A Collective Extension of Relational Grammar

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Abstract

Relational grammar was proposed in Suppes (1976) as a semantical grammar for natural language. Fragments considered so far are restricted to distributive notions. In this article, relational grammar is extended to collective notions.

Keywords: Collectivity, distributivity, relational algebra, Peirce algebra, relational grammar, algebraic semantics, variablefree semantics.

1 Introduction

The notion of a relational grammar was proposed by Suppes as a grammar with a model-theoretic semantics for natural languages.[1] It maps natural language expressions onto terms of relational algebra. The construction of algebraic terms from constants by operations fits the constituent structure of a natural language expression much better than conventional quantifier variable structures. Being rich enough to express classical mathematics,[2] the language of relational algebra turns out to be a powerful and elegant tool for natural language semantics.

Relational grammars so far have been presented mostly for English fragments: A fragment for sentences occurring in the syllogism was given in Suppes (1976). The fragment was extended to attributive constructions and possessive NP constructions in Suppes and Macken (1978) and to intonation in Suppes (1979). A fragment for English imperatives was presented in Böttner (1992a). An analysis of copulative terms is given in Böttner (1994). In Böttner (1992b) a proposal was made for the analysis of English constructions with anaphorical pronouns. No precise rules of grammar had been given. The purpose of this paper is to provide a relational grammar for the constructions in question. Whereas in the earlier paper the thrust was on a variable-free semantics for certain anaphoric structures, the main result of this paper is that all the structures mentioned in Böttner (1992b) can indeed be accommodated in a relational grammar.

All the notions considered in various fragments satisfy the following property: if they hold of a class of individuals they also hold of each individual of that class. Notions of this kind are called *distributive*. It is well-known that not all natural language notions are of that kind. An example is *John and Mary are a couple*. Verb phrases of this kind are called *collective*. In this paper, an extension of relational grammar for collectives will be proposed and extended to anaphora.

[1] Suppes (1976).

[2] Tarski and Givant (1987).

Our procedure will be as follows: We start from the calculus of sets and relations. On the basis of standard operations we define some new operations. We then show the usefulness of these operations for the analysis of certain English constructions, in particular for collective verb phrases and certain anaphoric constructions that go with them. The notions from the calculus of sets and binary relations are introduced in 1. The notion of a relational grammar is illustrated by some example grammar in 2. In 3 this grammar is extended to capture collective notions. In 4 the grammar is extended to capture some constructions with anaphoric pronouns. In a final section 5, some problems of the analysis are pointed out.

2 Calculus of Sets and Binary Relations

All we presuppose for the following is elementary set theory. Let us assume some set D and consider subsets of D and binary relations on D. I assume familiarity with the operations of set theory like *intersection*, *union*, *complement*, or *difference*. We use $\emptyset$ for both the empty set and the empty relation, I for the *identity relation* on D. We use the *conversion* of a relation R defined

$$\breve{R} = \{< x, y > \mid < y, x > \in R\},$$

and *composition* of two binary relations R and S defined

$$R; S = \{< x, y > |(\exists z)(xRz \wedge zSy)\}.$$

There are two variants of the composition operation depending on whether one of the arguments is a subset of D. If the first argument of composition is a subset of D, this leads to the operation of *upper image* of a set A under a relation R defined

$$R``A = \{y|(\exists x)(x \in A \wedge xRy)\}.$$

Familiar from set theory is also the operation of the *Cartesian Product* of sets A and B:

$$A \times B = \{< x, y > |x \in A \wedge y \in B\}.$$

From the elementary theory of binary relations the following theorems are known to hold:

THEOREM 2.1

1. $A \times -B \subseteq -(A \times B)$
2. $R; (S; T) = (R; S); T$
3. $(R, S)\breve{} = \breve{S}; \breve{R}$
4. $(R \cap S)\breve{} = \breve{R} \cap \breve{S}$
5. $(A \times B) \cap R \neq \emptyset \leftrightarrow A \cap (\breve{R}``B) \neq \emptyset$
6. $(-R)\breve{} = -\breve{R}$

