Cosmological perturbations from full quantum gravity

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The early universe provides an opportunity for quantum gravity to connect to observation by explaining the large-scale structure of the Universe. In the group field theory (GFT) approach, a macroscopic universe is described as a GFT condensate; this idea has already been shown to reproduce a semiclassical large universe under generic conditions, and to replace the cosmological singularity by a quantum bounce. Here we extend the GFT formalism by introducing additional scalar degrees of freedom that can be used as a physical reference frame for space and time. This allows, for the first time, the extraction of correlation functions of inhomogeneities in GFT condensates: in a way conceptually similar to inflation, but within a quantum field theory of both geometry and matter, quantum fluctuations of a homogeneous background geometry become the seeds of cosmological inhomogeneities. We compute the power spectrum of scalar cosmological perturbations and find that it is naturally approximately scale invariant, with a naturally small amplitude. This confirms the potential of GFT condensate cosmology to provide a purely quantum gravitational foundation for the understanding of the early universe.

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INTRODUCTION

Cosmology provides the most promising avenue for connecting quantum gravity to observable physics; this has motivated much work in particular on models replacing the big bang with a bounce [1]. Since our Universe is very well described on large scales by a simple Friedmann-Lemaître-Robertson-Walker (FLRW) metric with small (linear) perturbations, one then looks for a manageable approximation or truncation of quantum gravity to nearly homogeneous and isotropic universes.

In the last years, a new promising approach has emerged. In the group field theory (GFT) formalism for quantum gravity [2] (itself a second quantised formalism for loop quantum gravity (LQG) [3] and an enrichment of random tensor models [4] by group theoretic data), in which space and time are fundamentally made up of discrete "atoms of geometry," one can describe a macroscopic, homogeneous universe as a *condensate*, a highly coherent configuration of many such atoms. Condensates realise a natural quantum notion of homogeneity – the condensation of many quanta into a single microscopic quantum state – and the idea that spacetime could be a type of Bose-Einstein condensate had been considered earlier [5]. In GFT, such condensates do describe spatially homogeneous universes [6]. By coupling to a massless scalar (clock) field, it was shown such universes satisfy the Friedmann dynamics of classical general relativity (GR) in a semi-classical regime [7]; the semi-classical regime is reached for generic initial conditions [8]. In addition, at high curvatures such condensates undergo a bounce similar to the one seen in loop quantum cosmology. For some GFT models, this bounce can be followed by a long lasting phase of acceleration, without the need to introduce an inflaton [9, 10].

An open question in the study of GFT condensates (as in other approaches deriving cosmology from full quantum gravity) has been to extend these results from exactly homogeneous to inhomogeneous universes, *i.e.* to physically realistic and testable situations. In previous studies [11, 12], a major obstacle was the localisation of perturbations in a fully background-independent context, without the structures of GR (manifold, coordinates, etc.). Ideas from conventional quantum cosmology such as a Born-Oppenheimer approximation for inhomogeneities [13] are not directly applicable to GFT where no separation of perturbation modes, *e.g.* as eigenmodes of a Laplacian, is readily available.

Our starting point is to realise that this is the generalisation of the problem of "localising events in time" that was solved in GFT condensate cosmology (as in quantum cosmology) by introducing a scalar field, used as a "clock" to label evolution of the geometry. We then solve the general problem of "localising events in spacetime" by coupling not one but four scalar fields (in four spacetime dimensions) to gravity, using these four scalars as relational clocks and rods, *i.e.* as a physical coordinate system. This idea has a long history in classical and quantum gravity, the most famous example perhaps being Brown-Kuchař dust [14]. In such models, one can solve the constraints of canonical GR and define observables on a physical phase space; models of this type have had numerous applications in LQG [15].

We define a class of GFT models for gravity coupled to four scalar fields ϕ^I, $I = 0, \dots, 3$, making a few assumptions that any such dynamics should satisfy, generalising Refs. [7]. This allows to define observables that correspond to a local volume element at each point in spacetime, and hence capture (scalar) inhomogeneities.

Working in the mean-field approximation to the full quantum GFT dynamics, the effective dynamics for geometric observables can be easily extracted by the same methods as in Ref. [7]. We then assume that the background continuum geometry is homogeneous, which corresponds to a condensate state independent of the "rod" degrees of freedom, to reproduce the usual setup for perturbations in early universe cosmology.

