Visiting Some Relatives of Peirce's *

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Abstract

The notion of a relational grammar is extended to ternary relations and illustrated by a fragment of English. Some of Peirce's terms for ternary relations are shown to be incorrect and corrected.

Binary relations have been studied extensively by Peirce and Schröder in the nineteenth century, and in this century by Tarski and his students. No comparable attention has been paid to ternary relations. This is surprising, for Peirce had already dealt with ternary relations on various occasions. But Schröder strictly confined himself to binary relations, and it is the topic of binary relations that has become the focus of interest for Tarski and his students. For notable exceptions see Carnap (1929), Copi and Harary (1953), and Aubert (1955) and the theory of relational database systems.

Peirce has illustrated his ternary relational terms by natural language examples. For instance, the equation

$$(b^a)^m = b^{(am)}$$

where b = *betray*, a = *enemy*, and m = *man* was explained in English as follows:

> "those individuals each of which stand to every man in the relation of betrayer to every enemy of his are identical with those individuals each of which is a betrayer to every enemy of a man of that man."[1]

This may be hard to swallow and even Peirce himself had some problems here as we shall see. Therefore I think that Peirce's discussion of ternary relations can best be studied in a framework that is as rigorous with respect to syntactic structure as it is to semantic structure. Such a method is provided by relational grammar. Relational grammar was proposed in Suppes (1976).

The purpose of this paper is therefore both to extend the notion of a relational grammar by adding ternary relations and to apply this extension to a better understanding of Peirce's writings about relations, or "relatives" to use his own term. Our focus is therefore more on the establishment of mapping natural language into the language of relation algebra than on the development of the algebra of ternary relations. The paper continues my work on relational grammar and builds especially on previous results on anaphoric pronouns in Böttner (1992, 1994, 1997).

The paper is organized as follows: in section 1, the notion of a relational grammar is introduced. In section 2, this notion is extended in order to account for ternary relations. In section 3, an example of a ternary relational grammar is given. In section 4, our analyses are compared to Peirce's examples. In section 5, our results are discussed and put in perspective.

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[1] Peirce (1870: 379).

1 Relational Grammar

A *denoting grammar* is a context-free phrase structure grammar that provides a semantic function for each production rule.[2] A *relational grammar* is a denoting grammar with the restriction that denotations are elements of an extended relation algebra over some set D.[3] An *extended relation algebra* over D is any collection of subsets of and binary relations over D that is closed with respect to *union*, *complement*, *conversion*, *composition* and *Peirce product*.[4] In line with Brink, Kahl and Schmidt (1997) we use the following notation:

- *union*: $A \cup B$
- *complement*: $\overline{A}$
- *intersection*: $A \cap B$
- *Cartesian product*: $A \times B$
- *conversion*: $\breve{R}$
- *composition*: $R;S$
- *Peirce product*: $R : A$

In addition, we shall use the following operations that can be defined in terms of the previous operations:

- *domain*: $domR = \{x \in D | (\exists y)(xRy)\}$
- *(progressive) involution*: $R^A = \overline{(\overline{R} : A)}$
- *range-restriction of R by A*: $R \restriction A = R \cap (D \times A)$

We refer to the identity relation by I. We refer to the maximal subset of D by the constant U and to the maximal binary relation over D by the constant V.

An example of a relational grammar is the following:

PRODUCTION RULE		*SEMANTIC FUNCTION*	
$NP \rightarrow$	$TN + P + EQ + NP$	$[NP] =$	$[TN] : [NP]$
$NP \rightarrow$	$TN + P + UQ + NP$	$[NP] =$	$[TN]^{[NP]}$
$NP \rightarrow$	N	$[NP] =$	$[N]$

The symbols abbreviate the names of conventional grammatical categories: NP = noun phrase, N = common noun, TN = transitive noun, P = preposition, UQ = universal quantifier, EQ = existential quantifier. A lexicon for this grammar would be as follows:

P	*of, to*
EQ	*some, a, an*
N	*flower, lady, horse, ...*
UQ	*each, every*
TN	*owner, enemy, lover, woman, ...*

[2]Suppes (1973).
[3]Suppes (1976).
[4]Suppes (1976).

