Blandford-Znajek process in vacuo and its holographic dual

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Blandford and Znajek discovered a process by which a spinning black hole can transfer rotational energy to a plasma, offering a mechanism for energy and jet emissions from quasars. Here we describe a version of this mechanism that operates with only vacuum electromagnetic fields outside the black hole. The setting, which is not astrophysically realistic, involves either a cylindrical black hole or one that lives in 2+1 spacetime dimensions, and the field is given in simple, closed form for a wide class of metrics. For asymptotically Anti-de Sitter black holes in 2+1 dimensions the holographic dual of this mechanism is the transfer of angular momentum and energy, via a resistive coupling, from a rotating thermal state containing an electric field to an external charge density rotating more slowly than the thermal state. In particular, the entropy increase of the thermal state due to Joule heating matches the Bekenstein-Hawking entropy increase of the black hole.

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Prodigious amounts of energy wind up in the rotation of spinning black holes. Although nothing can escape from inside a black hole, that energy can be extracted via a dynamo effect, the *Blandford-Znajek (BZ) process* [1], when a black hole is threaded by a magnetic field and surrounded by plasma. The mechanism of the BZ effect is a type of *Penrose process* [2]: Near a spinning black hole is an *ergosphere* where it is impossible to remain stationary relative to a distant observer, because the local inertial frames are dragged around faster than the speed of light. A system in the ergosphere can carry negative values of the conserved global energy quantity, yet have positive ordinary energy relative to a local observer. (In this paper the word "energy" will refer to the globally conserved quantity, which agrees with "local energy" only far from the black hole, in its rest frame.) Total energy can therefore be globally conserved, while positive energy flows away and negative energy flows into the black hole.

Although the principles of the BZ process are fully understood (e.g. [3, 4]), and it is well studied analytically, semi-analytically, and numerically (e.g. [1, 5–9]), it remains difficult to develop an intuitive picture of how the flow of electromagnetic energy and charge current is organized in space, and how and where the negative energy originates. The difficulty is in part due to the three dimensionality, the number of different field quantities, and the complexity of the Kerr metric which describes the spacetime geometry of a spinning black hole. In an effort to boil down the process to a "toy model," we began looking for a version in a 2+1 dimensional spacetime. We assumed that, as in the original BZ solution, the plasma is "force-free", i.e. that it has a (nonzero) charge 4-current density orthogonal to the field strength, but discovered that there is no such solution. Much to our surprise, however, we found that there is a *purely electromagnetic* version of the BZ process, in which plasma plays no role.

We begin here with the cylindrical BH examples, since they are easier to visualize, and the analogy with the astrophysical case is closer. The 2+1 version will then be obtained as a special case of the 3+1 cylindrical one. Of particular interest is the spinning BTZ black hole background in 2+1 dimensions [10], since it is an exact solution of general relativity with a negative cosmological constant, is locally maximally symmetric, and is related by AdS/CFT duality [11] to a thermal state of a two dimensional conformal field theory. Thus one can also examine the CFT dual of a BZ process. A similar idea was previously pursued in Ref. [12], with force-free plasma in the Kerr-AdS background, but no energy extracting solution was found. It was suggested there that this might be due to the existence of a globally timelike Killing vector outside the event horizon, but this implies only that positive values of the corresponding conserved quantity cannot be extracted. Indeed, the spinning BTZ black hole also has a globally timelike Killing vector, yet we find that positive energy extraction is possible.

In the BZ process [1] a stationary plasma, around a spinning black hole threaded by a magnetic field, radiates a Poynting flux of energy to infinity. Energy conservation implies that the radial component of the Poynting vector $\vec{E} \times \vec{B}$ must fall with radius inversely as the surface area grows, so that the flux of energy is the same through any surface surrounding the black hole. To simplify the geometry, and to eliminate the need for charge outside the black hole, we enhance the axisymmetry to cylindrical symmetry. Since the azimuthal circumference grows as the cylindrical radius r, the radial component of the Poynting vector must then fall as $1/r$. For example, if there is a uniform electric field parallel to the cylindrical axis, then the magnetic field must have an azimuthal, circular component that falls as $1/r$. Ampère's law then implies that there must be an enclosed electric current parallel to the axis. In order for the Poynting flux to be

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radially outward, the flow of current must be opposite to the external electric field. The energy required to drive this current ends up in the outgoing Poynting flux.

