Embedding Standard Model Symmetries into $K(E_{10})$

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Following up on our earlier work [1] where we showed how to amend a scheme originally proposed by M. Gell-Mann to identify the 48 spin-$\frac{1}{2}$ fermions of $N=8$ supergravity that remain after complete breaking of $N=8$ supersymmetry with the 48 quarks and leptons of the Standard Model, we further generalize the construction to account for the full $SU(3)_c \times SU(2)_w \times U(1)_Y$ assignments, with an additional family symmetry $SU(3)_f$ which does not commute with the electroweak symmetries. Our proposal relies in an essential way on embedding the SU(8) R-symmetry of $N=8$ supergravity into the 'maximal compact' subgroup $K(E_{10})$ of the conjectured duality symmetry E_{10} of M theory. As a by-product, it predicts fractionally charged and strongly interacting massive gravitinos.

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It is generally believed that neither the particle content nor the dynamics of maximal $N = 8$ supergravity [2, 3] can be matched with the Standard Model (SM) of particle physics. In this Letter we re-examine this widely accepted wisdom on the basis of recent work [1, 4] which further develops an old proposal of Gell-Mann's [5] to identify the 48 quarks and leptons of the SM with the 48 spin-$\frac{1}{2}$ fermions that remain after complete breaking of $N=8$ supersymmetry. As shown in [1] the mismatch $\pm\frac{1}{6}$ of electric charges in the original Gell-Mann scheme can be rectified by introducing a new vectorlike generator $\mathcal{I}$ (see (9) below) not contained in the SU(8) R-symmetry group of $N = 8$ supergravity. However, in subsequent work [4] it was shown that this new generator does belong to $K(E_{10})$, the maximal compact subgroup of the hyperbolic Kac–Moody group E_{10} conjectured to be a symmetry underlying M theory [6] [20]. Here we take this construction one step further by exploring how the full $SU(3)_c \times SU(2)_w \times U(1)_Y$ symmetry of the SM might be incorporated into this scheme, together with an additional family symmetry $SU(3)_f$ that does not commute with the electroweak symmetries.

As the present proposal is very different from more established approaches to derive the SM fermions from a Planck scale unified theory of quantum gravity, let us first explain our basic 'philosophy'. Our considerations here are based on the central conjecture of [6] according to which the full E_{10} symmetry conjectured to underlie M theory manifests itself in a 'near singularity limit'. This conjecture itself is based on a BKL-type analysis of cosmological singularities, where the causal decoupling of spatial points entails an effective dimensional reduction to one (time) dimension [7]. In this pre-geometric regime, the spatial dependence of the fields is supposed to 'de-emerge' near the singularity, in the sense that it gets 'spread' over the Lie algebra, as outlined in [6]. We emphasize that this picture remains highly conjectural: explaining the emergence of a space-time based quantum field theory in this scheme is still an unsolved problem. Similar comments apply to the emergence of space-time symmetries and the distinction between global and local symmetries (which is why we do not further comment on these issues below).

Subsequent analysis of the fermionic sector of the theory [8] has revealed that the relevant symmetry acting on the fermions is the 'maximal compact' subgroup $K(E_{10})$ of E_{10}, *alias* the conjectured *R-symmetry of M theory*. Here as well, one restricts attention in a first step to the fermionic fields *at a given spatial point*, with the idea that the spatial dependence would emerge in a more complete description based on more faithful realizations of $K(E_{10})$. So far, however, only finite-dimensional *unfaithful* representations of $K(E_{10})$ are known [8, 9], which are in one-to-one correspondence with the fermions of maximal supergravity. The action of $K(E_{10})$ on these representations can be conveniently described in terms of the quotient group $\mathfrak{G}_{\mathcal{R}} = K(E_{10})/\mathfrak{N}_{\mathcal{R}}$ where $\mathfrak{N}_{\mathcal{R}}$ is the normal subgroup associated with the ideal spanned by those $K(E_{10})$ generators that annihilate all elements of the given representation space $\mathcal{R}$. For the Dirac representation corresponding to the supersymmetry parameter we have $\mathfrak{G}_D = \mathrm{Spin}(32)/\mathbb{Z}_2$, while for the Rarita-Schwinger representation describing the physical fermionic degrees of freedom we have $\mathfrak{G}_{RS} = \mathrm{Spin}(288,32)/\mathbb{Z}_2$ [10]; the latter group also contains transformations acting chirally on the $N=8$ fermions. We expect there to exist bigger and less unfaithful representations which can possibly capture more of the spatial dependence, see *e.g.* [11].

