

Inconsistencies of the New No-Boundary Proposal

Comment on “Damped Perturbations in the No-Boundary State”

by Diaz Dorronsoro *et al.* (arXiv:1804.01102)

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In previous works, we have demonstrated that the path integral for *real, Lorentzian* four-geometries in Einstein gravity yields sensible results in well-understood physical situations, but leads to unstable fluctuations when the “no boundary” condition proposed by Hartle and Hawking is imposed. In order to circumvent our result, new definitions for the gravitational path integral have been sought, involving specific choices for a class of *complex* four-geometries to be included. In their latest proposal, Diaz Dorronsoro *et al.* [1] advocate integrating the lapse over a complex circular contour enclosing the origin. In this note we show that, like their earlier proposal, this leads to mathematical and physical inconsistencies and thus cannot be regarded as a basis for quantum cosmology.

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I. INTRODUCTION

The no-boundary proposal of Hartle and Hawking [2] has been an influential idea in theoretical cosmology for more than three decades, and with good reason: it puts forth a proposal for the initial state of the universe, from which everything else is supposed to follow. If true, it would do no less than explain the origin of space and time. What is more, the proposal involves only semi-classical gravity, *i.e.*, a theoretical framework already within reach of contemporary physics, without requiring the development of a full theory of quantum gravity. Given the magnitude of this claim, it should be analyzed with care. In previous works [3, 4] we attempted to put the no-boundary proposal on a sound mathematical footing by defining the gravitational path integral more carefully. Unfortunately, we found as a consequence that the no-boundary proposal leads to a universe with large fluctuations which are out of control. Our work led Diaz Dorronsoro *et al.* to propose a new definition of the no-boundary proposal, involving an inherently complex contour in the space of four-metrics, *i.e.*, one which cannot be deformed to an integral over real four-metrics and hence has no geometrical interpretation. In particular, they chose to integrate the lapse N over a complex contour running below the origin in the complex N -plane, from negative to

positive infinite real values [5]. In follow-up work [6], we demonstrated the inconsistency of this proposal. Very recently, Diaz Dorronsoro *et al.* have proposed yet another definition of the no-boundary proposal, this time in a particular truncation of Einstein gravity and taking instead a complex contour for the lapse which encircles the origin [1]. In this note we show that this latest incarnation of the no-boundary idea also leads to physical and mathematical inconsistencies.

II. PHYSICAL MOTIVATION

The path integral over four-geometries provides a well-motivated framework for the study of semi-classical quantum gravity. In analogy with Feynman's path integral formulation of quantum mechanics, one attempts to define transition amplitudes between two three-geometries $h_{ij}^{(0)}, h_{ij}^{(1)}$ by summing over all four-geometries that interpolate between the initial $h_{ij}^{(0)}$ and final boundary $h_{ij}^{(1)}$, *i.e.*,

$$G[h_{ij}^{(1)}, h_{ij}^{(0)}] = \int_{\partial g = h_{ij}^{(0)}}^{\partial g = h_{ij}^{(1)}} \mathcal{D}g e^{iS[g]/\hbar}, \quad (1)$$

where g denotes the four-metric. In this note, as in the work of Diaz Dorronsoro *et al.*, we study a simplified model in which $S[g]$ is taken to be the usual action for Einstein's theory of gravity plus a positive cosmological constant Λ .

In our previous works [3, 6] we demonstrated that, somewhat to our surprise, the path integral, over *real, Lorentzian* four-geometries yields well-defined and unique results *as it stands*, when evaluated semiclassically and in cosmological perturbation theory, *i.e.*, when we treat the four-geometry as a homogeneous, isotropic background with small, but otherwise generic, perturbations. In contrast, we found the path integral over Euclidean four-geometries (as originally advocated by Hartle and Hawking [2]), even at the level of the homogeneous, isotropic background, to be a meaningless divergent integral. The key to our work was the use of Picard-Lefschetz theory, a powerful mathematical framework that allows one to rewrite highly oscillatory and only conditionally convergent integrals (such as (1) turns out to be) as absolutely convergent integrals. To do so, one regards the integral (1) over real four-metrics as being taken over a real subspace in the space of complex four-metrics. Cauchy's theorem then allows one to deform the original, real integration

domain into a complex domain consisting of one or more steepest descent integrals, which are each absolutely convergent and whose sum yields the same result as the original integral. Note that Picard-Lefschetz theory, and the analytical continuation to complex metrics, are merely a convenient (although very powerful) calculational tool, used to evaluate the original, uniquely defined integral over real, Lorentzian four-geometries, in a saddle point approximation.