Next we compute quantum fluctuations of local volume observables in such a background quantum state, staying within the full quantum gravity framework, but in a hydrodynamic approximation. We propose their two-point function as the relevant quantity in order to compare to standard cosmology and observation, and show that this is non-vanishing for a homogeneous condensate, very similar to how inhomogeneities arise from quantum vacuum fluctuations in inflation. Remarkably, we find naturally a scale-invariant power spectrum. The results outline a concrete, workable formalism for deriving a power spectrum of cosmological perturbations directly from a proposed theory of quantum gravity, and bring quantum gravity substantially closer to observational tests.

RELATIONAL CLOCKS AND RODS

We introduce physical reference frames and define relational dynamics first in classical GR, to later implement these ideas in the quantum GFT formalism.

First, consider a single massless, free scalar field, used as a relational clock in a flat FLRW metric, with action

$$S_\phi = -\frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = V_0\int \mathrm{d}t\,\frac{a^3}{2N}\dot{\phi}^2 \quad (1)$$

where $V(t) = a^3(t)V_0$ is the 3-volume of space given in terms of a fiducial volume V_0. The conjugate momentum $\pi_\phi := V\dot{\phi}/N$ is a conserved quantity; for any choice of time variable t, then, $\dot{\phi}(t)$ is strictly monotonic (unless $\pi_\phi = 0$, which has to be excluded). Hence ϕ is a good clock, and the dynamics of the Universe can be expressed in terms of ϕ; the Friedmann equation becomes

$$\left(\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}\phi}\right)^2 = 9\left(\frac{\dot{a}}{aN}\frac{V}{\pi_\phi}\right)^2 = 12\pi G\,. \quad (2)$$

The solutions to the Friedmann equations are

$$V(\phi) = \alpha\exp(\pm\sqrt{12\pi G}\phi)\,, \quad (3)$$

where the sign depends on the choice of time orientation. This final expression for $V(\phi)$ would be valid as $\pi_\phi \to 0$. However, this is misleading; this limit simply corresponds to Minkowski spacetime.

The scalar action (1) has additional symmetries:

- translation symmetry $\phi(t) \mapsto \phi(t) + \phi_0$,
- and time reversal $\phi(t) \mapsto -\phi(t)$.

This construction can be straightforwardly generalised: given four scalars ϕ^I, one can identify the points $\{p\}$ of an open (connected) region by the values $\phi^I(p)$ if the gradients of ϕ^I are everywhere non-degenerate, $\det(\partial_\alpha\phi^I) \neq 0$.

Similarly to a clock scalar field, one has to impose symmetries on the dynamics of these four scalars to be used as a material reference frame. For instance, consider the class of models in Ref. [15], with action

$$S_{\mathrm{m}} = -\frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,(g^{\mu\nu}\,[\rho\,\partial_\mu T\,\partial_\nu T + A(\rho)V_\mu\,V_\nu + 2B(\rho)\partial_\mu T\,V_\nu] + \Lambda(\rho))\,, \quad V_\mu := W_k\partial_\mu Z^k \quad (4)$$

defined in terms of eight scalars (T, Z^k, ρ, W_k). The dynamical fields T and Z^k give a local reference frame for space and time. Depending on $A(\rho)$, $B(\rho)$ and $\Lambda(\rho)$, Eq. (4) can reduce to Brown–Kuchař dust or null, non-rotational or Gaussian dust. Eq. (4) is invariant under constant shifts in T and Z^j, sign reversal of all four fields,

$$T(x) \mapsto T(x) + T_0\,, \quad Z^j(x) \mapsto Z^j(x) + Z^j_0\,, \quad (5)$$

$$(T(x) \mapsto -T(x)\,,\; Z^j(x) \mapsto -Z^j(x))\,, \quad (6)$$

as well as O(3) transformations

$$Z^k(x) \mapsto O^k{}_j\,Z^j(x)\,, \quad W_k(x) \mapsto (O^{-1})_k{}^j W_j(x)\,. \quad (7)$$

Eq. (7) implements isotropy of space: rotating the "rods" will define another, equivalent set of "rods." We will assume all these transformations are symmetries of our reference scalar matter; they would also be symmetries of the coordinates of a good reference frame.