Although the present scheme necessarily transcends $N = 8$ supergravity, this theory nevertheless continues to play a crucial role in 'guiding' our proposal, in that we take the hints from the known vacuum structure of gauged maximal supergravities and insist on remaining compatible with the orginal scheme of [5]. While SO(8) gauged maximal supergravity has long been known to admit a stationary point which breaks SO(8) to a residual $SU(3) \times U(1)$ symmetry [12, 13], new gaugings with similar breaking (and sometimes even Minkowski vacua) have been constructed more recently [14, 15]. The group $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{em}$ is believed to be the gauge symmetry that survives to the lowest energies in the SM, but a naive identification of the supergravity SU(3) with the color group $\mathrm{SU}(3)_c$ does not work, as is immediately obvious from (6) below. For this reason, M. Gell-Mann introduced an additional family symmetry $\mathrm{SU}(3)_f$ that acts between the three particle families (generations) and proposed to identify the residual SU(3) of supergravity with the *diagonal subgroup* of color and family [5]. This scheme 'almost' works in the sense that, after the removal of eight Goldstinos there is complete agreement of the SU(3) assignments, but there remains a systematic mismatch in that the U(1) charges of the supergravity fermions are systemically off by $\pm\frac{1}{6}$ from the electric charges of the quarks and leptons. As noticed in [1] this mismatch can be remedied by means of the extra generator (9) which 'deforms' the SU(3) × U(1) group. Although this deformation is no longer contained in SU(8), it *is* contained in $\mathrm{K}(\mathrm{E}_{10})$ [4]. Evidently, this requires part of $\mathrm{K}(\mathrm{E}_{10})$ to become dynamical, in a partial realization of a conjecture already made in [2] for the R-symmetry SU(8), but now for a much bigger group, and with a very different mechanism for extracting space-time fields. One notable feature here is that even though accompanied by a vast enlargement of the R-symmetry, our proposal makes do with the original 56 spin-$\frac{1}{2}$ fermions, whereas more conventional schemes would require correspondingly larger multiplets for larger groups.

Accordingly, we now focus on the fermionic sector of $N = 8$ supergravity, which consists of eight gravitinos ψ^i_μ transforming in the $\mathbf{8}$, and a tri-spinor of spin-$\frac{1}{2}$ fermions χ^{ijk} transforming in the $\mathbf{56}$ of SU(8), where χ^{ijk} is fully antisymmetric in the SU(8) indices i, j, k. We adopt the conventions and notations of [3], where complex conjugation raises (or lowers) indices, such that for instance $\chi^{ijk} = \big(\chi_{ijk}\big)^*$, and where upper (lower) position of the SU(8) indices indicates positive (negative) chirality. Hence the chiral SU(8) transformations act as $\chi^{ijk} \to U^i{}_l U^j{}_m U^k{}_n \chi^{lmn}$, $\chi_{ijk} \to U_i{}^l U_j{}^m U_k{}^n \chi_{lmn}$ with $U \in \mathrm{SU}(8)$, $U_i{}^j \equiv \big(U^i{}_j\big)^*$ and $U^i{}_k U_j{}^k = \delta^i_j$ (for the maximal vectorlike subgroup SO(8) ⊂ SU(8) there would be no need to distinguish between upper and lower indices).