One frequently raised question is the range over which the lapse N should be integrated over in the path integral. The Lorentzian four-geometries we consider may be parameterized with the line element $-N^2(t, x)dt^2 + h_{ij}(t, x)dx^i dx^j$, where $0 \leq t \leq 1$ is a good time-like coordinate, *i.e.*, a one to one, invertible map from the manifold into the closed unit interval. The lapse N accounts, for example, for the proper time interval τ between two spacetime points (t_1, x^i) and (t_2, x^i) , both at fixed x^i : one has $\tau = \int_{t_1}^{t_2} N(t, x^i)dt$. Note that the coordinate t already defines an orientation for the integral: the lapse N is simply a local rescaling, which must therefore be taken strictly positive as long as the coordinate chart and the manifold are both nonsingular. Stated more generally, assigning a non-singular coordinate system to a four-manifold already introduces an orientation, allowing one to define integrals such as the action or measures of volume, area or length. Writing the metric as usual by $g_{\mu\nu} = e_\mu^A e_\nu^B \eta_{AB}$, with e_μ^A the frame field and η_{AB} the Minkowski metric, only one continuously connected component of non-singular frame fields e_μ^A – for example the component with strictly positive eigenvalues – is needed in order to describe a general, nonsingular four-geometry. To sum over additional components (for example to sum over both positive and negative lapse functions N while taking the determinant h to be positive) is not only unnecessary, it represents an overcounting which is unjustified from a geometrical point of view. Furthermore, although arbitrarily small N should be allowed, one should *not* include the point $N = 0$ in the sum since it does not describe a four-geometry. Finally, integrating over all Lorentzian four-geometries requires only *real* (and positive) values of N . If that fundamental, geometrical definition can be deformed into a mathematically equivalent integral over complex metrics which is easier to calculate, as Picard-Lefschetz theory and Cauchy’s theorem allow, that is all well and good. But it makes little geometrical sense to take an integral over complex lapse functions N as a fundamental definition of the theory.

In their most recent paper, Diaz Dorronsoro *et al.* [1] misrepresent our work by stating that we “have recently advanced a larger class of wavefunctions that extend the original”

no boundary wavefunction. Quite to the contrary, what we explained in our earlier papers is that the integral over Lorentzian four-geometries is actually *unique*! This allowed us to compute *the only* geometrically meaningful “no boundary wavefunction.” The fact that calculation failed to give an observationally acceptable result is not the fault of the path integral for gravity, but rather that of imposing the “no boundary” idea in this particular model, attempting to describe the beginning of the universe in the context of inflationary scenarios.

In fact, it is Diaz Dorronsoro *et al.*, not us, who are “advancing a larger class of wavefunctions” in an attempt to rescue the no boundary proposal. As we have explained, there is no geometrical justification for taking an integral over complex metrics as a starting point for the theory. Yet this is exactly what they propose [1]. They consider metrics of the axial Bianchi IX form

$$ds^2 = -\frac{N^2}{q}dt^2 + \frac{p}{4}(\sigma_1^2 + \sigma_2^2) + \frac{q}{4}\sigma_3^2, \quad (2)$$

where $p(t), q(t)$ are time dependent scale factors and $\sigma_1 = \sin \psi d\theta - \cos \psi \sin \theta d\varphi$, $\sigma_2 = \cos \psi d\theta + \sin \psi \sin \theta d\varphi$, and $\sigma_3 = -(d\psi + \cos \theta d\varphi)$ are differential forms on the three sphere with $0 \leq \psi \leq 4\pi$, $0 \leq \theta \leq \pi$, and $0 \leq \phi \leq 2\pi$. For real $N > 0$, this metric describes Bianchi IX spacetimes on the axes of symmetry. In the well-known notation of Misner [7] this corresponds to the line $\beta_- = 0$. Diaz Dorronsoro et al. now propose to define the gravitational path integral as a sum over real values of p and q , supplemented by a sum over values of the lapse function N , taken along a *complex* circular contour enclosing the origin.