From our list of primitive operations many more operations can be defined, like, for instance:

$$\begin{aligned}
dom(R) &= \breve{R}``D \\
E(A) &= dom(D \times A) \\
U(A) &= -E(-A) \\
N(A) &= -E(A) \\
e(R, A) &= \breve{R}``A \\
u(R, A) &= -e(-R, A) \\
Refl(R) &= dom(R \cap I) \\
I_A &= I \cap (A \times A) \\
Poss(A) &= P``A \\
Poss(R, A) &= -dom((P; I_A) \cap -R) \\
Col_e(R, A) &= R; I_A; \breve{R} \\
Col_u(R, A) &= u(R, A) \times u(R, A) \\
sym(R) &= R \cap \breve{R} \\
Rec(R) &= sym(R) - I \\
RecPoss(R, A) &= sym(-(-R; I_A; \breve{P})) - I \\
Id(R, A) &= -(R; I_A; -\breve{R}) \cap -(-R; I_A; \breve{R}) \\
Div(R, A) &= -(R; I_A; \breve{R}) \cap -(-R; I_A; -\breve{R})
\end{aligned}$$

The operations dom, E, U, N, e, u, $Refl$ and $Poss$ return a set, the other operations return a binary relation. More specifically, dom returns the *domain* of its argument relation. The operation $e(R, A)$ is called the *upper counterimage* of set A under relation R, also known by the name of *Peirce Product* of relation R to set A.[3] The operation $u(R, A)$ had already been used by de Morgan and is known under the term of *ordinary involution*.[4] I_A is the identity relation over set A.[5] The operation $sym(R)$ returns the greatest symmetric subrelation of its argument relation R. In the definition of $Poss$ a relation P is used that does not occur in the parameters of the operation $Poss$, since we think of P as a constant denoting the relation of possession in all models.[6]

THEOREM 2.2

$$E(A) = \begin{cases} D & \text{if } A \neq \emptyset, \\ \emptyset & \text{otherwise.} \end{cases}$$

PROOF. i. $A \neq \emptyset \Rightarrow dom(D \times A) = D$.
ii. $A = \emptyset \Rightarrow dom(D \times A) = dom(D \times \emptyset) = dom(\emptyset) = \emptyset$. Q.E.D.

■

E is a two-valued operation. It returns D in case its argument is not the empty set, and returns the empty set otherwise.

From elementary set theory we have the following theorem:

[3] cf. Brink (1981).

[4] cf. Brink (1978).

[5] cf. Suppes (1960).

[6] This was suggested in Suppes and Macken (1978).

Theorem 2.3
$A \neq \emptyset \rightarrow u(R, A) \subseteq e(R, A)$.

A binary relation R is called *symmetrical* if $\breve{R} \subseteq R$.

Theorem 2.4
sym(R) is symmetrical.

Proof. $sym(R) = (R \cap \breve{R})\breve{} = \breve{R} \cap \breve{\breve{R}} \subseteq \breve{R} \cap R = R \cap \breve{R} = sym(R)$. ■

Theorem 2.5
I is symmetrical.

Theorem 2.6
If R is symmetrical and S is symmetrical, then $R \cap S$ is symmetrical.

Proof. Follows from definition and Theorem 2.2. ■

Theorem 2.7
The relation Rec(R) is symmetrical.

Proof. Follows from Theorem 2.4, 2.5, and 2.6. ■

Theorem 2.8
$RecPoss(R)$ is symmetrical.

Proof. Follows from Theorem 2.4, 2.5, and 2.6. ■

Theorem 2.9
$Id(R, A)$ is symmetrical.

Proof. $(-(R; I_A; -\breve{R}) \cap -(-R; I_A; \breve{R}))\breve{}$ (by Theorem 2.2)
$= -(R; I_A; -\breve{R})\breve{} \cap -(-R; I_A; \breve{R})\breve{}$ (by Theorem 2.2)
$= -(-R; I_A; \breve{R}) \cap -(R; I_A; -\breve{R})$(by Theorem 1, iii)
$= -(R; I_A; -\breve{R}) \cap -(-R; I_A; \breve{R})$.
■

A binary relation R is called *transitive* if $R; R \subseteq R$.

Theorem 2.10
$Id(R, A)$ is transitive.

Proof. $< x, y > \in Id(R, A)$, i.e. $(\forall u)(u \in A \rightarrow (xRu \leftrightarrow yRu)$ (premise 1)
$< y, z > \in Id(R, A)$, i.e. $(\forall u)(u \in A \rightarrow (yRu \leftrightarrow zRu)$ (premise 2)
Let $u \in A$. Then $xRu \leftrightarrow yRu$ (from premise 1) and $yRu \leftrightarrow zRu$ (from premise 2), hence, $xRu \leftrightarrow zRu$, i.e. $< x, z > \in Id(R, A)$. ■

R is an *equivalence relation* iff R is transitive and symmetrical.