GROUP FIELD THEORY WITH REFERENCE SCALAR MATTER

We now define GFT models for gravity coupled to four reference scalar fields, a straightforward generalisation of GFT models for gravity and a single (clock) field [7, 16]. The basic ingredient of any GFT is a quantum field on an abstract group manifold [2], whose excitations are interpreted as quanta of geometry, labelled by data in the domain space of the GFT field. We picture these as tetrahedra, equipped with a discrete SU(2) connection given by parallel transports across the four faces and with labels corresponding to the scalar field degrees of freedom. These are the variables associated to an LQG spin network vertex with four open links for gravity coupled to four scalar fields [3]. We work with a complex field on $\mathrm{SU}(2)^4 \times \mathbb{R}^4$, invariant under right multiplication

$$\varphi(g_I, \phi^J) = \varphi(g_I h, \phi^J) \quad \forall\, h \in \mathrm{SU}(2)\,. \tag{8}$$

One can then define the quantum GFT in the path integral or operator formalism; the latter is well suited for the study of GFT condensates [3, 6]. Here one postulates canonical commutation relations

$$\left[\hat\varphi(g_I, \phi^J), \hat\varphi^\dagger(g'_I, \phi'^J)\right] = \int \mathrm{d}h\, \delta^4(g'_I h g_I^{-1})\delta^4(\phi^J - \phi'^J) \tag{9}$$

while two $\hat\varphi$ or two $\hat\varphi^\dagger$ operators commute.

The Fock space is defined starting from a "no-space" vacuum $|\emptyset\rangle$, annihilated by $\hat\varphi$; $\hat\varphi^\dagger$ creates bosonic excitations over this vacuum, interpreted as geometric tetrahedra or LQG spin network vertices. Eq. (8) is then gauge invariance at the vertex where the four open links meet, as in lattice gauge theory. A state describing a macroscopic, approximate continuum geometry contains a very large number (potentially infinite) of excitations.

The GFT dynamics is governed by an action

$$S[\varphi,\bar\varphi] = -\int \mathrm{d}g\, \mathrm{d}\phi\; \bar\varphi(g_I, \phi^J)\mathcal{K}\varphi(g_I, \phi^J) + \mathcal{V}[\varphi,\bar\varphi] \tag{10}$$

where the kernel $\mathcal{K}$ is assumed to be local and contain derivatives with respect to g_I and ϕ^J. $\mathcal{K}$ and the interaction $\mathcal{V}$ can then be chosen such that the GFT Feynman amplitudes correspond to spin foam amplitudes, *i.e.* discrete path integrals for gravity coupled to matter fields discretised on the (simplicial) lattice dual to a Feynman diagram, with discrete boundary data [2, 3]. $\mathcal{V}$ then involves a convolution of five fields corresponding to gluing five tetrahedra to form a 4-simplex; other interactions are possible, and suggested by work on random tensor models and GFT renormalisation.

We are interested in models that include scalar fields used as reference matter. Following the above discussion, we assume that the GFT dynamics is invariant under

- arbitrary (constant) shifts in ϕ^I,
- the parity/time-reversal transformation $\phi^I \mapsto -\phi^I$,
- rotations $\phi^i \mapsto O^i{}_j\phi^j$ where $i, j = 1, 2, 3$.

The first of these forbids explicit dependence on ϕ^I.

We then work in the approximation developed in Refs. [7, 16], considering a standard effective field theory/hydrodynamic expansion of $\mathcal{K}$ in derivatives with respect to the ϕ^J variables [16],

$$\mathcal{K} = \mathcal{K}^0 + \mathcal{K}^{IJ}\frac{\partial}{\partial\phi^I}\frac{\partial}{\partial\phi^J} + \ldots \tag{11}$$

This leads to an effective low-energy GFT dynamics that can be compared with cosmology on large scales, where one can truncate $\mathcal{K}$ to second derivatives. First derivatives would violate parity/time-reversal; there is no dependence on ϕ^J. Rotational symmetry implies the form

$$\mathcal{K} = \mathcal{K}^0 + \mathcal{K}^1\Delta_{\phi^i} + \tilde{\mathcal{K}}^1\frac{\partial^2}{\partial(\phi^0)^2} + \ldots\,,\;\; \Delta_{\phi^i} = \sum_{i=1}^{3}\frac{\partial^2}{\partial(\phi^i)^2}\,. \tag{12}$$

We assume that the potential $\mathcal{V}$ satisfies the same symmetries as $\mathcal{K}$, but make no assumptions about its precise form. We will employ a weak-coupling approximation in which the effect of $\mathcal{V}$ on the dynamics is negligible.