In our view this proposal is quite arbitrary, as it is not motivated by any fundamental physical principle. What does it mean to integrate over metrics with complex proper time intervals? In [1], this sum over specific complex metrics is regarded not merely as a calculational device, but as the starting definition of the theory. Furthermore, this definition seems context dependent. Such a definition will neither allow one to calculate meaningful transition amplitudes between two large three-geometries nor to understand how quantum field theory on curved space-time emerges when the scale factor evolves classically. It is unclear how this definition should be implemented for more general metrics – for instance, in some contexts the lapse integral will contain poles at various locations, leading to an ambiguity about which of these poles the proposed circular contour should enclose [8]. Given its poor

motivation, we find it unsurprising that this definition ultimately leads to mathematical and physical inconsistencies, as we shall explain in the remainder of this note.

III. NORMALIZABILITY

With the metric (2), the action for gravity plus a cosmological constant Λ , in units where $8\pi G = 1$, is given by

$$S/(2\pi^2) = \int_0^1 dt \left[-\frac{1}{4N} \left(\frac{q\dot{p}^2}{p} + 2\dot{p}\dot{q} \right) + N \left(4 - \frac{q}{p} - p\Lambda \right) \right], \quad (3)$$

where the integrals over the angular directions yields a factor of $16\pi^2$. We here evaluate the classical action and subsequently apply Picard-Lefschetz theory to approximate the path integral. We finish with a discussion on the normalizability of the resulting “wavefunction”.

A. The classical action

The equations of motion corresponding to the variation of q and p are given by

$$2p\ddot{p} - \dot{p}^2 = 4N^2, \quad (4)$$

$$\ddot{q} + \frac{\dot{p}}{p}\dot{q} = N^2 \left(2\Lambda - \frac{4q}{p^2} \right). \quad (5)$$

The constraint following from the variation of N is given by

$$\frac{1}{4} \left(\frac{q}{p}\dot{p}^2 + 2\dot{p}\dot{q} \right) + N^2 \left(4 - \frac{q}{p} - p\Lambda \right) = 0. \quad (6)$$

The equation of motion for p , with the boundary conditions $p(t=0) = p_0, p(t=1) = p_1$, has two solutions [1]

$$p(t) = p_0 - 2 \left(p_0 \pm \sqrt{p_0 p_1 - N^2} \right) t + \left(p_0 + p_1 \pm 2\sqrt{p_0 p_1 - N^2} \right) t^2. \quad (7)$$

In the case of no-boundary conditions, we assume the initial volume of the universe to vanish

and set $p_0 = 0$. The corresponding classical action takes the form [1]

$$S/(2\pi^2) = \int_0^1 dt \left(-\frac{1}{2N} \frac{d}{dt} (\dot{p}q) + N(4 - p\Lambda) \right) \quad (8)$$

$$= \frac{1}{N} \left(-p_1 q_1 \mp \sqrt{-N^2} q_1 \right) + N \left(4 - \frac{\Lambda}{3} p_1 \pm \frac{\Lambda}{3} \sqrt{-N^2} \right), \quad (9)$$

with the boundary conditions $q(t=0) = 0, q(t=1) = q_1$.

