$\begin{aligned} \mathcal{L}_{ZN\nu} &= iZ_\mu (\mathcal{F}_{ji}^{(Z)} \bar{N}_\alpha^j \bar{\sigma}^{\mu\dot{\alpha}\alpha} \nu_\alpha^i + \mathcal{F}_{ji}^{(Z)*} N^{j\alpha} \sigma_{\alpha\dot{\alpha}}^\mu \bar{\nu}^{i\dot{\alpha}}) \\ &\equiv iZ_\mu \bar{\psi}_N^j \gamma^\mu (\mathcal{F}_{ji}^{(Z)} P_L + \mathcal{F}_{ji}^{(Z)*} P_R) \psi_\nu^i, \end{aligned} \quad (23)$$

$$\begin{aligned} \mathcal{L}_{WNe} &= i(W_\mu^1 - iW_\mu^2) \mathcal{F}_{ji}^{(W)} \bar{N}_\alpha^j \bar{\sigma}^{\mu\dot{\alpha}\alpha} e_\alpha^i + \text{H.c.} \\ &\equiv i(W_\mu^1 - iW_\mu^2) \mathcal{F}_{ji}^{(W)} \bar{\psi}_N^j \gamma^\mu P_L \Psi_E^i + \text{H.c.}, \end{aligned} \quad (24)$$

where for clarity we also give the result in standard 4-spinor notation, and where  $P_{L/R} \equiv \frac{1}{2}(1 \mp \gamma^5)$  are the usual chiral projectors. The matrices  $\mathcal{F}^{(Z,W)}$  follow immediately from Eq. (16)

$$\mathcal{F}^{(Z)} = -\frac{i}{2}(g_w^2 + g_y^2)^{1/2} \mathbb{X}_2^\dagger \mathbb{X}_1, \quad (25)$$

$$\mathcal{F}^{(W)} = -\frac{i}{2} g_w \mathbb{X}_2^\dagger. \quad (26)$$

To avoid confusion we also use calligraphic letters to denote the couplings between the new fields  $N/\nu$  and the scalars  $S = h, \varphi, a$ , to wit,

$$\begin{aligned} \mathcal{L}_{SN\nu} &= -S(\mathcal{Y}_{ji}^{(S)} N^{j\alpha} \nu_\alpha^i + \mathcal{Y}_{ji}^{(S)*} \bar{N}_\alpha^j \bar{\nu}^{i\dot{\alpha}}) \\ &\equiv -S \bar{\psi}_N^j (\mathcal{Y}_{ji}^{(S)} P_L + \mathcal{Y}_{ji}^{(S)*} P_R) \psi_\nu^i, \end{aligned} \quad (27)$$

where the leading terms in  $m_D$  read

$$\begin{aligned} \mathcal{Y}^{(h)} &= +i \left\{ \frac{c_\beta}{v_H} - \frac{s_\beta}{v_\phi} \right\} V_0^\top m_D U_0, \\ \mathcal{Y}^{(\varphi)} &= -i \left\{ \frac{s_\beta}{v_H} + \frac{c_\beta}{v_\phi} \right\} V_0^\top m_D U_0, \\ \mathcal{Y}^{(a)} &= \frac{1}{v_\phi} V_0^\top m_D U_0, \end{aligned}$$

(as said,  $U_0 = 1$ ). Because a main postulate behind the CSM is the presumed absence of any intermediate scales between the electroweak scale and the Planck scale  $M_{\text{PL}}$ , the scale of lepton-number symmetry breaking  $v_\phi$  is assumed to lie in the TeV range. With  $Y_M \sim 1$ , the masses

of heavy neutrinos are relatively small, and the light neutrino data [39] indicate that  $Y^\nu$  is of order  $Y^\nu \sim 10^{-6}$ . To allow for baryon number generation despite the low masses of heavy neutrinos, the mechanism of ‘‘resonant leptogenesis’’ was proposed and explored in [29–32]. This mechanism is based on the observation that  $CP$  violation (a crucial ingredient in dynamically generated baryon asymmetry [47]) is enhanced whenever the masses of heavy neutrinos are approximately degenerate. Accordingly, we assume that the Yukawa Majorana matrix is in fact *proportional to the unit matrix*, that is,

$$Y_{ij}^M = y_M \delta_{ij} \quad (28)$$

with  $y_M \sim \mathcal{O}(1)$ . Consequently there is an approximate  $SO(3)$  symmetry in the heavy neutrino sector, which is only very weakly broken by the Yukawa couplings  $Y^\nu$ . For definiteness, we assume Eq. (28) to hold at the electroweak scale, for the  $\overline{\text{MS}}$  renormalization scale  $\mu = M_{\text{top}}$ . In turn, the mass splitting of heavy neutrinos is entirely due to the seesaw mechanism, Eq. (19). [As emphasized in [48] the  $SO(3)$  symmetry ensures that (28) is stable against quantum corrections in a good approximation; nonetheless, when (28) holds instead at high Renormalization Group (RG) scale  $\mu_* \sim M_{\text{PL}}$ , then  $Y^\nu$ -induced RG-splitting of  $y_M$ 's yields splitting of heavy neutrino masses that is of similar order as the seesaw one, see e.g. [49].] It should be stressed here that, due to the degeneracy (28), the  $V_0$  matrix in Eq. (17) is clearly *not* an  $\mathcal{O}(Y^\nu)$  perturbation of the identity matrix; this is technically similar to (though physically different from) the Dashen's vacuum realignment condition [50] (see also [34]).

## B. Canceling quadratic divergences

We stress again the presence of explicit scalar mass terms in (3), in contrast to the original model of [6] which relied on the CW mechanism [28] to break electroweak symmetry. Our main reason for this is that the CW mechanism does not eliminate quadratic divergences, and thus the low-energy theory would remain sensitive to Planck scale corrections.

At one loop the coefficients of the quadratic divergences  $\Lambda^2$  for the two scalar fields are [25]

$$\begin{aligned} 16\pi^2 f_1^{\text{quad}}(\lambda, g, y) &= 6\lambda_1 + 2\lambda_3 + \frac{9}{4}g_w^2 + \frac{3}{4}g_y^2 - 6y_t^2 \\ 16\pi^2 f_2^{\text{quad}}(\lambda, g, y) &= 4\lambda_2 + 4\lambda_3 - 3y_M^2. \end{aligned} \quad (29)$$

Here  $g_w$  and  $g_y$  are the  $SU(2)_L \times U(1)_Y$  gauge couplings, while  $y_t$  is the top quark Yukawa coupling. For simplicity (and without much loss in precision) we neglect all other Yukawa couplings. Note that Eq. (29) is independent of the details of the cutoff regularization, as long as the regulator (here assumed to be provided by the quantum theory of

gravity) acts in the same way on all fields. Of course, another crucial assumption here is that we can neglect contributions of graviton loops to (29); this assumption is based on the hypothesis that the UV finite theory of quantum gravity effectively screens these contributions from low-energy physics.

An obvious question at this point is the following. One would at first think that Eq. (29) depends on the renormalization scale  $\mu$  via the RG running of the couplings, a well-known issue in the context of Veltman's conditions [51]. This is, however, only apparent, since when all higher corrections are included, the functions  $f_1, f_2$  obey appropriate renormalization group equations, in such a way that the implicit scale dependence is exactly canceled by the explicit presence of  $\log(\mu)$  introduced by higher loop corrections. Therefore, the all-order coefficients  $f_1, f_2$  are in fact  $\mu$ -independent (and  $\Lambda$ -independent) functions of the bare couplings  $\lambda_B$  (which themselves depend on the cutoff  $\Lambda$ , as the latter is varied). Thus, the couplings appearing on the right-hand side of (29) are  $\lambda_B(\Lambda)$  etc., rather than  $\lambda(\mu)|_{\mu=M_{\text{top}}}$ .<sup>6</sup> Nonetheless, employing running couplings  $\lambda(\mu)$  is convenient also in the present context, as these allow for a resummation of leading logarithms in the relation between the bare couplings  $\lambda_B(\Lambda)$  and the renormalized ones  $\lambda_R = \lambda(\mu)|_{\mu=M_{\text{top}}}$ , via the usual renormalization group improvement (see e.g. [52–54]). In fact, in a minimal-subtraction-type scheme based on cutoff regularization [25] (below called  $\Lambda$ -MS), the bare couplings  $\lambda_B(\Lambda)$  coincide with the running couplings  $\lambda(\mu)$  corresponding to  $\mu = \Lambda$ ,

$$\lambda_B(\Lambda) \equiv \lambda(\mu)|_{\mu=\Lambda}, \quad (30)$$

see also [55] for a discussion of the issues appearing in cutoff regularized gauge theories.

The appearance of bare couplings in (29) can also be motivated and understood from the point of view of constructive QFT (see e.g. [56]), although we are, of course, aware that there is no rigorous construction of the SM. There is one attempt to rigorously construct a functional measure for interacting QFTs. This requires the introduction of both UV and IR (i.e. finite volume) regulators. For the regularized theory one then introduces counterterms as functions of the *bare* parameters  $\lambda_B(\Lambda)$  and tries to adjust the latter as functions of the UV cutoff  $\Lambda$  in such a way that the theory gives well-defined physical answers in the limit  $\Lambda \rightarrow \infty$  (in which the bare couplings usually assume singular values). In particular, for a given value of the cutoff one can thus impose the vanishing of the

<sup>6</sup>In writing these equations we suppress a reference scale  $\mu_0$  needed to render the arguments dimensionless. The latter can be chosen as  $\mu_0 = M_{\text{top}}$ , or alternatively as  $\mu_0 \equiv \mu$ , in which case all couplings would depend only on the ratio  $\mu/\Lambda$  where the cutoff  $\Lambda$  is kept fixed (which is the case we consider below).

coefficient of the quadratic divergence as a single condition on the bare parameters. In that framework running couplings  $\lambda(\mu)$  play no role; they are merely an auxiliary device to conveniently parametrize the scale dependence of correlation functions.