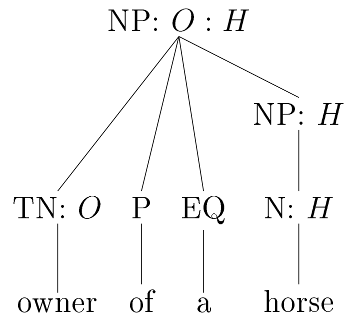

Figure 13.

This grammar derives semantic trees for terms like, e.g., *owner of a horse* or *owner of every horse*. A semantic tree is a derivation tree in the sense of the theory of formal languages where the nodes of the tree, in addition to their category labels, bear denotations as semantic labels. An example of a semantic tree is given in Figure 1.

2 Ternary Relations

Relational grammar is restricted to subsets of some domain D and binary relations over D. This would not be sufficient to provide meanings for sentences like, e.g., *Mary is sitting between John and Bill* or *John gives Mary a book* since *between* denotes a ternary relation and so does *give*. We therefore shall extend our ontology by ternary relations.

One way to introduce ternary relations is to introduce them as Cartesian products of a binary relation over D and a subset of D. This definition, however, has the following drawback. One and the same ternary relation gets two representations that need to be identified by stating separately

$$< a, < b, c >> = << a, b >, c > .$$

We therefore prefer to start from ordered triples and define a ternary relation as a set of ordered triples.

Relational operations have been defined for binary relations. Adding ternary relations requires a slight redefinition of our relational operations. In the case of union and intersection it is understood that both operands should be of the same type, i.e. either D, D^2 or D^3. In the case of complement of X we understand the complement with respect to either D, D^2, or D^3 depending on the type of X.

We assume two operations R^t and R^c as primitive: R^t switches the last two places of a ternary relation and R^c moves the first place of a ternary relation to the end. The operation that reverses a ternary relation R can be expressed by the composition of a transposition and a cyclic permutation: R^{tc}.

Since binary relations are sets, the operation of a *Peirce product* can be generalized to ternary relations provided that $m < n$. Let R denote an n-ary relation on D and let S denote an m-ary relation. Then the *generalized Peirce product* of R and S is defined

$$R : S := \{< x_1, ..., x_{n-m} > |(\exists x_{n-m+1})...(\exists x_n)(Sx_{n-m+} \wedge Rx_1...x_n)\}. \qquad (31)$$

This definition looks rather complicated but in fact captures only three cases: either R is binary and S is unary, or R is ternary and S is unary, or R is ternary and S is binary. If in particular R is a ternary relation over D and $S \subseteq D$, then $R : S$ is a binary relation over D, and if R is a ternary relation over D and S is a binary relation over D, then $R : S \subseteq D$.

In a similar fashion, the operation of *range-restriction* is generalized

$$R \upharpoonright S := \{< x_1, ..., x_n > \in R | < x_{n-m}, ..., x_n > \in S\} \tag{32}$$

where R is an n-ary relation and S is an m-ary relation. If R is a binary relation and S some subset of the domain the operation coincides with the operation defined in section 1. If R is a ternary relation and S is a subset of the domain $R \upharpoonright S$ denotes a binary relation over D. If R is a ternary relation and S is a binary relation over the domain $R \upharpoonright S$ denotes a subset of D.

Composition is defined as an operation on the set of binary relations. We extend this operation to pairs of a ternary relation R and a binary relation S like this:

$$R \circ_{3,2} S = \{< x, y, z > |(\exists u)(Rxyu \wedge uSz)\}. \tag{33}$$

Therefore

$$(R \circ_{3,2} S)xyz \text{ iff } (\exists u)(Rxyu \wedge uSy\}.$$

Since *dom* can be defined in terms of Peirce product, it shares this ambiguity with it: if R is a binary relation, then $domR$ is a set, and if R is a ternary relation, then $domR$ is a binary relation.