We thus obtain a classical action, up to an ambiguity, as we must decide which classical solution for p dominates the integral. For large $|N|$ the classical action is dominated by the last term. With the convention that $\sqrt{-N^2} = +iN$, we are forced to choose the upper sign solution of equation (7) in order to obtain a convergent path integral or the integration domain $(0, \infty)$ for N , yielding

$$S_{conv}/(2\pi^2) = -\frac{p_1 q_1}{N} - i q_1 + N \left(4 - \frac{\Lambda}{3} p_1 \right) + i \frac{\Lambda}{3} N^2. \quad (10)$$

This choice is in conflict with the ‘‘momentum constraint’’ imposed in [1] (in fact it corresponds to the opposite ‘‘momentum constraint’’), but it is mandatory, as the original integral over real fields was already convergent. In order to be able to comment on some aspects of the calculations in [1] we will also consider their choice of solution for p , *i.e.*, the lower sign in (7), which leads to the complex conjugate of the action shown in (10),

$$S_{div}/(2\pi^2) = -\frac{p_1 q_1}{N} + i q_1 + N \left(4 - \frac{\Lambda}{3} p_1 \right) - i \frac{\Lambda}{3} N^2. \quad (11)$$

B. Picard-Lefschetz theory

Having reduced the path integral to an ordinary integral over the lapse function N , we are now in a position to evaluate it in the saddle point approximation. Figures 1 and 2 show the locations of the saddle points and steepest ascent/descent lines emanating from them for the two choices of the action given in (10) and (11) respectively. It is straightforward to see that the integral over real Lorentzian metrics (in Fig. 1) can be deformed into the steepest descent contour \mathcal{J}_1 passing through saddle point 1. The location of this saddle point for various values of p_1 and fixed q_1 is shown in Fig. 3. For large anisotropies it moves closer

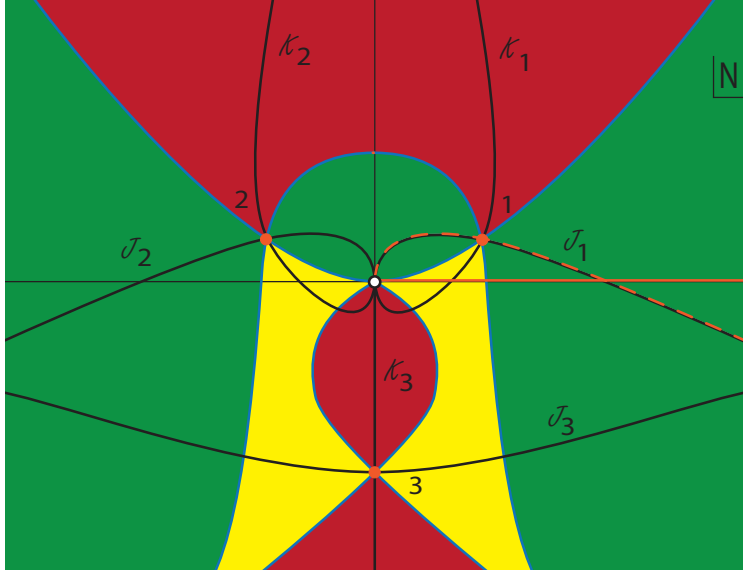


Figure 1: The location of the saddle points and flow lines for the action we advocate, Eq. (10), for which the Lorentzian integral is convergent. The saddle points are indicated by the orange dots. Green regions have a lower magnitude of the integrand than at the adjacent saddle point, red regions have a higher magnitude and yellow regions have a magnitude in between two saddle point values. If N approaches the singular point at infinity or the essential singularity at $N = 0$ along a contour in a green region, we obtain a convergent integral. Conversely, if N approaches these points along a contour in a red region, the integral diverges.