In summary, the coefficients of quadratic divergences (29) are *calculable* functions of the cutoff scale  $\Lambda$ , provided that all low-energy parameters  $\lambda_1(\mu)|_{\mu=M_{\text{top}}}$  etc. are fixed by experiment. To determine the evolution of the couplings from  $\mu = M_{\text{top}}$  up to  $\Lambda$  (where they are identified with the bare couplings) in the leading logarithmic (LL) approximation we need only the one-loop beta functions [25] [we use the notation  $\tilde{\beta} \equiv 16\pi^2\beta$ ; furthermore we make use of (28)]

$$\begin{aligned} \tilde{\beta}_{\lambda_1} &= 24\lambda_1^2 + 4\lambda_3^2 - 3\lambda_1(3g_w^2 + g_y^2 - 4y_t^2) \\ &\quad + \frac{9}{8}g_w^4 + \frac{3}{4}g_w^2g_y^2 + \frac{3}{8}g_y^4 - 6y_t^4 \\ \tilde{\beta}_{\lambda_2} &= 20\lambda_2^2 + 8\lambda_3^2 + 6\lambda_2y_M^2 - 3y_M^4 \\ \tilde{\beta}_{\lambda_3} &= \frac{1}{2}\lambda_3\{24\lambda_1 + 16\lambda_2 + 16\lambda_3 \\ &\quad - (9g_w^2 + 3g_y^2) + 6y_M^2 + 12y_t^2\} \end{aligned} \quad (31)$$

$$\begin{aligned} \tilde{\beta}_{g_w} &= -\frac{19}{6}g_w^3, & \tilde{\beta}_{g_y} &= \frac{41}{6}g_y^3, & \tilde{\beta}_{g_s} &= -7g_s^3, \\ \tilde{\beta}_{y_t} &= y_t\left\{\frac{9}{2}y_t^2 - 8g_s^2 - \frac{9}{4}g_w^2 - \frac{17}{12}g_y^2\right\}, \\ \tilde{\beta}_{y_M} &= \frac{5}{2}y_M^3, \end{aligned} \quad (32)$$

which show in particular how the  $SU(3)_c$  gauge coupling  $g_s$  affects the evolution of  $y_t$  so no Landau pole develops for  $y_t$ . This effect is also seen in the other expressions where bosonic and fermionic contributions balance each other in such a way that the theory remains perturbatively under control up to  $M_{\text{PL}}$  (with appropriate initial values).

At this point it should be stressed that all the ingredients necessary to find the coefficients  $f_1, f_2$  with resummed next-to-leading logarithms are at our disposal. In particular, the two-loop beta functions in  $\Lambda$ -MS together with the two-loop coefficients in a generic renormalizable model are given in [55] (one can also find there the generic one-loop relation between renormalized parameters in  $\Lambda$ -MS and their counterparts in the conventional  $\overline{\text{MS}}$  scheme of dimensional regularization). However, as most of the parameters of CSM are still unknown, we are content here with resummation of the leading logarithms only. The rationale behind this restriction, is that the *one-loop* RG evolution in gauge-Yukawa sector is independent of quartic scalar couplings, which significantly simplifies the scan over the parameter space; in particular  $y_t$  and gauge couplings at the Planck scale are known. Recall that the one-loop beta functions reflect the structure of nonlocal

terms in one-particle-irreducible effective action  $\Gamma[\cdot]$ , and thus are universal across different regularizations (at least in the class of mass independent renormalization schemes [57], to which  $\Lambda$ -MS belongs). Therefore the RG-improved coefficients (29) at the LL order are independent of the details of cutoff regularization as well.

We note that the cutoff dependence of coefficients of quadratic divergences in the pure SM was already analyzed in [58] where it was found that they cancel for  $\Lambda \approx 10^{24}$  GeV, and thus (logarithmically speaking) not so far from the Planck scale. This observation motivated our proposal that the vanishing of quadratic divergences at the Planck scale, and thus stabilization of the electroweak scale, may be achieved by means of a “small” modification of the SM like the one proposed here. Ultimately, the cancellation of quadratic divergences would be due to a still unknown quantum gravity induced mechanism which is different from low-energy supersymmetry (but which could still involve Planck scale supersymmetry in an essential way).

From the perspective of effective field theory (EFT), valid for energies  $E \lesssim M_{\text{PL}}$ , we have a clear distinction between low-energy ( $N=1$ ) supersymmetry and the present proposal. In supersymmetric models, the underlying mechanism of quantum gravity appears via the (super)symmetry of the EFT itself, and thus the cancellation holds independently of the value of the cutoff

$$f_{\text{SUSY}}^{\text{quad}}(\Lambda) = 0, \quad \forall \Lambda.$$

By contrast, in the present context the absence of quadratic divergences (and thus the stabilization of the electroweak scale) manifests itself via the existence of a distinguished value  $\Lambda_*$  of the cutoff (close to the Planck scale) such that  $f(\Lambda_*) = 0$ . Importantly, the question whether or not such a scale exists for which both coefficients (29) vanish, can in principle be answered provided that all CSM parameters can be measured with sufficient accuracy.

We therefore assume that such a distinguished value close to  $M_{\text{PL}}$  exists, so we can impose the conditions

$$f_1^{\text{quad}}(\lambda, g, y) = f_2^{\text{quad}}(\lambda, g, y) = 0 \quad (33)$$

on the running couplings with  $\mu$  equal to the (reduced) Planck scale; from a low-energy perspective these can be considered as an RG-improved version of Veltman’s conditions [51]. Disregarding the other SM couplings this condition restricts the four-dimensional space of parameters  $(\lambda_1, \lambda_2, \lambda_3, y_M)$ , cf. Eq. (8), to a two-dimensional submanifold.<sup>7</sup> To implement our conditions in practice we then evolve the couplings along this submanifold from

<sup>7</sup>Because the conditions (33) are RG invariant, our approach bears also some resemblance to Zimmerman’s reduction of couplings [59,60].

$M_{\text{PL}}$  back down to the electroweak scale  $\mu = M_{\text{top}}$  and calculate the masses and mixing angle using Eqs. (8)–(9). Moreover, to ensure perturbativity we demand that all running couplings (including  $y_M$ ) remain small over the whole range of energies between  $M_{\text{top}}$  and  $M_{\text{PL}}$  (more concretely, for our numerical checks we demand  $0 < \lambda_1, \lambda_2, y_M < 2$ , and  $-2 < \lambda_3 < 2$ , see also the next subsection; in practice for all points in Table I scalar self-couplings at the electroweak scale are smaller than 0.25). It should be stressed that this approach is consistent because the values of the gauge and Yukawa couplings at the Planck scale are independent of the values of quartic couplings, as far as leading logarithms are concerned.

### C. Stability of electroweak vacuum

One of the very few “weak spots” of the pure SM is the metastability of the electroweak vacuum [33]. Namely, the effective potential of the SM (with appropriately resummed large logarithms) develops a new deeper minimum for  $H \gtrsim 10^{10}$  GeV, thus implying an instability of the electroweak vacuum via quantum-mechanical tunneling. This can be seen also more heuristically, by following the RG evolution  $\lambda = \lambda(\mu)$  of the scalar self-coupling and noticing that for  $\mu \sim 10^{10}$  GeV the function  $\lambda(\mu)$  dips below zero due to the large negative contribution from the top quark [33] (but becomes positive again for yet larger values of  $\mu$ ). For values of the fields that are much larger than the electroweak scale, the full effective potential of the SM is well approximated by the quartic term

$$V_{\text{eff}}(H) \approx \tilde{\lambda}(H^\dagger H)^2. \quad (34)$$

However, here one cannot simply substitute the self-coupling at the electroweak scale; rather, in order to avoid huge logarithmic corrections on the right-hand side, the correct value of the quartic coupling  $\tilde{\lambda}$  in the above formula is obtained by substituting the running coupling evaluated at the appropriate energy scale of the order of  $\sqrt{H^\dagger H}$ , i.e.

$$\tilde{\lambda} = \lambda(\mu)|_{\mu=\sqrt{H^\dagger H}}, \quad (35)$$

rather than  $\tilde{\lambda} = \lambda(\mu)|_{\mu=M_{\text{top}}}$ . For theories like the SM, in which the effective potential depends only on a single field (up to the orbits of symmetry group), this somewhat heuristic reasoning can be put on firmer grounds, by resumming large logarithms via the renormalization group improvement [52–54].

For the CSM there are now *two* scalar fields (up to symmetries of  $V_{\text{eff}}$ ) and the situation is more complicated, basically because with more than one scalar field, the RG improvement cannot simultaneously determine the resummation of logarithms in all directions in field space. For this reason we have to rely on the more heuristic argument, by demanding that the positivity conditions (5) be satisfied not



only at the electroweak scale  $\mu = M_{\text{top}}$ , but also for the running couplings at all intermediate scales  $M_{\text{top}} < \mu < M_{\text{PL}}$ . This provides a strong indication that the electroweak vacuum (4) in the CSM remains the global minimum of the full effective potential, at least in the region  $|\phi|^2 < M_{\text{PL}}^2$ ,  $H^\dagger H < M_{\text{PL}}^2$ , in which EFT is valid. Thus, following the RG evolution from the Planck scale, where the conditions (33) are imposed, down to the electroweak scale we impose the inequalities

$$\lambda_1(\mu) > 0, \quad \lambda_2(\mu) > 0, \quad \lambda_3(\mu) > -\sqrt{\lambda_1(\mu)\lambda_2(\mu)} \quad (36)$$

in addition to the conditions enunciated at the end of the foregoing subsection. These extra stability conditions lead to further restrictions on the parameters. It is therefore a nontrivial fact that parameter ranges exist which satisfy all these conditions and restrictions.