Since involution can be defined in terms of Peirce product and complement, a notion of *generalized involution* can be defined

$$R^S := \overline{(\overline{R} : S)}. \tag{34}$$

Many more operations can of course be defined in the context of ternary relations. But since our main focus is on the interaction of ternary relations with either binary relations or sets, so-called exterior operations will be more important than interior operations involving just the set of ternary relations. We have therefore refrained from defining various types of composition since we have not found them exemplified in any construction of English.

Definition 1 *Let D be some nonempty set. An extended ternary relation algebra of sets over D is any subset of*

$$\mathcal{P}(D) \cup \mathcal{P}(D^2) \cup \mathcal{P}(D^3)$$

that is closed with respect to

i. *union*

ii. *complement*

iii. *transposition*

iv. *cycle*

v. *composition*

vi. *composition of a a ternary relation with a binary relation*

vii. *generalized Peirce product*

viii. *generalized domain-restriction.*

Notice that this list of operations appears to lack conversion. But in fact it occurs twice because both transposition and cycle coincide with conversion in the case of binary relations. Notice that we do not have composition of two ternary relations because this would return a quaternary relation. This does not mean that quaternary relations of this kind do not arise in natural language. Peirce himself has given the example *praiser of – to a maligner of – to –*.[5]

Some simple arithmetical properties of operations of ternary relations are listed below.

Theorem 1 *Let $R \subseteq D^3$.*

i. $R^{tt} = R$.

ii. $R^{ccc} = R$.

iii. If X, Y are either both subsets of D or both binary relations over D, then

$$R : (X \cup Y) = (R : X) \cup (R : Y).$$

iv. If A and B are arbitrary subsets of D, then

$$(R^t : A) : B = (R : B) : A.$$

v. If A and B are subsets of D, then

$$((R^t)^B) : A \subset (R : A)^B.$$

Proof of Theorem 1.

i. This is simply an extension of the binary case.

ii. This follows from the fact that a cyclic transposition of a set with thre elements needs to be applied three times to return the original set.

iii. Obvious.

iv. The left hand side is equivalent to

$$(\exists z)(z \in B \wedge (\exists y)(y \in A \wedge R^t xzy)).$$

The right hand side is equivalent to

$$(\exists y)(y \in A \wedge (\exists z)(z \in B \wedge Rxyz)).$$

Since

$$R^t xzy \leftrightarrow Rxyz,$$

both expressions are equivalent.

v. By (34), the equation can be reduced to

$$\overline{\overline{R^t} : B} : A \subset \overline{\overline{R : A} : B}.$$

The left hand side is equivalent to

$$(\exists z)(z \in A \wedge (\forall y)(y \in B \rightarrow Rxyz)).$$

The right hand side is equivalent to

$$(\forall y)(y \in B \rightarrow (\exists z)(z \in A \wedge Rxyz)).$$

Since the second follows from the first the theorem is proved. Note that this property cannot be strengthened to equality, since both expressions are not equivalent.

[5] Peirce (1902).

PRODUCTION RULE		SEMANTIC FUNCTION
$VP \to$	$TVP + EQ + NP$	$[VP] = [TVP] : [NP]$
$VP \to$	$TVP + UQ + NP$	$[VP] = [TVP]^{[NP]}$
$TVP \to$	TV	$[TVP] = [TV]$
$TVP \to$	$DV + EQ + NP + P$	$[TVP] = [DV] : [NP]$
$TVP \to$	$DV + UQ + NP + P$	$[TVP] = [DV]^{[NP]}$
$VP \to$	$DV + EQ + NP + P + EQ$ $+TN + P + Dem + NP$	$[VP] = dom(([DV];[TN]) \cap (D^3 \upharpoonright I)) : [NP]$
$VP \to$	$DV + EQ + NP + P + EQ$ $+TN + P + Pers$	$[VP] = dom(([DV];[TN]) \cap (D^3 \upharpoonright I)) : [NP]$
$VP \to$	$DV + EQ + NP + P + UQ$ $+TN + P + Pers$	$[VP] = dom(([DV];[TN]) \cap (D^3 \upharpoonright I)) : [NP]$
$VP \to$	$DV + EQ + NP + UQ + NP'$	$[VP] = ([DV] : [NP'])^{[NP]}$
$VP \to$	$DV + UQ + NP + EQ + NP'$	$[VP] = ([DV]^{[NP']}) : [NP]$
$VP \to$	$DV + EQ + NP + EQ + NP'$	$[VP] = ([DV] : [NP']) : [NP]$
$VP \to$	$DV + UQ + NP + UQ + NP'$	$[VP] = ([DV]^{[NP']})^{[NP]}$