Consider this model first in flat, empty spacetime. We can imagine shrinking the current down to the axis, so that the solution is vacuum everywhere except on the axis where it is singular. In this limit, the energy emerges from the axis and propagates radially outward as Poynting flux. Of course a line source of Poynting flux doesn't much resemble the BZ process, so let us introduce a cylindrical black hole, and hide the line source inside the black hole. A solution essentially like the one just described can be placed on this black hole spacetime but, since nothing can escape from a black hole, we expect that energy cannot be extracted from the source behind the horizon. Indeed, if we try to construct such a solution, we find that the fields are singular on the horizon. In effect, the horizon becomes a singular source of energy, which again does not resemble the BZ process.

However, suppose that we now set the black hole spinning with angular velocity Ω_H. Then outgoing Poynting flux exists with fields that are regular on the horizon, provided there is a nonzero magnetic flux through the horizon, that is related in a particular way to the electric field, the current, and Ω_H. The electromagnetic dual of this solution is also energy extracting. It has a uniform magnetic field parallel to the cylindrical axis, a radial electric flux through the horizon, and an azimuthal electric field. This reduces to a $2+1$ dimensional solution to Maxwell's equations when the axial direction is omitted. We turn now to a precise demonstration of these conclusions.

We adopt cylindrical spacetime coordinates (t, r, ϕ, z), and assume the spacetime line element takes the form

$$ds^2 = -\alpha^2 dt^2 + \alpha^{-2} dr^2 + r^2(d\phi - \Omega dt)^2 + dz^2, \quad (1)$$

where the functions α and Ω depend only on r, and we use units with $c = 1$. The spacetime is then stationary and cylindrically symmetric: ds^2 is invariant under translation of t, ϕ, and z. We refer to the t coordinate as "time". This line element describes a wide class of spacetimes. If there is a radius r_H where $\alpha(r_H) = 0$, (1) describes a "black cylinder" or "black string," i.e. a black hole with a cylindrical horizon at the outermost root r_H. If $\Omega_H := \Omega(r_H) \neq 0$, the black cylinder is spinning with angular velocity Ω_H, and has an ergosphere where the time translation is spacelike, i.e. where $\alpha^2 < r^2\Omega^2$. The BTZ black string metric [13] is conformal to (1). Since Maxwell's equations are conformally invariant, the solutions we discuss here are also solutions in the background of the BTZ black string.

To describe the electromagnetic field we employ the language of differential forms, which greatly simplifies computations, especially in curved spacetime. (For a concise review of differential forms and their application to electromagnetism see Appendix A of [14].) The electromagnetic field strength 2-form is denoted by F, an index-free notation for the antisymmetric tensor F_{ab}. (For example, $F = E\, dx \wedge dt + B\, dx \wedge dy$ describes an electric field strength E in the x direction and magnetic field strength B in the z direction, in flat spacetime with Cartesian coordinates.) Maxwell's equations are $dF = 0$ (absence of magnetic monopoles and Faraday's law) and $d * F = J$ (Gauss and Ampère-Maxwell laws), where d is the (metric independent) exterior derivative operator, J is the current 3-form, and $*$ is the Hodge dual operation that sends a p form ω to the orthogonal $4-p$ form $*\omega$ with the same magnitude. We adopt the orientation of $dt \wedge dr \wedge d\phi \wedge dz$ for defining the dual ($\omega \wedge *\omega = \omega^2 \epsilon$, where ω^2 is the squared norm of ω and ϵ is the unit 4-form with the adopted orientation). Note that the metric (1) is a sum of squares of 1-forms, hence those 1-forms are orthonormal, which facilitates the computation of duals and norms.