Theorem 2.11
The relation $Id(R, A)$ is an equivalence relation.

Proof. Follows from Theorem 2.9 and Theorem 2.10. ■

Theorem 2.12
$Div(R, A) = -Id(R, A)$.

Proof. We show

1. $Div(R, A) \subseteq -Id(R, A)$
2. $-Div(R, A) \subseteq Id(R, A)$

1. $$\begin{aligned} & Div(R, A) \cap Id(R, A) \\ & = -(R; I_A; \breve{R}) \cap -(-R; I_A; \breve{-R}) \cap -(R; I_A; -\breve{R}) \cap -(-R; I_A; \breve{R}) \\ & = -(R; I_A; \breve{R}) \cap -(R; I_A; -\breve{R}) \cap -(-R; I_A; -\breve{R}) \cap -(-R; I_A; \breve{R}) \\ & = -((R; I_A \breve{R}) \cup (R; I_A; -\breve{R}) \cap -((-R; I_A; -\breve{R}) \cup (-R; I_A; \breve{R})) \\ & = -((R; I_A; (\breve{R} \cup -\breve{R})) \cap -(-R; I_A; (-\breve{R} \cup \breve{R}))) \\ & = -(R; I_A; (D \times D)) \cap -(-R; I_A; (D \times D)) \\ & = -((R; I_A; D \times D) \cup (-R; I_A; (D \times D))) \\ & = -((R \cup -R); I_A; (D \times D)) \\ & = -((D \times D); I_A; (D \times D)) \\ & = -(D \times D) \\ & = \emptyset \end{aligned}$$
2. $$\begin{aligned} & -Div(R, A) \cap -Id(R, A) \\ & = -(R; I_A; \breve{R}) \cap -(-R; I_A; -\breve{R}) \cap -(R; I_A; -\breve{R}) \cap -(-R; I_A; \breve{R}) \\ & = -(R; I_A; \breve{R}) \cap -(R; I_A; -\breve{R}) \cap -(-R; I_A; -\breve{R}) \cap -(-R; I_A; \breve{R}) \\ & = -((R; I_A; \breve{R}) \cup (R; I_A; -\breve{R})) \cap -((-R; I_A; -\breve{R}) \cup (-R; I_A; \breve{R})) \\ & = -(R; I_A; (\breve{R} \cup -\breve{R})) \cap -(-R; I_A; (-\breve{R} \cup \breve{R})) \\ & = -(R; I_A; (D \times D)) \cap -(-R; I_A; (D \times D)) \\ & = -((R; I_A; (D \times D)) \cup (-R; I_A; (D \times D))) \\ & = -((R \cup -R); I_A; (D \times D)) \\ & = -((D \times D); I_A; (D \times D)) \\ & = -(D \times D) \\ & = \emptyset. \end{aligned}$$

∎

Theorem 2.13
$R; I_A; \breve{R} \subseteq e(R, A) \times e(R, A)$.

Proof.
$$\begin{aligned} R; I_A; \breve{R} & = R; I_A; I_A; \breve{R} \\ & \subseteq dom(R; I_A) \times dom((I_A; \breve{R})\breve{}) \\ & = dom(R; I_A) \times dom(R; I_A) \\ & = e(R, A) \times e(R, A) \text{ since } R; S \subseteq dom(R) \times dom(\breve{S}) \end{aligned}$$

∎

Theorem 2.14
$Col_u(R, A) \subseteq Id(R, A)$.

Proof. 1. Let $A = \emptyset$. Then $Col_u(R, A) = D \times D$ and $Id(R, A) = D \times D$. Hence $Col_u(R, A) = Id(R, A)$.

2. Let $A \neq \emptyset$. Therefore $u(R, A) \subseteq e(R, A)$. From this we get:
(a) $$\begin{aligned} & u(R, A) \times u(R, A) \\ & = -e(-R, A) \times e(R, A) \text{ (by Definition of } E) \\ & \subseteq -e(-R, A) \times e(R, A) \text{ (by Theorem 2.3)} \\ & \subseteq -(e(-R, A) \times e(R, A)) \text{ (by Theorem 2.2)} \\ & = -(-R; I_A; \breve{R}) \text{ (by Theorem 2.13)} \end{aligned}$$

(b) $u(R, A) \times u(R, A)$
$\subseteq e(R, A) \times u(R, A)$ (by Theorem 2.3)
$= e(R, A) \times -e(-R, A)$ (by Definition of E)
$\subseteq -(e(R, A) \times e(-R, A)$ (by Theorem 2.2)
$\subseteq -(R; I_A; -\breve{R})$ (by Theorem 2.13)
Since $Col_u(R, A) = u(R, A) \times u(R, A)$,
and $Id(R, A) = -(R; I_A; -\breve{R}) \cap -(-R; I_A; \breve{R})$,
assertion follows.