EFFECTIVE COSMOLOGICAL DYNAMICS

The main proposal of GFT condensate cosmology [6] is that a macroscopic, nearly homogeneous universe is well approximated by a GFT condensate state, in which the field acquires a nonvanishing expectation value, $\sigma(g_I, \phi^J) := \langle\hat\varphi(g_I, \phi^J)\rangle \neq 0$. In the *mean-field approximation*, this condition is implemented by choosing the coherent state

$$|\sigma\rangle \equiv N(\sigma)\exp\left(\int \mathrm{d}g\,\mathrm{d}\phi\; \sigma(g_I, \phi^J)\hat\varphi^\dagger(g_I, \phi^J)\right)|\emptyset\rangle \tag{13}$$

and all dynamical information is determined by the mean field $\sigma(g_I, \phi^J)$. This corresponds to the Gross-Pitaevskii approximation for weakly interacting Bose-Einstein condensates [17]. One then considers the expectation value

$$\langle\sigma|\frac{\delta S[\varphi,\bar\varphi]}{\bar\varphi(g_I,\phi^J)}|\sigma\rangle = \frac{\delta S[\sigma,\bar\sigma]}{\bar\sigma(g_I,\phi^J)} = 0 \tag{14}$$

which is the GFT analogue of the Gross-Pitaevskii equation for a Bose-Einstein condensate and, using Eqs. (10) and (12), takes the form

$$\left(\mathcal{K}^0 + \mathcal{K}^1\Delta_{\phi^i} + \tilde{\mathcal{K}}^1\partial^2_{\phi^0} + \ldots\right)\sigma(g_I, \phi^J) - \frac{\delta\mathcal{V}[\sigma,\bar\sigma]}{\bar\sigma(g_I,\phi^J)} = 0\,. \tag{15}$$

We neglect higher than second derivatives, and use an approximation in which the contribution of $\mathcal{V}$ is neglected. The latter is compatible with the simple state (13), *i.e.* weak correlations between GFT quanta, and, tentatively, with spatial gradients of the effective geometry being very small. Including interaction terms is possible [9], but this approximation already allows for an interesting cosmological dynamics, as we now show.

With our approximations, the GFT equation of motion for the mean field σ becomes

$$\left(\mathcal{K}^0 + \mathcal{K}^1\Delta_{\phi^i} + \tilde{\mathcal{K}}^1\partial^2_{\phi^0}\right)\sigma(g_I, \phi^J) = 0\,. \tag{16}$$

We further restrict the mean field σ to isotropic excitations (equilateral tetrahedra) [7] (again this can be relaxed [18]), with

$$\sigma(g_I, \phi^J) = \sum_{j=0}^{\infty} \sigma_j(\phi^J)\, \mathbf{D}^j(g_I)\,; \tag{17}$$

the sum is over irreducible SU(2) representations j and the fixed $\mathbf{D}^j(g_I)$ is a convolution of Wigner D-matrices with SU(2) intertwiners that encodes the shape of the tetrahedra. σ now only depends on a single j, which determines volume information and thus the cosmological scale factor. The volume can be computed within full GFT as a second quantised operator, whose expectation value we compute in the state (13) below.

The isotropic mean field $\sigma_j(\phi^J)$ then satisfies

$$\left(-B_j + A_j \partial^2_{\phi^0} + C_j\, \Delta_{\phi^i}\right) \sigma_j(\phi^J) = 0\,; \tag{18}$$

we have rewritten $\mathcal{K}^0$, $\mathcal{K}^1$ and $\tilde{\mathcal{K}}^1$ as j-dependent couplings with no further derivatives, and used orthogonality of the Wigner D-matrices.