A set of exemplary points consistent with all our restrictions is given in Table I.  $\Gamma_{h,\varphi}$  denote decay width of the Higgs particle  $h$  and its ‘‘heavy brother’’  $\varphi$ .  $\text{Br}(\varphi \rightarrow [\text{SM}])$  is the branching ratio for SM-like decay channels of  $\varphi$ , while non-SM-like decay channels of  $h$  are negligible for all points in the Table.  $\mathcal{Y}_{B0}$  denotes the current baryon number density to entropy density ratio calculated on assumptions specified in Sec. III. In particular, the table displays a viable range of mass values for both the new scalar and the heavy neutrinos. For all points the heavy neutrinos are heavier than the new scalar field  $\varphi$ , and thus their decays are the main source of lepton asymmetry. Note also the relatively large values of  $v_\phi$  which are necessary for successful leptogenesis. This comes about because  $y_M$  must remain sufficiently small so as to allow for the departure of heavy neutrinos from thermal equilibrium, while their masses  $\sim y_M v_\phi$  should be large enough so that the departure takes place when baryon-number violating processes are still fast. Importantly, the values of dimensionless couplings corresponding to all points in the Table are small while masses of new states are comparable to the electroweak scale; thus one can trust that radiative corrections to the tree-level masses etc. are small.

### III. RESONANT LEPTOGENESIS

By assumption the lepton-number symmetry  $L$  of the CSM is spontaneously broken by the nonvanishing vacuum expectation value  $\langle \phi \rangle \sim \mathcal{O}(1 \text{ TeV})$ . The proper quantity to study is therefore the lepton-number density  $L'$  of the SM under which heavy neutrinos have vanishing charges.<sup>8</sup> The individual lepton-number symmetries  $L'_i$ ,  $i = e, \mu, \tau$ , of the

<sup>8</sup>In the following we adopt the convention that primed quantities refer to the pure SM, while unprimed letters refer to the CSM with its enlarged set of fields. For instance,  $L'_i$  is the lepton number of species  $i$  in the pure SM, which thus excludes the contributions of  $N_i$  and the new scalar  $\phi$ .

SM (with  $L' = \sum_i L'_i$ ) are only weakly broken by  $Y^\nu$  effects as well as by gauge anomalies.

In the framework of leptogenesis [61] the baryon number density  $n_B$  in the Universe [39,62]<sup>9</sup>

$$n_B = (6.05 \pm 0.07) \times 10^{-10} n_\gamma, \quad (37)$$

(where  $n_\gamma$  denotes the number density of photons) is produced by nonperturbative SM interactions that break baryon and lepton-number symmetries down to the non-anomalous combination  $(B - L')$ , and generate baryons from nonvanishing lepton-number density  $n_{L'}$  via the usual sphaleron mechanism [63]. Thus the problem can be reduced to that of explaining the lepton asymmetry  $n_{L'}$ , which itself is produced in lepton-number and  $CP$  violating out-of-equilibrium decays of heavy neutrinos, as they occur in the CSM. In this way all the Sakharov conditions [47] can be satisfied.<sup>10</sup>

To achieve the correct order of  $CP$  violation despite small  $Y^\nu$  values, we rely on the mechanism of resonant leptogenesis [29–32], which can be naturally realized within the present scheme as a consequence of the assumed degeneracy of the Yukawa matrix  $Y^M$ , cf. Eq. (28). The baryon number density  $n_B$  can then be calculated by solving the relevant Boltzmann equations (see e.g. [64]).

#### A. $CP$ violation

The  $CP$  asymmetries relevant for calculation of  $n_B$  are

$$\varepsilon_{ji}^{(h\nu)} \equiv \frac{\Gamma(N_j \rightarrow h\nu_i) - \Gamma(N_j \rightarrow h\bar{\nu}_i)}{\Gamma(N_j \rightarrow h\nu_i) + \Gamma(N_j \rightarrow h\bar{\nu}_i)}, \quad (38)$$

$$\varepsilon_{ji}^{(Z\nu)} \equiv \frac{\Gamma(N_j \rightarrow Z\nu_i) - \Gamma(N_j \rightarrow Z\bar{\nu}_i)}{\Gamma(N_j \rightarrow Z\nu_i) + \Gamma(N_j \rightarrow Z\bar{\nu}_i)}, \quad (39)$$

together with their counterparts with additional scalars (or  $W$ -bosons and charged leptons) in the final states. The tree-level contributions to the decay widths in the formulas above follow immediately from the vertices in Eqs. (27) and (23)–(24),<sup>11</sup> while nonzero contributions to  $\varepsilon_{ji}$  originate from the interference between these tree-level vertices and loop diagrams describing the correction to proper vertices and external lines [65]. Generically, both kinds of corrections are of the same order [65], and are way too small to ensure a successful leptogenesis for  $Y^\nu$  having the

<sup>9</sup>This number is often given by normalizing with respect to the entropy density, see (61) and (72) below.

<sup>10</sup>See also [49] for a discussion of resonant leptogenesis for a CSM-like model with *gauged*  $(B - L)$  symmetry.

<sup>11</sup>Here we can neglect the light neutrino masses in very good approximation. Thus the field  $P_L \psi_\nu \simeq \nu_\alpha$  in (27) annihilates neutrinos  $\nu$  and creates antineutrinos  $\bar{\nu}$ , while  $P_R \psi_\nu \simeq \bar{\nu}^\alpha$  does the opposite, see also the appendix for an explicit description of the neutrino operators in the  $SL(2, \mathbb{C})$  basis.

matrix elements of the order of  $10^{-6}$ . However, the external line corrections are resonantly enhanced for (approximately) degenerate masses of heavy neutrinos [29–32].

In fact, calculation of “external” line corrections (especially in the resonant regime) requires some care, since the incoming states correspond to unstable particles. In [66], see also [29,67,68], the  $CP$  asymmetry  $\varepsilon_{ji}^{(X\ell)}$  was calculated without any references to the external lines of unstable states. Instead, the amplitudes of associated scattering processes in which unstable heavy neutrinos appear only as *internal* lines were studied; the resulting prescription for  $\varepsilon_{ji}^{(X\ell)}$  can be summarized as follows [66]. Consider the interaction (27) between a Hermitian scalar field  $h$ , heavy neutrinos  $N_j$  (described in terms of Majorana fields  $\psi_N^j$ ), and (approximately) massless SM (anti)neutrinos, described in terms of Majorana fields  $\psi_\nu^i$ . Suppose that the matrix of propagators of heavy neutrino Majorana fields  $\psi_N^j$  has the following form ( $\mathcal{C}$  is the charge-conjugation matrix)<sup>12</sup>

$$\hat{\mathcal{G}}(p) = i\hat{\zeta}[p^2 - m^2]^{-1}[\not{p} + m]\hat{\zeta}^\top \mathcal{C}^{-1} + [\text{nonpole part}], \quad (40)$$

where the matrix of pole masses,

$$m = \text{diag}(m_1, m_2, m_3), \quad (41)$$

is diagonal with positive real parts  $\text{Re}(m_a) > 0$ , while its imaginary part gives the total decay widths. The residue matrices  $\hat{\zeta}$  can be written as

$$\begin{aligned} \hat{\zeta} &= \zeta_L \otimes P_L + \zeta_R \otimes P_R, \\ \hat{\zeta}^\top &= \zeta_L^\top \otimes P_L + \zeta_R^\top \otimes P_R, \end{aligned} \quad (42)$$

with  $3 \times 3$  matrices  $\zeta_{L,R}$  carrying only family indices, and chiral projections  $P_{L,R}$ ; clearly, at tree level, in the basis of mass eigenstates one has  $\zeta_L = \zeta_R = \mathbb{1}$ . If these matrices are known, the  $CP$  asymmetry (38) can then be calculated with the aid of the following formula [66]:

$$\varepsilon_{ji}^{(h\nu)} = \frac{|\mathcal{Y}_{ji}^R|^2 - |\mathcal{Y}_{ji}^L|^2}{|\mathcal{Y}_{ji}^R|^2 + |\mathcal{Y}_{ji}^L|^2}, \quad (43)$$

with

$$\mathcal{Y}_{ji}^L = \mathcal{Y}_{ki}^{(h)}(\zeta_L)^k_j + \dots, \quad (44)$$

$$\mathcal{Y}_{ji}^R = \mathcal{Y}_{ki}^{(h)\star}(\zeta_R)^k_j + \dots, \quad (45)$$

where the ellipses indicate contributions of corrections to external lines of  $h$  and  $\psi_\nu$  fields, as well as loop corrections

<sup>12</sup>With apologies to the reader for the proliferation of different fonts; unlike the tree-level masses pole masses  $m$  are in general complex.

to the 1PI vertices (which are negligible in TeV-scale leptogenesis). If heavy neutrinos were stable, the matrix  $\zeta_R$  would be the complex conjugate of  $\zeta_L$ . In that case Eqs. (44)–(45) are nothing more than the ordinary Lehmann-Symanzik-Zimmermann reduction rules for calculating the  $S$ -matrix elements, see e.g. [69]. Similarly, the  $CP$  asymmetry  $\varepsilon_{ji}^{(Z\nu)}$  can be calculated with the aid of Eq. (43), with the following replacements:

$$\begin{aligned} \mathcal{Y}_{ji}^L &= \mathcal{F}_{ki}^{(Z)}(\zeta_R)^k_j + \dots, \\ \mathcal{Y}_{ji}^R &= \mathcal{F}_{ki}^{(Z)\star}(\zeta_L)^k_j + \dots, \end{aligned}$$

(the change of chirality is caused by  $\gamma^\mu$ ). The enhancement effect that underlies resonant leptogenesis is due to the  $\zeta_{L,R}$  matrices which contain the factors  $\sim(m_1^2 - m_2^2)^{-1}$  etc. (see below).

