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Simultaneous Analysis of Space-Time Variability: Principal Oscillation Patterns and Principal Interaction Patterns with Applications to the Southern Oscillation

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With 3 Figures

Summary: The merits of the analysis techniques "POP" (Principal Oscillation Pattern) and "PIP" (Principal Interaction Pattern) are demonstrated by applying them to a combined data set of surface wind and sea surface temperature along the equator.

The POP analysis detects a dominant low-frequency oscillatory mode with a time scale of about 2 to 3 years which can be identified as the Southern Oscillation. With respect to SST, the mode behaves approximately as a standing pattern with maximum extremes in the Central and Eastern Pacific. In the zonal surface wind, however, an eastward propagating feature can be detected. The meridional wind component is less organized, except for the occurrence of southerly winds in the Eastern Pacific during the "peak phase" of El Niño.

The result of the PIP analysis is essentially the same as that of the POP analysis, except for a seasonal dependence of the system's stability: disturbances tend to be amplified during April to July and damped throughout the rest of the year.

Zusammenfassung: Die Möglichkeiten der Analyseverfahren „POP" (Principal Oscillation Pattern) und „PIP" (Principal Intercation Pattern) werden diskutiert anhand eines Beispiels. Der analysierte Datensatz enthält Monatsmittel des oberflächennahen Windes sowie der Ozeanoberflächentemperatur längs des Äquators im Indischen und Pazifischen Ozean.

Das POP-Verfahren filtert aus den Daten ein dominantes Schwankungsmuster mit einer charakteristischen Zeit von 2 bis 3 Jahren heraus. Es kann mit der Southern Oscillation identifiziert werden. In bezug auf die Ozeantemperatur erscheint das Signal als ein stehendes Muster mit Extrema im zentralen und östlichen Äquatorialpazifik. Im Zonalwind analysiert das POP-Verfahren ein nach Osten wanderndes Signal, das im Indischen Ozean zuerst erscheint und im östlichen Pazifik dissipiert. In der Meridionalkomponente des Windes erscheint kein klares Signal abgesehen vom Südwind im östlichen Pazifik während der "peak phase" eines El Nino Ereignisses.

Das PIP-Verfahren interpretiert die Daten in fast gleicher Weise wie das POP-Verfahren. Im PIP-Verfahren wird allerdings zusätzlich eine saisonale Modulation der Dämpfungseigenschaften des betrachteten Systems berücksichtigt. Demnach werden Störungen, die im April bis Juli auftreten, verstärkt. Andere Störungen werden gedämpft.

1. Introduction

"PIP" (Principal Interaction Pattern) and "POP" (Principal Oscillation Pattern) analyses are multivariate techniques to infer empirically the characteristics of the space-time variations of a complex system in a high-dimensional space. The basic ansatz is to specify a low-order system with a few free parameters which are fitted to the data. Then, the space-time characteristics of the low-order system are regarded as being the same as those of the full system. The POP analysis is in operational use at the Max-Planck Institut für Meteorologie and has been used to analyze the tropical 30–60 day wave in a multi-year GCM run (*Storch* et al., 1988), and to design a statistical forecast scheme to predict the Southern Oscillation (*Xu* and *Storch*, 1989).

The PIP ansatz (*Hasselmann*, 1988) is a fairly general approach which allows for a large variety of complex scenarios. It may be seen as a particular case of "state space models" (*Honerkamp* and *Weese*, 1989). POPs (*Storch* et al., 1988) may be understood in two conceptually different ways, namely as being normal modes of a linear system and those parameters are inferred from a vector time series, or as a somewhat trivial case of PIPs.

The purpose of the present paper is to describe both approaches (sec. 2 and 4) and to show results which were obtained by the simple POP (sec. 3), and the more sophisticated PIP analysis (sec. 5) when applied to the same combined anomaly data set, equatorial zonal and meridional components of the surface (10 m) wind and equatorial sea surface temperature (SST).