As already pointed out, a special role is played by the subgroup SU(3) × U(1), which is contained in the vectorlike gauge group SO(8) ⊂ SU(8) via the real embedding SU(3) × U(1) ⊂ SO(6) × SO(2) ⊂ SO(8) [5, 13]. To study the relevant decompositions we introduce boldface indices $\mathbf{1}, ..., \mathbf{4}$, and barred indices $\bar{\mathbf{1}}, ..., \bar{\mathbf{4}}$ for the conjugate representations as in [1, 13], but now with unit normalization so that $v^{\mathbf{1}} \equiv 2^{-1/2}(v^1 + iv^2), v^{\mathbf{2}} \equiv 2^{-1/2}(v^3 + iv^4)$, *etc.* and $v^{\bar{\mathbf{1}}} \equiv 2^{-1/2}(v^1 - iv^2), v^{\bar{\mathbf{2}}} \equiv 2^{-1/2}(v^3 - iv^4)$, *etc.* Similar rules apply to the fermions; for instance,

$$\varphi^{\mathbf{12\bar{4}}} \equiv \frac{1}{2\sqrt{2}}\Big(\chi^{137} + i\chi^{237} + i\chi^{147} - i\chi^{138} - \chi^{247} + \chi^{238} + \chi^{148} + i\chi^{248}\Big), \quad (1)$$

and so on. It is important here that the spinor $\varphi^{\bar{\mathbf{1}}\bar{\mathbf{2}}\mathbf{4}}$ with barred and unbarred indices interchanged is *not* the complex conjugate of $\varphi^{\mathbf{12\bar{4}}}$ because the spinor components χ^{ijk} are themselves complex. Writing (1) in the schematic form $\varphi = \mathrm{S} \circ \chi$ it is easy to see that S decomposes into 12 blocks of 2-by-2 matrices and four blocks of 8-by-8 matrices, see [16] for explicit formulas.

For the further analysis we single out the first three indices with labels $\mathbf{a}, \mathbf{b}, \cdots = \mathbf{1}, \mathbf{2}, \mathbf{3}$, on which the SU(3) subgroup acts. There is furthermore a two-parameter subgroup U(1) × U(1) whose associated Lie algebra is embedded as follows into SO(8)

$$Y(\alpha,\beta) = \begin{pmatrix} 0 & -\alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta & 0 \end{pmatrix}, \quad (2)$$

This matrix commutes with U(4) ⊂ SO(8) for all α, β, and thus defines an SU(3) × U(1) subgroup of SO(8) for each choice of α and β. As shown in [5] we must take

$$\alpha = \frac{1}{6}\ , \quad \beta = \frac{1}{2}\,, \quad (3)$$

in order to match the $N = 8$ fermions with those of the Standard Model (and also to be in accord with the structure of the $N = 2$ AdS supermultiplets [13]). With this choice one easily reads off the SU(3) × U(1) assignments for the gravitinos

$$\psi^{\mathbf{a}}_\mu \in (\mathbf{3}\,,\,\frac{1}{6})\,, \quad \psi^{\bar{\mathbf{a}}}_\mu \in (\bar{\mathbf{3}}\,,\,-\frac{1}{6})\,,$$
$$\psi^{\mathbf{4}}_\mu \in (\mathbf{1}\,,\,\frac{1}{2})\,, \quad \psi^{\bar{\mathbf{4}}}_\mu \in (\mathbf{1}\,,\,-\frac{1}{2}) \quad (4)$$

The 56 spin-$\frac{1}{2}$ fermions are split into 6+2 Goldstinos

$$\varphi^{\mathbf{a}} \equiv \varphi^{\mathbf{a4\bar{4}}} \in (\mathbf{3}\,,\,\frac{1}{6})\,, \ \ \varphi^{\bar{\mathbf{a}}} \equiv \varphi^{\bar{\mathbf{a}}\mathbf{4}\bar{\mathbf{4}}} \in (\bar{\mathbf{3}}\,,\,-\frac{1}{6})\ ,$$
$$\varphi^{\mathbf{4}} \equiv \varphi^{\mathbf{abc}} \in (\mathbf{1}\,,\,\frac{1}{2})\,, \ \ \varphi^{\bar{\mathbf{4}}} \equiv \varphi^{\bar{\mathbf{a}}\bar{\mathbf{b}}\bar{\mathbf{c}}} \in (\mathbf{1}\,,\,-\frac{1}{2}) \quad (5)$$

and the remaining 48 spin-$\frac{1}{2}$ fermions:

$$\varphi^{\mathbf{ab4}} \in (\bar{\mathbf{3}}\,,\,\frac{5}{6})\,, \quad \varphi^{\mathbf{ab\bar{4}}} \in (\bar{\mathbf{3}}\,,\,-\frac{1}{6})\,, \quad (6)$$
$$\varphi^{\bar{\mathbf{a}}\bar{\mathbf{b}}\mathbf{4}} \in (\mathbf{3}\,,\,\frac{1}{6})\,, \quad \varphi^{\bar{\mathbf{a}}\bar{\mathbf{b}}\bar{\mathbf{4}}} \in (\mathbf{3}\,,\,-\frac{5}{6})$$
$$\varphi^{\mathbf{ab\bar{c}}} \in (\mathbf{3},\frac{1}{6}) \oplus (\bar{\mathbf{6}},\frac{1}{6})\,, \ \varphi^{\bar{\mathbf{a}}\bar{\mathbf{b}}\mathbf{c}} \in (\bar{\mathbf{3}},-\frac{1}{6}) \oplus (\mathbf{6}\,,-\frac{1}{6})$$
$$\varphi^{\mathbf{a\bar{b}4}} \in (\mathbf{8}\,,\,\frac{1}{2}) \oplus (\mathbf{1}\,,\,\frac{1}{2})\,, \ \varphi^{\mathbf{a\bar{b}\bar{4}}} \in (\mathbf{8}\,,-\frac{1}{2}) \oplus (\mathbf{1}\,,-\frac{1}{2})$$

To be sure, in order to properly identify the Goldstinos, we must allow for a possible mixing between the representations with the same SU(3) × U(1) content, as is also obvious from the mass eigenstates at the SU(3) $\times$ U(1) stationary point [13]. However, the precise form of the mixing depends on the dynamics which is expected to deviate from $N=8$ supergravity, and which at this point is unknown. It is an essential assumption here that even with such mixing we can bring the fermions to the form with the labeling exhibited above, possibly by means of a chiral redefinition in SU(56). We then assume that all supersymmetries are broken at a high scale, such that all gravitinos acquire a very large mass.

Now, as first pointed out in [5], with the above choice of Goldstinos the three generations of SM fermions can be matched with the remaining 48 spin-$\frac{1}{2}$ fermions from (6), provided one identifies the supergravity SU(3) with the diagonal subgroup of color $\mathrm{SU}(3)_c$ and a new family symmetry $\mathrm{SU}(3)_f$, *viz.*

$$\mathrm{SU}(3) \equiv \big[\mathrm{SU}(3)_c \times \mathrm{SU}(3)_f\big]_{\rm diag} \tag{7}$$

and furthermore allows for a spurion charge $\pm\frac{1}{6}$ to correct the U(1) charges in (6) so as to recover the known electric charges of quarks and leptons. More specifically, the assignments are as follows [5]

$$\begin{array}{lll r}
(u,c,t)_L & \mathbf{3}_c \times \bar{\mathbf{3}}_f \;\rightarrow\; \mathbf{8} \oplus \mathbf{1} & \frac{2}{3} = \frac{1}{2} + q & \\
(\bar{u},\bar{c},\bar{t})_L & \bar{\mathbf{3}}_c \times \mathbf{3}_f \;\rightarrow\; \mathbf{8} \oplus \mathbf{1} & -\frac{2}{3} = -\frac{1}{2} - q & \\
(d,s,b)_L & \mathbf{3}_c \times \mathbf{3}_f \;\rightarrow\; \mathbf{6} \oplus \bar{\mathbf{3}} & -\frac{1}{3} = -\frac{1}{6} - q & \\
(\bar{d},\bar{s},\bar{b})_L & \bar{\mathbf{3}}_c \times \bar{\mathbf{3}}_f \;\rightarrow\; \bar{\mathbf{6}} \oplus \mathbf{3} & \frac{1}{3} = \frac{1}{6} + q & \\
(\nu_e,\nu_\mu,\nu_\tau)_L & \mathbf{1}_c \times \bar{\mathbf{3}}_f \;\rightarrow\; \bar{\mathbf{3}} & 0 = -\frac{1}{6} + q & \\
(\bar{\nu}_e,\bar{\nu}_\mu,\bar{\nu}_\tau)_L & \mathbf{1}_c \times \mathbf{3}_f \;\rightarrow\; \mathbf{3} & 0 = \frac{1}{6} - q & \\
(e^-,\mu^-,\tau^-)_L & \mathbf{1}_c \times \mathbf{3}_f \;\rightarrow\; \mathbf{3} & -1 = -\frac{5}{6} - q & \\
(e^+,\mu^+,\tau^+)_L & \mathbf{1}_c \times \bar{\mathbf{3}}_f \;\rightarrow\; \bar{\mathbf{3}} & 1 = \frac{5}{6} + q & (8)
\end{array}$$

