Validation of gyrokinetic simulations with measurements of electron temperature fluctuations and density-temperature phase angles on ASDEX Upgrade

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Measurements of turbulent electron temperature fluctuation amplitudes, $\delta T_{e\perp}/T_e$, frequency spectra and radial correlation lengths, $L_r(T_{e\perp})$, have been performed at ASDEX Upgrade using a newly upgraded Correlation ECE diagnostic in the range of scales $k_r < 1.4$ cm$^{-1}$, $k_r < 3.5$ cm$^{-1}$ ($k_r \rho_s < 0.28$ and $k_r \rho_s < 0.7$). The phase angle between turbulent temperature and density fluctuations, $\alpha_{n,T}$, has also been measured by using an ECE radiometer coupled to a reflectometer along the same line of sight. These quantities are used simultaneously to constrain a set of ion-scale non-linear gyrokinetic turbulence simulations of the outer core ($\rho_{i0} = 0.75$) of a low density, electron heated L-mode plasma, performed using the gyrokinetic simulation code, GENE. The ion and electron temperature gradients were scanned within uncertainties. It is found that GK simulations are able to match simultaneously the electron and ion heat flux at this radius within the experimental uncertainties. The simulations were performed based on a reference discharge for which $\delta T_{e\perp}/T_e$ measurements were available, and $L_r(T_{e\perp})$ and $\alpha_{n,T}$ were then predicted using synthetic diagnostics prior to measurements in a repeat discharge. While temperature fluctuation amplitudes are underestimated by > 50% for all simulations within the sensitivity scans performed, good quantitative agreement is found for $L_r(T_{e\perp})$ and $\alpha_{n,T}$. A validation metric is used to quantify the level of agreement of individual simulations with experimental measurements, and the best agreement is found close to the experimental gradient values.

I. INTRODUCTION

Understanding the turbulent driven heat flux in a tokamak remains one of the key goals of fusion research. Anomalous transport up to two orders of magnitude above what one would expect from neoclassical theory is observed and this is now understood to be caused by turbulent fluctuations in the plasma density, temperature and potential, originating from drift-wave like instabilities [1, 2]. The most complete, tractable models of this turbulence are currently given by numerical approaches to the gyrokinetic (GK) equations [3]. Much progress has been made over the past few decades in the development of gyrokinetic approaches and in validating non-linear gyrokinetic codes against experiment. In some cases, validation studies performed with these codes are able to match the experimental heat flux for both ions and electrons within experimental error bars [4–9]. However, there remain many examples where this is not the case, with the simulations unable to match the ion heat flu, $Q_i$ [10, 11] or the electron heat flux, $Q_e$ [12–15]. In the cases where ion scale simulations are unable to match the electron heat flux, it has been suggested that electron scale turbulence may be playing a role, either directly [16, 17] or through non-linear multi-scale coupling of ion and electron turbulence [18–20]. Since this latter case requires simulations which are presently almost prohibitively computationally expensive, it is of a very high importance to identify the region of parameter space where single scale simulations suffice.

For the situations where the models are able to reproduce the experimental heat fluxes, it is important to understand whether they do this with a physically realistic model. This can only be done by quantitatively comparing the simulation to one or more experimentally measured properties of the turbulence, known in the community as a validation study [21]. At ASDEX Upgrade (AUG), in order to probe the low wavenumber ($k$) temperature fluctuations contributing to the electron heat flux channel and to compliment the existing turbulence diagnostics used for GK validation [22], a Correlation Electron Cyclotron Emission (CECE) diagnostic has been installed [23, 24]. The principles of this diagnostic are briefly outlined in Section III. Using a unique channel comb, this diagnostic can measure high radial resolution $\delta T_{e\perp}/T_e$ and radial correlation length $L_r(T_{e\perp})$ profiles. The CECE diagnostic also shares a line of sight to the plasma with a number of reflectometers [25–27], which are sensitive to the fluctuating density component of the turbulence. This allows simultaneous, co-located measurements of density and temperature fluctuations and their corresponding cross-phase angle $\alpha_{n,T}$.

Measurements of $\delta T_{e\perp}/T_e$, frequency spectra, $L_r(T_{e\perp})$ and $L_r(T_{e\perp})$, have been performed in an ASDEX Upgrade (AUG) L-mode plasma. These quantities are used simultaneously to constrain a set of ion-scale non-linear gyrokinetic turbulence simulations of the outer core of a low density, electron heated L-mode plasma, performed using the gyrokinetic simulation code, GENE. The gradient scale lengths of the $T_i$ and the $T_e$ profile were varied.
within uncertainties to assess the sensitivity of the simulations to these parameters. It is found that the GK simulations are able to match simultaneously the $Q_i$ and $Q_e$ at this radius within the experimental uncertainties. While temperature fluctuation amplitudes are overestimated by > 50% for all simulations within the sensitivity scans performed, good quantitative agreement is found for $L_r(T_{e\perp})$ and $\alpha_{nT}$. A validation metric is used to quantify the level of agreement of individual simulations with experimental measurements, and the best agreement is found close to the experimental gradient values.