2. POPs as Normal Modes

2.1. Normal Modes

The normal modes of a linear discretized system

$$\boldsymbol{x}(t+1)=\boldsymbol{A}\cdot\boldsymbol{x}(t) \tag{2.1}$$

are the eigenvectors $\boldsymbol{p}$ of the matrix $\boldsymbol{A}$. In general, $\boldsymbol{A}$ is not symmetric and some or all of its eigenvalues λ and eigenvectors $\boldsymbol{p}$ are complex. However, since $\boldsymbol{A}$ is a real matrix, the conjugate complex quantities λ^* and $\boldsymbol{p}^*$ satisfy also the eigen-equation $\boldsymbol{A}\cdot\boldsymbol{p}^*=\lambda^*\boldsymbol{p}^*$. In most cases, all eigenvalues are different and the eigenvectors form a linear basis. So, each state $\boldsymbol{x}$ may be expressed in terms of the eigenvectors,

$$\boldsymbol{x}=\sum_j z_j\boldsymbol{p}_j \tag{2.2}$$

Since the basis of eigenvectors is complete, this expansion is unique, i.e.

$$0=\sum_j z_j\boldsymbol{p}_j \Leftrightarrow z_j=0 \quad \text{for all} \quad j\,. \tag{2.3}$$

The coefficients of the pairs of conjugate complex eigenvectors are conjugate complex, too. Inserting (2.2) in (2.1) and using (2.3) leads to a set of time evolution equations for the coefficients $z_j(t)$:

$$z_j(t+1)=\lambda_j z_j(t) \quad \text{for all} \quad j\,. \tag{2.4}$$

If the eigenvalue λ is complex, (2.4) may be written in terms of real quantities by introducing the notation $z=z^1+iz^2$, $\lambda=\lambda^1+i\lambda^2$, $\varrho=|\lambda|$ and $\omega=\tan^{-1}(\lambda^2/\lambda^1)$:

$$\begin{pmatrix} z^1(t+1)\\ z^2(t+1)\end{pmatrix}=\varrho\begin{pmatrix}\cos(\omega) & \sin(\omega)\\ -\sin(\omega) & \cos(\omega)\end{pmatrix}\begin{pmatrix} z^1(t)\\ z^2(t)\end{pmatrix}. \tag{2.5}$$

As sketched in Figure 1, the system (2.5) performs a damped rotation in the two-dimensional subspace span-

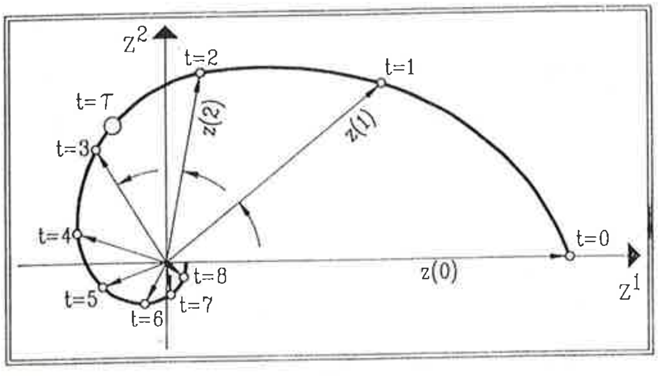


Fig. 1. Schematic diagram of the time evolution of POP coefficients

Abb. 1. Schematische Darstellung der zeitlichen Entwicklung von POP-Koeffizienten.

ned by the real and imaginary part of the eigenvector $\boldsymbol{p}=\boldsymbol{p}^1+i\boldsymbol{p}^2$. From the complex eigenvalue λ two characteristic numbers can be derived, the oscillation period $T=2\pi/\tan^{-1}(\lambda^2/\lambda^1)$ and the e-folding time $\tau=\ln(|\lambda|)^{-1}$.

In the original $\boldsymbol{x}$-space the trajectory given by the complex normal mode $\boldsymbol{p}$ is given by

$$z^1(t)\cdot\boldsymbol{p}^1+z^2(t)\cdot\boldsymbol{p}^2\,, \tag{2.6}$$

which together with (2.5) leads to this interpretation: if at a certain time t_0 the system is in state $\boldsymbol{p}^1$, it will be at $t_0+T/4$ in state $-\boldsymbol{p}^2$, at time $t_0+T/2$ in state $-\boldsymbol{p}^1$, at $t_0+3T/4$ in $\boldsymbol{p}^2$, and eventually at time $t=t_0+T$ back in state $\boldsymbol{p}^1$. Or, the system is generating the cyclic sequence of patterns

$$\cdots\rightarrow\boldsymbol{p}^1\rightarrow-\boldsymbol{p}^2\rightarrow-\boldsymbol{p}^1\rightarrow\boldsymbol{p}^2\rightarrow\boldsymbol{p}^1\rightarrow\cdots \tag{2.7}$$