The first vacuum field we shall discuss is given by

$$F = \left[\frac{I}{2\pi r \alpha^2} dr + \psi_z (d\phi - \Omega_F dt)\right] \wedge dz, \quad (2)$$

where I, ψ_z, and Ω_F are all constant. The dual of the field (2) is

$$*F = \frac{I}{2\pi} d\phi \wedge dt + \psi_z \left[\frac{1}{r} dt + \frac{r(\Omega - \Omega_F)}{\alpha^2}(d\phi - \Omega dt)\right] \wedge dr, \quad (3)$$

It is clear by inspection that Maxwell's equations $dF = 0 = d * F$ are satisfied (since $d^2 = 0$ and $dr \wedge dr = 0$), so this is indeed a vacuum solution, except at the axis where $d\phi$ is singular. The field (2) is the wedge product of two 1-forms, which implies that $F \wedge F = 0$. Equivalently, the Lorentz-invariant scalar $\vec{E} \cdot \vec{B}$ vanishes, as it does for ideal plasmas since $\vec{E}$ vanishes in the rest frame of a perfect conductor. A field with this algebraic property is called *degenerate*.

The constant parameters (and their notation) are directly analogous to functions that appear in standard treatments of stationary axisymmetric plasma magnetospheres (e.g. [14, 15] and references therein). The singularity on the axis carries a line current I in the $-z$ direction, sourcing the azimuthal magnetic field, and a magnetic monopole line charge density $2\pi\psi_z$ sourcing the radial magnetic field. ($2\pi\psi_z$ is the z-derivative of the magnetic flux through a cylinder ending at coordinate z.) The quantity Ω_F is the "angular velocity of the magnetic field lines," in the sense that the electric field vanishes in the local frame that rotates that way: $(\partial_t + \Omega_F \partial_\phi) \cdot F = 0$. This is a standard concept in ideal plasma physics, where the plasma determines the preferred frame. It is meaningful here without the plasma only because the field is degenerate. We are interested in the solution (2) on a rotating black hole background, but it is worth noting that it could also be terminated on a cylindrical conductor rotating with angular velocity Ω_F.

The Maxwell action is given by $-\frac{1}{2}\int F \wedge *F$, from which it follows that the conserved Noether current associated with the t-translation symmetry of the spacetime is [14]

$$\mathcal{J}_E = -(\partial_t \cdot F) \wedge *F + \tfrac{1}{2}\partial_t \cdot (F \wedge *F). \tag{4}$$

As is usual, we call this conserved quantity simply "energy". The outgoing energy flux is given by the integral of the 3-form $\mathcal{J}_E$ over a surface of constant r, which by virtue of the conservation law is the same as the flux across any other surface of constant r. Replacing $\partial_t \to -\partial_\phi$ in (4), one obtains the angular momentum current $\mathcal{J}_L$ associated with the ϕ-translation symmetry.

The energy current outward through a surface of constant r arises from the part of $\mathcal{J}_E$ that contains no dr factor (so the second term on the right hand side of (4) does not contribute), which for the field (2) is given by

$$\mathcal{J}_E|_r = \frac{1}{2\pi}\Omega_F \psi_z I \, dt \wedge d\phi \wedge dz. \tag{5}$$

The outward flux of this current over a constant r cylinder of length Δz and time interval Δt is $\Omega_F \psi_z I \Delta t \Delta z$, so the energy and angular momentum fluxes per unit proper length per unit t-coordinate time at any radius are given by

$$\mathcal{E}_r = \Omega_F \psi_z I, \quad \mathcal{L}_r = \psi_z I. \tag{6}$$

The outgoing energy flux is positive if I has the same sign as $\Omega_F \psi_z$, which means that the current is opposite to the electric field. This corresponds to the situation described in the introduction, where the line current driven against the electric field produces an outgoing Poynting flux.