The paper is organised as follows: Section II contains a description of the plasma discharges used for the validation study, Section III details the experimental techniques used to measure the fluctuating quantities which are subsequently compared to the simulations, Section IV describes the simulations of the plasmas and the synthetic diagnostics used to make the quantitative comparisons, Section V describes the simulations of the plasmas and the synthetic diagnostics used to make the quantitative comparisons. Section IV E describes an attempt to use a validation metric to quantitatively compare the simulations within the sensitivity scans and Section V is a summary of the observations and presents some concluding remarks.

II. L-MODE PLASMA USED FOR GK VALIDATION

In this study measurements of the temperature fluctuation amplitude, $\delta T_{e\perp}/T_e$, the radial correlation length, $L_r(T_{e\perp})$, and the cross-phase angle between temperature and density fluctuations, $\alpha_{nT}$ are used. These measurements were taken during two discharges in the ASDEX Upgrade (AUG) experiment, a reference discharge (#33585) in which only $\delta T_{e\perp}/T_e$ is available from the pilot CECE diagnostic [23], and a repeat discharge (#34626) where all the aforementioned fluctuation quantities are available from a newly upgraded CECE diagnostic [24].

The gyrokinetic simulations presented in this paper are all based on the reference discharge. This discharge is a low density, predominantly electron heated L-mode plasma, with magnetic field on axis $B = 2.5 \text{T}$, plasma current $I_p = 1.0 \text{ MA}$. The applied heating was Ohmic and Electron Cyclotron Resonance Heating (ECRH) with a power $P_{\text{ECRH}} = 0.7 \text{ MW}$. The reference L-mode plasma is stationary for a period of 2 s and it is over this time that the profile measurements are averaged. The repeat discharge is stationary for 3.3 s allowing a longer period for averaging the fluctuation measurements. For the reference discharge, Figure 1 shows as a function of the normalised toroidal flux radius, $\rho_{\text{tor}}$, the electron temperature measured with a profile radiometer [28], electron density measured with Thomson Scattering [29, 30] and ion temperature and toroidal rotation profile measured using a charge exchange recombination spectroscopy (CXRS) system combined with periodic neutral beam blips [31]. The effective plasma charge $z_{\text{eff}}$, is calculated from the Bremsstrahlung radiation measured by the CXRS diagnostic and is $1.6+/−0.2$ for the reference case. The uncertainty on this value is significantly higher for the repeat discharge, but consistent with the reference. Characteristic uncertainties for these parameters can be found in the respective publications, however at the radial location of interest the uncertainties $\sigma_{T_e} \approx 15\%$, $\sigma_{n_e} \approx 10\%$, $\sigma_{T_i} \approx 20\%$ and $\sigma_{\omega} \approx 50\%$. The profile fits to these data are also shown as solid lines. Panel b) of Figure 1 shows the profiles of the normalised scale lengths of these quantities and the dashed line indicates the radius for which the gyrokinetic simulations were performed. Thomson and ECE profile data are not available over the radial range $\rho_{\text{tor}} = 0−0.2$, however this is not thought to affect the profile fits at the simulation radius.

In this paper we define the inverse gradient scale length of a quantity $X$, $1/L_X$, as $−d/d\rho_{\text{tor}}(\ln X)$. This is sometimes referred to in the literature as $\omega_X$. Uncertainties in the scale lengths come from a combination of the inherent scatter in the data and the method used to calculate the profile fits. It was estimated using a Monte Carlo approach that the normalized gradient scale length uncertainties were 20% for $1/L_{T_e}$, 30% for $1/L_{T_i}$, and 30% for $1/L_{n_e}$ at $\rho_{\text{tor}} = 0.75$. The profiles and scale lengths in the repeat discharge match the reference discharge within uncertainties from $\rho_{\text{tor}} = 0.25 − 1.00$. Further, $\delta T_{e\perp}/T_e$ is also in agreement within uncertainties.

![FIG. 1. Measured kinetic profiles a) and scale lengths b) for the reference discharge AUG 33585.](image)

Power balance transport analysis for the reference discharge considered in this study was performed with the transport code TRANSP [32]. The input uncertainties were propagated through the governing transport equations solved by TRANSP to obtain uncertainties on the output heat fluxes. This calculation leads to conductive heat flux values of $Q_i = (0.33 ± 0.10) \text{ MW}$ and $Q_e = (0.8 ± 0.13) \text{ MW}$ to an uncertainty of 26% for $Q_i$ and 14% for $Q_e$ and at $\rho_{\text{tor}} = 0.75$. 

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*Note: The image with the graph is not directly translatable into text.*
III. TURBULENCE MEASUREMENT TECHNIQUES

A. Correlation ECE measurements of $\delta T_{e\perp}$ fluctuations

Under typical tokamak conditions, core turbulent electron temperature fluctuations are sufficiently broadband ($\sim 0.5$ MHz) and low-amplitude ($\sim 1\%$) that a conventional radiometer is fundamentally unable to detect them [33]. However, correlation techniques can further reduce the intrinsic noise and extract the small, wideband turbulent fluctuation signals [33–35]. For a review of CECE types and techniques see [36]. The AUG CECE diagnostic [23] is of the spectral decorrelation type, meaning it is an ECE radiometer with channels sensitive to ECE radiation in distinct, non-overlapping narrow frequency bands, which are spaced sufficiently closely that neighbouring pairs are sensitive to the same turbulent temperature fluctuations. Correlation analysis between channel pairs then allows the uncorrelated thermal noise of each channel to be significantly reduced, allowing the measurement of the turbulent fluctuations. The diagnostic operates in the frequency range 110-130 GHz and each channel is 100 or 200 MHz in bandwidth ($B_{\text{IF}}$).