2.2. Estimation of Normal Modes: Principal Oscillation Patterns

All information used so far was the existence of a linear equation (2.1) with some matrix $\boldsymbol{A}$. No assumption was made where this matrix originates from. In dynamical theory, the origin of (2.1) are linearized and discretized differential equations. In case of the POP analysis the relationship

$$\boldsymbol{x}(t+1)=\boldsymbol{A}\cdot\boldsymbol{x}(t)+noise \tag{2.8}$$

is hypothesized. Multiplication of (2.8) from the right-hand side by the transposed $\boldsymbol{x}(t)$ and taking expectations, $\langle\cdot\rangle$, leads to

$$\boldsymbol{A}=\langle\boldsymbol{x}(t+1)\cdot\boldsymbol{x}^T(t)\rangle\cdot\langle\boldsymbol{x}(t)\cdot\boldsymbol{x}^T(t)\rangle^{-1}. \tag{2.9}$$

The eigenvectors of (2.9), or, the normal modes of (2.8) are called *Principal Oscillation Patterns*. Their time evolution is given by (2.5–6), superimposed by noise.

To estimate $\boldsymbol{A}$ the lag covariance matrix $\langle\boldsymbol{x}(t+1)\times\times\boldsymbol{x}^T(t)\rangle$ and the covariance matrix $\langle\boldsymbol{x}(t)\cdot\boldsymbol{x}^T(t)\rangle$ are estimated in the usual way.

Criteria to decide whether a POP contains useful information or if it should be regarded as reflecting merely sample properties are given by *Storch* et al. (1988). The most important rule-of-thumb is related to the cross spectrum of the POP coefficients z^1 and z^2: at the POP period T, or at least in the neighbourhood of T, the two time series should be significantly coherent and 90° out-of phase, according to the interpretation given in (2.7).

3. Example of POP Analysis

3.1. Data

A POP analysis was carried out for monthly mean equatorial sea surface temperature (SST) and the zonal and meridional components of surface (10 m) wind (u and v). The data sets were collected from ship data for the period 1951–1986 and have been averaged on a 10°×10° latitude/longitude grid centered along the equator extending over the Indic and Pacific oceans, i.e., from 45° E to 85° W.

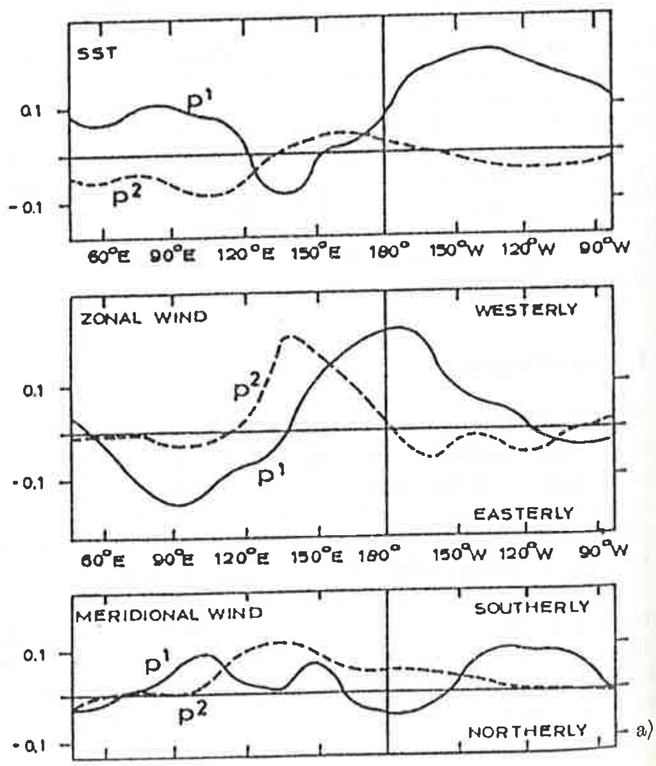
The purpose of the analysis was to identify patterns characteristic of the El Niño/Southern Oscillation phenomenon. To focus on that process the seasonal cycle is removed and the data are time-filtered prior to the analysis: all variations on time scales shorter than 12 months and longer than 80 months were suppressed. To provide a simultaneous analysis of SST, u and v, the three variables were scaled to have equal spatially averaged variance.

