

# Probing the existence of ultralight bosons with a single gravitational-wave measurement

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**We demonstrate that gravitational waves from binary systems can provide smoking gun evidence for ultralight bosons (such as ultralight axions). If ultralight bosons exist, they will form “clouds” by extracting rotational energy from astrophysical black holes of size comparable to the boson Compton wavelength through superradiant instabilities [1–3]. The properties of the cloud are intimately related with those of the black hole, and they are encoded in the gravitational waves emitted by compact objects orbiting the black hole/cloud system. We show that a single measurement of these waves yields at least three independent ways to estimate the mass of the boson from the cloud. Gravitational wave observations by the Laser Interferometric Space Antenna (LISA) could either confirm the existence of ultralight bosons and measure their mass via “consistency tests” similar to the general relativity tests routinely performed with binary pulsars, or rule out the cloud’s existence.**

Light bosonic particles, such as ultralight axions, are a popular dark matter candidate proposed to address problems ranging from fundamental physics to cosmology [4–6]. A runaway instability due to superradiance [1–3] can convert the rotational energy of astrophysical black holes to a non-axisymmetric cloud of bosonic matter when the boson Compton wavelength  $\lambda = \hbar/(m_s c)$  is comparable with the black hole’s Schwarzschild radius  $R = 2GM/c^2$ , i.e. when  $R/\lambda = 0.15(M/10^6 M_\odot)(m_s c^2/10^{-17} \text{eV}) \sim 1$ .

Superradiance sets the geometry of the “host” black hole by extracting mass and angular momentum until the black hole/cloud system reaches equilibrium at some critical values of the black hole mass  $M$  and dimensionless spin  $j = a/M = J/M^2$ ; vice versa, for a given boson mass  $\mu_s \equiv m_s/\hbar$ , the black hole geometry sets the shape of the boson cloud [1, 2] (from now on we will use geometrical units,  $G = c = 1$ ). The gravitational field generated by this cloud can be computed using standard techniques [2] and it affects the inspiral of compact objects, such as stellar-mass black holes.

Since superradiant instabilities make specific predictions on the shape of the bosonic cloud, light boson masses can be measured (or excluded) by looking at the extreme mass ratio inspiral (EMRI) of small compact objects into the black hole [7]. Ultralight bosons leave a characteristic imprint in the gravitational radiation emitted as the object spirals into the central black hole, which is quite different from the effect

of dark matter minispikes [8–10] or perturbations due to astrophysical accretion disks [11, 12]. This radiation is emitted at the low frequencies accessible by the space-based interferometer LISA, which will therefore allow us to map *both* the matter distribution and the black hole geometry.

Here we show that a single gravitational-wave measurement yields at least *three independent ways* to measure the boson mass. To allow for the measurement, our working hypothesis throughout is that superradiant instability has occurred and the black hole/cloud system is in equilibrium during a measurement. Comparisons of these independent boson mass measurements can be used as a self-consistency test to either confirm the existence of ultralight bosons and measure their masses or (as in binary pulsar experiments) to rule out the hypothesis. This is because gravitational waves allow us to measure to exquisite accuracy the host black hole properties ( $M, a$ ), which – given our hypothesis – gives us a model prediction for the boson cloud profile. We can also directly measure the properties of the boson cloud profile (as encoded in two “shape parameters”  $A$  and  $B$ , defined below) and confirm that they match the model predictions in a way that does not allow for free model parameters.

The superradiant instability quickly extracts rotational energy from the black hole, leading the black hole/cloud system to equilibrium on a so-called “Regge trajectory” [1], where the rotational frequency of the boson (which from now on, for simplicity, we assume to be a scalar field) is comparable to the black hole rotational frequency [1, 2]:

$$\mu_s^{(1)} \simeq \frac{a}{2Mr_+}, \quad (1)$$

where  $r_+ = \sqrt{M^2 - a^2} + M$  is the outer horizon of the rotating black hole. This occurs on the superradiant instability time scale  $\tau_{\text{inst}} \sim 10^5 \text{yr} j^{-1} (10^6 M_\odot/M)^8 (10^{-17} \text{eV}/\mu_s)^9$  [13, 14]. Once the (non-axisymmetric) boson cloud has grown, it dissipates through gravitational waves on a time scale  $\tau_{\text{GW}} \sim 5 \times 10^{11} \text{yr} j^{-1} (10^6 M_\odot/M)^{14} (10^{-17} \text{eV}/m_s)^{15}$ . For a typical LISA observation time,  $T_{\text{obs}} \sim 1 \text{yr} \ll \tau_{\text{inst}} \ll \tau_{\text{GW}}$ . Therefore, our hypothesis that the superradiant instability has occurred (on  $\tau_{\text{inst}} \ll \tau_{\text{GW}}$ ) and that the black hole/cloud system is in equilibrium during a measurement ( $T_{\text{obs}} \ll \tau_{\text{GW}}$ ) is justified. The black hole/cloud system will remain in equilibrium even if there is accretion, because the (Salpeter) accretion timescale  $\tau_{\text{acc}} \gg T_{\text{obs}}$  [2].

To a good approximation, the scalar field profile in the equilibrium configuration is well described by  $\varphi(t, r, \theta, \phi) = ABre^{-Br/2} \cos(\phi - \omega_R t) \sin \theta$  [2, 7]. Here  $A$  is the scalar field amplitude,  $B = M\mu_s^2$  is a “scale” parameter (note that

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the radial profile of the cloud has a maximum at  $r_{\max} = 2/B$ , and  $\omega_R \simeq \mu_s$ . Both  $A$  and  $B$  are determined through independent physical processes:  $A$  is set by the evolution of the black hole/cloud system, while  $B$  is set by the black hole geometry when the black hole/cloud system is in equilibrium. For typical black hole/cloud systems of interest  $M\mu_s \sim 1$ , so that the field oscillation time scale  $\sim 1/\omega_R$  is of the order of seconds (hence much shorter than the LISA observation time  $T_{\text{obs}}$ ) when  $M \sim 10^6 M_\odot$ . Therefore we can time-average the gravitational potential generated by the cloud. This leads to a time-averaged gravitational potential in the equatorial plane of the form  $\Phi_b(r) = \Phi_b(A, B, M, r)$  [7], which is given explicitly in Eq. (7) of the Supplementary Material. By imposing that  $\Phi_b(r) \sim -M_s/r$  at large  $r$ , where  $M_s$  is the total mass in the boson cloud, we can relate the scalar field amplitude to its mass:  $A = (\mu_s^2 M^{3/2} M_s^{1/2})/[8\pi(4 - \mu_s^2 M^2)]^{1/2}$ . Therefore, if we can measure the amplitude  $A$  and scale  $B$  of the scalar cloud we get two more estimates of the boson mass:

$$\mu_s^{(2)} = (M/B)^{-1/2} \quad (2)$$

and

$$\mu_s^{(3)} \simeq 2 \left[ \frac{\pi A^2}{M M_s} \left( \sqrt{1 + \frac{2M_s}{A^2 M \pi}} - 1 \right) \right]^{1/2}. \quad (3)$$

To infer  $\mu_s^{(3)}$ , we require an estimate for the enclosed boson mass  $M_s$ . Although  $M_s$  can be obtained from the evolution of the black hole/cloud system [2] given  $\mu_s^{(1)}$  measured from Eq. (1), the host's initial spin and accretion rate are unknown, and therefore only a maximum value for  $M_s$  can be set. We assume that  $M_s$  can be anywhere between  $M_s \in [0, M_s^{\max}]$ , and we fix  $M_s^{\max}$  by assuming that the initial black hole spin (pre-superradiant amplification) is maximal. Even under this conservative estimate, we find that the ultralight boson hypothesis can be correctly confirmed (or ruled out) in the two cases we consider below.