The new CECE diagnostic on AUG allows for more detailed measurements of the ion scale electron temperature fluctuations than has been achieved before, due both to the large number of channels (28) and their arrangement in a comb pattern [24]. Rather than relying on individual pairs, as used in the previous AUG CECE diagnostic [23], where the number of radial points is $n/2$, the comb arrangement allows $n - 1$ radial points in a $\delta T_{e\perp}/T_e$ fluctuation profile with a radial resolution of 2 - 4 mm, as shown in Figure 2 a). This arrangement also allows the possibility to measure the correlation length by correlating one channel with others at increasing separation, Figure 2 b), presenting a distinct advantage over the usage of tune-able filters, in that only a single shot is required to measure both $\delta T_e/T$ and $L_e(T_e)$ profiles with high sensitivity.

The AUG CECE diagnostic is sensitive to low-$k$ fluctuations in the range $k_{\perp} < 1.4 \text{ cm}^{-1}$, $k_r < 3.5 \text{ cm}^{-1}$, which equates to the range $k_{\perp}\rho_s < 0.28$ and $k_r\rho_s < 0.7$, (where $\rho_s$ is the ion gyro-radius evaluated at the local ion sound speed and magnetic field) for a typical AUG L-mode plasma, measures in the ion-scale range. Fluctuations in this range are expected to dominate both the electron and ion heat flux ($Q_e$ and $Q_i$) under most circumstances, however, it has been shown that electron scales can contribute directly to $Q_e$ if the Electron Temperature Gradient (ETG) mode growth rate is above $\sqrt{m_i/m_e} \approx 60$ [17] and through the effects of cross-scale coupling [19].

Fluctuation amplitudes are calculated using Equation 1, where $\gamma_e$ is the complex coherence [37] between channel pairs, $\gamma_{bg}$ is the background coherence estimated from a region of frequency space separated from the fluctuations and $\Re$ denotes taking the real part. For a more in-depth discussion of the signal analysis techniques, error estimation and sensitivity limits used in this paper, see Ref [24]

$$\left( \frac{\delta T_{e\perp}}{T_e} \right)^2 = \frac{2}{B_{\text{IF}}} \int_{f_0}^{f_1} \Re \{ \gamma_e(f) - \gamma_{bg} \} \, df. \tag{1}$$

The temperature fluctuation profile measured in the repeat discharge, together with two subsequent discharges where the channel position was scanned, are shown in Figure 3. The outer region of the plasma ($\rho_{\text{cor}} > 0.87$) has an optical depth $\tau$ below 2. Radii smaller than this are considered optically thick for our purpose, with $\tau > 3$ for all further measurements presented in this paper. The sensitivity limit is $0.13\%$ for the data averaged over a 3 second period.

**FIG. 2.** Two examples of correlation patterns used to make measurements of $\delta T_{e\perp}/T_e$. a) is a pattern allowing a fine resolution radial profile of $\delta T_{e\perp}/T_e$ and b) allows the measurement of the radial correlation length in the traditional sense.

**FIG. 3.** $\delta T_{e\perp}/T_e$ profiles measured for the reference discharge 34626 and two subsequent discharges where the channel position was scanned radially inwards and outwards. The region of low optical depth $\tau$ is marked.

Figure 4 shows Equation 1 evaluated for different channel spacings, calculated from the cold resonance positions. The figure contains data from two sets of CECE channels, one with $B_{\text{IF}} = 100$ MHz, spaced at 125 MHz ($\sim 2$ mm) and another with $B_{\text{IF}} = 200$ MHz, spaced at 250 MHz ($\sim 4$ mm). The two sets agree with each
The estimated correlation length for the \( T_{\text{rad}} \) fluctuations, \( L_{\text{meas}} \) is 8.4 mm (5.2\( \rho_s \)) in this discharge, at \( \rho_{\text{tor}} = 0.78 \). Since each ECE channel has a finite radial width (\( \approx 5 \) mm), the measured correlation length \( L_r (T_e, \text{meas}) \) is moderately larger than \( L_r (T_e) \). The necessary modeling for the interpretation of this measurement will be discussed in section IV in the context of the comparison of radial correlation lengths to gyrokinetic simulations via a synthetic diagnostic.