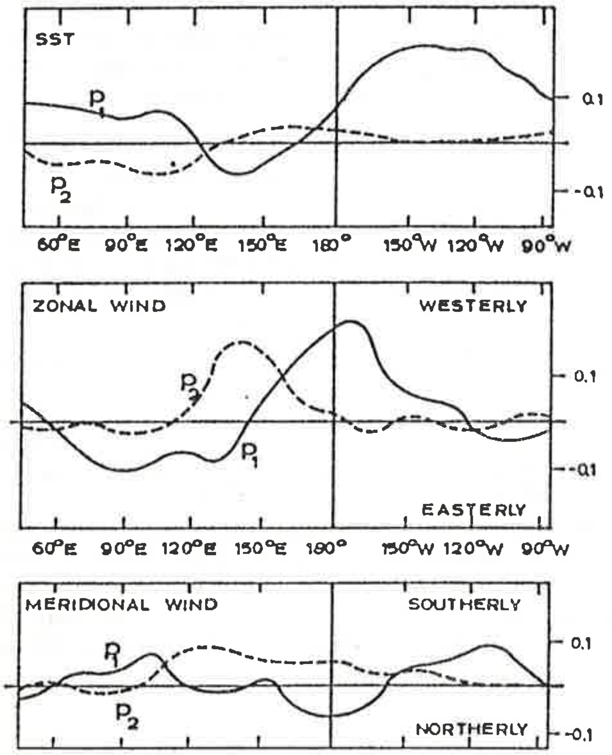
To reduce the noise the POP analysis was made in the subspace spanned by the first eight EOFs which explain 72 % of the variance.

3.2. Results

One complex pattern with the characteristic times: oscillation period $T=32$ months, e-folding time $\tau=64$ months could be identified by POP analysis. The damping time should be considered with care, because it is certainly contaminated by the time filtering. The cross-correlation function of the POP coefficients z^1 and z^2 has its maximum value, 0.72, at a lag of 6 months which indicates that a quarter of the oscillation period is 6 months. This number is consistent with $T/4=8$ months derived from the POP analysis.

The patterns $\boldsymbol{p}^1$ (solid line) and $\boldsymbol{p}^2$ (dashed line) are





b)

Fig. 2. Simultaneous analysis of SST anomalies, and the zonal and meridional component of equatorial surface wind anomalies along the equator (45° E to 85° W). Upper diagram: SST, middle diagram: zonal wind; bottom diagram: meridional wind. Dimensionless units.

a) POPs. Characteristic times: oscillation period $T=32$ months, e-folding time $\tau=64$ months. The characteristic POP cycle is: $p^1 \to -p^2 \to -p^1 \to p^2$.

b) PIPs according to (5.1.2). Characteristic times: oscillation period $T=40$ months, $\varkappa_0=-0.02$, $\varkappa_a=0.04$, $t_p=5.3$ months. The characteristic PIP cycle is: $p_1 \to -p_2 \to -p_1 \to p_2$.

Abb. 2. Ergebnisse einer simultanen Analyse von Anomalien dreier mit der „Southern Oscillation“ verknüpfter physikalischer Parameter längs des Äquators (45° E–85° W): Ozeanoberflächentemperatur („SST“) sowie Zonal- und Meridionalkomponente des oberflächennahen Windes („u-“ und „v-Wind“). Die drei Parameter sind normiert, so daß die räumlich gemittelten Varianzen von SST, u- und v-Wind identisch sind. Oben: SST; Mitte: u-Wind; unten: v-Wind.

a) Principal Oscillation Patterns – die charakteristischen Zeiten sind: Rotationszeit $T=32$ Monate, Dämpfungszeit: $\tau=64$ Monate. Der charakteristische POP-Zyklus ist: $p^1 \to -p^2 \to -p^1 \to p^2$

b) Principal Interaction Patterns nach (5.1.2) – die charakteristischen Zahlen sind: $T=40$ Monate, $\varkappa_0=-0{,}02$, $\varkappa_a=0{,}04$, $t_p=5{,}3$ Monate. Der charakteristische PIP-Zyklus ist: $p_1 \to -p_2 \to -p_1 \to p_2$

shown in Fig. 2a. In $\boldsymbol{p}^1$ there are positive SST anomalies over the Central and Eastern Equatorial Pacific and the Indian Ocean, and negative SST anomalies over Indonesia. These anomalous SSTs are associated with westerly (easterly) wind anomalies in the Central Pacific (Indian Ocean). The maximum of the warming lies to the east of the maximum westerly wind. The meridional wind component shows weak northerly (southerly) anomalies in the Central (Eastern and Western) Pacific. The pattern $\boldsymbol{p}^2$ describes south-westerly wind anomalies in the Australian sector, but hardly any SST anomalies except for some weak negative ones in the Indian Ocean.