From an observational standpoint there is no reason why three independent measurements of the boson mass using Eqs. (1), (2) and (3) should yield the same result, unless the superradiant instability hypothesis is correct. The gravitational waveform emitted by the EMRI of a small compact object orbiting the black hole/cloud system encodes both the host geometry and the gravitational potential of the cloud, making it possible to either confirm this hypothesis if the measurements are self-consistent, or rule it out if they aren't. In other words, a measurement of either  $\mu_s^{(i)}$  ( $i = 1, 2, 3$ ) gives the boson mass only *if* the boson cloud exists. However, a self-consistent measurement of *more than one*  $\mu_s^{(i)}$  confirms the existence of the cloud.

EMRI observations by LISA can measure both the mass and spin of the host black hole to better than 1% accuracy [15]. Matter effects may be resolved when the density of the surrounding material is sufficiently high: in fact, such matter effects are resolvable even when the density is much smaller than expected from boson clouds [8]. Therefore, as we show below, the tests we just outlined can be performed

with LISA EMRI observations. For illustration: if the mass  $M = 10^5 M_\odot$  and spin  $a = 0.6M$  can be resolved to within 1% error,  $A$  and  $B$  may be measured to an accuracy 10%, taking the 95% confidence interval of  $M_s \in [0, 0.1M]$ . Then the three estimates of the ultralight boson particle  $\mu_s^{(1)}$ ,  $\mu_s^{(2)}$  and  $\mu_s^{(3)}$  would have measurement errors 12%, 8% and 69%, respectively.

As a proof of principle, we conduct a simple simulation to show that it is indeed possible to confirm ultralight bosons' existence with LISA. We construct an EMRI gravitational-wave template in the black hole/cloud potential of Eq. (7), where  $A$  and  $B$  are free parameters. Following [8], to compute the evolution we include the lowest post-Newtonian (PN) order in the phasing as well as the leading order contribution from matter effects. We also add spin-dependent PN corrections to the *inspiral* waveform (as implemented in [16]), which allows us to estimate the black hole spin [17–19]. Since the waveform includes matter effects, an EMRI observation allows us to infer *both* the boson cloud and host black hole properties: in particular, by matched filtering we can recover the masses and (aligned) spin of the central black hole, as well as the boson cloud amplitude and steepness parameters ( $A$  and  $B$ ).

To be specific, we consider gravitational waves from a stellar-mass black hole ( $m = 60 M_\odot$ ,  $a' = 0$ ) inspiralling into a supermassive black hole ( $M = 10^5 M_\odot$ ,  $a = 0.6M$ ) surrounded by a cloud generated by bosons of mass  $\mu_s = 2.26 \times 10^{-16}$  eV, with total cloud mass  $M_s = 0.05M$ , one year observation time and a LISA signal-to-noise ratio  $(h, h)^{1/2} = 97$  which corresponds to redshift  $z \sim 1$  [15, 20]. We use a nested sampling Markov-Chain Monte Carlo algorithm [21] to recover three independent posteriors  $\mu_s^{(i)}$  ( $i = 1, 2, 3$ ) from measurements of  $M$ ,  $a$ ,  $A$  and  $B$ . Figure 1 (top panel) shows that in this case we confirm the ultralight boson hypothesis, because all three measurements overlap. In the bottom panel of Figure 1 we consider instead the gravitational wave signal produced by a small compact object falling into a black hole surrounded by a dark matter minispikes with  $\rho_{\text{sp}} = 4.0 \times 10^5 M_\odot/\text{au}^3$ ,  $\alpha = 1$  and  $r_{\text{sp}} = 6M$  (see "Methods" for the motivation for the parameters) [8, 9, 22]. In this case we can rule out ultralight bosons as a source of the matter distribution, because the recovered ultralight boson masses do not overlap.

Astrophysical EMRIs can probe a wide range of boson particle masses. We simulated 1 year of LISA EMRI observations for different boson masses, finding that the growth of the cloud can have measurable effects across the mass range  $10^{-17}$  eV  $\lesssim \mu_s \lesssim 10^{-14}$  eV. The lower bound can be improved by extending the observation time. However,  $\mu_s \lesssim 10^{-20}$  eV result in small (less than one gravitational-wave cycle) and probably unmeasurable effects on the waveform even if the binary and observation parameters are fine tuned. Black hole/cloud systems with  $\mu_s \gtrsim 10^{-14}$  eV would produce EMRI signals outside of the LISA band.