\[ \alpha_{nT} = \arg \{ S_{T_{e \perp}} S_{n_e} \}, \quad (2) \]

which has a statistical uncertainty given in radians by [41]:

\[ \sigma_{\alpha_{nT}} = \sqrt{\frac{1}{2n_d} \left( \frac{1}{|\gamma_e|^2} - 1 \right)}, \quad (3) \]

where \( n_d \) is the number of independent records used to calculate \( \gamma_e \). For the AUG CECE, the channel comb continuously covers a sufficiently large radius, that typically only one shot is required to find the peak sensitivity of the reflectometer with one of the ECE channels. The experimental strategy is to launch the reflectometer at fixed frequency (cut-off position) and correlate the reflectometer signal amplitude with all of the available ECE channels. This approach is demonstrated in Figure 5, which shows the average coherence in the range 0-100 kHz between the reflectometer amplitude and several ECE radiometer channels (squares) taken over a period of 1 s. The x-axis represents the ECE channel position given in mm from the outermost channel (0 mm), increasing along the line of sight towards the plasma core. A clear maximum peak in coherence can be distinguished at 40 mm, indicating that reflectometer ‘alignment’ has been found. This corresponds to \( \rho_{\text{tor}} = 0.744 \). The triangles represent the average CECE coherence with reference to the ECE channel which gives the highest coherence with the reflectometer. One can see that after 44 mm, the coherence between reflectometer and CECE falls off approximately as fast as the CECE coherence falls, whereas the coherence between reflectometer and ECE before 40 mm falls somewhat slower, suggesting the reflectometer cut-off has been captured. Figure 6 shows the coherence between two adjacent CECE channels in panel a) for two radii, which correspond to the maximal average coherence between reflectometer and CECE in two cases. Panel b) shows the coherence between reflectometer and CECE radiometer channel for these cases and panel c) shows the cross phase spectra. One can see that the phase has a relatively steady value where the coherence is high (below 50 kHz) and is random where the coherence is low (above 50 kHz and below 10 kHz).

It has previously been shown that \( \alpha_{nT} \) varies continuously with increasing/decreasing electron temperature gradient [42, 43]. This makes \( \alpha_{nT} \) a particularly interesting quantity to use in the validation of mixed turbulent mode plasmas, since it can provide a strong constraint on the mix of electron and ion modes in the saturated turbulent state, thus constraining the ratio of electron and ion mode drives as valid inputs to the simulations.
IV. GYROKINETIC VALIDATION

A. Linear Gyrokinetic Simulations

Linear simulations were first performed with the GENE [17] linear eigenvalue solver in order to inform the non-linear simulations. Figure 7 shows the growth rate and frequency of the two most unstable eigenmodes in simulations for 0.9, 1.0 and 1.2 times the experimentally measured ion temperature gradient scale length, $1/L_{T_i,\text{nom}}$. At the lowest value for $1/L_{T_i}$, the dominant linear mode is the hybrid Trapped Electron Mode - ETG (TEM-ETG), so called in this instance, because with increasing $k$, there is no discontinuity in growth rate or frequency between ion scales and electron scales, making a definition of TEM and ETG difficult. As $1/L_{T_i}$ is increased the TEM-ETG mode peak is suppressed and the Ion Temperature Gradient (ITG) mode grows, as expected, to become the fastest growing mode.

The figure shows that, within the error-bars of the experimental $1/L_{T_i,\text{nom}}$, the plasma at this radial position could either be classified as a linearly ITG or TEM. However, the non-linear mix of modes may well be different, as the ITG or TEM-ETG may be subject to different saturation mechanisms and non-linearly interact with each other.

Not shown in Figure 7, are the high $k$ values. The ratio of high and low $k$ peak growth rates $\gamma_{\text{high } k}/\gamma_{\text{low } k} = 35$. Previously it has been suggested that this figure of merit should be as high as $\sqrt{m_i/m_e} \approx 60$ for the heat flux carried at the electron scales to be significant. Therefore it was decided to simulate only the ion scales in this study.

B. Non-linear Gyrokinetic Simulations

Non-linear gyrokinetic simulations were performed with GENE for the reference discharge (profiles shown in Figure 1) at $\rho_{\text{tor}} = 0.75$). The GK simulations were ion-scale, local flux tube simulations, including electromagnetic effects, using a realistic ion/electron mass ratio and using an experimentally reconstructed magnetic geometry from the CLISTE equilibrium code. Collisions were modeled via a linearized Landau-Boltzmann operator and impurities included through a $Z_{\text{eff}}$ term in this operator. The experimental input (nominal) values are presented in Table I. Sensitivity scans are performed around these nominal values to explore the agreement / disagreement with TRANSP calculated heat fluxes. Both the ion and electron temperature gradient scale lengths are varied with the experimental error bars, in two separate sets of scans.