The interpretation (2.7) applies to each observed variable (SST and both components of surface wind). If at some time the pattern $\boldsymbol{p}^1$ is observed–with westerlies at the date-line, southerlies in the eastern equatorial Pacific and above normal SST in most of the Pacific-, it is likely that it will be replaced by $-\boldsymbol{p}^2$ after $T/4=8$ months with almost no SST anomalies, but north-easterlies north of Australia. After another 8 months, $-\boldsymbol{p}^1$ describes the state of atmosphere and ocean: the easterly wind anomalies are shifted eastward to the date-line and the ocean is cooler than normal. 8 months later, pattern $\boldsymbol{p}^2$ emerges which eventually will be replaced by $\boldsymbol{p}^1$ again.

The anomalies described by this cycle suggest that the zonal wind signal propagates eastward, whereas the SST anomalies are a standing oscillation. The characteristics of $\boldsymbol{p}^1$ are typical for the "peak phase" of ENSO: see, e.g., the August–October composite of *Rasmusson* and *Carpenter* (1982; see Fig. 20). Pattern $\boldsymbol{p}^2$ describes an intermittent stage of the ENSO cycle: it can be compared with the "onset phase" anomaly composed by *Rasmusson* and *Carpenter* (1982; see Fig. 18).

That the patterns $\boldsymbol{p}^1$ and $\boldsymbol{p}^2$ describe the ENSO signal can be supported by a cross-correlation analysis of the POP coefficients and the SST Southern Oscillation Index, SOI (Wright, 1984). The maxima of the cross-correlation functions are $\mathrm{Corr}_{\tau=0}(\mathrm{SOI}, z^1)=0.64$ and $\mathrm{Corr}_{\tau=6}(\mathrm{SOI}, z^2)=0.53$.

4. PIPs

4.1. State Space Models

Many complex dynamical systems, $\boldsymbol{x} \in \mathbb{R}^n$, may conveniently be approximated as being driven by a simpler dynamical system, $z \in \mathbb{R}^m$, with a reduced number of degrees of freedom, $m \leqq n$. Mathematically, this may be described by a *state space model* which consists of a system equation

$$\boldsymbol{z}\,(t+1)=\mathcal{F}[\boldsymbol{z}(t), \alpha, t]+noise\,, \tag{4.1}$$

for the dynamical variables $\boldsymbol{z}=(z_1, \ldots, z_m)$ and an observation equation

$$\boldsymbol{x}(t)=\boldsymbol{P}\boldsymbol{z}(t)+noise=\sum_j z_j(t)\,\boldsymbol{p}_j+noise \tag{4.2}$$

for the observed variables $\boldsymbol{x}$. $\mathcal{F}[\boldsymbol{z}(t), \alpha, t]$ belongs to a class of models which can be non-linear in the dynamical variables $\boldsymbol{z}$ and which depends additionally on a set of free parameters $\alpha=(\alpha_1, \alpha_2, \ldots)$. Both equations are disturbed by an additive noise.

Clearly, the columns of the matrix $\boldsymbol{P}=(\boldsymbol{p}_1, \ldots, \boldsymbol{p}_m)$ may be interpreted as patterns in the full $\boldsymbol{x}$-space. Since $m \leqq n$, the time coefficient $z_j(t)$ of a pattern $\boldsymbol{p}_j$ at a time t are not uniquely determined by the $\boldsymbol{x}(t)$. Instead, it may be obtained by a least-square fit, i.e.

$$\boldsymbol{z}(t)=(\boldsymbol{P}^T\boldsymbol{P})^{-1}\,\boldsymbol{P}^T\boldsymbol{x}(t)\,. \tag{4.3}$$

The intriguing aspect of state space models is that the dynamical behaviour of complex systems often appears to be dominated by the interaction of only a few characteristic patterns $\boldsymbol{p}_j$. That is, even if the dynamics of the full system are restricted to the subspace spanned by the columns of $\boldsymbol{P}$, its principal dynamical properties are represented.