∎

In our calculations we have made use of laws from the theory of sets and binary relations. The laws of set-theory are captured under the notion of a Boolean algebra. Many laws referring to binary relations can be captured under the notion of a relation algebra. An equational definition of relation algebra was first given by Tarski.[7] An equational definition of the underlying structure was given under the notion of a *Peirce algebra*.[8]

3 Relational Grammar

A relational grammar is a denoting grammar. A denoting grammar is a context-free grammar with a model-theoretic semantics.[9] With each production rule, a semantic function is associated. The characteristic feature of a relational grammar is that its denotations are restricted to subsets of the domain D and to binary relations on D, i.e. to the hierarchy

$$\mathcal{H}(D) = \mathcal{P}(D) \cup \mathcal{P}(D \times D) \tag{3.1}$$

where D is some non-empty set.[10]

An example of a denoting grammar is the following:

$$\begin{array}{rll} S & \to PN + VP & [S] = E([PN] \cap [VP]) \\ S & \to UQ + N + VP & [S] = U(-[N] \cup [VP]) \\ S & \to EQ + N + VP & [S] = E([N] \cap [VP]) \\ S & \to NQ + N + VP & [S] = N([N] \cap [VP]) \\ S & \to CPNP + VP & [S] = U(-[CPNP] \cup [VP]) \\ CPNP & \to PN + CC + PN' & [CPNP] = [PN] \cup [PN'] \\ CPNP & \to PN + CPNP & [CPNP] = [PN] \cup [CPNP] \\ NP & \to PN + PossInf + RN & [NP] = e([RN], [PN]) \\ NP & \to PN + PossInf + N & [NP] = Poss([PN]) \cap [N] \\ VP & \to Cop + NP & [VP] = [NP] \\ VP & \to TV + PN & [VP] = e([VP], [PN]) \\ VP & \to TV + UQ + N & [VP] = u([TV], [N]) \\ VP & \to TV + EQ + N & [VP] = e([TV], [N]) \\ VP & \to TV + Refl & [VP] = Refl([TV]) \\ VP & \to TV + Poss + N & [VP] = Poss([TV], [N]) \\ VP & \to TV + CPNP & [VP] = U([TV], [CPNP]) \end{array}$$

[7] Tarski (1941).

[8] Brink, Britz, and Schmidt (1994).

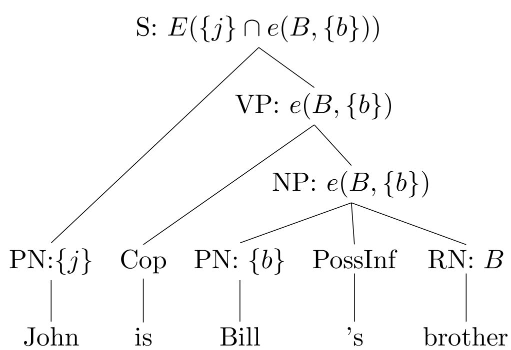
[9] Suppes (1973).

[10] Suppes (1976).

where S (sentence), NP (noun phrase), VP (verb phrase), and CPNP (conjoined proper noun phrase) refer to phrasal categories. We distinguish the following lexical categories:

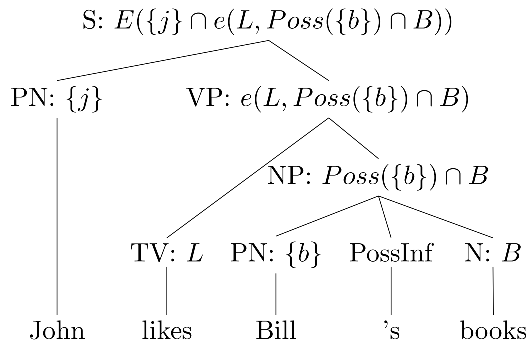
EQ	existential quantifier	*some*
UQ	universal quantifier	*all*
NQ	negative quantifier	*no*
PN	proper noun	*John, Mary, Bill, Dick, Harry*
N	common noun	*books, houses, toys, students, guests*
RN	relational noun	*brother, brothers*
TV	transitive verb	*own, like*
CC	conjoining conjunction	*and*
Cop	copula	*is, are*
Refl	reflexive pronoun	*himself, herself, themselves*
Poss	possessive pronoun	*his, her, their*
PossInf	possessive inflexion	*'s*