Figure 8 shows the GK conductive heat fluxes in the electron and ion channels compared to the conductive heat flux values inferred by the TRANSP transport analysis code (shaded regions) for the $1/L_{T_i}$ scan. Both the $Q_i$ and $Q_e$ are in agreement within uncertainties with those estimated from TRANSP, for the nominal gradient.
FIG. 7. The linear growth rate (a) and mode frequency (b) for the ion (blue) and electron (red) modes. The modes are shown for 0.9/L_{T_i,\text{nom.}} (circles), 1.0/L_{T_i,\text{nom.}} (triangles) and 1.2/L_{T_i,\text{nom.}} (pentagons). The growth rate peak is either an electron or ion mode within the experimental uncertainty and is thus classed as a mixed mode plasma.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>nominal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>I_p [MA]</td>
<td>1.0</td>
</tr>
<tr>
<td>$\rho_{\text{tor}}$</td>
<td>0.75</td>
</tr>
<tr>
<td>$q$</td>
<td>2.28</td>
</tr>
<tr>
<td>$\hat{s}$</td>
<td>1.923</td>
</tr>
<tr>
<td>$\hat{s}<em>{EB}L</em>{\text{ref}}/c_s$</td>
<td>0.0168</td>
</tr>
<tr>
<td>$1/L_{T_i} = -d/d\rho_{\text{tor}}(\ln T_i)$</td>
<td>5.11</td>
</tr>
<tr>
<td>$1/L_{T_e} = -d/d\rho_{\text{tor}}(\ln T_e)$</td>
<td>2.65</td>
</tr>
<tr>
<td>$1/L_{n_e} = -d/d\rho_{\text{tor}}(\ln n_e)$</td>
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</tr>
<tr>
<td>$T_e$ [keV]</td>
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<tr>
<td>$n_e$ [10^{19} m^{-3}]</td>
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</tr>
<tr>
<td>$T_i/T_e$</td>
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</tr>
<tr>
<td>$Z_{\text{eff}}$</td>
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</tr>
<tr>
<td>$\beta_e$ [%]</td>
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</tr>
<tr>
<td>$v_{cs}/(L_{\text{ref}}/c_s)$</td>
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</tr>
<tr>
<td>$R_{\text{axis}}$ [m]</td>
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<td>$a$ [m]</td>
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</tr>
<tr>
<td>$B_{\text{ref}}$ [T]</td>
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</tr>
<tr>
<td>$L_{\text{ref}}$ [m]</td>
<td>0.6523</td>
</tr>
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</table>

TABLE I. The input values for the nominal case for both the linear and non-linear simulations. $\hat{s}$ is the magnetic shear and $\hat{s}_{EB}$ is the $ExB$ shearing rate.

Critical to any direct quantitative comparison of the $T_e$ fluctuation measurements to GK simulations requires the use of a sufficiently accurate synthetic diagnostic to apply to the simulation output data [10]. This synthetic diagnostic ideally should not be a source of error for the quantitative comparison. In our case, GK fluctuation data has the toroidal velocity as measured from CXRS imposed on it and is then mapped to R,Z coordinates. The perpendicular velocity of the structures is then found to agree well with that measured by the poloidal correlation reflectometer [44] at this radius. There are several effects that are significant for the fluctuation amplitude calculated by the synthetic diagnostic.

C. CECE synthetic diagnostic
Of prime importance is the accurate modeling of the spatial region to which the CECE is sensitive. The finite volume from which the measurements are made results in a spatial smoothing, which dictates the \( k \) range to which the diagnostic is sensitive [45]. Since, in the simulations, there is significant fluctuation power above the measured \( k \) range, this can have a significant impact on the predicted fluctuation amplitude. A 2D Gaussian function in the \( R, z \) plane is used to model the finite volume of the real measurements. The radial size of the Gaussian function is taken from a fit to the emissivity function [46], determined using a standard ECE radiation transport model (ECFM) [47]. This model takes into account the finite bandwidth of the IF filters, the relativistic broadening and ECE frequency downshift, the Doppler broadening at finite toroidal angle and beam refraction in the plasma. This synthetic diagnostic currently uses a fitted Gaussian with \( 1/e \) radial width of 4.7 mm.

The \( 1/e \) width of the Gaussians in the \( z \) direction, \( w_z \), are set by the quasi-optical beam width and are critical parameters for the synthetic diagnostic. The beam width was measured using a near-field measuring technique ex-situ. This technique measures the complex field in two polarizations in the near field of the diagnostic antenna and then extrapolates the field in space using a Fast Irregular Antenna Field Transformation Algorithm [48]. The advantage of this measurement technique is that the beam waist may be extrapolated reliably to arbitrary distances from the CECE optics. Two sets of measurements were made over the range 75 - 105 GHz. Firstly, the antennas were measured alone and a Gaussian beam was fit to the results. This was then used as input to a Gaussian Optics (GO) formalism [49] to propagate the beam through the focusing elements. Secondly, the beam from the complete CECE optics was measured and compared directly to the GO predictions. Figure 9 shows the results of this comparison (blue line - GO, green stars - measurement). The trends in the antenna waist and position vs. frequency were then extrapolated to the measurement frequency of 117.5 GHz and a new beam was calculated (solid red line in Figure 9). Since this extrapolation comes with some uncertainty, the results for \( \pm 20\% \) of the extrapolated antenna waist size are also shown (red dashed and dotted respectively). The CECE measurements compared to GO simulations in this paper were made in the AUG plasma at a position 515 mm from the mirror.