where we made use of the fact that right-chiral particles can be equivalently described by their left-chiral anti-particles, and where the spurion shift is $q = -\frac{1}{6}$ (resp. $q = +\frac{1}{6}$) for triplets (resp. anti-triplets) of $\mathrm{SU}(3)_f$.

The main advance reported in [1] was to show that with the representation (6) of the $N=8$ spin-$\frac{1}{2}$ fermions the spurion shift can be accounted for by means of the $\mathrm{U}(1)_q$ transformations $\exp\left(\frac{1}{6}\omega\mathcal{I}\right)$, where

$$\mathcal{I} := \frac{1}{2}\Big(T{\otimes}\mathbf{1}{\otimes}\mathbf{1} + \mathbf{1}{\otimes}T{\otimes}\mathbf{1} + \mathbf{1}{\otimes}\mathbf{1}{\otimes}T + T{\otimes}T{\otimes}T\Big) \tag{9}$$

with $T \equiv Y(1,1)$ (cf. (2)). Because T acts as $+i$ (or $-i$) on an unbarred (barred) index it is easy to see that $\mathcal{I}$ gives $(+i)$ on φ's with no or only one barred index, and $(-i)$ on φ's with two or three barred indices. Importantly, the term with $T \otimes T \otimes T$, and hence the generator $\mathcal{I}$ are *not* elements of SU(8). However, in [4] it was shown that this new generator does belong to $\mathrm{K(E_{10})}$, and that furthermore the action of the generator $T \otimes T \otimes T$ can be extended to the gravitinos, from which it follows that $\mathcal{I} \circ \psi = T \circ \psi$. This implies that the action of $\frac{1}{6}\mathcal{I}$ adds $\pm\frac{1}{6}$ to the U(1) charges of the gravitinos in (4). As a consequence, ψ_μ^{a} has electric charge $\frac{1}{3}$, and $\psi_\mu^{\mathbf{4}}$ has electric charge $\frac{2}{3}$; $\psi_\mu^{\bar{\mathsf{a}}}$ and $\psi_\mu^{\bar{\mathbf{4}}}$ have the opposite electric charges.

While the new generator $\mathcal{I}$ commutes with the original SU(3) $\times$ U(1), and therefore merely deforms it but does not enlarge it, this is not so with the remaining generators of SU(8). As shown in [10], repeated commutation of $\mathcal{I}$ with the elements of SU(8) generates a much bigger group acting on χ^{ijk}, namely SU(56). Because $\mathcal{I} \in \mathrm{K(E_{10})}$, the latter group is also contained in $\mathrm{K(E_{10})}$, and should consequently be viewed as a subgroup of the quotient group $\mathfrak{G}_{RS} = \mathrm{Spin}(288{,}32)/\mathbb{Z}_2$. The fact that this SU(56) group acts chirally motivates us to look for a realization of the full SM symmetries within $\mathrm{K(E_{10})}$.

To this aim we now 'take the tri-spinor apart' into its 8 + 48 components, and write out the correspondence more explicitly for the left-chiral particles