The case of Refs. [7] arises as the limit of our model in which the mean field takes the form

$$\sigma_j(\phi^J) \equiv \sigma^0_j(\phi^0)\,, \tag{19}$$

with a relational 3-volume operator at "time" ϕ^0

$$\hat{V}(\phi^0) = \int \mathrm{d}g\, \mathrm{d}g'\; \hat{\varphi}^\dagger(g_I, \phi^0) V(g_I, g'_I) \hat{\varphi}(g'_I, \phi^0)\,. \tag{20}$$

$V(g_I, g'_I)$ are matrix elements of the LQG volume operator between single-vertex spin network states.

Given a GFT state, $\langle \hat{V}(\phi^0)\rangle$ gives its total 3-volume at relational time ϕ^0. This appears in the Friedmann equation (2), which connects GFT condensates to cosmology.

Solutions to this homogeneous case, for generic initial conditions, lead to a semiclassical regime in which the Universe expands to macroscopic size [7, 8]; in this regime the 3-volume follows the classical Friedmann solution (3). At small volumes, the Universe undergoes a bounce and the classical singularity is avoided [7].

In the simplest example in which only a single spin j_0 is excited, the 3-volume behaves as

$$\langle \hat{V}(\phi^0)\rangle \overset{\phi^0 \to \pm\infty}{\sim} |\sigma^\pm|^2 \exp\left(\pm 2\sqrt{\frac{B_{j_0}}{A_{j_0}}}\phi^0\right) \tag{21}$$

for generic initial conditions ($\sigma^\pm \neq 0$), if $B_{j_0}/A_{j_0} > 0$; this is precisely the classical GR result (3) if one identifies $B_{j_0}/A_{j_0} =: 3\pi G$. $V(\phi)$ interpolates between the classical contracting and expanding solutions, and only ever vanishes for special initial conditions [7, 8, 10]. Including interactions can affect this cosmological evolution in several ways, prolonging the phase of accelerated expansion after the bounce and causing a later recollapse, producing a cyclic cosmology [9].

COSMOLOGICAL PERTURBATIONS IN GFT CONDENSATES

Our GFT model has enough degrees of freedom to describe inhomogeneous quantum geometries and their dynamics. Here we consider situations relevant for fundamental cosmology: we study quantum fluctuations of the local 3-volume around a nearly homogeneous background, seeking a quantum gravitational mechanism for explaining the origin of inhomogeneities (cosmic structure), in a similar spirit to the inflationary paradigm, where this mechanism is the imprint of quantum fluctuations in the homogeneous vacuum of the inflaton [19]. We show how such mechanism, natural in any quantum field theory for gravity and matter, is realised by GFT condensates, without the need to introduce an inflaton.

We start with the generalisation of Eq. (20) for a GFT for gravity coupled to four reference scalar fields ϕ^I,

$$\hat{V}(\phi^J) = \int \mathrm{d}g\, \mathrm{d}g'\; \hat{\varphi}^\dagger(g_I, \phi^J) V(g_I, g'_I) \hat{\varphi}(g'_I, \phi^J)\,. \tag{22}$$

Here all four ϕ^J take fixed values: $\hat{V}(\phi^J)$ defines a local volume element at the spacetime point specified by values of the reference fields. The total 3-volume at the clock value ϕ^0 is obtained by integrating over the "rods" ϕ^i,

$$\hat{V}(\phi^0) \equiv \int \mathrm{d}\phi^i\; \hat{V}(\phi^0, \phi^i)\,. \tag{23}$$

In a simple coherent state of the form (13), the expectation value of $\hat{V}(\phi^J)$ can be evaluated immediately,

$$\langle \hat{V}(\phi^J)\rangle = \int \mathrm{d}g\, \mathrm{d}g'\; \bar{\sigma}(g_I, \phi^J) V(g_I, g'_I) \sigma(g'_I, \phi^J)\,. \tag{24}$$

For a homogeneous mean field that only depends on ϕ^0, the integral over ϕ^i in Eq. (23) must be regularised; for the isotropic wavefunction (19), we obtain

$$\langle \hat{V}(\phi^J)\rangle = \sum_{j=0}^{\infty} V_j |\sigma^0_j(\phi^0)|^2\,, \tag{25}$$

with eigenvalues $V_j \sim V_{\mathrm{Pl}}\, j^{3/2}$ of the volume operator. The local and total 3-volume coincide up to regularisation, as expected in a homogeneous geometry.