The plasma considered in this study is sufficiently optically thick (\( \tau > 3 \)) that contributions from density fluctuations are negligible. Further, since the CECE is only sensitive to the perpendicular temperature fluctuations [36], the synthetic diagnostic only acts on those. In the non-linear simulations performed in the study presented here, the \( \Delta T_{e\perp} \) is found to be a factor of 3.8 higher than \( \Delta T_{e\parallel} \). Since \( \Delta T_{\text{tot}} = 2/3\Delta T_{\perp} + 1/3\Delta T_{\parallel}, \Delta T_{e\perp} \) is thus 33\% higher than \( \Delta T_{e\text{tot}} \). There has only been one other gyrokinetic validation study thus far where this effect has been taken into account, significantly improving agreement between the code and the experimental measurements in that case [6].

For the simulation of the measurement of \( \alpha_nT_e \), the CECE synthetic diagnostic Gaussian filters are applied simultaneously to the \( \Delta T_{e\perp} \) and \( \Delta n_e \) fields and cross-correlations are performed on the time series resulting from co-located Gaussian filters in a similar manner to the synthetic diagnostic used previously on GYRO simulations [39]. The justification is that the CECE limits the \( k \)-range over which the measurements are made more than the reflectometer in the poloidal direction. Reflectometer physics is not explicitly included and the Born approximation is assumed to apply to the fluctuation amplitude. There is experimental evidence for this assumption, both from our measurements and in previous experiments at DIII-D [40].

D. Comparison of simulations to measured fluctuations

Comparing \( \Delta T_{e\perp}/T_e \) from GK simulations of the reference discharge to the measurements shows that the temperature fluctuations are systematically over-estimated, even when the ion and electron heat fluxes are matched within error bars. Two example spectra are shown in Figure 10, with the experimental measurements in black and the synthetic diagnostic result in orange. The GK spectrum is both higher in amplitude and broader in frequency than in the experiment with \( (\Delta T_{e\perp}/T_e)_{\text{expt}} = 0.70\% \) and \( (\Delta T_{e\perp}/T_e)_{\text{GK}} = 1.14\% \). Increasing the CECE beam width by 30\%, which is greater than the experimentally measured uncertainty, cannot account for the discrepancy, in fact a factor of 2 is required to find agree-
ment between experiment and simulation.

In contrast to this, the correlation function is found to agree well in shape. Correlation lengths are calculated by fitting Gaussian functions to both the experimental data and GK data. In this case $L_e(T_e \perp)_{\text{GK}} = 10.5 \text{ mm}$ and $L_e(T_e \perp)_{\text{expt}} = 9.8 \text{ mm}$. There is of course some ambiguity as neither the GK correlation function nor the experimentally measure one are strictly Gaussian, both possessing elevated tails. The data in Figure 4, which is made at higher resolution, slightly further out of an identical plasma shows this more clearly for the case of the experiment. It is these fitted values which are used in the validation metric in Section IV E, with a representative error of 1.5 mm.

For the same simulation there is also quantitative agreement between measured and simulated $\alpha_{nT}$. Figure 12 shows the cross-phase frequency spectra measured in the repeat discharge and the GK simulated result. In the region where there is high coherence between reflectometer and CECE (10-50kHz), there is quantitative agreement in the cross-phase.

The $\alpha_{nT}$ cross-phase angle can be a very useful discriminator between simulations which differ in the ion and electron mode content. This can be seen in Figure 13, which shows the synthetic diagnostic results for selected points in the sensitivity scans performed. The squares show the result of an $1/L_T$ scan with $1/L_T$ fixed at the nominal value. The triangles are two points of an $1/L_T$ scan with $1/L_T$ fixed at the nominal value. The black point is the experimentally measured $\alpha_{nT}$. For reference in this figure, artificially setting $1/L_T = 0$ in the simulations (ITG like) gives $\alpha_{nT} = -135^\circ$ and setting $1/L_T = 0$ (TEM like) gives $\alpha_{nT} = -15^\circ$. It can be seen that increasing $1/L_T$ smoothly increases the magnitude of the phase angle towards the ITG like value as the turbulence becomes more ITG like in nature. In contrast, increasing $1/L_T$, decreases the magnitude of $\alpha_{nT}$, becoming more TEM like. The experimental point is in quantitative agreement for the simulations with a stronger ITG drive compared to TEM drive.

The direct comparisons of fluctuating quantities presented in this section in Figures 10, 11 and 12 come exclusively from a single GK simulation using $1.0/L_{T_i,\text{nom.}}, 0.87/L_{T_e,\text{nom.}}$. This simulation gives the closest agreement for each of the turbulent quantities and is within experimental uncertainties for $Q_i$ and $Q_e$. The metrics which justify this simulation choice for the comparisons are discussed in section IV E.
E. Validation metric

It is useful to adopt a metric that allows an assessment of the overall quality of the simulations when all the experimental comparisons are considered. The parameters constraining the simulations in this study are: $Q_i$, $Q_e$, $\delta T_e/T_e$, $L_r(T_e \perp)$ and $\alpha_{nT}$. The choice of validation metric is an ongoing discussion in the field of validation with various mathematical formulations proposed \cite{21, 50, 51} and a recent review article \cite{52} on the subject. In this study, the formulation laid out by Ricci et al \cite{51} has been employed. First an "uncertainty-normalized distance", $d_i$, is calculated for each observable. This quantifies the distance between the simulated quantity, $s_i$, and the experimental quantity, $e_i$, in units of the uncertainties $\Delta s_i$ and $\Delta e_i$:

$$d_i = \sqrt{\frac{(s_i - e_i)^2}{\Delta s_i^2 + \Delta e_i^2}}$$  \hspace{1cm} (4)

where the sum over instances has been dropped, since in this case we want to calculate values for each individual simulation and there is only one measurement of each experimental quantity. The values of $d_i$ for each of the observable for each of the simulations performed can be found in Table II and bar charts of $d_i$ can be seen in Figure 14 for the 4 flux matching simulations.