To find the  $\zeta_{L,R}$  matrices we use the prescription given in [70], to which we also refer for further details. Adopting some renormalization scheme, let  $\tilde{\Gamma}(-p, p)$  be the matrix of renormalized 1PI two-point functions (inverse propagators) of the Majorana fields  $\psi_N^j$

$$\begin{aligned} \tilde{\Gamma}(-p, p) &= \mathcal{C}\{(\not{p}\mathcal{Z}_L(p^2) - \mathcal{M}_L(p^2))P_L \\ &\quad + (\not{p}\mathcal{Z}_R(p^2) - \mathcal{M}_R(p^2))P_R\}, \end{aligned} \quad (46)$$

where matrices  $\mathcal{M}_{L,R}$  and  $\mathcal{Z}_{L,R} = \mathbb{1} + \mathcal{O}(\hbar)$  carry only family indices. Now let  $\mathbb{M}_L^2(p^2)$  be the following matrix (with  $s \equiv p^2$ ):

$$\mathbb{M}_L^2(s) \equiv \mathcal{Z}_L(s)^{-1}\mathcal{M}_L(s)\mathcal{Z}_R(s)^{-1}\mathcal{M}_L(s). \quad (47)$$

Then the propagator of  $\psi_N^j$  has the form (40) where the (complex) pole masses  $m$  are solutions to

$$\det(s\mathbb{1} - \mathbb{M}_L^2(s))|_{s=m_a^2} = 0, \quad (48)$$

while the columns of  $\zeta_{L,R}$  matrices are given by vectors  $\zeta_{L,R[a]}$

$$\zeta_X = [[\zeta_{X[1]}][\zeta_{X[2]}][\zeta_{X[3]}]], \quad X = L, R,$$

which are obtained in the following way. Let  $\xi_{[a]}$  be an eigenvector of  $\mathbb{M}_L^2(m_a^2)$ , with eigenvalue  $m_a^2$

$$\mathbb{M}_L^2(m_a^2)\xi_{[a]} = m_a^2\xi_{[a]}, \quad (49)$$

and obeying the following normalization condition:

$$\xi_{[a]}^\top \mathcal{M}_L(m_a^2)\xi_{[a]} = m_a, \quad (50)$$

then

$$\zeta_{L[a]} = \mathcal{N}(a)\xi_{[a]}, \quad (51)$$



FIG. 1. One-loop contributions to  $\mathcal{M}_{L,R}$  and  $\mathcal{Z}_{L,R}$  in the Landau gauge. In CSM external lines represent heavy neutrino Majorana fields. Solid internal lines represent all fermions (with the exception of quarks), with dashed/springy lines denoting scalars/massive vectors.

with a normalizing factor

$$\mathcal{N}(a) = \left\{ 1 - \frac{1}{m_a} \xi_{[a]}^\top \mathcal{M}_L(m_a^2) \mathbb{M}_L^2(m_a^2) \xi_{[a]} \right\}^{-1/2}, \quad (52)$$

where  $\mathbb{M}_L^2(s) \equiv d\mathcal{M}_L^2(s)/ds$ , and

$$\zeta_{R[a]} = \frac{1}{m_a} \mathcal{Z}_R(m_a^2)^{-1} \mathcal{M}_L(m_a^2) \zeta_{L[a]}. \quad (53)$$

For heavy neutrinos  $m_a \neq 0$  and the corresponding eigenspaces are one dimensional. Thus the above prescription is all we need to calculate the  $CP$  asymmetries  $\varepsilon_{ji}$  (for a generalization to massless or Dirac fermions, as well as the discussion of reality properties of  $\zeta_{L,R}$  matrices, see [70]). To obtain the required numerical values of  $\mathcal{Z}_{L,R}$  and  $\mathcal{M}_{L,R}$  matrices, we use the one-loop formulas given in [70]; these are valid for a general renormalizable model in the Landau gauge and correspond to the diagrams shown in Fig. 1. Since  $\zeta_{L[a]}$  is an eigenvector, the ordinary quantum-mechanical perturbation theory for discrete spectra (more precisely, its generalization to non-Hermitian matrices) indicates that the components of  $\zeta_L$  are enhanced whenever masses of fermions are approximately degenerate. This in turn causes the enhancement of the  $CP$  asymmetry (43), and thus lepton asymmetry, dubbed “resonant leptogenesis” [29].

Some remarks are in order. While the above prescription for finding  $\zeta_{L,R}$  matrices is, in principle, independent of the choice of basis in the space of fields, we apply it in the basis of tree-level mass eigenstates in which loop calculations are done. In this basis the  $\zeta_{L,R}$  matrices [unlike the  $V_0$  matrix that diagonalizes the tree-level mass matrix itself, cf. Eq. (17)] are numerically small perturbations of the identity matrix, for all cases studied below. It is also worth stressing that we completely neglect the masses of light neutrinos circulating in loops as in Fig. 1; this is justified since contributions of these masses are subdominant in  $Y^\nu$ , as can be easily checked from the mentioned generic one-loop formulas. In light of this fact, our choice  $U_0 = \mathbb{1}$  (rather than  $U_0 = U_{\text{MNS}}$ ) in Eq. (17) for light neutrino states is self-consistent.

## B. Boltzmann equations

To determine the lepton-number asymmetry one has to solve the Boltzmann equations in the context of an expanding universe [64]. For the CSM the full set of equations would be close to unmanageable due to the large number of degrees of freedom and possible processes involved, and one therefore has to resort to several simplifying assumptions. A first such assumption is that the elastic processes are fast, so that all species are in kinetic equilibrium, having the occupancy given by the Fermi-Dirac/Bose-Einstein distributions

$$f(\mathbf{p}) = \{\exp[(E(\mathbf{p}) - \mu)/T] \pm 1\}^{-1}. \quad (54)$$

Second, in order to reduce the large number of independent distribution functions  $f$  (or, equivalently, the associated chemical potentials  $\mu$ ), we assume that all the interactions described by the Lagrangian density (1), with the exception of those triggered by  $Y^\nu$  or  $Y^M$ , are in chemical equilibrium.<sup>13</sup> Note that in TeV-scale leptogenesis this assumption is justified for the SM Yukawa couplings [71]. A further simplification is achieved by assuming that the nonperturbative SM interactions that violate  $B$  and  $L'_i$  symmetries down to the combinations  $B - L'$  and  $L'_i - L'_j$  are also in equilibrium; direct analysis of these processes [48,63] indicates that this assumption is reasonable for  $T \gtrsim 80$  GeV. Note that, although  $X \equiv (B - L')$  unlike  $(B - L)$  or  $(L'_i - L'_j)$  has a  $X - X - X$  anomaly, it does not have anomalies in the presence of the SM gauge field background, and thus it is preserved by sphalerons. In other words,  $B - L'$  and  $L'_i - L'_j$  are violated only by  $Y^\nu$  induced interactions, and only these interactions contribute to the Boltzmann equations for the densities of these *differences*, see Eqs. (63) and (64), i.e. spaleronic interactions cancel out [clearly, these equations must then still be supplemented by the ones for the heavy neutrino densities, see (68)].

Under these circumstances, there are four independent chemical potentials for the SM species, which correspond to these global symmetries, and, in addition, after the electroweak phase transition, to the electric charge; however, the electric neutrality of the Universe allows us to express the charge potential as linear combinations of the remaining ones [72]. For our purposes, it is convenient to choose the light neutrinos' potentials as independent ones

$$\mu_{\nu_i} \equiv \mu_{e_i} + \mu_{W_+}. \quad (55)$$

Neglecting masses of SM particles,  $\mu_{\nu_i}$  can be expressed in terms of individual SM lepton-number

<sup>13</sup>Recall that, when the reaction  $i + j \rightarrow k + l$  between particles  $i, j, k$  and  $l$  is in chemical equilibrium, then the corresponding chemical potentials obey the relation  $\mu_i + \mu_j = \mu_k + \mu_l$ .

densities  $n_{L'_i}$  in the broken phase of the SM, which simplifies to<sup>14</sup>

$$\frac{\mu_{\nu_i}}{T} = \frac{166n_{L'_i} + 16(n_{L'_j} + n_{L'_k})}{75T^3}, \quad (56)$$

where  $i \neq j \neq k \neq i$ . Similarly, the lepton-number density can be expressed in terms of the  $B - L'$  density as follows [72]:

$$\sum_i n_{L'_i} = -\frac{25}{37} \left[ n_B - \sum_i n_{L'_i} \right]. \quad (57)$$

To simplify the BEQs for the number densities, we approximate the occupancies (54) with the Maxwell-Boltzmann distributions

$$f \approx \exp[-(E - \mu)/T]. \quad (58)$$

This allows us to perform some of the momentum integrals analytically (and should not lead to errors bigger than 20% [66]). In this approximation, the distribution of heavy neutrinos  $N_i$  can be written as

$$f_{N_i} = \exp(-E/T) \frac{n_{N_i}}{n_{N_i}^{\text{EQ}}}, \quad (59)$$

where  $n_{N_i}$  is the number density of  $N_i$ , while  $n_{N_i}^{\text{EQ}}$  is the value  $n_{N_i}$  in the chemical equilibrium (i.e. the one corresponding to the vanishing chemical potential, as indicated by various  $y_M$ -induced annihilation processes, e.g.  $N_i N_i \rightarrow \bar{t} \bar{t}$ , see Fig. 2) that reads (see e.g. [73])

$$n_{N_i}^{\text{EQ}} = \frac{m_N^2 T}{\pi^2} K_2(m_N/T), \quad (60)$$

with  $K_\ell(z)$  denoting the modified Bessel functions of the second kind [similarly, thermally averaged decay widths lead to the appearance of  $K_1(z)$  in BEQs below] and  $m_N = y_M v_\phi / \sqrt{2}$  being the Majorana mass (the mass splitting due to  $Y^\nu$  is negligible as far as the distributions of heavy neutrinos are concerned). Since the Majorana mass in the present model originates from the vacuum expectation value of  $\phi$ , we assume below that the baryon asymmetry is produced after spontaneous breaking of  $B - L$ , from initially symmetric state. The analysis of phase transition will be given elsewhere.