In cosmology the key observables encoding the pattern of cosmic structure are correlation functions for geometric observables. Here we focus on local volume fluctuations $\langle \hat{V}(\phi^J)\hat{V}(\Phi^J)\rangle$, computed in a mean field state (13), which depend crucially on the one-body matrix elements $V^2(g_I, g'_I)$ of the *squared* volume operator. Using "squared matrix elements" to compute a spectrum of perturbations has been suggested before [12], but without "rods" only global information was obtained. Here, we can use the ϕ^I to extract statistical information about cosmological perturbations; Fourier transforming from ϕ^i

to their momenta k_i introduces a notion of wavenumber, defined with respect to the reference matter.

We can then obtain, within the full quantum gravity formalism, a power spectrum of cosmological perturbations. Instead of working with a generic inhomogeneous mean field, we consider a situation of interest for the study of cosmological perturbations and consider a mean field perturbed around exact homogeneity,

$$\sigma_j(\phi^J) = \sigma_j^0(\phi^0)(1 + \epsilon\, \psi_j(\phi^J))\,. \tag{26}$$

We then find for the quantum fluctuations of the volume

$$\begin{aligned} &\langle \hat{\tilde{V}}(\phi^0, k_i)\hat{\tilde{V}}(\Phi^0, K_i)\rangle - \langle \hat{\tilde{V}}(\phi^0, k_i)\rangle\langle \hat{\tilde{V}}(\Phi^0, K_i)\rangle \\ &= \delta(\phi^0 - \Phi^0) \sum_j V_j^2\, |\sigma_j^0(\phi^0)|^2 \big[(2\pi)^3\delta^3(k_i + K_i) \\ &\quad + \epsilon\, (\tilde{\psi}_j(\phi^0, k_i + K_i) + \overline{\tilde{\psi}_j(\phi^0, -k_i - K_i)}\big]\,, \end{aligned} \tag{27}$$

where we have Fourier transformed $\hat{V}$ and ψ_j; the delta function in ϕ^0 arises because $\hat{V}(\phi^J)$ is a density on scalar field space. This power spectrum is a genuine quantum correlation in the GFT condensate.

Let us illustrate the main features of this expression. Remarkably, the dominant part of the power spectrum

$$(2\pi)^3\delta^3(k_i + K_i)\delta(\phi^0 - \Phi^0) \sum_j V_j^2\, |\sigma_j^0(\phi^0)|^2 \tag{28}$$

is naturally scale-invariant: it only depends on ϕ^0. This property follows from computing cosmological perturbations on an exactly homogeneous background. Representing quantum fluctuations, even in this case the right-hand side of Eq. (27) is not zero: it must then be scale-invariant, with scale defined by the reference matter. Within our mean-field approximation, scale invariance and translational invariance, as expressed by the momentum delta function in Eq. (27), are necessarily connected.

Deviations from exact scale invariance are encoded in the last line of Eq. (27). They arise from inhomogeneous fluctuations around the exactly homogeneous GFT condensate, which should generically be present, although maybe small. Approximate scale invariance is intrinsically linked, in this framework, to such GFT fluctuations being small. Further deviations will come from relaxing the mean-field approximation, *i.e.* from using more refined quantum states. Importantly, such deviations from scale invariance depend both on the coupling of inhomogeneities with the homogeneous background and on their own dynamics, as expected on physical grounds and in agreement with the standard theory of cosmological perturbations. They are fully determined by the GFT perturbation density field, itself a solution to the perturbed mean field equations. A more detailed study of solutions of such perturbed equations, in particular their initial conditions, would be crucial to identify the precise form of these deviations. In particular, the influence of the bouncing dynamics for the background on the power spectrum should also be studied.