$$\begin{aligned}
U_L^{\mathsf{a}\bar{\mathfrak{a}}} &\equiv (u^{\mathsf{a}}, c^{\mathsf{a}}, t^{\mathsf{a}})_L \equiv \big(\varphi^{\mathsf{a}\bar{\mathbf{1}}\mathbf{4}}, \varphi^{\mathsf{a}\bar{\mathbf{2}}\mathbf{4}}, \varphi^{\mathsf{a}\bar{\mathbf{3}}\mathbf{4}}\big) \\
D_L^{\mathsf{a}\mathfrak{a}} &\equiv (d^{\mathsf{a}}, s^{\mathsf{a}}, b^{\mathsf{a}})_L \equiv \big(\varphi^{\mathsf{a}\bar{\mathbf{2}}\bar{\mathbf{3}}}, \varphi^{\mathsf{a}\bar{\mathbf{3}}\bar{\mathbf{1}}}, \varphi^{\mathsf{a}\bar{\mathbf{1}}\bar{\mathbf{2}}}\big) \\
N_L^{\bar{\mathfrak{a}}} &\equiv (\nu_e, \nu_\mu, \nu_\tau)_L \equiv \big(\varphi^{\mathbf{23}\bar{\mathbf{4}}}, \varphi^{\mathbf{31}\bar{\mathbf{4}}}, \varphi^{\mathbf{12}\bar{\mathbf{4}}}\big) \\
E_L^{\mathfrak{a}} &\equiv (e^-, \mu^-, \tau^-)_L \equiv \big(\varphi^{\bar{\mathbf{2}}\bar{\mathbf{3}}\bar{\mathbf{4}}}, \varphi^{\bar{\mathbf{3}}\bar{\mathbf{1}}\bar{\mathbf{4}}}, \varphi^{\bar{\mathbf{1}}\bar{\mathbf{2}}\bar{\mathbf{4}}}\big)
\end{aligned} \tag{10}$$

and for the left-chiral *anti*-particles,

$$\begin{aligned}
\bar{U}_L^{\bar{\mathsf{a}}\mathfrak{a}} &\equiv (\bar{u}^{\bar{\mathsf{a}}}, \bar{c}^{\bar{\mathsf{a}}}, \bar{t}^{\bar{\mathsf{a}}})_L \equiv \big(\varphi^{\bar{\mathsf{a}}\mathbf{1}\bar{\mathbf{4}}}, \varphi^{\bar{\mathsf{a}}\mathbf{2}\bar{\mathbf{4}}}, \varphi^{\bar{\mathsf{a}}\mathbf{3}\bar{\mathbf{4}}}\big) \\
\bar{D}_L^{\bar{\mathsf{a}}\bar{\mathfrak{a}}} &\equiv (\bar{d}^{\bar{\mathsf{a}}}, \bar{s}^{\bar{\mathsf{a}}}, \bar{b}^{\bar{\mathsf{a}}})_L \equiv \big(\varphi^{\bar{\mathsf{a}}\mathbf{23}}, \varphi^{\bar{\mathsf{a}}\mathbf{31}}, \varphi^{\bar{\mathsf{a}}\mathbf{12}}\big) \\
\bar{N}_L^{\mathfrak{a}} &\equiv (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)_L \equiv \big(\varphi^{\bar{\mathbf{2}}\bar{\mathbf{3}}\mathbf{4}}, \varphi^{\bar{\mathbf{3}}\bar{\mathbf{1}}\mathbf{4}}, \varphi^{\bar{\mathbf{1}}\bar{\mathbf{2}}\mathbf{4}}\big) \\
\bar{E}_L^{\bar{\mathfrak{a}}} &\equiv (e^+, \mu^+, \tau^+)_L \equiv \big(\varphi^{\mathbf{234}}, \varphi^{\mathbf{314}}, \varphi^{\mathbf{124}}\big)
\end{aligned} \tag{11}$$