Due to the expansion of the Universe it is convenient to write the BEQs for densities normalized to the entropy density  $s(T) \propto T^3$  (see e.g. [64])

$$\mathcal{Y}_X = \frac{n_X}{s(T)}, \quad (61)$$

as functions of the following ‘‘time’’ variable

$$z = z(T) = \frac{m_N}{T}. \quad (62)$$

With these approximations, it is fairly easy to write BEQs for the densities of (approximately) conserved charges. In particular, the symmetries  $B - \sum_i L'_i$  and  $L'_i - L'_j$  are violated only by the  $Y^\nu$ -induced interactions (but not by anomaly induced instanton processes). Denoting by  $D^{L'_i}$  the appropriate combinations of averaged squared amplitudes of  $Y^\nu$ -induced processes that violate  $L'_i$ , one can write the relevant BEQs for the densities of these nonanomalous charges in the following form:

$$s(T)H(T)z \frac{d}{dz} [\mathcal{Y}_{L'_i} - \mathcal{Y}_{L'_j}] = D^{L'_i} - D^{L'_j}, \quad (63)$$

$$s(T)H(T)z \frac{d}{dz} \left[ \mathcal{Y}_B - \sum_i \mathcal{Y}_{L'_i} \right] = -\sum_i D^{L'_i}, \quad (64)$$

where  $H(T) \propto T^2$  is the expansion rate of the universe [64]. The BEQs for individual  $\mathcal{Y}_{L'_i}$  follow then immediately from (57). The dominant contributions to  $D^{L'_i}$  come from decays and inverse decays of heavy neutrinos (as well as the subtraction of their real intermediate states from the associated scattering processes, the latter has been taken care of by following the approach of [66]; in particular decays of heavy neutrinos with equilibrium distributions  $\mathcal{Y}_{N_i} = \mathcal{Y}_{N_i}^{\text{EQ}}$  do not contribute to  $D^{L'_i}$  given below, in agreement with the Sakharov conditions [47]). To the first order in small parameter (56),  $D^{L'_i}$  have the form (for a discussion of thermally averaged rates, see e.g. [73])

$$D^{L'_i} = \frac{m_N^3}{\pi^2 z} K_1(z) \sum_j \left\{ \left[ \frac{\mathcal{Y}_{N_j}}{\mathcal{Y}_{N_j}^{\text{EQ}}} - 1 \right] \Delta_{ji} - \frac{\mu_{\nu_i}}{T} \Sigma_{ji} \right\},$$

with

$$\Sigma_{ji} = \sum_{X,\ell} [\Gamma(N_j \rightarrow X\ell_i) + \Gamma(N_j \rightarrow \bar{X}\bar{\ell}_i)], \quad (65)$$

$$\Delta_{ji} = \sum_{X,\ell} [\Gamma(N_j \rightarrow X\ell_i) - \Gamma(N_j \rightarrow \bar{X}\bar{\ell}_i)], \quad (66)$$

where the summation runs over different decay channels with  $\ell_i \in \{e_i, \nu_i\}$  denoting a charged or neutral lepton of  $i$ th flavor. Clearly,  $\Sigma_{ji}$  can be calculated with a good accuracy at the tree level. In calculating  $\Delta_{ji}$ , the  $CP$  asymmetries introduced in the previous section are crucial [cf. Eq. (38)]

<sup>14</sup>This result can be easily obtained by repeating the analysis of [72] without the assumption that  $\mu_{\nu_i} \equiv \mu_\nu$  for all flavors  $i$ . Note that in the present context there are no rapid flavor-mixing interactions; in particular matrix elements of  $Y^\nu$  are small.

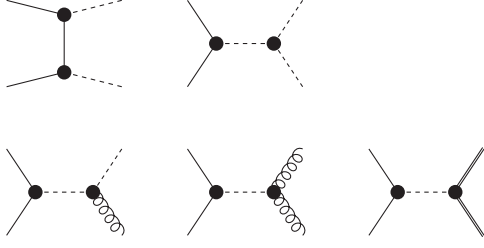


FIG. 2. Annihilation diagrams  $N_i N_i \rightarrow XY$  induced by  $y_M$ . Standard Model fermions are depicted as double solid lines.

$$\Delta_{ji} = \sum_{X,\ell} \varepsilon_{ji}^{(X\ell)} \times [\Gamma(N_j \rightarrow X\ell_i) + \Gamma(N_j \rightarrow \bar{X}\bar{\ell}_i)]. \quad (67)$$

For given  $\mathcal{Y}_{N_j} = \mathcal{Y}_{N_j}(z)$ , Eqs. (63)–(64) form a system of three equations for three independent functions  $n_{L'_i}$ , cf. Eqs. (56) and (57). They have to be supplemented with three more equations for  $\mathcal{Y}_{N_j}$

$$s(T)H(T)z \frac{d\mathcal{Y}_{N_j}}{dz} = D_j^N + A_j^N + S_j^N, \quad (68)$$

where  $D_j^N$  represents the effects of  $Y^\nu$ -induced decays of a heavy neutrino  $N_j$

$$D_j^N = -\Gamma_{N_j} \frac{m_N^3}{\pi^2 z} K_1(z) \left[ \frac{\mathcal{Y}_{N_j}}{\mathcal{Y}_{N_j}^{\text{EQ}}} - 1 \right], \quad (69)$$

where the total decay width is determined via  $\Gamma_{N_j} \approx -\text{Im}(m_j/2)$  with the pole mass obtained from Eq. (48). While this is the standard contribution to BEQs that occurs also in “minimal” leptogenesis scenarios, the other two contributions labeled  $A_j^N$  and  $S_j^N$  are absent in such a minimal framework, as they represent contributions arising from the new scalar field  $\phi$ . More specifically,  $A_j^N$  describes the rates of  $y_M$ -induced annihilation processes of heavy neutrinos (see Fig. 2), and  $S_j^N$  represents the rates of inelastic scatterings shown in Fig. 3.

$$A_j^N = -\frac{m_N}{64\pi^4 z} \mathcal{K}[\sigma_j] \left\{ \left[ \frac{\mathcal{Y}_{N_j}}{\mathcal{Y}_{N_j}^{\text{EQ}}} \right]^2 - 1 \right\},$$

$$S_j^N = -\frac{m_N}{64\pi^4 z} \sum_{i \neq j} \mathcal{K}[\sigma_{j \rightarrow i}] \left\{ \left[ \frac{\mathcal{Y}_{N_j}}{\mathcal{Y}_{N_j}^{\text{EQ}}} \right]^2 - \left[ \frac{\mathcal{Y}_{N_i}}{\mathcal{Y}_{N_i}^{\text{EQ}}} \right]^2 \right\},$$

where

$$\mathcal{K}[\sigma] = 2 \int_{(2m_N)^2}^{\infty} ds \sqrt{s} (s - 4m_N^2) K_1(\sqrt{s}/T) \sigma(s). \quad (70)$$

Here  $\sigma_j(s)$  denotes the total cross-section for the processes  $N_j N_j \rightarrow XY$  shown in Fig. 2, while  $\sigma_{j \rightarrow i}(s)$  is the cross-section for the processes  $N_j N_j \rightarrow N_i N_i$  (Fig. 3). [Note that

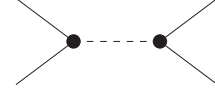


FIG. 3.  $\mathcal{O}(y_M^2)$  inelastic processes  $N_i N_i \rightarrow N_j N_j$ .

these cross sections are summed, rather than averaged, over initial spin states, and that the form of the lower limit in (70) appears because heavy neutrinos turn out to be the heaviest particles in the model; for a discussion of thermally averaged cross sections see e.g. [73].] Despite the huge hierarchy between  $y_M$  and  $Y^\nu$ , both classes of processes are equally important, since  $D_j^N$  and  $A_j^N$  ( $S_j^N$ ) have different dependencies on  $z$ . The importance of  $y_M$ -induced processes was emphasized in [74] (see also [49,75] for a discussion in the context of local  $B - L$  models). Their presence is a main difference between models with spontaneous lepton-number violation and the “minimal leptogenesis,” in which  $Y^\nu$  is the sole source of nonconservation of both,  $B - L'$  as well as the number of heavy neutrinos. In particular, they keep heavy neutrinos in thermal equilibrium at early times (see also the discussion in the next subsection).

### C. Results

For  $Y^\nu = \sqrt{2}m_D/v_H$  we use the Casas-Ibarra parametrization (20), assuming inverted ordering ( $m_{\nu 3} < m_{\nu 1} < m_{\nu 2}$ <sup>15</sup>) with central values of all neutrino oscillation parameters, including the Dirac phase of the matrix  $U_{\text{MNS}}$ , given in [39] (Table 14.7 on page 252). For both unconstrained Majorana phases in  $U_{\text{MNS}}$  we take the value  $2\pi/5$ , while for the lightest neutrino we assume  $m_{\nu 3} = 1.08 \times 10^{-3}$  eV. As to the complex angles of Casas-Ibarra matrix, a set of values (in the standard CKM-like parametrization) that works is the following:

$$\alpha = \frac{9\pi}{25} + \frac{33i}{25}, \quad \beta = \frac{6\pi}{5} + \frac{18i}{25}, \quad \gamma = \frac{4\pi}{5} + \frac{11i}{25}, \quad (71)$$

so that

$$\mathbf{R}_{\text{CI}} \approx \begin{pmatrix} -0.15 + 2.0i & -2.2 + 0.1i & -0.75 - 0.63i \\ 2.2 + 0.13i & 0.02 + 1.9i & -0.49 + 0.68i \\ -0.18 - 0.06i & -0.31 - 0.42i & 1.0 - 0.13i \end{pmatrix}.$$

While the matrix elements of  $\mathbf{R}_{\text{CI}}$  are of order  $\mathcal{O}(1)$ , the above form of  $\mathbf{R}_{\text{CI}}$  ensures that the decay width of one of the heavy neutrinos is suppressed in comparison with the other two. This allows for a sufficient departure from equilibrium in the range of temperatures in which  $B$ -violating processes are still fast. Let us emphasize that

<sup>15</sup>The inverted ordering of light neutrino masses is just a choice that we made for the scan over remaining parameters, but not necessarily a prediction of our model.