Table 5. Ternary Extension of Relational Grammar

3 Grammar Extension

To derive semantic trees for English expressions we propose the grammar of Table 5. Familiar grammatical categories are referred to by the following additional symbols: TVP = transitive verb phrase, DV = ditransitive verb, Dem = demonstrative pronoun, and Pers = personal pronoun.

A lexicon for the extended grammar would be as follows:

P	*to*
Dem	*that*
DV	*give, betray, ...*
Pers	*him, her, it, them*

Ditransitive verbs differ from monotransitive verbs like, e.g., *own* by having two objects rather than one. One object is called the *direct object* (DO), the other object is called the *indirect object* (IO). A paradigm ditransitive verb is *give*. In the verb phrase *gives a flower to some lady* the direct object is *a flower* and the indirect object is *(to) a lady*. According to our extended grammar, the semantic tree for this verb phrase would be the one shown in Figure 14 where F and L are subsets of D denoted by the noun *flower* and *lady*, respectively, and $G = \{< x, y, z >\}$ is a ternary relation on D denoted by the ditransitive verb *give* where x denotes the giver, y denotes the receiver, and z denotes the object given.

For the expression *betray a woman to a man* our grammar derives the denotation

$$(B : W) : M$$

where M is the subset of D denoted by *man*, W is the subset of D denoted by *woman* and B is a ternary relation on D denoted by *betray*. That this denotation provides the correct denotation follows from the fact that it is equivalent to the set

$$\{x | (\exists y)(y \in M \land (\exists z)(z \in W \land Bxyz)\}. \quad (35)$$

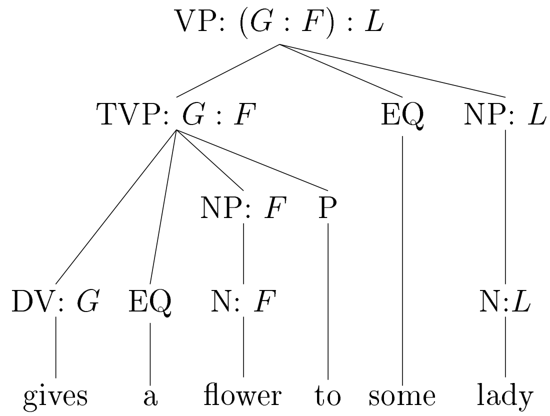

Figure 14.

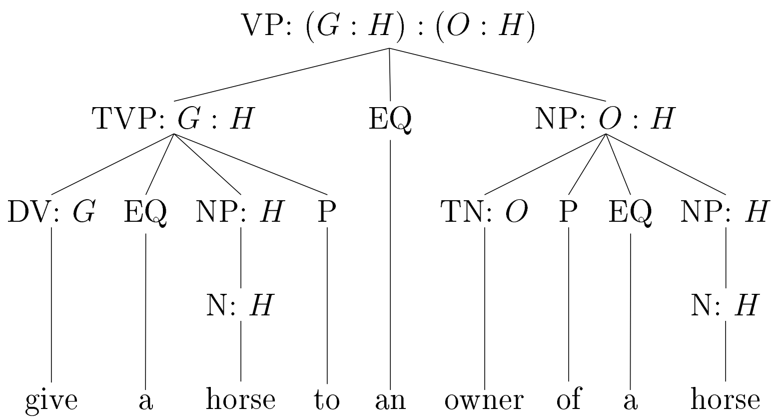

Figure 15.