The line element (1) indicates that $\alpha\, dt$ and dr/α are unit 1-forms, so that dt and dr/α^2 are both singular at $\alpha = 0$ in a black cylinder spacetime. Therefore the I and Ω_F terms in the field (2) are both singular at the horizon. However, it should certainly be possible to *at least* have a regular solution with positive energy flowing inward across the horizon, so there must be a way to remove these divergences. Indeed, the forms dt and dr become proportional on the horizon, and their divergences can cancel. To analyze this divergence balancing act we can use the three Killing vectors, ∂_t, ∂_ϕ, and ∂_z, which are all tangent to the horizon. Their contractions with F are finite as long as the constants ψ_z and Ω_F are finite. For the fourth regular basis vector we use the 4-velocity k of null ($k \cdot k = 0$), zero angular momentum ($k \cdot \partial_\phi = 0$) and zero longitudinal momentum ($k \cdot \partial_z = 0$) geodesics, with affine parameter fixed by the choice $k \cdot \partial_t = -1$. These conditions (which are conserved along the geodesic) determine k uniquely up to the choice of whether the geodesic is ingoing or outgoing. The ingoing one is

$$k = \alpha^{-2}(\partial_t + \Omega \partial_\phi) - \partial_r. \tag{7}$$

The field (2) is regular if all the scalars formed by contracting it with these four vectors are finite. In particular, finiteness of $k \cdot \partial_z \cdot F$ requires

$$\frac{\psi_z(\Omega - \Omega_F) - I/2\pi r}{\alpha^2} < \infty. \tag{8}$$

A necessary condition for finiteness at the horizon is therefore

$$I = 2\pi r_H \psi_z (\Omega_H - \Omega_F). \tag{9}$$

This condition is also sufficient, provided the denominator α^2 in (8) has nonvanishing first derivative at r_H. The relation (9) is known as the *Znajek condition* [16].

If the regularity condition (9) holds, then the outgoing energy flux (6) becomes

$$\mathcal{E}_r = 2\pi(\psi_z)^2 \, r_H \Omega_F (\Omega_H - \Omega_F). \tag{10}$$

If the black cylinder is not rotating ($\Omega_H = 0$), then the outward energy flux is always negative. This means that, as expected, while energy can be sent in, it cannot be extracted. However, if the black cylinder is rotating, and if $0 < \Omega_F < \Omega_H$, then positive energy can be radiated outward. The energy flux formula (10) is precisely analogous to that for a force-free plasma on a Kerr black hole spacetime [1, 14, 15].

The vacuum Maxwell equations are invariant under electric-magnetic duality, $F \to *F$, as is the energy momentum tensor and Noether current (4), so the dual (3) of the field (2) provides another solution that extracts energy from the black cylinder. The dual solution has a uniform magnetic field in the z direction, and azimuthal and radial electric field components. The radial electric field is sourced by an electric line charge, and the azimuthal electric field can be sourced by either a magnetic monopole current or a magnetic flux line with uniformly changing flux.

The dual field strength (3) has no dz factor, and is invariant under z translations, so it descends to a solution $F_3 := *_4 F$ in three-dimensional spacetime. The three-dimensional Hodge dual $*_3 F_3$ is a 1-form, which is just the negative of the first factor of our original solution, (2). In the remainder of this article, we shall focus on this example. Maintaining the convention that the vector potential is an inverse length, in three spacetime dimensions the Maxwell action written above should be multiplied by a constant with dimensions of length, $1/g^2$. We adopt units here with $g = 1$.

Dualizing interchanges electric and magnetic quantities, so we change our notation for the parameters accordingly: $Q := -2\pi\psi_z$ is the electric charge of the solution, and $\dot{\Phi} := -I$ is the t derivative of the magnetic flux through a loop encircling the origin (or the magnetic monopole current). The constant Ω_F is now the "angular velocity of the electric field lines", that is, the angular velocity of the frame in which the magnetic field (which in 2+1 dimensions is a pseudoscalar) vanishes (presuming the vector $\partial_t + \Omega_F \partial_\phi$ is timelike, which is here the case for large enough r). The three dimensional field and its dual are given by

$$F_3 = -\frac{\dot\Phi}{2\pi}\,d\phi\wedge dt - \frac{Q}{2\pi r}dt\wedge dr - \frac{Qr(\Omega-\Omega_F)}{2\pi\alpha^2}(d\phi-\Omega dt)\wedge dr, \quad (11)$$

$$*_3\,F_3 = \frac{\dot\Phi}{2\pi\alpha^2 r}\,dr + \frac{Q}{2\pi}(d\phi - \Omega_F\,dt). \quad (12)$$

This form includes all stationary, axisymmetric vacuum solutions to Maxwell's equations in 2+1 dimensions.