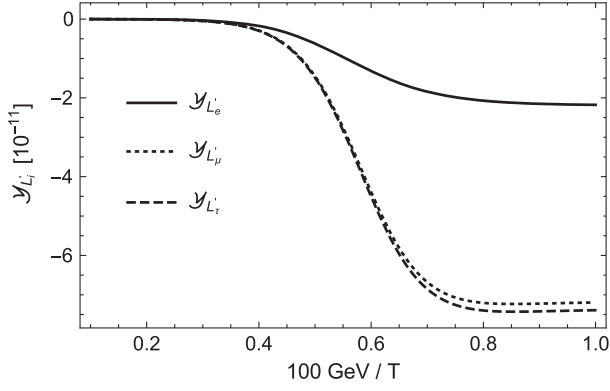


FIG. 4. Leptonic asymmetries  $\mathcal{Y}_{L_i}$  for different flavors as a function of inverse temperature for the first point in Table I.

there is nothing unique about this choice of parameters, which we have adopted here simply because it does give the right order of magnitude for the lepton asymmetry; there may thus exist other viable ranges of parameters.

For the integration of the BEQs, we assume that for  $T = 10m_N$  heavy neutrinos are in equilibrium ( $\mathcal{Y}_{N_i} = \mathcal{Y}_{N_i}^{\text{EQ}}$ ) while all leptonic asymmetries  $\mathcal{Y}_{L_i}$  vanish. The resulting baryon asymmetry  $\mathcal{Y}_B$  (for  $T = 100$  GeV, when  $B$ -violating interactions decouple) is given in Table I. It corresponds directly to the present value  $\mathcal{Y}_{B0}$  predicted by CMS, under assumption of entropy conservation. Using the present entropy to photon ratio  $s \approx 7n_\gamma$  [64], the baryon-to-photon ratio (37) translates into

$$\mathcal{Y}_{B0} \approx 8.6 \times 10^{-11}, \quad (72)$$

we thus see that the values in Table I agree quite well with the data. Although our input value for  $s$  does not include the contribution from Majorons to the total entropy, their inclusion would not affect our results in any essential way.

The integral curves of Boltzmann equations are illustrated, for the first point in Table I, in Figs. 5 and 4. Note that, due to fast  $y_M$ -induced interactions, heavy neutrinos depart from equilibrium for relatively small temperatures; this behavior was also observed in [74]. Nonetheless, our analysis shows that successful *resonant* leptogenesis is possible. We note that the  $y_M$ -induced processes justify our assumption about initial thermal abundance of heavy neutrinos. In fact, the present baryon asymmetry is essentially independent of the distribution of heavy neutrinos for  $T \gg m_N$ . This can be seen in Fig. 6, where the dynamically generated lepton asymmetry for thermal initial distribution of heavy neutrinos (solid line) is compared with its counterpart for vanishing initial abundance (dashed line). We also stress that the effects of thermal corrections to particles' masses were neglected here, and will be discussed in a separate publication, where also issues related to the phase transition will be addressed.

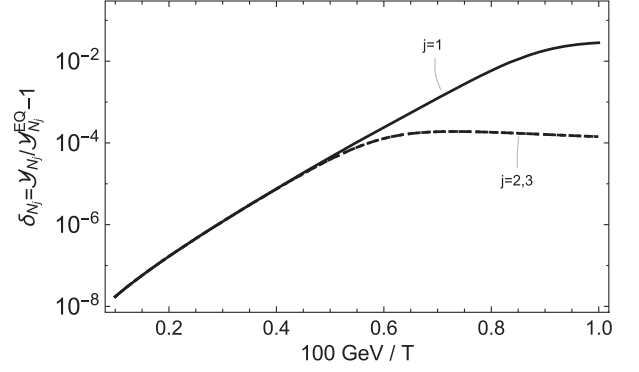


FIG. 5. Departure  $\delta_{N_j} \equiv \mathcal{Y}_{N_j}/\mathcal{Y}_{N_j}^{\text{EQ}} - 1$  of different flavors of heavy neutrinos from thermal equilibrium as a function of inverse temperature for the first point in Table I. Only one flavor has a significant departure in the range of temperatures in which  $B$ -violating processes are fast.

Let us also mention that a similar analysis can be performed for the model with an extended scalar sector that was proposed in our previous work [17]. The result is that resonant leptogenesis does not work in that case, even though for that model Eq. (28), and thus the near degeneracy of heavy neutrino masses, is an automatic consequence of spontaneous symmetry breaking. The reason is that with this extended scalar sector, the minimization condition for the pseudo-Goldstone boson potential (Dashen's condition [34,50]) requires  $R_{\text{CI}}$  to be real, whence the unitary matrix  $U_{\text{MNS}}$  in (20) is the sole source of  $CP$  violation. This  $CP$  breaking turns out to be insufficient to overcome the rapid  $y_M$ -induced interactions that keep heavy neutrinos in thermal equilibrium: the processes of the type  $A$  and  $S$  above are generically faster in the presence of more scalar fields.

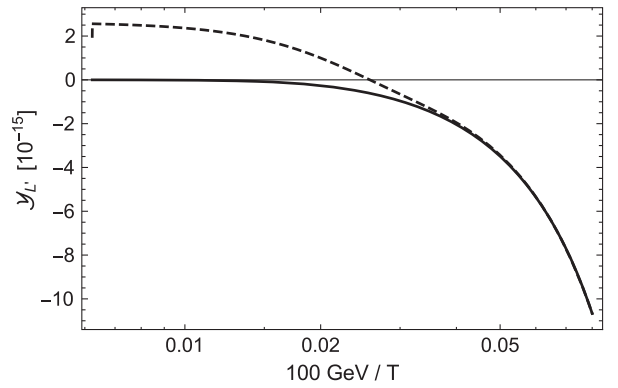


FIG. 6. Total leptonic asymmetry  $\mathcal{Y}_{L_i}$  for the first point in Table I. The solid line corresponds to thermal initial abundance of heavy neutrinos (for  $T_0 = 10m_N$ ). The dashed line corresponds to vanishing initial abundance of heavy neutrinos (for  $T_0 = 10m_N$ ). In both cases vanishing of initial asymmetries  $\mathcal{Y}_{L_i}$  is assumed.

#### IV. DARK MATTER

We next turn attention to the Goldstone boson that accompanies the spontaneous breaking of  $(B - L)$  symmetry. We note that this particle comes “for free” with our model, and provides a natural “habitat” for lepton-number violation, a feature that we exploited already in the previous section. However, spontaneous symmetry breaking is not enough for a possible explanation of dark matter, because for that the Goldstone boson must acquire (an albeit tiny) mass by a separate mechanism.

##### A. Explicitly breaking $(B - L)$ symmetry

As already evident from the previous section an important feature of the CSM in its unbroken phase is the lepton-number symmetry under which also the new scalar  $\phi$  transforms nontrivially. However, rather than focusing on this symmetry separately, we will now consider the  $(B - L)$  transformations which likewise leave the Lagrangian (1) invariant

$$\begin{aligned} (L_\alpha^i(x), \bar{E}_\alpha^i(x), \bar{N}_\alpha^i(x)) &\rightarrow e^{-i\omega}(L_\alpha^i(x), \bar{E}_\alpha^i(x), \bar{N}_\alpha^i(x)) \\ (Q_\alpha^i(x), \bar{U}_\alpha^i(x), \bar{D}_\alpha^i(x)) &\rightarrow e^{i\omega}(Q_\alpha^i(x), \bar{U}_\alpha^i(x), \bar{D}_\alpha^i(x)) \\ \phi(x) &\rightarrow e^{-2i\omega}\phi(x). \end{aligned}$$

The appearance of both barred and unbarred spinors here is dictated by demanding invariance of the Yukawa interactions (10); clearly, the resulting transformations of the Dirac fields (11) and (12) are indeed nonchiral. The reason for considering  $(B - L)$  rather than just lepton number is that this symmetry is anomaly free (see e.g. [34]), which ensures that after spontaneous symmetry breaking  $\phi(x)$  contains, in addition to a real massive scalar, also a Goldstone boson that remains massless to all orders in perturbation theory thanks to the vanishing  $(B - L)$  anomaly. Some of these properties are better visible in the exponential parametrization

$$\phi(x) = \frac{1}{\sqrt{2}}(v_\phi + R(x)) \exp(2iA(x)) \quad (73)$$

where we split the complex field  $\phi(x)$  into a modulus  $R(x) + v_\phi$  and a phase  $A(x)$ . The latter can be absorbed into a redefinition of the fermions

$$\begin{aligned} \chi_\alpha(x) &\rightarrow \chi_\alpha(x)^{\text{NEW}} \equiv \exp[-i(b - \ell)A(x)]\chi_\alpha(x), \\ \bar{\chi}_\alpha(x) &\rightarrow \bar{\chi}_\alpha(x)^{\text{NEW}} \equiv \exp[+i(b - \ell)A(x)]\bar{\chi}_\alpha(x), \end{aligned} \quad (74)$$

where  $\chi_\alpha(x)$  is any CSM Weyl field, and  $(b - \ell)$  is its charge under  $U(1)_{B-L}$ . Due to the exact  $(B - L)$  invariance the field  $A(x)$  then appears in the new Lagrangian with redefined fields only via derivative couplings originating from the kinetic terms of  $\phi$  and  $\chi_\alpha$ 's, to wit,

$$\mathcal{L}_{\text{int}} \propto (b - \ell) \bar{\chi}_\alpha \bar{\sigma}^{\mu\alpha\beta} \chi_\beta \partial_\mu A \quad (75)$$

(we drop the label NEW) and the kinetic term

$$\mathcal{L}_{\text{kin}}^A = 2v_\phi^2 \partial^\mu A \partial_\mu A + \dots \quad (76)$$

which is not canonically normalized; the dots stand for couplings of  $A(x)$  to the real scalar  $R(x)$ . In this picture the fact that  $A(x)$  couples only via derivatives is completely manifest.<sup>16</sup>

The above parametrization in terms of redefined fields will be referred to as the “exponential picture” (as opposed to the “linear picture” introduced in Sec. II A). In particular  $R(x)$  is the counterpart  $\sqrt{2}\text{Re}(\phi)$  from Sec. II A, that is, it describes mainly the extra massive scalar boson  $\phi$  with a small admixture of the SM-like particle  $h$ , cf. Eq. (6), while  $A(x)$  corresponds to the Goldstone mode  $a(x)$  up to normalization. The shift symmetry in the Goldstone field,  $A(x) \rightarrow A(x) + \text{const}$ , is manifest in the exponential picture, but the price to pay is that manifest renormalizability is lost.