For the expression *betray every woman to every man* our grammar derives the denotation

$$(B^W)^M. \tag{36}$$

Applying our definition, we have

$$(B^W)^M = \overline{(\overline{B^W} : M)} = \overline{(\overline{\overline{(\overline{B} : W)}} : M)} = \overline{(\overline{B} : W) : M}. \tag{37}$$

An element z of this set fulfils the condition

$$(\forall z)(z \in M \rightarrow (\forall y)(y \in W \rightarrow Bxyz)), \tag{38}$$

and this captures the intuitive meaning of the verb phrase in question.

Our grammar also derives semantic trees for expressions with binary relations occurring in argument position. An example would be the tree in Figure 15 where H is a set denoted by *horse*, O is a binary relation denoted by *owner* and G is a ternary relation as in (14). For the expression

betray each man to an enemy of every man (39)

our grammar derives the term

$$(B^M) : (A^M) \tag{40}$$

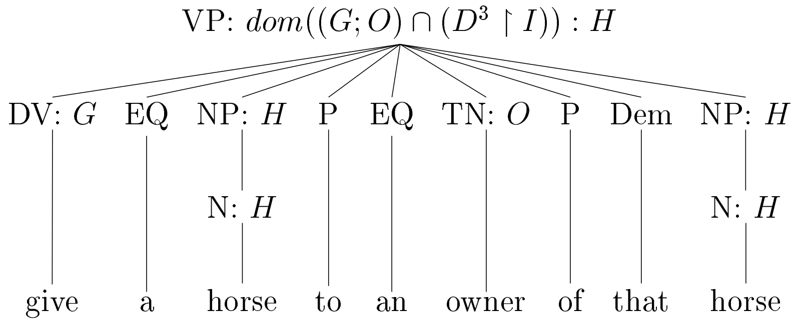

Figure 16.

where B is the ternary relation denoted by the ditransitive verb *betray*, A is a binary relation denoted by the transitive noun *enemy*, and M is the set denoted by the common noun *man*. By definition, this term is equivalent to

$$\overline{(\overline{B} : M)} : \overline{(\overline{A} : M)}. \tag{41}$$

By simple computation this expression will turn out to be equivalent to

$$\{x|(\exists y)(\forall z)(z \in M \rightarrow yAz \wedge Bxyz)\}, \tag{42}$$

which is an appropriate translation of (39).

Our grammar also derives semantic trees for expressions with anaphoric pronouns in Figure 16. Notice that the Peirce product and conversion operations are not sufficient here and some additional operation is required. In a similar fashion, a semantic tree can be derived for the expression *betray a man to an enemy of him.* The root denotation of this tree is

$$dom((B;A) \cap (D^3 \restriction I)) : M. \tag{43}$$

Any element x of this set fulfils the condition

$$(\exists y)(y \in M \wedge (\exists z)(Bxyz \wedge zAy)) \tag{44}$$

which captures the intuitive meaning of the verb phrase. Correspondingly, the expression

$$\textit{betray a man to every enemy of him} \tag{45}$$

would by our grammar be assigned the denotation

$$dom(\overline{(\overline{B};A) \cap (D^3 \restriction I)}) : M. \tag{46}$$

This term is equivalent to the set

$$\{x|(\exists y)(y \in M \wedge (\forall z)(zAy \rightarrow Bxyz))\}, \tag{47}$$

which is in line with our intuition about the meaning of (45).

In Table 1 a grammar was given for a fragment of English that is large enough to derive many of Peirce's English examples to illustrate his operations and their use to construct complex terms. In the next section we use our fragment to check Peirce's constructions.