and closer to the real N line, without however ever reaching it. The induced weighting is shown by the blue curve of the left panel in Fig. 4, where it can be seen that the isotropic boundary conditions (here $p_1 = q_1 = 10000$) receive the lowest weighting. In other words, the model is unstable, as more anisotropic geometries are favoured. This is in agreement with our earlier findings regarding small fluctuations around the isotropic background solution [4, 6]. It is evident that for very small p_1 the weighting becomes unbounded. However, at large p_1 the behaviour is also pathological, as the weighting tends to a constant. One can derive simple analytical approximations to the saddle point location and the corresponding action in the limit where both p_1 and q_1 are large, (directly related expressions were already presented in [1]),

$$N_1 \approx \sqrt{\frac{3q_1}{\Lambda}} + i\frac{3}{\Lambda} \frac{q_1}{p_1}, \quad p_1, q_1 \gg \frac{3}{\Lambda} \quad (12)$$

$$S_{conv}(N_1) \approx -2\sqrt{\frac{\Lambda}{3}} \sqrt{q_1 p_1^2} + 4i\frac{3}{\Lambda} \frac{q_1}{p_1}, \quad p_1, q_1 \gg \frac{3}{\Lambda}. \quad (13)$$

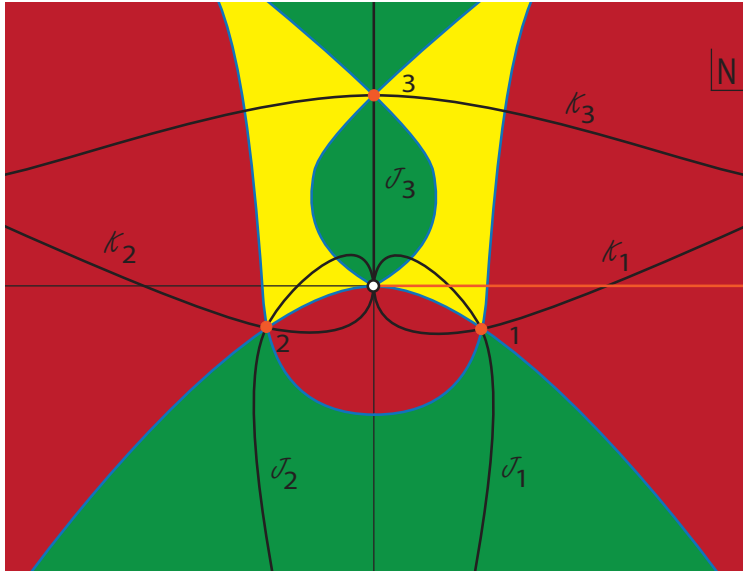


Figure 2: The location of the saddle points and flow lines for the action in Eq. (11), for which the Lorentzian path integral diverges, but which is chosen by Diaz Dorronsoro *et al.* [1]. For a description of the colour scheme, see the caption of Fig. 1. Note that, as this Figure shows, it would also have been possible to define a purely Euclidean contour along the positive imaginary axis for this choice of action, and this would have led to only saddle point 3 contributing. This latter saddle point leads to a purely Euclidean geometry, without any classical Lorentzian evolution.

At large p_1 these expressions confirm that the relevant Lorentzian saddle point becomes almost real, and that the imaginary part of the action approaches zero. Hence an integral of the weighting $e^{-2\text{Im}(S_{conv})/\hbar}$ over p_1 is unbounded, so that one obtains a non-normalizable probability distribution. Note that the saddle point approximation becomes more and more accurate at large p_1 , as the second derivative around the saddle point becomes larger – see the right panel in Fig. (4). Thus, we can reliably conclude that the theory is not only unstable, but is also non-normalizable. We attribute this pathology to a failure of the no-boundary condition, similar to that we have previously identified, while the authors of [1] attribute this pathology to a wrong choice of integration contour for the lapse function together with a wrong choice in the solution (7). We have already argued in favour of our choices above, but it is instructive to see that the very same issue of non-normalizability also occurs for the choices made by Diaz Dorronsoro *et al.* [1].