Our work is meant to be a proof-of-principle demonstration that binary pulsar-like tests of ultralight dark matter are possible through EMRI observations with LISA, but the practical implementation of this program will require further work.

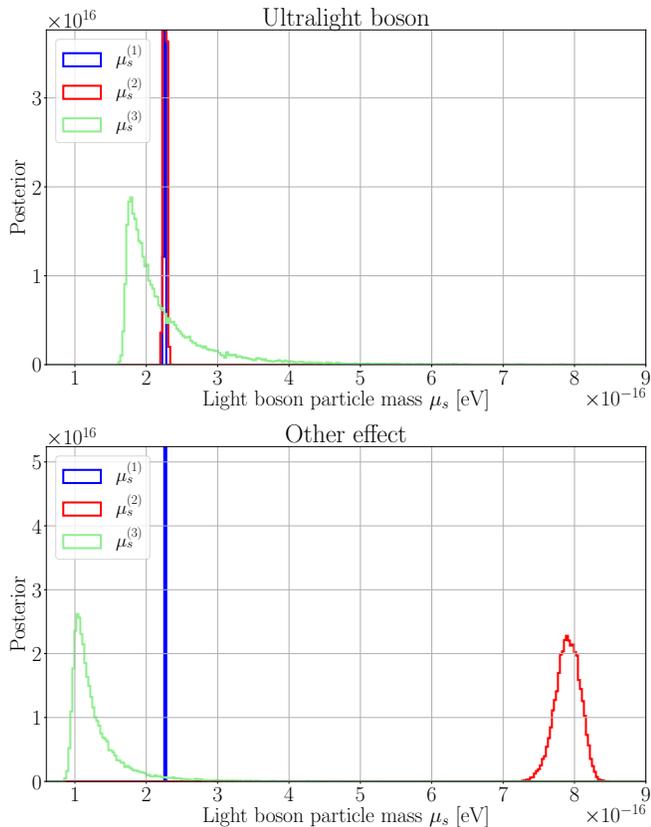


FIG. 1. Three independent posterior distribution measurements of ultralight boson particle masses  $\mu_s^{(1)}$  (red),  $\mu_s^{(2)}$  (blue) and  $\mu_s^{(3)}$  (green) [see Eqs. (1)-(3)] from a single gravitational-wave observation with LISA. Top: the signal is produced by an EMRI into a black hole/cloud system with  $\mu_s = 2.26 \times 10^{-16}$  eV. All three measurements overlap with each other, providing smoking-gun evidence for ultralight bosons. Bottom: the black hole has the same properties, but the “cloud” is produced by a dark matter minispikes. Measurements do not overlap, ruling out the boson cloud hypothesis.

Gravitational waves from EMRIs can be computed to high accuracy for astrophysical massive black holes in isolation, which are characterized only by their mass and spin, but surrounding matter can back-react on the binary. Back-reaction effects are small for the high mass ratios considered here, but they can be relevant for comparable-mass binaries. Furthermore, it will be interesting to take into account the possibility of “mode mixing”: perturbations due to the small orbiting companion can mix superradiating modes with “dumping” (infalling) levels of the cloud, causing the cloud to collapse before the binary can trace its properties [23]. If this occurs, our test would yield a null result with non-overlapping distributions for the  $\mu^{(i)}$ ’s, as we would not measure the effects of the cloud. This is an active area of research: recent work suggests that the boson cloud would in fact survive mode mixing in the high mass ratio scenario studied in this paper when the small object is in a corotating orbit [24], and then a measurement would be possible.

The cloud’s potential could, in principle, look degenerate

with the potential of a more massive Kerr black hole. We have accounted for possible degeneracies between the binary parameters and matter effects using nested sampling to simultaneously infer the binary properties (masses, spins), the matter properties and the merger time (maximizing over phase of the wave). Figure 1 shows that the consistency test can be performed despite such degeneracies for signal-to-noise ratios expected for LISA [15].