Although the field  $A(x)$  thus cannot acquire a mass term within the framework of relativistic QFT in flat spacetime, we now recall a folklore theorem (still based on somewhat heuristic reasoning, cf. [76–78]) according to which there cannot exist exact *continuous* global symmetries in a quantum theory of gravity. This then leaves two options: either  $(B - L)$  is gauged, in which case there is an extra massive  $Z'$  boson, or otherwise the  $(B - L)$  symmetry is broken explicitly by quantum gravity effects.

The former possibility has been studied both within a GUT context (in which case  $Z'$  would be very heavy) or in a “low-energy” realization with a  $Z'$  boson whose mass is  $\propto v_\phi^2$ ; a possible realization of the latter scenario within the CW context was investigated in detail in [8]. Although we will not further consider this possibility here, let us note that for the CSM, gauging  $(B - L)$  would give a very definite prediction for the mass of the  $Z'$  vector boson. From the gauged kinetic term for  $\phi$

$$(\partial_\mu \phi + 2iq_{BL}Z'_\mu \phi)^\dagger (\partial_\mu \phi + 2iq_{BL}Z'_\mu \phi) \quad (77)$$

we would get (after spontaneous symmetry breaking)

$$m_{Z'} = 2\sqrt{2}q_{BL}\langle\phi\rangle = 2q_{BL}v_\phi. \quad (78)$$

The potential discovery of  $\phi$  and knowledge of  $v_\phi$  and  $q_{BL}$  would thus severely constrain the possible range of mass values for  $Z'$ , such that existing lower bounds on the mass of  $Z'$  (that now exceed 4 TeV [2]) could already exclude this possibility.

<sup>16</sup>Due to the small mass term to be introduced below, cf. (80), there will also arise *nonderivative* effective couplings to SM fields which are very small [45].

Because there is so far no evidence for a low lying  $Z'$  vector boson, and because we wish to exploit the presence of the Goldstone boson in a different way by exploring its possible role as a dark matter candidate, we will here consider the second option, invoking (as yet unknown) quantum gravity effects, possibly in the form of a non-perturbative self-regularization of IR divergences, to generate a mass for the Goldstone boson. The nonperturbative breaking of  $(B - L)$  symmetry via quantum gravity was already considered in [79] which also invokes a gravity induced mass for the Majoron to derive limits on its mass from the requirement that it should not lead to overclosure of the Universe. Although that work invokes a dimension 5 operator rather than a dimension 6 operator, as we do here, and does not appear to consider possible connections with dark matter, we note that, interestingly, it also arrives at the conclusion that the scale of  $(B - L)$  symmetry breaking must not exceed  $\mathcal{O}(10 \text{ TeV})$ . See also [80] for a proposal along these lines with gauged  $U(1)_{B-L}$  and an extra scalar field as the dark matter candidate.

To implement the explicit symmetry breaking, we thus postulate the mass term

$$\mathcal{L}_A = \frac{v^4}{M_{\text{PL}}^2} \phi^2 + \text{H.c.} \quad (79)$$

which breaks  $U(1)_{B-L}$  symmetry *explicitly* to its discrete subgroup  $\mathbb{Z}_2$ . Unlike continuous symmetries, *discrete* symmetries are generally believed to be compatible with quantum gravity, which is our reason for excluding dimension 5 operators, as the  $\mathbb{Z}_2$  symmetry of the CSM is thus preserved. Here  $v$  is assumed to be of the same order of magnitude as  $v_\phi$ , and the above mass term should thus be treated on a par with the tree-level Lagrangian. The inverse factor of  $M_{\text{PL}}^2$  in (79) is included because this term is expected to be the low-energy effective operator originating from quantum gravity. Importantly, (79) breaks  $(B - L)$  symmetry only softly, and thus does not entail new quadratic divergences, nor  $(B - L)$  breaking dimensionless couplings, in analogy with the soft terms in MSSM-like models.

Without spontaneous symmetry breaking the above mass term is completely negligible. When  $(B - L)$  symmetry is spontaneously broken, however, this term will manifest itself in the form of a violation of the Goldstone theorem, by endowing the Goldstone boson with a tiny mass and, in fact, a periodic potential for the Goldstone field  $A(x)$ . The Goldstone theorem is a well-known result of flat space QFT, but there is no reason to expect it to hold in the presence of a curved spacetime background or in the context of quantum gravity, and this is a possibility we wish to exploit here. A possible breakdown of the Goldstone theorem in the framework of curved space QFT has already been discussed in the literature, see e.g. [81–83] all of which reach the conclusion that in a curved background

such as de Sitter space the Goldstone boson acquires a (small) mass. Choosing  $v \sim 1 \text{ TeV}$  in formula (79) as an example we get

$$m_A = \frac{2v^2}{M_{\text{PL}}} \sim 10^{-3} \text{ eV}. \quad (80)$$

With the assumed small quantum gravity induced mass and because of its very small couplings to SM particles, we name the associated pseudo-Goldstone particle *minoron*.

Importantly, the operator equation (79) in the exponential picture not only generates a mass term for the minoron, but also induces very small (and calculable) *nonderivative* couplings for the scalar field  $A(x)$ . In particular the continuous shift symmetry  $A(x) \rightarrow A(x) + \text{const}$  is now reduced to a symmetry under discrete shifts, which implies that the induced potential for  $A(x)$  must be a periodic function.

## B. Minorons as dark matter candidates

The mass estimate (80) lies very well within the range of mass values generally accepted (or even desired) for dark matter constituents. Of course, in any such model we have to ensure that the dark matter candidate cannot decay early on in the history of the Universe, and therefore we assume that  $m_A < 2m_\nu$ .<sup>17</sup> In this section we briefly discuss the potential prospects for the minoron to be a viable dark matter candidate. In addition to its stability to decays, this requires that minorons must be created in sufficient amounts and in such a fashion that they can clump (as opposed to being thermally distributed like the CMB). There are obviously many analogies between the present proposal and axionic dark matter scenarios [87,88], as the axion is also a pseudo-Goldstone particle. On the other hand, there are also differences—in particular, there is no immediate link between the minoron and the strong interactions, unlike for the usual axion, although axionlike couplings can be generated via higher loop corrections [45].

A main feature of any dark matter model concerns the possible interactions with SM matter which must be small. The coupling between the minoron and photons is of the loop origin. After summation over the helicity states of final photons, the amplitude for the processes  $a \rightarrow \gamma\gamma$  can be bounded above by the following estimate:

$$\widetilde{\mathcal{M}} = \left\{ \sum_{\text{spin}} |\mathcal{M}|^2 \right\}^{1/2} \lesssim \frac{1}{F} \frac{y_M e^2}{(4\pi)^2} p^2, \quad (81)$$

where  $F$  is at least of the order of masses of particles circulating in the loops. Clearly gauge invariance of the  $a\gamma\gamma$

<sup>17</sup>For a discussion of  $m_A > 2m_\nu$  case in the context of dark matter, see [84–86].



vertex requires at least one momentum for each photon; when the minoron is on-shell we have  $p^2 \sim m_A^2$ . Taking  $F = 100$  GeV and  $m_A = 10^{-3}$  eV we get

$$\Gamma(a \rightarrow \gamma\gamma) = \frac{\widetilde{\mathcal{M}}^2}{16\pi m_A} \lesssim 10^{-48} \text{ GeV}. \quad (82)$$

Comparing this with the age of the Universe ( $H_0 \sim 10^{-42}$  GeV) we see that the minorons can easily survive to the present epoch. Nonetheless we should note that the decay width in Eq. (82) is, in fact, overestimated by many orders of magnitude. First, the amplitude (81) originates from multiloop diagrams of the type discussed in [45], while in (81) we have included only coupling from vertices to which external lines are attached, as well as a single loop-suppression factor. Second, Goldstone bosons of *nonanomalous* symmetries have derivative couplings to *gauge-invariant* operators (see e.g. [34]), thus additional powers of  $p^2/v_\phi^2 \sim m_A^2/v_\phi^2$  should appear on the right-hand side of (81); while the minoron is pseudo-Goldstone boson, the explicit breaking of  $(B-L)$  would itself introduce an additional factor  $m_A^2/v_\phi^2$ .

The minoron abundance is more difficult to estimate, and we can offer only some preliminary heuristic arguments at this point. The contribution to the density can come from three sources: particles, strings and domain walls. Minorons, being lighter than light neutrinos, can decay only into photons but their lifetime is longer than the age of the Universe, so they are effectively stable—therefore they pose no problem for the galaxy formation, nor for the nucleosynthesis. At the present time the relic thermal density of minorons is negligible. The minoron potential becomes relevant when the field  $\phi$  acquires its vacuum expectation value, and the minoron field decouples from other fields (its interaction with neutrinos is too weak to maintain equilibrium). The field starts to be dynamical when  $3H \sim m_A$ ;  $\langle\phi\rangle$  thus starts to differ from zero at  $T \sim 1$  TeV. In this case both quantities are of the order of  $10^{-3}$  eV. The initial density of coherent oscillations is  $\rho_{\text{osc}} \sim m_A^2 v^2$  and after dilution it gives a negligible contribution now. So we are left with strings (that decay very fast) and domain walls as the most important possible source of dark matter in the late history of the Universe relevant for the present day [80,89–91].

We can write the approximate Lagrangian for  $A(x)$  as

$$\mathcal{L}_{\text{minoron}} = 2v_\phi^2 g^{\mu\nu} \partial_\mu A \partial_\nu A - \frac{1}{4} v_\phi^2 m_A^2 [1 - \cos(4A)]. \quad (83)$$

Since  $2A$  is a phase [see (73)] the period equals  $\pi$  (and not  $2\pi$ ), and this is important for the stability of domain walls. For axion dark matter scenarios this stability is usually a problem, as it could lead to an overclosure of the Universe, but it is *not* a problem for the present scheme because the domain walls start to have a significant effect only in the

present era, when the cosmological constant starts to dominate the evolution of the Universe.