A bounded error metric $R$ can then be defined

$$R_i = \frac{1 + \tanh[(d_i - d_0)/\lambda]}{2}$$  \hspace{1cm} (5)

where $R_i = 0$ indicates perfect agreement and $R_i = 1$ indicates complete disagreement. The free parameter $d_0$ defines the level at which transition from agreement to disagreement and is set to 1.5, meaning that a $d_i$ of 1.5 maps to an $R_i$ of 0.5. The parameter $\lambda$ defines the sharpness of the transition from agreement to disagreement and is set to 1.5, meaning that a $d_i$ of 1.5 maps to an $R_i$ of 0.5. The parameter $\lambda$ defines the sharpness of the transition from agreement to disagreement and is set to 1.5, meaning that a $d_i$ of 1.5 maps to an $R_i$ of 0.5.

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The validation metric allows one to compare the simulations in a defined and quantitative manner. Simulations with $\chi < 0.5$ are defined to be in overall agreement with the experiment and with $\chi > 0.5$ in disagreement \cite{51}. Figure 15 shows $\chi$ for the simulation set presented in this paper. A minimum $\chi$ (best agreement) is found for the simulation performed at $1.0/L_{T_{i,nom.}}, 0.87/L_{T_e,nom.}$, which is also the best overall match to the turbulence measurements and this is
a different simulation to that which best matches the heat fluxes \((0.95/L_{T_e,\text{nom}}, 1.0/L_{T_e,\text{nom}})\). Although both simulations are in agreement with the heat fluxes, caution should be exercised in concluding that the best heat flux match is the most physically realistic simulation and highlights the importance of having as many measurements as possible to constrain the simulations. There is a clearly defined minimum in \(\chi\) as a function of \(1/L_{T_i}\), with only a small improvement when reducing \(1/L_{T_e}\) by 12%. In contrast, increasing \(1/L_{T_e}\) by 12% brings a significant penalty to the overall agreement. This is brought about by a large discrepancy arising in all three of \(Q_e\), \(\alpha_{nT}\) and \(L_r(T_e)\). Similar to Ricci, we also find that conclusions based primarily on the relative ordering of the \(\chi\) of the simulations, i.e. which simulation proved a closer match to experiment is not affected by the exact choice of \(d_0\) and \(\lambda\) within the stated ranges.

![Graph](image)

**FIG. 15.** A validation metric, \(\chi\), for assessing the quality of the simulations. The dashed line at \(\chi = 0.5\) marks the boundary between overall agreement and disagreement.

Table II. Uncertainty normalised distance, \(d\), and validation metric, \(\chi\), for all simulations in the \(1/L_{T_i}\) and \(1/L_{T_e}\) scans considered in this study.

<table>
<thead>
<tr>
<th>(L_{T_i,\text{nom}}/L_{T_i})</th>
<th>(L_{T_e,\text{nom}}/L_{T_e})</th>
<th>(d_{Q_i})</th>
<th>(d_{Q_e})</th>
<th>(d_{\delta T/T})</th>
<th>(d_{L_e})</th>
<th>(d_{\alpha})</th>
<th>(\chi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>1.00</td>
<td>1.76</td>
<td>2.62</td>
<td>2.66</td>
<td>0.02</td>
<td>5.60</td>
<td>0.71</td>
</tr>
<tr>
<td>0.90</td>
<td>1.00</td>
<td>0.98</td>
<td>0.40</td>
<td>3.07</td>
<td>0.15</td>
<td>2.17</td>
<td>0.51</td>
</tr>
<tr>
<td>0.95</td>
<td>1.00</td>
<td>0.20</td>
<td>0.30</td>
<td>3.66</td>
<td>0.27</td>
<td>1.28</td>
<td>0.33</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>0.88</td>
<td>0.50</td>
<td>5.61</td>
<td>0.89</td>
<td>0.84</td>
<td>0.30</td>
</tr>
<tr>
<td>1.00</td>
<td>1.12</td>
<td>0.78</td>
<td>2.52</td>
<td>3.94</td>
<td>0.90</td>
<td>2.82</td>
<td>0.65</td>
</tr>
<tr>
<td>1.20</td>
<td>1.00</td>
<td>2.55</td>
<td>1.21</td>
<td>5.21</td>
<td>0.05</td>
<td>0.37</td>
<td>0.40</td>
</tr>
<tr>
<td>1.00</td>
<td>0.87</td>
<td>0.49</td>
<td>1.01</td>
<td>2.67</td>
<td>0.46</td>
<td>0.13</td>
<td>0.27</td>
</tr>
</tbody>
</table>