As shown by Diaz Dorronsoro *et al.*, when choosing a circular contour around the essential singularity at $N = 0$ for the action (11), this contour can be deformed to a sum over the two steepest descent paths \mathcal{J}_1 and \mathcal{J}_2 in Fig. 2. These saddle points lie respectively at the

complex conjugate and negative values of the Lorentzian saddle point 1 in Fig. 1 (and whose asymptotic location at large p_1 is given by (12)). The weighting of these saddle points is just the inverse of the weighting of the Lorentzian saddle point, and is shown by the orange curve in the left panel of Fig. 4. For these the isotropic configuration $p_1 = q_1$ is indeed the configuration with the highest weighting. However, having a maximum is not enough to ensure normalizability. Indeed, just as for the Lorentzian saddle point, the weighting of these saddle points tends to a constant at large values of p_1 (the inverse of a constant being another constant), so that again an integral of the weighting $e^{-2Im(S_{div})/\hbar}$ over p_1 is unbounded, and the corresponding wavefunction is non-normalizable. Thus if one regards normalizability as a crucial criterion, the new circular contour must also be discarded.

For reasons that are not clear to us, the authors of [1], even though they also noticed the unboundedness of the integral, simply chose to truncate the inconvenient integral by hand. The stated reason was that the approximations involved in the calculation break down. We find this statement puzzling, as the axial Bianchi IX model is attractive precisely because it allows one to calculate the action analytically, and moreover the saddle point approximation becomes better and better at large p_1 (see again the right panel of Fig. 4, which also applies to the saddle points in question). Thus the implied non-normalizability seems robust to us, to the extent that normalizability can be rigorously established at all. In this context it should be emphasized however that normalizability is in fact rather difficult to define precisely, until much more becomes known about measures in quantum gravity. For now, we simply record that the the new no-boundary proposal offers no advance in this regard.

IV. MATHEMATICAL AND PHYSICAL CONSISTENCY

We now come to what we regard as the biggest flaw in the proposal of Diaz Dorronsoro *et al.*, namely that it seems to us to fail some simple tests of physical and mathematical consistency. When we take the limit in which the final three-geometry is isotropic, it seems reasonable to expect that we should recover the result of the truncated, isotropic theory, at least in the semi-classical limit where quantum backreaction is negligible. Likewise, if we add an additional metric perturbation mode to the final three-geometry, for example one of an inaccessibly small wavelength, this should not immediately lead to inconsistent results. We will discuss these two tests of their proposal, in turn.

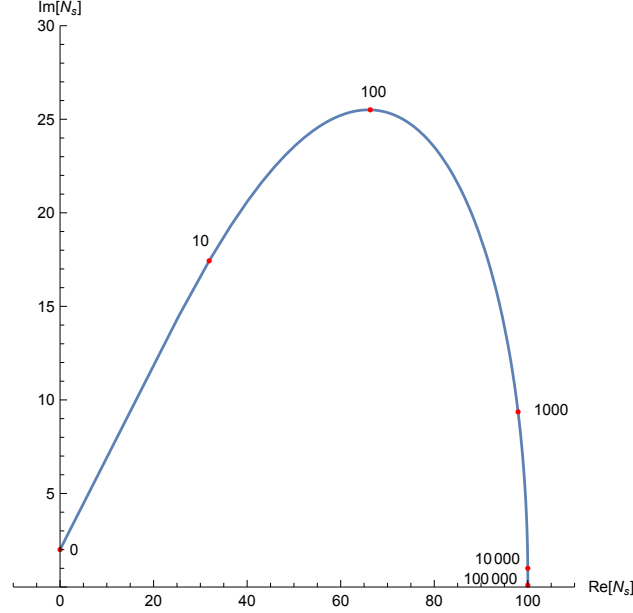


Figure 3: The location of the relevant saddle point, for fixed $q_1 = 10000$ and as a function of $0 < p_1 < 100000$. Some indicative values of p_1 are shown next to the curve. At large values of p_1 the saddle point remains complex but moves very close to the real N line.