Following [2, 7], we focus on the most unstable mode as it accounts for most of the matter distribution. Higher modes are unlikely to be observed, because they would only become unstable on much longer timescales ( $\tau_{\text{inst}}^{\text{HM}} \sim 10^7$  years for the case considered here) [14]. This is appropriate for the present order-of-magnitude estimate of the effect of the matter distribution. For simplicity we assumed that the small compact objects is in an equatorial orbit and we computed the gravitational potential for real scalar fields, however our results also apply to complex scalar fields where the potential is stationary (see e.g. [25]). In the same spirit we used PN waveforms with aligned spins, as opposed to more realistic, fully precessing EMRI waveforms with eccentricity [26–28]. These corrections will matter in LISA data analysis, but they only contribute a fraction of the total phase shift, and so they can be omitted for order-of-magnitude estimates. We have also checked the convergence of the PN expansion by comparing the accumulated phase shift of the highest term relative to the next-to-highest term, finding the difference to be negligible at the percent level. Another interesting effect is that, because the potential of a boson cloud is not spherically symmetric, orbital resonances could result in angular momentum transfer between the companion and the cloud with an increase in orbital eccentricity [7]. These resonances are an interesting topic for future study, but we verified that they do not occur for the orbital parameters considered here.

In conclusion, the possibility to obtain three independent measurements of ultralight boson masses which can be cross-compared for consistency is largely unaffected by the corrections listed above. We have demonstrated that LISA EMRI observations can be used to confirm (or rule out) the formation of ultralight boson condensates around astrophysical black holes. More accurate waveform models and more accurate treatments of superradiance (including higher-order modes and possible transitions among superradiant states) will be needed for an implementation of this idea in LISA data analysis.

## METHODS

**Posterior estimation.** We consider a LISA EMRI signal from a black hole/cloud system and use Nested Sampling (as implemented in `MultiNest` [21, 29, 30]) to evaluate the posterior distribution of our measurement, with the likelihood defined for colored gaussian noise following the LISA power spectral density (PSD) [31]

$$\log \mathcal{L} = (s, h(\vec{\theta})) - \frac{1}{2}(h(\vec{\theta}), h(\vec{\theta})). \quad (4)$$

Here  $s$  is the injected signal, which we assume to be noiseless (this is approximately true at high signal-to-noise (SNR), as in our chosen scenario) and  $h(\vec{\theta})$  is the gravitational-wave template at 3.5 PN order [17–19] for a detector oriented optimally for the plus polarized wave, which we use in our parameter estimation to sample over both binary and matter parameters  $\vec{\theta}$ . The gravitational potential due to the cloud is included at lowest PN order, following [8]:

$$h(f) = h(f)e^{i(\psi(f) + \Delta\psi_{\text{matter}}(f) + 2\pi ft_c + 2\phi_c)}, \quad (5)$$

where  $h(f)$ ,  $\psi(f)$  are the amplitude and phase of the gravitational wave,  $\Delta\psi_{\text{matter}}(f)$  is the phase shift due to matter effects, and  $(t_c, \phi_c)$  are the time and phase of coalescence.

The inner product  $(a, b)$  is defined as

$$(a, b) = 4\Re \left[ \int_0^\infty \frac{a(f)b^*(f)}{S_n(f)} df \right], \quad (6)$$

where  $S_n(f)$  is the LISA PSD [31]. We take the absolute value of the inner product for the purpose of maximizing over phase of coalescence (see [32, 33]). In our parameter estimation, we simulate a single gravitational-wave event with given parameters and sample over binary masses  $(m_1, m_2)$ , aligned spins  $(s_1, s_2)$ , time of coalescence  $t_c$ , and boson cloud parameters  $(A, B)$ : see Eq. (7).