The notion of a denoting grammar gives rise to the notion of a semantic tree. A semantic tree of a denoting grammar G is a derivation tree of the context-free grammar G with the root node having a denotation that is derived from the denotations of its daughter nodes by semantic functions. The leaves of the semantic tree get their denotation (in some model) by a function into the hierarchy $\mathcal{H}(D)$. For purposes of illustration let us give a couple of examples of semantic trees that can be derived in our grammar.



(3.2)

Recall from Theorem 2.2 that E is a two-valued operation. We can identify the two values with the truth values: A sentence is called *true* with respect to a model structure if the root node denotation is D otherwise *false*.



(3.3)

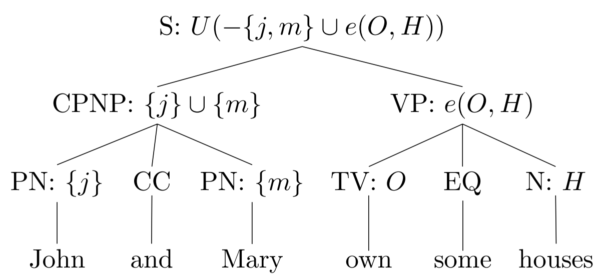
Our solution for *Bill's books* is in line with the solution proposed by Suppes and Macken.[11] In the case of *Bill's brother* that would be treated similarly by Suppes and Macken, our solution does not use the possessive relation P. We think that the words *books* and *brother* belong to different categories: *books* belongs to the category N of a classificatory common noun, *brother* belongs to the category RN of a relational common noun.

Our grammar derives the following constructions with proper noun coordinations:

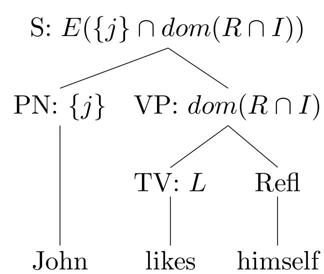
John and Mary are students	$U(-(\{j\} \cup \{m\}) \cup S)$
like John and Mary	$u(L, \{j\} \cup \{m\})$

where *John* denotes $\{j\}$, *Mary* denotes $\{m\}$ and S is the set of students and L is the relation denoted by the verb *like*.

Notice that the language generated by these rules is not restricted to just two constituents but can derive copulative terms with any number of constituents.

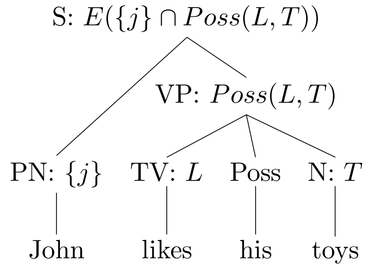


(3.4)



(3.5)

[11] Suppes and Macken (1978).



(3.6)

According to our grammar, the phrase *likes his toys* denotes the term

$$-dom((P; I_T) \cap -L). \tag{3.7}$$

That (3.7) corresponds to the intuitive meaning of *like his toys* follows from the fact that (3.7) is equivalent to the quantifier expression

$$\{x \in D : (\forall y)(y \in T \wedge xPy \rightarrow xLy)\}. \tag{3.8}$$

4 Collective Extension

There are many constructions that cannot be analyzed with our grammar developed so far. Our focus here will be on constructions like the following one:

$$\textit{John and Bill are brothers} \tag{4.1}$$

Although the structure of (4.1) closely resembles the structure of

$$\textit{John and Mary are students} \tag{4.2}$$

its meaning is equivalent to that of (3.2).