The amplitude of volume fluctuations relative to the homogeneous background, *i.e.* of $\widehat{\delta\tilde{V}}(\phi^0, k_i) \equiv \hat{\tilde{V}}(\phi^0, k_i)/\langle\hat{V}(\phi^0)\rangle$, is obtained by dividing Eq. (27) by the squared background volume $\langle\hat{V}(\phi^0)\rangle^2 \equiv (\int \mathrm{d}\phi^i \sum_j V_j |\sigma_j^0(\phi^0)|^2)^2$. This amplitude is of order $1/N$, for $N \gg 1$ quanta in the condensate. For instance, considering only the scale invariant contribution and with only a single spin j_0 excited, the power spectrum of such perturbations is

$$\mathcal{P}_{\delta V}(k) = \frac{V_{j_0}^2\, |\sigma_{j_0}^0(\phi^0)|^2}{(\int \mathrm{d}\phi^i\; V_{j_0} |\sigma_{j_0}^0(\phi^0)|^2)^2} = \frac{V_{j_0}}{(\int \mathrm{d}\phi^i) V(\phi^0)}\,, \tag{29}$$

with $V(\phi^0) = N(\phi^0)V_{j_0}$. A small amplitude of scalar perturbations, decreasing as the universe expands, arises naturally from the simplest GFT condensates.

For $C_j/B_j < 0$ in Eq. (18), inhomogeneous perturbations decay relative to the homogeneous background at large volumes; one approaches scale invariance even more closely, further suppressing the deviations coming from the inhomogeneous term. If GFT interactions produce a long-lasting accelerated expansion after the bounce regime [9], this leads to an even stronger suppression of the deviations from scale invariance. This would be basically the inflationary mechanism without an inflaton, purely driven by quantum gravity dynamics.

The choice of vacuum, *e.g.* as made in inflation, is replaced by the GFT condensate state (13) that refers to both quantum geometric and matter degrees of freedom. This is because such fluctuations are computed directly within the complete quantum gravity formalism, which also defines the ultraviolet completion of the theory.

CONCLUSIONS

By introducing in the GFT formalism scalar field degrees of freedom that can be used as physical reference frames, we could extend the mean field approximation for GFT condensates beyond homogeneity. This approximation has already been shown to provide an effective cosmological dynamics in which not only a semiclassical large Friedmann universe is reproduced under generic conditions, but also the cosmological singularity is replaced by a quantum bounce, followed by an accelerated phase of expansion of pure quantum gravity origin that, depending on the GFT interactions, can be long lasting. We then considered the typical setup of early universe cosmology within this full quantum gravity framework: we computed the power spectrum of quantum fluctuations of the local volume, *i.e.* scalar cosmological perturbations, in a homogeneous background geometry perturbed by small inhomogeneities. We found that this is naturally approximately scale invariant, with a small amplitude that decreases as the volume of the universe grows. This confirms the potential of the GFT condensate cosmology framework to provide a solid, purely quantum gravitational foundation for the understanding of the early universe and for physical cosmology.

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[1] M. Novello and S. E. P. Bergliaffa, "Bouncing Cosmologies," *Phys. Rept.* **463** (2008) 127–213, arXiv:0802.1634; M. Bojowald, "Loop quantum cosmology," *Living Rev. Rel.* **11** (2008), 4; A. Ashtekar and P. Singh, "Loop quantum cosmology: a status report," *Class. Quant. Grav.* **28** (2011) 213001, arXiv:1108.0893; J.-L. Lehners, "Ekpyrotic and cyclic cosmology," *Phys. Rept.* **465** (2008), 223–263, arXiv:0806.1245; B. Craps, T. Hertog, and N. Turok, "Quantum resolution of cosmological singularities using AdS/CFT correspondence," *Phys. Rev. D* **86** (2012), 043513, arXiv:0712.4180.

[2] D. Oriti, "The microscopic dynamics of quantum space as a group field theory," arXiv:1110.5606; A. Baratin and D. Oriti, "Ten questions on Group Field Theory (and their tentative answers)," *J. Phys. Conf. Ser.* **360** (2012) 012002, arXiv:1112.3270; T. Krajewski, "Group field theories," *PoS QGQGS* **2011** (2011) 005, arXiv:1210.6257.

[3] D. Oriti, "Group field theory as the 2nd quantization of Loop Quantum Gravity," *Class. Quant. Grav.* **33** (2016), 085005, arXiv:1310.7786; D. Oriti, "Group Field Theory and Loop Quantum Gravity," arXiv:1408.7112.