This solution can be placed in particular on the rotating BTZ black hole background [10], which is a three-dimensional solution of Einstein's equations with a negative cosmological constant, $\Lambda_3 = -\ell^{-2}$. The corresponding line element (in Boyer-Lindquist-like coordinates) is (1) without dz^2, and with

$$\alpha^2 = \frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2\ell^2}, \qquad \Omega = \frac{r_- r_+}{r^2\ell} \quad (13)$$

where $0 < r_- < r_+$ are the inner and outer horizon radii. On this spacetime, the invariant square of the field strength is

$$F_3^2 = \frac{Q^2}{4\pi^2}\frac{\ell^2\Omega_F^2 - 1}{r^2 - r_-^2}. \quad (14)$$

In the energy extracting case ($0 < \Omega_F < \Omega_H$) we have $\ell\Omega_F < \ell\Omega_H \leq 1$, so this field is electric dominated everywhere outside the inner horizon, and is singular at the inner horizon. (For the case $\Omega_F = \Omega_H$ this singularity has been previously noted [17].)

We now turn to the AdS/CFT dual interpretation of the vacuum BZ process on the spinning BTZ black hole background. A number of previous studies have employed this duality with Einstein-Maxwell theory in three spacetime dimensions (see e.g. [18–23]), sometimes with other fields included, to study conductivity properties of the dual 1+1 dimensional system. The black hole is dual to a thermal state of a CFT at the Hawking temperature, rotating with angular velocity Ω_H [24]. To extract rotational energy from this thermal state, it must be coupled to another system, and that coupling is described by the boundary conditions on the electromagnetic field. In the limit of large radius r, the field F_3 can be derived (as $F_3 = dA_3$) from the potential

$$A_3 \xrightarrow{r\to\infty} \frac{\dot\Phi}{2\pi}t\,d\phi - \frac{Q}{2\pi}\ln r\,(dt - \ell^2\Omega_F\,d\phi), \quad (15)$$

which corresponds, via the AdS/CFT dictionary [19, 25], to a boundary CFT current j and gauge field a,

$$j = \frac{Q}{2\pi\ell}(\partial_t + \Omega_F\partial_\phi), \qquad a = \frac{\dot\Phi}{2\pi}\,t\,d\phi. \quad (16)$$

One dynamically consistent boundary condition fixes j [25], which fixes the black hole charge and the asymptotic magnetic field. The transfer of energy between the CFT and the (unspecified) degrees of freedom carrying the current then entails Ohmic dissipation.

The conductivity σ of the CFT is the ratio of the current to the electric field, figured in the rest frame of the thermal state. That is, $\sigma = (s\cdot j)/(u\cdot s\cdot f)$, where $f = da = (\dot\Phi/2\pi)dt\wedge d\phi$ is the boundary electromagnetic field strength, $u = \gamma(\partial_t + \Omega_H\partial_\phi)$ is the velocity 2-vector of this rotating frame, and $s = \gamma(\ell\Omega_H\partial_t + \ell^{-1}\partial_\phi)$ is the orthogonal, spatial unit 2-vector (with respect to the boundary metric, $ds^2 = -dt^2 + \ell^2 d\phi^2$), with $\gamma = (1-\ell^2\Omega_H^2)^{-1/2}$. After using the Znajek horizon regularity condition (9), which in the present notation is $\dot\Phi = Qr_+(\Omega_H - \Omega_F)$, the dependence on the electromagnetic field parameters drops out, and we find

$$\sigma = \frac{\gamma\ell}{r_+} = \frac{1}{\gamma\kappa\ell} = \frac{\hbar}{2\pi\gamma T_H}, \quad (17)$$

where κ is the surface gravity of the BTZ black hole, and $T_H = \hbar\kappa/2\pi$ is the Hawking temperature. In the nonrotating limit, σ is inversely proportional to the Hawking temperature, in agreement a prior result [22]. The conductivity depends on angular momentum of the black hole only via the Lorentz boost factor γ, and γT_H is the temperature in the corotating frame of the thermal state [26], so in fact the rotating conductivity is the same as in the nonrotating case, when expressed in terms of the corotating temperature.