4 Peirce's Relatives

Our grammar is able to derive semantic trees for most of the terms with ternary relations considered in Peirce (1870). A term expressing a ternary relation is called a "conjugate term" by Peirce. Peirce illustrates his relational terms by examples from everyday English. Peirce made occasional blunders in his notation as had been pointed out before.[6]

In order to be able to correctly assess the terms proposed by Peirce we need to explain some of his notation. Peirce uses juxtaposition and exponentiation in case the first term is a relation and the second term is relational or absolute. So xy may correspond to standard relational composition in case both x and y are binary relations, or to the Peirce product in case y is an absolute term and x is a binary relation. Similarly, x^y may correspond to standard involution if x is a binary relation and y is an absolute term, or to generalized involution in case x is a ternary relation and y is a binary relation or absolute.

Relations as Arguments Peirce also considers the case of a binary relation occurring in argument position like, e.g., in

giver of a horse to an owner of a horse. (48)

In Peirce's notation, this corresponds to the term *gohh*. This is equivalent to the root denotation of the tree of Figure 15. But in the case of

betrayer of each man to an enemy of every man (49)

Peirce appears to have got it wrong. The term he proposed is ba^m. On our account, the denotation would be (40). Notice that the respective denotations are not equivalent.

This term can be analyzed either by $(ba)^m$ or by $b(a^m)$. Recall that by juxtaposition of two terms x and y, Peirce denotes either relational composition[7] or Peirce product.[8] Assume juxtaposition denotes composition. Then ba denotes a ternary relation and $(ba)^m$ denotes a binary relation. This cannot be correct, since (4) is an absolute term and should denote a set. Assume therefore that juxtaposition denotes the Peirce product. Then $b(a^m)$ denotes a binary relation too. Consider now the possibility that ba denotes the Peirce product. Then ba denotes a set. And if ba denotes a set then $(ba)^m$ is not defined. Notice that a^m always denotes a set. But if a^m denotes a set, b a ternary relation and $b(a^m)$ denotes the Peirce product of b and a^m, then $b(a^m)$ denotes a binary relation. But this is not correct, since $b(a^m)$ is supposed to denote a set. Similar remarks hold for other terms with three quantifiers proposed by Peirce.

Anaphora Some of Peirce's terms involve anaphoric pronouns. For instance, for the expression *betrayer of a man to every enemy of him*, the term $b^a m$ is proposed by Peirce.[9] This is not correct. For $b^a m$ is equivalent to

$$\{x|(\exists y)(y \in M \wedge (\forall z)(xAz \to Bxzy)\}, \tag{50}$$

and (50) is not equivalent to (47).

[6] Cf. Brink (1978) and Martin (1978).
[7] cf. Brink (1978: 288).
[8] cf. Martin (1978: 27).
[9] Peirce (1870: 378 and 426).

Peirce proposed the term *goh* as a denotation for[10]

$$\textit{giver of a horse to the owner of that horse.} \tag{51}$$

Martin pointed out correctly that this is wrong but did not give a correct term for (51).[11] Recall that our grammar derives a semantic tree in Figure 16 for a structure that is closely related. If we assume the denotation for *own* to be a left-unique binary relation, then the tree in Figure 16 would also be a semantic tree for (51).

Scope Peirce sharply distinguishes two notions of *give*:[12]

g_1: *giver of —- to —-*
g_2: *giver to —- of —-*

This distinction corresponds to a difference in syntactic structures with g_1 occurring in a structure with the direct object preceding the indirect object like in, e.g.

$$\textit{give a flower to every lady,} \tag{52}$$

and g_2 occurring in a structure with the indirect object preceding the direct object like in, e.g.,

$$\textit{give every lady a flower.} \tag{53}$$

More important than the relative order of the direct and indirect objects is the scope of direct and indirect objects. In principle, two situations can be distinguished: either the direct object is in the scope of the indirect object as is the case in (52) or the indirect object is in the scope of the direct object. The first situation is called the patient analysis. The second situation is called the recipient analysis. It is often claimed that (53) has the same meaning as (52).[13]

According to Peirce the meaning of bm^w would be *betrayer of all women to a man.*[14] Notice that bm^w is equivalent to

$$\{x|(\forall y)(y \in W \rightarrow (\exists z)(z \in M \wedge Bxyz))\}. \tag{54}$$