4.2. Fitting State Space Modells: Principal Interaction Patterns

When fitting the state space model (4.1.2) to a time series, the following entities have to be specified: the class of models $\mathfrak{F}$, the patterns $\boldsymbol{P}$, the free parameters α and the dimension of the reduced system m. The class of models $\mathfrak{F}$ has to be selected a priori on the basis of physical reasoning. Also, the number m might be specified a priori. The parameters α and the patterns $\boldsymbol{P}$ are fitted simultaneously to a time series by requesting them to minimize

$$\varepsilon[\boldsymbol{P}; \alpha] = \langle \| \boldsymbol{x}(t+1) - \boldsymbol{x}(t) - \boldsymbol{P}\{\mathfrak{F}[\boldsymbol{z}(t), \alpha, t] - \boldsymbol{z}(t)\}\|^2 \rangle . \quad (4.4)$$

$\varepsilon[\boldsymbol{P}; \alpha]$ is the mean square error of the approximation of the (discretized) time derivative of the observations $\boldsymbol{x}$ by the state space model. The patterns $\boldsymbol{P}$, which minimize (4.4) are called *Principal Interaction Patterns*. If only a finite time series of observations $\boldsymbol{x}$ is available, the expectation $\langle\cdot\rangle$ is replaced by a summation over time.

In general, the solution of (4.4) is not unique. In particular, the set of patterns $\boldsymbol{P}' = \boldsymbol{P} \cdot \boldsymbol{L}$ with any non-singular matrix $\boldsymbol{L}$ will minimize (4.4), if $\boldsymbol{P}$ does, as long as the corresponding model $\mathfrak{F}' = \boldsymbol{L}^{-1}\mathfrak{F}$ belongs to the model class specified a priori. This problem may be solved by requesting the solution to fulfill some constraints, e.g. that the linear term in the Taylor expansion of $\mathfrak{F}$ is a diagonal matrix.

The minimization problem can be solved by minimizing iteratively with respect to the patterns $\boldsymbol{P}$ and the parameters α, where the $\boldsymbol{z}(t)$ (4.3) are given by the $\boldsymbol{P}$ of the last iterative step. Then the minimization with respect to $\boldsymbol{P}$ is reduced to a set of linear equations, and the minimization with respect to α can be solved by an quasi-Newton algorithm available in standard libraries, since the number of parameters α is small.

4.3. POPs as Particular Case of PIPs

The Principal Oscillation Patterns can be understood as a kind of simplified Principal Interaction Patterns. For that let us assume $m = n$. Then, the patterns $\boldsymbol{p}$ span the full $\boldsymbol{x}$-space, and their choice does not affect $\varepsilon[\boldsymbol{P}; \alpha]$. Also, let $\mathfrak{F}$ be a linear model $\mathfrak{F}[\boldsymbol{z}(t), \alpha] = \boldsymbol{A} \cdot \boldsymbol{z}(t)$, where the parameters α are the entries of $\boldsymbol{A}$. Equation (2.9) is obtained by minimization of $\varepsilon[\boldsymbol{P}; \alpha]$ (4.4), so that the dynamical equation (4.1) is identical to (2.8). The constraint mentioned above leads to the eigenvectors of $\boldsymbol{A}$ as being the PIPs of the particular, admittedly simplified, state space model.

5. Example of PIP Analysis

5.1. Model Design

In this section we present results of a PIP analysis of the equatorial sea surface temperature and the zonal and meridional component of equatorial surface wind. The same data set was examined by means of POP analysis in section 2. After having found one relevant oscillatory POP in section 3, we choose as dimension of the reduced system $m = 2$. As model class $\mathfrak{F}$ we take

$$\mathfrak{F}[\boldsymbol{z}(t), \alpha] = e^{\varkappa} \begin{pmatrix} \cos(\omega) & \sin(\omega) \\ -\sin(\omega) & \cos(\omega) \end{pmatrix} \cdot \boldsymbol{z}(t) \quad (5.1)$$

which describes a damped oscillation at a period of $T = 2\pi/\omega$. Since ENSO is known to be more persistent in summer and autumn than in winter and spring the damping factor $\varkappa$ is set to oscillate seasonally:

$$\varkappa(t) = \varkappa_0 + \varkappa_a \cdot \cos \omega_0 (t - t_p) \quad (5.2)$$

with $\omega_0 = 2\pi/(1 \text{ year})$. The set of free parameters is now $\alpha = (\omega, \varkappa_0, \varkappa_a, t_p)$.