We emphasize again that the spinors (11) are not simply complex conjugates of (10), but independent fields (as would be completely obvious if we had written them as right-chiral particles). Thus, we now have color indices $\mathsf{a}, \mathsf{b}, ... \equiv \mathbf{1}, \mathbf{2}, \mathbf{3}$ and family indices $\mathfrak{a}, \mathfrak{b}, ... \equiv \mathbf{1}, \mathbf{2}, \mathbf{3}$ (or $([\bar{\mathbf{2}}\bar{\mathbf{3}}], [\bar{\mathbf{3}}\bar{\mathbf{1}}], [\bar{\mathbf{1}}\bar{\mathbf{2}}])$), thereby disentangling the diagonal SU(3) subgroup into its factors $\mathrm{SU}(3)_c$ and $\mathrm{SU}(3)_f$. Accordingly, we stipulate that $\mathrm{SU}(3)_c$ acts on indices $\mathsf{a}, \mathsf{b}, ...$ as $D^{\mathsf{a}\mathfrak{a}} \rightarrow i\lambda^{\mathsf{a}}{}_{\mathsf{b}} D^{\mathsf{b}\bar{\mathfrak{a}}}$, $U^{\mathsf{a}\bar{\mathfrak{a}}} \rightarrow i\lambda^{\mathsf{a}}{}_{\mathsf{b}} U^{\mathsf{b}\bar{\mathfrak{a}}}$ where λ is any of the (hermitean) Gell-Mann matrices. Similarly, the $\mathrm{SU}(3)_f$ acts on the indices $\mathfrak{a}, \mathfrak{b}, ...$, such that for instance $D^{\mathsf{a}\mathfrak{a}} \rightarrow i\lambda^{\mathfrak{a}}{}_{\mathfrak{b}} D^{\mathsf{a}\mathfrak{b}}$ and $U^{\mathsf{a}\bar{\mathfrak{a}}} \rightarrow -i(\lambda^*)^{\bar{\mathfrak{a}}}{}_{\bar{\mathfrak{b}}} U^{\mathsf{a}\bar{\mathfrak{b}}}$. The indices $\mathbf{4}$ and $\bar{\mathbf{4}}$ are unaffected by these SU(3) actions.

For the electroweak $\mathrm{SU}(2)_w$ we treat $(U, D)_L$ and $(N, E)_L$ as doublets, *viz.*

$$\begin{pmatrix} U^{\mathsf{a}\bar{\mathfrak{a}}} \\ D^{\mathsf{a}\mathfrak{a}} \end{pmatrix} \rightarrow i\boldsymbol{\omega}\cdot\boldsymbol{\tau} \begin{pmatrix} U^{\mathsf{a}\bar{\mathfrak{a}}} \\ D^{\mathsf{a}\mathfrak{a}} \end{pmatrix} \;,\quad \begin{pmatrix} N^{\bar{\mathfrak{a}}} \\ E^{\mathfrak{a}} \end{pmatrix} \rightarrow i\boldsymbol{\omega}\cdot\boldsymbol{\tau} \begin{pmatrix} N^{\bar{\mathfrak{a}}} \\ E^{\mathfrak{a}} \end{pmatrix} \;, \tag{12}$$

with the Pauli isospin matrices $\boldsymbol{\tau} \equiv (\tau^1, \tau^2, \tau^3)$ and isospin parameters $\boldsymbol{\omega}$. By contrast, the left-chiral anti-particles $(\bar{U}, \bar{D}, \bar{N}, \bar{E})$ are treated as $\mathrm{SU}(2)_w$ *singlets*, and this is perfectly consistent because these spinors are independent, as we already said. There is no need to discuss $\mathrm{U}(1)_Y$ separately, as the associated hypercharges can be recovered from the standard formula $Q = T^3 + Y$. It is an unaccostumed feature that the electroweak $\mathrm{SU}(2)_w$ does *not* commute with $\mathrm{SU}(3)_f$, as the upper and lower components of the would-be electroweak doublets are assigned to opposite representations of $\mathrm{SU}(3)_f$, such that the $\mathrm{SU}(3)_f$ is realized via the block matrix

$$\begin{pmatrix} (\mathcal{U}^*)^{\bar{\mathfrak{a}}}{}_{\bar{\mathfrak{b}}} & 0 \\ 0 & \mathcal{U}^{\mathfrak{a}}{}_{\mathfrak{b}} \end{pmatrix} \quad , \quad \mathcal{U} \in \mathrm{SU}(3)_f \tag{13}$$

in the electroweak isospin space. This prescription is somewhat reminiscent of old attempts to merge the spin rotation group SU(2) with the flavor symmetry SU(3) into SU(6), but we are here dealing with internal symmetries. The non-commutativity of $\mathrm{SU}(2)_w$ and $\mathrm{SU}(3)_f$ might explain why attempts at a group theoretical family unification have not met with success so far.