The response to weak fluxes of electromagnetic energy and angular momentum can be determined using the laws of black hole mechanics (which are derived from Einstein's equation). The flux formula (6) shows that $dE = \Omega_F dL$ for the electromagnetic fluxes of energy and angular momentum, and conservation laws imply that infinitesimal changes of the black hole mass M and angular momentum J are related in the same way, $dM = \Omega_F dJ$, with $dJ = -dL$. The First Law of black hole mechanics [27], gives $dM - \Omega_H dJ = (\kappa/8\pi G)dA$, where κ and A are the surface gravity and area of the horizon, respectively, hence we have $(\Omega_F - \Omega_H)dJ = (\kappa/8\pi G)dA$. The Second Law of black hole mechanics [28] states that $dA \geq 0$, in processes with matter satisfying the null positive energy condition (such as electromagnetic fields). Thus, if $\Omega_F < \Omega_H$, the black hole spins down, and loses or gains mass according as Ω_F is positive or negative. If instead $\Omega_F > \Omega_H$, the black hole spins up and gains mass. If the asymptotic electric and magnetic fields are held fixed, then Ω_F is fixed, and Ω_H evolves until it exponentially approaches the condition $\Omega_H = \Omega_F$, for which the fluxes vanish.

The black hole entropy $S_{\rm BH} = A/4\hbar G$ evolves as

$$\dot S_{\rm BH} = \frac{\dot M - \Omega_H\dot J}{T_H} = \frac{(\Omega_F - \Omega_H)\dot J}{T_H} = \frac{\dot\Phi Q(\Omega_H - \Omega_F)}{2\pi T_H}, \quad (18)$$

where the overdot signifies derivative with respect to the Killing time t, and $T_H = \hbar\kappa/2\pi$ is the Hawking temperature. [The first equality invokes the first law of black hole mechanics, while the second and third make use of (6) with the notational change indicated in the paragraph containing (11).] The rate of entropy generation due to Ohmic dissipation in the holographic dual field theory process is given by $u \cdot j \cdot F/\gamma T_H$ per unit spacetime volume, hence

$$\dot{S}_{\rm FT} = \frac{(u \cdot j \cdot F) 2\pi\ell}{\gamma T_H}, \tag{19}$$

which agrees with (18) when the previously given expressions for u, j and F are inserted. It is amusing to note that this dual description is a perfect electrical analogy for the mechanical analogy originally described by Blandford and Znajek [1]. That analogy involved a thermally conducting disk spinning with angular velocity Ω_H, coupled by friction to a concentric, thermally insulating ring spinning with a smaller angular velocity.

We have thus far treated the electromagnetic field as a test field on the background of a spinning BTZ black hole background, and inferred adiabatic evolution from the conservation laws. To more fully probe the duality one should solve the coupled Einstein-Maxwell equations. A similar study was carried out in [22] for the case of of electromagnetic energy flowing into a nonrotating black hole, allowing the nonlinear conductivity of the dual field theory to be found. In our case, a first step would be to use for the background the charged, spinning black hole solution [17, 29], which also has a magnetic field with $\Omega_F = \Omega_H$ in our notation. This is the solution to which the black hole would relax, and the azimuthal electric field alone could be treated as a test field. The next step would be to attempt to find an Einstein-Maxwell solution including the azimuthal electric field, which could describe nonlinear effects in the conductivity. Another avenue to investigate would be alternate boundary conditions at infinity, which can modify the nature of the dual field theory degrees of freedom and the external source that couples to them [19, 21, 25].

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