[4] R. Gurau and J. P. Ryan, "Colored Tensor Models – a review," *SIGMA* **8** (2012) 020, arXiv:1109.4812; R. Gurau, *Random Tensors* (Oxford University Press, 2017).

[5] B. L. Hu, "Can spacetime be a condensate?," *Int. J. Theor. Phys.* **44** (2005), 1785–1806, gr-qc/0503067; D. Oriti, "Group field theory as the microscopic description of the quantum spacetime fluid: A New perspective on the continuum in quantum gravity," *PoS QG* **-PH** (2007), 030, arXiv:0710.3276; T. Koslowski and H. Sahlmann, "Loop quantum gravity vacuum with nondegenerate geometry," *SIGMA* **8** (2012), 026, arXiv:1109.4688.

[6] S. Gielen, D. Oriti and L. Sindoni, "Cosmology from Group Field Theory Formalism for Quantum Gravity," *Phys. Rev. Lett.* **111** (2013), 031301, arXiv:1303.3576; S. Gielen, D. Oriti and L. Sindoni, "Homogeneous cosmologies as group field theory condensates," *JHEP* **1406** (2014), 013, arXiv:1311.1238; S. Gielen and L. Sindoni, "Quantum Cosmology from Group Field Theory Condensates: a Review," *SIGMA* **12** (2016), 082, arXiv:1602.08104.

[7] D. Oriti, L. Sindoni and E. Wilson-Ewing, "Emergent Friedmann dynamics with a quantum bounce from quantum gravity condensates," *Class. Quant. Grav.* **33** (2016), 224001, arXiv:1602.05881; D. Oriti, L. Sindoni and E. Wilson-Ewing, "Bouncing cosmologies from quantum gravity condensates," *Class. Quant. Grav.* **34** (2017), 04LT01, arXiv:1602.08271.

[8] S. Gielen, "Emergence of a low spin phase in group field theory condensates," *Class. Quant. Grav.* **33** (2016), 224002, arXiv:1604.06023.

[9] M. de Cesare, A. G. A. Pithis and M. Sakellariadou, "Cosmological implications of interacting Group Field Theory models: cyclic Universe and accelerated expansion," *Phys. Rev. D* **94** (2016), 064051, arXiv:1606.00352.

[10] M. de Cesare and M. Sakellariadou, "Accelerated expansion of the Universe without an inflaton and resolution of the initial singularity from Group Field Theory condensates," *Phys. Lett. B* **764** (2017) 49–53, arXiv:1603.01764.

[11] S. Gielen, "Perturbing a quantum gravity condensate," *Phys. Rev. D* **91** (2015), 043526, arXiv:1411.1077

[12] S. Gielen, "Identifying cosmological perturbations in group field theory condensates," *JHEP* **1508** (2015), 010, arXiv:1505.07479.

[13] J. J. Halliwell and S. W. Hawking, "The Origin of Structure in the Universe," *Phys. Rev. D* **31** (1985), 1777–1791; C. Kiefer and M. Krämer, "Quantum Gravitational Contributions to the CMB Anisotropy Spectrum," *Phys. Rev. Lett.* **108** (2012), 021301, arXiv:1103.4967.

[14] J. D. Brown and K. V. Kuchař, "Dust as a standard of space and time in canonical quantum gravity," *Phys. Rev. D* **51** (1995) 5600–5629, gr-qc/9409001.

[15] K. Giesel and T. Thiemann, "Scalar Material Reference Systems and Loop Quantum Gravity," *Class. Quant. Grav.* **32** (2015), 135015, arXiv:1206.3807.

[16] Y. Li, D. Oriti and M. Zhang, "Group field theory for quantum gravity minimally coupled to a scalar field," *Class. Quant. Grav.* in press (2017), arXiv:1701.08719.

[17] L. Pitaevskii and S. Stringari, *Bose-Einstein Condensation* (Oxford University Press, 2003).

[18] A. G. A. Pithis and M. Sakellariadou, "Relational evolution of effectively interacting group field theory quantum gravity condensates," *Phys. Rev. D* **95** (2017), 064004, arXiv:1612.02456.

[19] V. F. Mukhanov and G. V. Chibisov, "Quantum Fluctuations and a Nonsingular Universe," *JETP Lett.* **33** (1981) 532–535; see also astro-ph/0303077.