We now assume that the domain wall connects two consecutive minima of the potential (along the  $z$  direction), for example 0 and  $\pi$ . Neglecting time derivatives we have to solve the equation (where the prime denotes derivative with respect to the physical coordinate  $z$ )

$$A''(z) - \frac{m_A^2}{4} \sin(4A(z)) = 0 \quad (84)$$

with  $A(-\infty) = 0$  and  $A(\infty) = \pi/2$ . The solution reads

$$A(z) = \arctan(e^{m_A z}). \quad (85)$$

We can calculate the surface energy of the domain wall by

$$\sigma = \int_{-\infty}^{\infty} dz 2v_\phi^2 \left( \left( \frac{dA}{dz} \right)^2 + \frac{m_A^2}{8} (1 - \cos 4A) \right) \quad (86)$$

with the result

$$\sigma = 2m_A v_\phi^2. \quad (87)$$

Assuming  $m_A \sim 10^{-3}$  eV and  $v_\phi \sim 2$  TeV we get  $\sigma \sim 2 \times 10^{35}$  eV/m<sup>2</sup>. Assuming that these domain walls are very large, and that there is one wall per Hubble volume, the energy density thus comes out to be

$$\rho(t) \sim \sigma H(t) \sim \frac{t_0}{t} (\text{GeV/m}^3) \quad (88)$$

where  $t_0 \sim 4 \times 10^{17}$  s is our present time. Remarkably, we thus arrive at the right order of magnitude for the present density of dark matter

$$\rho_{\text{DM}} \approx 1 \text{ GeV/m}^3. \quad (89)$$

The presence of one or several large domain walls at the time of last scattering could have observable impact on the CMB spectrum especially for low  $\ell$  (quadrupole) so possibly the domain walls should start decaying into (cold) minorons before the last scattering. Then the above estimate should be slightly changed since the energy density of particles decreases faster than that of domain walls. However, this topic requires further study for a more precise analysis.

One can also note that the self-interaction of massive minorons via interactions with right-chiral neutrinos (via box diagrams *à la* Euler-Heisenberg) gives similar values as would be required by the Steinhardt-Spergel analysis of dark matter in the Abell cluster [92]. The values for the cross section are in the region  $\sigma \sim m_A \times 10^{-24 \pm 1} \text{ cm}^2 \text{ GeV}^{-1}$  which gives (for  $m_A \sim 10^{-3}$  eV)  $\sigma \sim 10^{-36 \pm 1} \text{ cm}^2$  i.e. in the region of cross sections mediated by

the exchange of heavy neutrinos. In conclusion, and subject to our assumptions on quantum gravity induced mass generation for the minoron we have shown that the CSM can offer a viable scenario for the explanation of dark matter.

## V. OUTLOOK

To conclude we summarize the main features of the CSM elaborated in this paper:

- (i) There is a range of parameter values for which the CSM is perturbative and the electroweak vacuum remains stable for all energies up to  $M_{\text{PL}}$ .
- (ii) The main prediction of the model is a new and almost sterile scalar resonance which comes with low mass heavy neutrinos, but nothing else.
- (iii) All new degrees of freedom are very weakly coupled to SM matter.
- (iv) There exist Casa-Ibarra matrices  $R_{\text{CI}}$  for which resonant leptogenesis is possible.
- (v) The pseudo-Goldstone boson associated with the breaking of  $(B - L)$  (minoron) is a possible dark matter candidate, whose nonvanishing mass is an indirect manifestation of quantum gravity.

We stress again that these properties set the CSM apart from many other current proposals (such as SUSY Higgs, two doublet models, vectorlike models) where neutral scalars are usually accompanied by other and “nonsterile” charged excitations, and which would all have to be produced together. So, barring the inconvenient possibility that a new scalar could escape detection because the associated resonance could be too narrow for the LHC energy bins, the acid test of the present model will be whether or not the new scalar shows up in future LHC searches with increased luminosity. In this way the model is eminently falsifiable.

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## APPENDIX: FIELD OPERATORS FOR MASSIVE AND MASSLESS NEUTRINOS

Since the use of the Weyl fields in  $S$ -matrix calculations is perhaps not so common, for readers’ convenience we here give the explicit decompositions of the corresponding field operators in terms of creation and annihilation operators [see e.g. [38] for an introduction to  $\text{SL}(2, \mathbb{C})$  spinor formalism]. These expressions can be derived for instance following Weinberg’s procedure [93,94] (although we use different normalization conventions), and they are the ones used in the computation of the matrix elements

$\langle h\nu | \mathcal{L}_Y | N \rangle$  and  $\langle h\bar{\nu} | \mathcal{L}_Y | N \rangle$  required for the determination of the  $CP$  asymmetries in (38).

For the massive case, and suppressing family indices, the field operator  $\bar{N}^\alpha(x)$  in the fundamental representation takes the form

$$\bar{N}^\alpha(x) = \sum_{r=\pm 1/2} \int \frac{d^3\mathbf{p}}{2(2\pi)^3 \sqrt{m^2 + \mathbf{p}^2}} \times \{ \bar{u}_r^\alpha(\mathbf{p}) b_r(\mathbf{p}) e^{-ip_\mu x^\mu} + \bar{v}_r^\alpha(\mathbf{p}) b_r^\dagger(\mathbf{p}) e^{ip_\mu x^\mu} \},$$

where  $p^\mu = (p^0, \mathbf{p})$  is on shell:  $p^0 = \sqrt{m^2 + \mathbf{p}^2}$ . The normalization of creation and annihilation operators can be read off from the anticommutator

$$[b_r(\mathbf{p}), b_s^\dagger(\mathbf{q})]_+ = 2\sqrt{m^2 + \mathbf{p}^2} \delta_{rs} (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{q}).$$

The two-component spinor wave functions  $\bar{u}_r^\alpha(\mathbf{p})$  and  $\bar{v}_r^\alpha(\mathbf{p})$  can be likewise read off as the column vectors

$$\bar{u}(\mathbf{p}) = B_{\mathbf{p}}, \quad \bar{v}(\mathbf{p}) = B_{\mathbf{p}} \epsilon^{-1} \quad (\text{A1})$$

from the  $2 \times 2$  matrix

$$B_{\mathbf{p}} \equiv \frac{1}{\sqrt{2(m + p^0)}} [p^\mu \bar{\sigma}_\mu + m \mathbb{1}], \quad (\text{A2})$$

where  $\epsilon^{-1}$  is the inverse antisymmetric metric, and where  $r, s = \pm 1/2$  label the eigenvalues of  $J_3$  in the rest frame. The conjugate Weyl spinor operator is obtained by taking the Hermitian conjugate  $N_\alpha \equiv \epsilon_{\alpha\beta} \bar{N}^{\beta\dagger}$ .

These formulas are, of course, in complete accord with textbook formulas in 4-spinor notation. More precisely, combining  $N_\alpha$  and  $\bar{N}^\alpha$  into a Majorana spinor as in (22) we reproduce the standard formula

$$\psi_N(x) = \sum_{r=\pm 1/2} \int \frac{d^3\mathbf{p}}{2(2\pi)^3 \sqrt{m^2 + \mathbf{p}^2}} \times \{ U_r(\mathbf{p}) b_r(\mathbf{p}) e^{-ip_\mu x^\mu} + V_r(\mathbf{p}) b_r^\dagger(\mathbf{p}) e^{ip_\mu x^\mu} \}, \quad (\text{A3})$$

with the 4-spinors  $U \equiv (v_\alpha, \bar{u}^\alpha)$  and  $V \equiv (u_\alpha, \bar{v}^\alpha)$  obeying the completeness relations

$$\begin{aligned} \sum_r U_r(\mathbf{p}) \bar{U}_r(\mathbf{p}) &= \not{p} + m \\ \sum_r V_r(\mathbf{p}) \bar{V}_r(\mathbf{p}) &= \not{p} - m, \end{aligned} \quad (\text{A4})$$

with the Weyl representation of Dirac matrices, see [38]. Although with this normalization the limit  $m \rightarrow 0$  is nonsingular, a more physical choice of basis for massless spinors corresponds to the helicity eigenstates, rather than

just the formal limit of the above expressions. In this basis we have for massless spinors

$$\bar{\nu}^{\dot{\alpha}}(x) = \int \frac{d^3\mathbf{p}}{2(2\pi)^3|\mathbf{p}|} \bar{w}^{\dot{\alpha}}(\mathbf{p}) \times \{a_+(\mathbf{p})e^{-ip_\mu x^\mu} - a_-^\dagger(\mathbf{p})e^{ip_\mu x^\mu}\}.$$

Because of the degeneracy of the Weyl operator  $p_\mu\sigma^{\mu}$  in the massless case there is now *only one* spinor wave function, unlike for the massive case where there are two. This helicity wave function satisfies the Weyl equation  $p_\mu\sigma_{\alpha\beta}^\mu\bar{w}^\beta(\mathbf{p}) = 0$  and obeys the completeness relation

$$\bar{w}^{\dot{\alpha}}(\mathbf{p})w^\beta(\mathbf{p}) = p_\mu\bar{\sigma}^{\mu\dot{\alpha}\beta}. \quad (\text{A5})$$

The helicity eigenstates are

$$|\nu(\mathbf{p})\rangle = a_-^\dagger(\mathbf{p})|0\rangle, \quad |\bar{\nu}(\mathbf{p})\rangle = a_+^\dagger(\mathbf{p})|0\rangle \quad (\text{A6})$$

whence  $a_-^\dagger(\mathbf{p})$  creates a helicity  $-1/2$  neutrino, while  $a_+^\dagger(\mathbf{p})$  creates a helicity  $+1/2$  antineutrino. Notice also, that the associated 4-spinors appearing in the resulting decomposition of the Majorana field  $\psi_\nu$  in (22) are consistent with the massless limit of (A4).

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