It has been proposed that denotations for collective predicates should be sets of sets of objects.[12] Within the confines of our set-theoretical hierarchy $\mathcal{H}(D)$ we cannot have properties of that kind. For any such property would be a set X of elements of $\mathcal{P}(D)$, i.e. $X \in \mathcal{P}(\mathcal{P}(D))$. I propose to construe collective predicates as binary relations.[13] Notice that our analysis of collectives stays within the confines of the restricted set-theoretical hierarchy $\mathcal{H}(D)$. This makes it different from proposals along the lines of Montague grammar[14], second order logic[15], or mereological ontology.[16]

For the derivation of collectives our grammar is extended by the following rules:

$$\begin{array}{rlrl}
S & \rightarrow PPNP + ColVP & [S] & = E([PPNP] \cap ([ColVP])) \\
S & \rightarrow UQ + N + ColVP & [S] & = U(-([N] \times [N] - I) \cup [ColVP]) \\
S & \rightarrow EQ + N + ColVP & [S] & = E([N] \times [N] \cap [ColVP]) \\
PPNP & \rightarrow PN + CC + PN' & [PPNP] & = [PN] \times [PN'] \\
ColVP & \rightarrow Cop + RN & [ColVP] & = [RN] \\
ColVP & \rightarrow TV + EQ + N & ColVP & = Col_E([TV], [N]) \\
ColVP & \rightarrow TV + UQ + N & ColVP & = Col_U([TV], [N])
\end{array}$$

[12] Hausser (1974), Hoeksema (1988), Keenan and Faltz (1985).

[13] This proposal can be traced back to Peirce (1880).

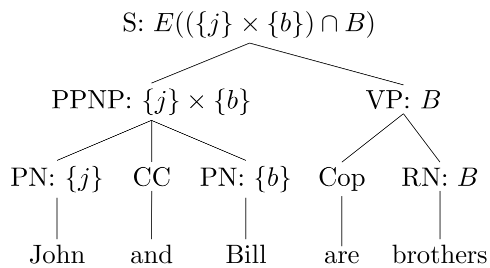
[14] Hausser (1974).

[15] Blau (1981).

[16] Link (1983).

where ColVP = collective verb phrase and PPNP = paired proper noun phrase.

We therefore have the following semantic tree for (4.1):



(4.3)

We are now able to prove that (4.1) is equivalent is to (3.2):

$$\begin{aligned} E(\{j\} \cap e(B, \{b\}) = D \quad & \leftrightarrow \{j\} \cap \breve{B}``\{b\} \neq \emptyset \text{ (Definition of } E) \\ & \leftrightarrow \{j\} \times \{b\} \cap B \neq \emptyset \text{ (by Theorem 2.1,v)} \\ & \leftrightarrow E(\{j\} \times \{b\} \cap B) \text{ (Definition of } E) \end{aligned}$$

The semantic function for the rule

$$S \rightarrow UQ + N + ColVP$$

may be surprising for the reason that it explicitly excludes identical pairs. The reason for doing so is that we want the sentence

$$\textit{All guests are brothers} \tag{4.4}$$

to be true in a model structure

$$\begin{aligned} & G = \{a, b, c\} \\ & B = \{<a, b>, <a, c>, <b, c>, <b, a>, <c, a>, <c, b>\} \end{aligned}$$

where set G is the denotation of *guests* and binary relation B is the denotation of *brother*. Without explicitly excluding I from $G \times G$, however, the sentence will be false, since $<a, a> \notin B$ for the relation denoted by *brother* is conventionally assumed to be irreflexive.

In departure from our general convention, the semantic function $Col_u(R, A)$ is not defined in line with the general transformation:

$$U(...) = -E(-...)$$

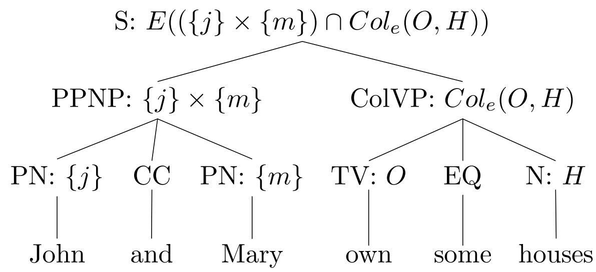
as one would expect. Applying this transformation to collective verb phrases would leave us with the relation

$$\{<x, y> | (\forall z)(z \in A \rightarrow (xRz \vee yRz))\} \tag{4.5}$$

as the denotation for the phrase *own all houses*. We think that this is not strong enough to account for the meaning of this phrase.

Some verb phrases can be used both distributively and collectively. Consider, for instance, the predicate denoted by *own a house*. In *John owns a house* it can only be used distributively. In *John and Mary own a house* it can be used both distributively and collectively. The distributive reading admits of the possibility that there are two houses: one belonging to John, the other belonging to Mary. The collective reading does not admit of this possibility but requires that the houses be owned jointly by John and Mary. The distinction between distributive and collective uses can be made explicit by adding *each* in the case of the distributive reading and *together* in the case of the collective reading.