**Waveform.** We estimate the waveform by perturbing the energy balance equation, following [8]. The gravitational potential produced by the boson cloud is [7]

$$\begin{aligned} \Phi_b(r) \simeq & [\pi A^2 e^{-Br} (-MB^6 r^5 - 2B^5 r^4 (M - 2r) \\ & - 12B^4 r^3 (M - 2r) + 8B^3 r^2 (10r - 3M) \\ & - 16B^2 r (M - 10r) + 16e^{Br} (B^3 M r^2 - 4B^2 r^2 \\ & + BM - 12) - 16B(M - 12r) + 192)] / (2B^4 M r^3). \end{aligned} \quad (7)$$

We expand the phase shift to first order in  $\Phi_b/\Phi_{\text{BH}}$ , where  $\Phi_{\text{BH}}$  is the gravitational potential in the absence of the cloud. This introduces a correction to Kepler’s law and to the energy balance, and changes the accumulated orbital phase shift. The phase shift due to matter can be computed from the orbital energy balance equation [8]

$$\frac{dE_{\text{orbit}}}{dt} + \frac{dE_{\text{gw}}}{dt} = 0, \quad (8)$$

where

$$\begin{aligned} \frac{dE_{\text{orbit}}}{dt} &= \frac{d}{dt} \left( \frac{1}{2} \mu v^2 + \mu \Phi(r) \right), \\ \frac{dE_{\text{gw}}}{dt} &= \frac{32\mu^2 r^4 \omega^6}{5}, \end{aligned} \quad (9)$$

and  $\Phi(r) = -M/r + \Phi_b(r)$ . Here  $\mu$  is the small compact object mass,  $v$  is the velocity of the companion and  $\omega$  is the orbital angular frequency. This gives the rate of change of the orbital radius

$$r'(t) = -\frac{32\mu^2 r^4 \omega^6}{5 [\mu v v'(r) + \mu \Phi'(r)]}, \quad (10)$$

which can be translated to the total gravitational-wave phase shift using the stationary phase approximation [34]:

$$\Delta\psi_{\text{matter}} = 2\pi f t(f) - 2\phi(f), \quad (11)$$

where the time and orbital phase are given by

$$\begin{aligned} t(f) &= \int \frac{1}{r'(t)} dr, \\ \phi(f) &= \int \frac{\omega}{r'(t)} dr. \end{aligned} \quad (12)$$

The mapping between the orbital radius and the gravitational-wave frequency may be solved by inverting the following relation for  $r$

$$\omega(f) = \pi f = \sqrt{\frac{\Phi'(r)}{\mu r}}, \quad (13)$$

and expanding to first order in  $\epsilon = \Phi_b/\Phi_{\text{BH}}$  [8]. We have included first-order corrections from matter effects, and verified that second-order corrections cause negligible phase corrections in the gravitational waveform at percent level in the case that we consider here.

**Dark matter minispikes.** To illustrate a case where our black hole/cloud test discriminates other effects from boson clouds, we consider a black hole surrounded by a dark matter minispikes (constructed loosely following Ref. [8]). We assume the minispikes density to follow a power-law

$$\rho(r) = \rho_{\text{sp}} \left( \frac{r}{r_{\text{sp}}} \right)^{-\alpha}, \quad (14)$$

where  $\rho_{\text{sp}}$  and  $r_{\text{sp}}$  are the density and radius normalization constants, and  $\alpha$  gives the steepness of the profile. We follow [8] to construct the orbital phase shift of the gravitational wave due to the minispikes. To assess whether the dark matter minispikes can mimic a boson cloud we first construct the orbital phase shift due to a dark matter spike, then we fit the results using a boson cloud, but treating the  $A$  and  $B$  parameters as free. We set  $\rho_{\text{sp}} = 3 \times 10^5 M_\odot/\text{au}^3$ ,  $\alpha = 1$  and  $r_{\text{sp}} = 6M$ , which causes an orbital shift of similar order as the boson cloud in our example scenario.

We chose the dark matter minispikes parameters to mimic boson cloud effects. If we had chosen the dark matter minispikes profile expected to form through adiabatic growth from a seed black hole in a typical cuspy dark matter environment with density  $\sim \text{GeV}/\text{cm}^3$  at 100 kpc [35], or if we had chosen different values of  $\alpha$ , the discriminatory power of our test would have improved even further.

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