With these settings, the deterministic solution of (4.1) is given by equation (2.5), but now with $\varrho = e^{\varkappa(t)}$. If at a certain time t_0 the strength of the signal is $\|z(t_0)\|$, it is at a later time t

$$\|\boldsymbol{z}(t)\| = e^{\varkappa(t-1)} \|\boldsymbol{z}(t-1)\| = e^{K(t,t_0)} \|\boldsymbol{z}(t_0)\| \quad (5.3)$$

with

$$K(t, t_0) = \sum_{\tau=t_0}^{t-1} \varkappa(\tau) = \varkappa_0 (t - t_0) + \varkappa_a \sum_{\tau=t_0}^{t-1} \cos \omega_0 (\tau - t_p) . \quad (5.4)$$

In order to keep the model stable, $K(t, t_0)$ has to be negative for large times $(t - t_0)$, i.e. $\varkappa_0 < 0$. The seasonal dependence of equation (5.2), however, allows the system to be unstable during some time of the year, namely if $\varkappa_a$ is sufficiently large compared with $|\varkappa_0|$, so that $\varkappa(t)$ is positive in part of the year.

5.2. Results

The PIPs $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ (Fig. 2b) show the same features as the patterns $\boldsymbol{p}^1$ and $\boldsymbol{p}^2$ of the POP analysis (Fig. 2a). The oscillation period $T = 40$ months and the long-term damping rate $\varkappa_0 = -0.02$ are almost the same, too (see sec. 3). Furthermore, the maximum cross correlation $\text{Corr}_{\tau=6}(z_1, z_2) = 0.66$ of the PIP coefficients is comparable to that one of the POP analyses. The PIP model, however, is somewhat more skillful in describing the ENSO signal: the maximum values of the cross-correlation functions of the PIP coefficients and the SST Southern Oscillation Index, SOI (*Wright*, 1984), $\text{Corr}_{\tau=0}(\text{SOI}, z_1) = 0.96$, $\text{Corr}_{\tau=6}(\text{SOI}, z_2) = 0.60$ are higher than those of the POP analysis. Whether this reflects the different number of dynamical parameters or a true improvement is unknown.

The major difference to the POP analysis is the fitting of the amplitude $\varkappa_a$ and the phase t_p of the seasonally oscillating damping rate $\varkappa(t)$ (5.2): $\varkappa_a = 0.04$, $t_p = 5.3$ months. The seasonal march of $\varkappa(t)$ is illustrated in Fig. 3. Hence, the process, described by $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, responds

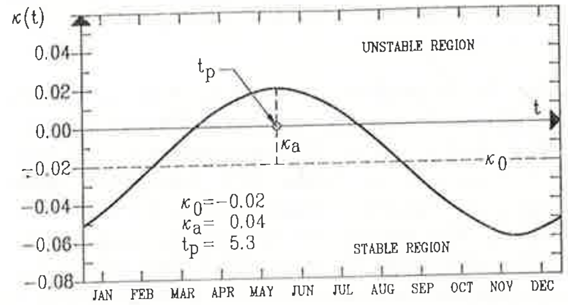


Fig. 3. Seasonally varying damping rate $\varkappa(t) = \varkappa_0 + \varkappa_a \cdot \cos \omega_0 \cdot (t - t_p)$ with $\varkappa_0 = -0.02$, $\varkappa_a = 0.04$, $t_p = 5.3$ months. $\varkappa(t) > 0$ indicates amplification and $\varkappa(t) < 0$ damping.

Abb. 3. Jahreszeitlich schwankende Dämpfungsrate $\varkappa(t) = \varkappa_0 + \varkappa_a \cos \omega_0 (t - t_p)$. $\varkappa(t) > 0$ zeigt Verstärkung von Störung an und $\varkappa(t) < 0$ deren Dämpfung.

with variable sensitivity to disturbances. From April to July, the damping rate is positive, i.e. the system is unstable, disturbances will be amplified. Throughout the rest of the year the system is stable with maximum damping at the end of November.

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