The tree for the collective reading is:



(4.6)

Recall from previous section that the distributive reading (3.4) is true if

$$\{j,m\} \subseteq e(O,H). \tag{4.7}$$

This is equivalent to the condition

$$(\exists u)(jOu \wedge u \in H) \wedge (\exists u)(mOu \wedge u \in H). \tag{4.8}$$

The collective reading is true if

$$< j,m > \in O; I_H; \breve{O}. \tag{4.9}$$

This is equivalent to the condition

$$(\exists z)(z \in H \wedge xOz \wedge yOz). \tag{4.10}$$

The collective reading is stronger than the distributive reading: (4.8) is a logical consequence of (4.10), but (4.10) is not a logical consequence of (4.8).

5 Anaphoric Extension

In this section our grammar is extended to derive semantic trees for constructions with anaphoric pronouns like *each other*, *same* and *different*. The rules are[17]

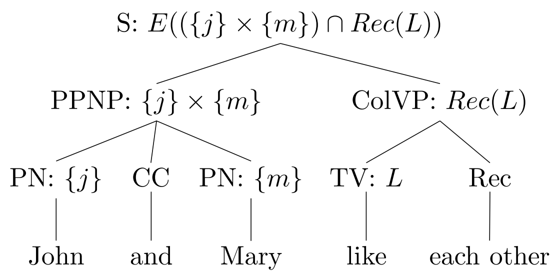
$$\begin{array}{llll}
ColVP & \rightarrow TV + Rec & [ColVP] & = Rec([TV]) \\
ColVP & \rightarrow TV + Rec + PossInf + N & [ColVP] & = RecPoss([TV],[N]) \\
CplVP & \rightarrow TV + DA + ID + N & [ColVP] & = Id([TV],[N]) \\
ColVP & \rightarrow TV + DIV + N & [ColVP] & = Div([TV],[N])
\end{array}$$

where lexical category abbreviations are as follows:

[17] The first rule corrects the analysis given in Böttner (1992b).

ID	identity pronoun	*same*
DIV	diversity pronoun	*different*
DA	definite article	*the*
Rec	reciprocal pronoun	*each other, one another*

Our grammar derives the following semantic tree:



(5.1)

Notice that it is necessary to have the verb phrase denote $Rec(R) = sym(R) - I$ rather than $sym(R)$. The reason for this is as follows: in a model structure with a singleton denoted by *students* and a singleton denoted by *like* like this

$$\begin{aligned} v(students) &= \{j\} \\ v(like) &= \{< j, j >\} \end{aligned}$$

the sentence *All students like each other* would be predicted to be true, since

$$[like\ each\ other] = sym([like]) = \{< j, j >\}.$$

But intuitively, we would not be willing to call this sentence true in such a situation, for we prefer to view *like each other* to involve at least two different individuals. Under the assumption

$$[like\ each\ other] = sym([like]) - I = \emptyset$$

the above sentence will be predicted to come out false.

For the sentence

$$All\ students\ like\ each\ other \tag{5.2}$$

our grammar derives the set-theoretical structure

$$S \times S - I \subseteq Rec(L). \tag{5.3}$$

(5.3) amounts to the following

$$(\forall x)(\forall y)(x \in S \wedge y \in S \wedge x \neq y \rightarrow xLy \wedge yLx \wedge x \neq y). \tag{5.4}$$

According to Langendoen's notion of strong reciprocity,[18] (5.2) would be true just in case the following condition holds:

$$(\forall x)(\forall y)(x \in S \wedge y \in S \wedge x \neq y \rightarrow xLz). \tag{5.5}$$

[18] Langendoen (1978).

Although our condition (5.4) may look more complicated in comparison with Langendoen's condition (5.5), the two conditions are in fact equivalent. Firstly, it is obvious that our condition implies Langendoen's condition, since both conditions have the same antecedents and Langendoen's succedent occurs as a constituent of the succedent of our condition. Secondly, our condition follows from Langendoen's condition, since

$$< x, y > \in s \times S \leftrightarrow < y, x > \in S \times S$$

from which both xRy and xRy follows.