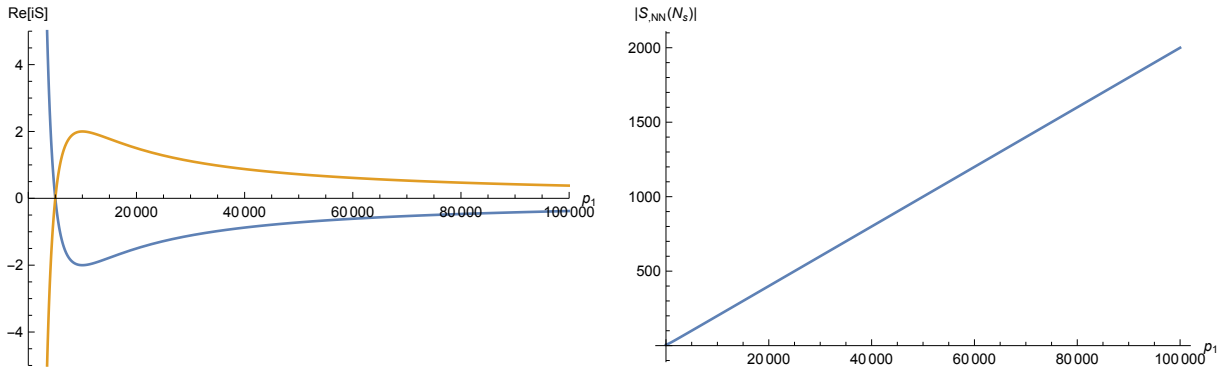


Figure 4: *Left:* The Morse function $-Im(S_{conv,div})$ for fixed $q_1 = 10000$ and as a function of $0 < p_1 < 100000$ both for the relevant Lorentzian saddle point (blue) and for one of the saddle points advocated by Diaz Dorronsoro *et al.* (orange). *Right:* The absolute value of the second derivative at the same saddle points for fixed $q_1 = 10000$ (with $\Lambda = 3$) and as a function of $0 < p_1 < 100000$. For large p_1 the saddle point approximation becomes better and better.

First, consider the isotropic limit, where $p_1 = q_1$. Here we would expect the axial Bianchi IX model to reproduce the results of an isotropic FLRW minisuperspace model, defined using the same integration contour for the lapse function. Certainly, for Lorentzian integrals, this is the case and the model implied by Eq. (10) indeed reproduces our earlier isotropic results

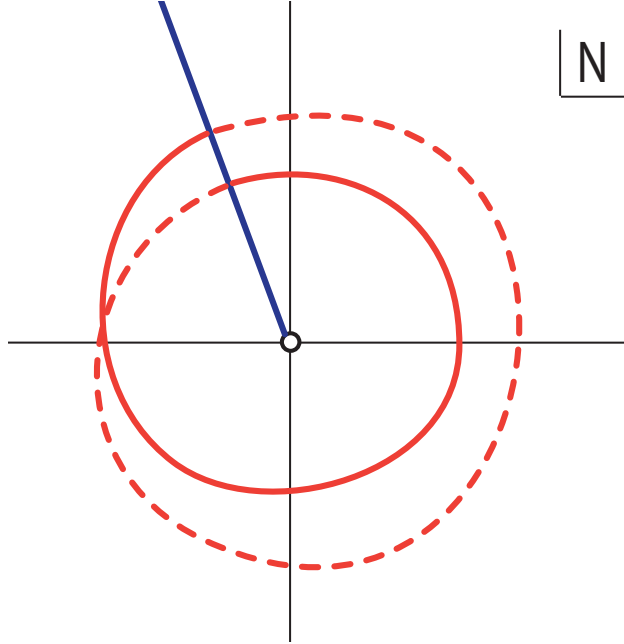


Figure 5: An example of a circular contour in the presence of a branch cut (in blue), for a two-sheeted integrand. The contour must complete two loops before it can close.

of [3]. When $p_1 = q_1$, the relevant saddle point of the action (10) is located at

$$N_{s1}^{iso} = \sqrt{\frac{3}{\Lambda}} \sqrt{q_1 - \frac{3}{\Lambda}} + i \frac{3}{\Lambda}, \quad (14)$$

i.e., it resides at the same value of N as for the isotropic model, where the action is given by a different function of N , namely [3, 9]

$$S^{iso}(N)/2\pi^2 = \left[N^3 \frac{\Lambda^2}{36} + N \left(3 - \frac{\Lambda}{2} q_1 \right) - \frac{3q_1^2}{4N} \right]. \quad (15)$$

Moreover, at the isotropic saddle point (14), the values of the axial Bianchi IX action (10) and the isotropic action (15) agree,

$$S_{conv}(N_{s1}^{iso}) = S^{iso}(N_{s1}^{iso}) = 2\pi^2 \left(-\frac{2\sqrt{3}}{\Lambda} (\Lambda q_1 - 3)^{3/2} + i \frac{6}{\Lambda} \right). \quad (16)$$

Thus we find a well-behaved isotropic limit, as we believe we should, since in the isotropic limit we are describing the same physical situation.