V. SUMMARY AND CONCLUSIONS

New measurements of turbulent electron temperature fluctuation amplitude \(\delta T_{e\perp}/T_e\), radial correlation length \(L_r(T_{e\perp})\) and density-temperature phase angle \(\alpha_{nT}\) have been made with an upgraded Correlation ECE diagnostic \([24]\) and nT-phase diagnostic on ASDEX Upgrade. The new diagnostic uses a closely spaced channel comb to allow measurement of high radial resolution \(\delta T_{e\perp}/T_e\) and \(L_r(T_{e\perp})\) profiles. The coupling of a reflectometer with the ECE radiometer allows the simultaneous measurement of density and temperature fluctuation fields and thus the measurement of \(\alpha_{nT}\). The CECE channel comb gives the advantage that one can reliably capture the maximal coherence between ECE and reflectometer, making the interpretation of the measurements easier.

These measurements have been used to simultaneously constrain a set of non-linear, ion-scale gyrokinetic simulations, modeling a ECRH heated L-mode plasma \((\#33585)\) at \(\rho_{tor} = 0.75\). The simulations are based on a reference discharge for which \(\delta T_{e\perp}/T_e\) measurements are available, and a repeat discharge was performed \((\#34626)\) to obtain measurements of \(L_r(T_{e\perp})\) and \(\alpha_{nT}\). These values were predicted ahead of the experiment, applying the synthetic diagnostic to the reference simulation set. Although \(\delta T_{e\perp}/T_e\) is found to be consistently over-predicted, simultaneous agreement is found for \(Q_i\), \(Q_e\), \(L_r(T_{e\perp})\) and \(\alpha_{nT}\) in two of the simulations performed, and the disagreement in \(\delta T_{e\perp}/T_e\) is minimised for the 1.0/\(L_{T_i,\text{nom}}\), 0.87/\(L_{T_e,\text{nom}}\) case.

The consistent disagreement between the measured and simulated \(\delta T_{e\perp}/T_e\) remains an outstanding issue. Initially it would seem that the simplest solution to the discrepancy is that the beam size \(w\) is incorrect with respect to the experiment. It can be shown using the syn-
thetic diagnostic that no significant effect of \( w_z \) is found on the measurement of \( \alpha_{nT} \) or \( L_T(T_e) \). However, careful measurements of the CECE beam pattern were performed, and sensitivity scans were made using the CECE synthetic diagnostic, varying \( w_z \) within the uncertainties of the measurements (20%). It was found that this alone cannot explain the discrepancy. In order to achieve quantitative agreement in \( n \delta T_{e\perp}/T_e \) between experiment and GK, a factor of 2.2 increase in \( w_z \) is required, 6 times greater than the measured uncertainty in \( w_z \).

The simulations display a high degree of realism, matching conductive heat fluxes to the experiment and demonstrating low (10-30 kW) convective heat fluxes consistent with the negligible convective heat fluxes in the experiment. However, although parameter scans have been performed, an exhaustive scan of all the inputs to the GK code within experimental uncertainties is not practically achievable. We cannot explicitly rule out a combination of inputs within uncertainties which may satisfy all the experimental constraints. This search could be guided in the future by reduced models.

We may also consider what additional experimental constraints would give insight into this discrepancy. For the simulations to match \( Q_z \) and be low on \( \delta T_e/T_e \) means that there must be other unmeasured quantities which are also not matched. For example, measurements of the \( \delta n_e \), fluctuation \( k \)-spectrum and high-\( k \) fluctuation amplitudes would experimentally constrain the importance of smaller scales. Measurements of \( \delta n_e/n_e \), which were not available for these discharges, would also provide a strong constraint in combination with \( \delta T_e/T_e \) as they are more closely related to the unmeasured potential fluctuations.

An uncertainty metric, as defined in [51] has been used to attempt to quantify the agreement of individual simulations in a sensitivity scan to the experimental constraints and it is found that a clear minimum exists at \( 1.0/L_{T_e,\text{nom}}, 0.87/L_{T_e,\text{nom}} \), with the simulations becoming increasingly unrealistic away from this point. For the best matched simulations, agreement is found with four of the five experimental constraints within the uncertainties.

Overall the Ricci parameter \( \chi \) provides a useful framework for assessing the relative agreement between these simulations. In particular, it clearly reveals that the heat fluxes, although important, may not be the best parameters with which to validate a gyrokinetic model. As an example, the simulation performed at \( 0.95/L_{T_e,\text{nom}}, 1.0/L_{T_e,\text{nom}} \) provides the best match for the experimental heat fluxes, and it thus tempting to say that it is the most physically realistic. However, it is not such a good match to the measured turbulence quantities as the simulation at \( 1.0/L_{T_e,\text{nom}}, 0.87/L_{T_e,\text{nom}} \). When we codify the relative importance of both the position in the hierarchy of the measured constraints and how uncertain the relative measurements are (the heat fluxes are both derived and relatively uncertain) we see that it is in fact not the best representation in this framework.

The GK data presented in this paper all come from the simulation with the lowest Ricci parameter and are not hand-picked for their agreement to any single quantity, demonstrating a rigorous and fair method for presenting the data.

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