While obviously motivated by the known symmetry assignments of quarks and leptons the above construction does not tell us how these symmetries act on the gravitinos and Goldstinos, and more specifically, whether the SU(3) acting on them should be identified with $\mathrm{SU}(3)_c$ or $\mathrm{SU}(3)_f$. It is clear that for consistency this action must be the same on (4) and (5). It then appears reasonable to demand the absence of gauge anomalies for quarks and leptons to persist with these fields. The simplest choice ensuring this is to assign (4) and (5) to transform as $\mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{1} \oplus \bar{\mathbf{1}}$ of $\mathrm{SU}(3)_c$, but as singlets under electroweak $\mathrm{SU}(2)_w$ and $\mathrm{SU}(3)_f$. Apart from possibly upsetting anomaly cancellations, the gravitinos could acquire masses only via electroweak symmetry breaking, if they were assigned to chiral representations of $\mathrm{SU}(2)_w$ (in this case the largest anomaly-free subgroup acting chirally on the gravitinos would be SU(2) × U(1) [17]). As a consequence we are led to predict *fractionally charged* and *strongly interacting* massive gravitinos, implying a novel menagerie of exotic Dark Matter candidates, possibly even fractionally charged massive bound states of gravitinos and quarks.

Having realized the action of all SM symmetries on all fermions φ, we can now express these actions directly in terms of the original tri-spinor χ^{ijk}, that is, in the form

$$\chi^{ijk} \rightarrow M^{ijk}{}_{lmn}(\mathrm{G})\chi^{lmn} \tag{14}$$

with

$$M(\mathrm{G}) = \mathrm{S}^{-1} \circ \mathrm{G} \circ \mathrm{S} \, \in \, \mathrm{SU}(56) \tag{15}$$

where $\mathrm{G} \in \mathrm{SU}(3)_c \times \mathrm{SU}(2)_w \times \mathrm{U}(1)_Y$ or $\in \mathrm{SU}(3)_f$. The matrix $M(\mathrm{G})$ is real [that is, $M \in \mathrm{SO}(56)$] for the vector-like rotations in $\mathrm{SU}(3)_c$, $\mathrm{SU}(3)_f$ and $\mathrm{U}(1)_{em}$, and complex (but still unitary) for chiral electroweak rotations; more generally, transformations that do not mix barred and unbarred indices are vectorlike, whereas transformations that do mix them are chiral. As there is no space here to present a complete list of 56-by-56 matrices we have collected explicit formulas in [16].

We have already pointed out that a realization of the present scheme would require part of $\mathrm{K}(\mathrm{E}_{10})$ to become dynamical (with composite vector bosons $W^{\pm}$ and Z). How this could happen dynamically, we do not know. Similarly, there remains the question why the theory should pick a vacuum where $\mathrm{K}(\mathrm{E}_{10})$ is broken to the SM symmetries in the way described above, and how supersymmetry is ultimately disposed of (it is an open question whether supersymmetry can be made compatible with $\mathrm{K}(\mathrm{E}_{10})$ at all [18]). Here we can at least refer back to the requirement that the residual gauge group that emerges after symmetry breaking should exhibit the standard cancellations of gauge anomalies for the SM fermions. Given the split-up of the 48 fermions into three families, this requirement would seem to fix the SM symmetry assignments, and thus single out the breaking pattern shown above almost uniquely, independently of the details of the dynamics. We recall that the original SU(8) R-symmetry of $N = 8$ supergravity is non-anomalous, provided one includes the contribution of the 28 electric and 28 magnetic vector fields [19].

The scheme proposed here is obviously very speculative, and much effort will be required to validate it, assuming there is any truth in it. Like for the bosonic sector [6], the main challenge remains to incorporate the spatial dependence into this framework. However, our proposal has two main advantages, namely (1) as far as it goes, it does agree with observation (confirming in particular the existence of only three generations of SM fermions), and (2) it would be directly and immediately falsifiable by the detection of any new fundamental spin-$\frac{1}{2}$ fermion, be it via the discovery of low energy $N = 1$ supersymmetry or of new sterile fermions. Moreover, it underlines the potential relevance of the so far elusive algebras E_{10} and $\mathrm{K}(\mathrm{E}_{10})$ for unification: it would be truly remarkable if these enigmatic structures were really needed to make contact between M theory and SM physics.

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