However, when we take the circular contour advocated by Diaz Dorronsoro *et al.*, a

problem arises. In the isotropic case, the path integral reduces to an ordinary integral over the lapse function of the form [3, 9]

$$G[q_1, 0] = \sqrt{\frac{3\pi i}{2\hbar}} \int \frac{dN}{\sqrt{N}} e^{iS^{iso}(N)/\hbar}. \quad (17)$$

The prefactor, which arises from the integral over the isotropic scale factor, contains a factor of $1/\sqrt{N}$, so that there is a branch cut in the integrand, emanating from the origin. This branch cut requires that a circular contour must complete two loops around the origin before it can close – see Fig. 5. However, on the second loop the factor $1/\sqrt{N}$ will acquire a minus sign relative to its value on the first loop, so that the contributions from the second loop exactly cancel those of the first loop. The result is that, for isotropic boundary conditions, a closed circular contour yields precisely zero! Hence there is blatant disagreement with the isotropic limit of the Bianchi IX model, although the physical situation being described is identical. (One may easily verify that the saddle points contributing to the path integral with final boundary $p_1 = q_1$ also have $p(t) = q(t)$ throughout the entire geometry $0 \leq t \leq 1$). Hence this choice of contour fails to satisfy our consistency check.

The second inconsistency manifests itself in the following, closely related manner. In minisuperspace models, when we include n deformations of the metric in addition to the lapse, the prefactor generally takes the form $1/N^{n/2}$ [9]. For n odd, the integrand will thus be taken around a branch point at $N = 0$ and a closed contour about the origin will again yield a vanishing result. But the results of our calculations should not depend on how many possible deformations we include as long as the same physical situation is described. One should be able to add a possible deformation and then consider boundary conditions in which this additional deformation is zero – and the results should, at this leading semi-classical level, be unchanged. A straightforward example is to use the full Bianchi IX metric and then restrict to boundary conditions corresponding to the axial Bianchi IX truncation studied in this paper. Once again this does not lead to consistent results, as the Bianchi IX metric contains one additional deformation, so that a closed contour enclosing the origin again leads to a vanishing wavefunction.

V. DISCUSSION

When constructing theories of the very early universe, the difficulty of making direct observations means that mathematical and physical consistency requirements must necessarily play a critical, guiding role. In our view, the new path integral for semi-classical gravity advocated by Diaz Dorronsoro *et al.* [1], involving a closed integral for the complexified lapse function, seems inadequate in this regard: it has no geometrical interpretation as it involves metrics with complex proper times. Likewise, it abandons any notion of causality from the outset. Furthermore, when describing the same physical situation using different truncations of the degrees of freedom in the spacetime metric, it yields vastly different results. The clearest example is provided by truncating the model to an isotropic, one-dimensional minisuperspace, for which a closed contour about the origin yields a vanishing “wavefunction.” More generally, such a closed contour fails to yield a meaningful wavefunction for any odd-dimensional truncation of minisuperspace – violating the seemingly reasonable requirement that including one additional mode, for example one with an inaccessibly small wavelength, should not change any physical result.

In contrast, the Lorentzian path integral for gravity stands out for its remarkable physical and mathematical consistency [3, 4, 6]. The fact that it leads to unstable fluctuations when no-boundary conditions are imposed in an attempt to define initial conditions for inflation, should be regarded as a failure of the no boundary proposal in this context, rather than one of the path integral for gravity.

Acknowledgments

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