On this analysis, the indirect object *man* falls inside the scope of the direct object *women*, which runs against common linguistic intuition. But the denotation of the phrase *betrayer of all women to a man* should rather be

$$\{x|(\exists z)(z \in M \wedge (\forall y)(y \in W \rightarrow Bxyz))\}. \tag{55}$$

Our grammar accounts for this fact by introducing the order DV DO IO in two steps, but introducing the order DV IO DO in one step and assigning

$$(B^W) : M \tag{56}$$

as a denotation for the phrase *betrayer to a man of all women.* This denotation is identical to the one provided for the phrase *betrayer of all women to a man.*

[10] Peirce (1870: 370).
[11] Martin (1978: 29).
[12] Peirce (1870: 370).
[13] Keenan and Faltz (1985: 193).
[14] Peirce (1870: 378).

5 Concluding Remarks

In this paper, we have (i) extended the notion of relational grammar such that it is able to accommodate ternary relations, (ii) illustrated this notion by a fragment of English that deals with transitive and ditransitive phrases, (iii) pointed out certain inadequacies in terms proposed by Peirce, and (iv) given correct interpretations for terms that had been pointed out to be flawed. In addition we would like to point out that our grammar extends the set of syllogisms considerably. For instance, it will be able to identify the argument

Some man gave every lady a rose
Every rose is a flower
___
Every lady was given a flower

as a valid syllogism of English. With additional rules introducing negative particles *no* and *not* we may end up with about 88 different syllogistic forms.

Our notion of an extended relation algebra as a structure closed with respect to certain operations resembles the notion of a "bonding algebra" proposed by Herzberger.[15] Herzberger proposed a structure closed with respect to relational composition, major permutation, minor permutation, bonding, and relative complement, where major permutation shifts the first argument into final position, minor permutation switches the first two arguments and bonding identifies the last two arguments of a relation. In line with Peirce, Herzberger does not distinguish between the operations of relational composition and Peirce product. This may be satisfactory in the case where only binary relations and sets are considered. However, the operations can be well distinguished: if R is a ternary relation and S is a binary relation, then $R;S$ will return a ternary relation but $R:S$ will return a set. Moreover, the operations turn out not to be sufficient. Some notion of union or intersection is required as is a notion of restriction. We would otherwise not be able to derive an appropriate structure for the tree in Figure 16.

Relational grammar is not compelled to distinguish two variants of a ternary relation depending on the order of their arguments. On the contrary, Peirce's assumption of two different notions for *give* is rather unnatural from the standpoint of natural English where one and the same form is used throughout. If we assume only one predicate for *give* we would then have to derive g_2 from g_1 or g_1 from g_2.[16]

Unlike most conventional linguistic approaches our grammar is semantically driven rather than syntactically driven. The sentences

give a horse to an owner of a horse
give a horse to an owner of that horse

exhibit an almost identical syntactic structure. The only difference is that one structure has an existential quantifier *a* where the other structure has the demonstrative pronoun *that*. Linguists have speculated that quantifiers and demonstrative pronouns belong to one and the same syntactic category of determiners. Under this assumption one should expect that the semantic trees for these expressions are very similar. But under our anlysis, this turns out not to be the case. The respective semantic trees are given in Figure 15 and in Figure 16. The semantic tree for the expression with the anaphoric pronoun *that* is much flatter than the tree for the expression without the anaphoric pronoun. But this is

[15] Herzberger (1981).
[16] This is in fact done in Böttner (To appear).

not surprising since the anaphoric reference requires information given at some location of the tree to be available at a distant location of the tree. It is an open question whether relational semantics has to stay with the flat tree of Figure 16 or can be tailored to fit better a more hierarchical structure.

The flat-tree problem is inherited by any standard one-dimensional representation. Peirce himself proposed a two-dimensional representation better known under the name of existential graphs. Existential graphs have become a major focus in the design of systems of knowledge represention in computer science under the name of conceptual graphs.[17] The problem will be to find uniform procedures to map the variant forms of a natural language syntax to two-dimensional graph structures.

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