For the sentence

$$\textit{Some students like each other} \tag{5.6}$$

our grammar derives the set-theoretical structure

$$E((S \times S) \cap Rec(L)) \tag{5.7}$$

which is true just in case

$$(\exists x)(\exists y)(x \in S \wedge y \in S \wedge x \neq y \wedge xLy \wedge yLx). \tag{5.8}$$

We think (5.8) to be in line with our intuition about the meaning of (5.6).

In contradistinction to the truth-condition for (5.4), no corresponding condition can be derived for (5.8) from Langendoen's definition of strong reciprocity. Bearing in mind the equivalence

$$(\exists x)\alpha = \neg(\forall x)\neg\alpha \tag{5.9}$$

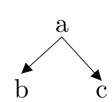
one could derive the following condition for particular affirmative sentences from Langendoen:

$$\neg(\forall x)\neg(\forall y)(x \in S \wedge y \in S \wedge x \neq y \rightarrow xLy) \tag{5.10}$$

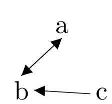
which is equivalent to

$$(\exists x)(\forall y)(x \in S \wedge y \in S \wedge x \neq y \rightarrow xLy). \tag{5.11}$$

But (5.11) is not equivalent to (5.8). Consider, for instance, a set of three students a, b, and c with a relation L (represented by the arrow) holding between them:



Then (5.11) would be true with respect to this model, since a is the element that fulfils the condition, but (5.8) would be false, since there is no pair of elements x and y such that xLy and yLx. On the other hand, the following model



would be true in (5.8), since the pair of a and b fulfils the condition, but false in (5.11), since there is no element from which every other element can "reached". So Langendoen's definition fails to account for particular sentences. Langendoen's definition suffers from being tailored for universal sentences.

The grammar predicts the following inferences to be valid:

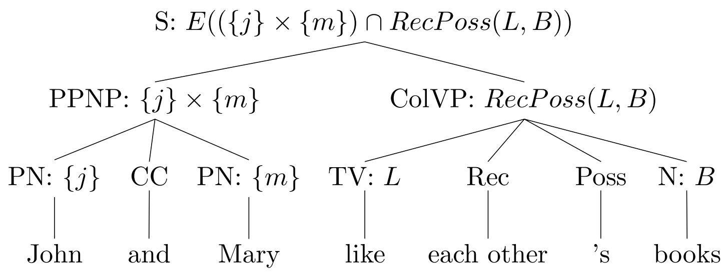
$$\frac{\textit{John and Mary like each other}}{\textit{Mary and John like each other}} \tag{5.12}$$

$$\frac{\textit{John and Mary like each other}}{\textit{John likes Mary}} \tag{5.13}$$

The proof for (5.12) follows from the fact that the relation denoted by the verb phrase *like each other* is symmetrical according to theorem 5:

$$(A \times B)\breve{} \cap R = (B \times A) \cap R$$

Our grammar derives the following semantic tree for combinations of possessive and reciprocal:



(5.14)

According to our grammar, the verb phrase *like each other’s books* denotes the following relation:

$$-(-L; I_B; \breve{P}) \cap -(P; I_B; \breve{-L}) \cap -I. \tag{5.15}$$

And this is what it is supposed to denote, since

$$< x, y > \in -(-L; I_B; \breve{P}) \cap -(P; I_B; \breve{-L})$$

is equivalent to

$$(\forall z)(z \in B \rightarrow ((yPz \rightarrow xLz) \wedge (xPz \rightarrow yLx))).$$

Our grammar predicts the following inference to be valid:

$$\frac{\textit{John and Mary like each other's books}}{\textit{Mary and John like each other's books}} \tag{5.16}$$

The proof follows from Theorem 8.

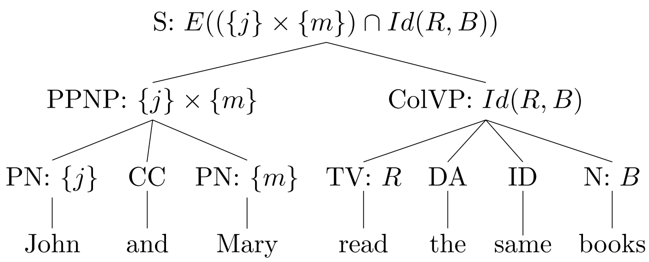
Notice that the sentence

$$\textit{John and Mary like each other's books} \tag{5.17}$$

is true if the denotation of *books* is the empty set for the same reason why universal sentences are true if the subject term denotes the empty set, which is in line with the convention of standard interpretation.

Contrary to our earlier pessimistic remarks,[19] constructions of the kind *John and Mary like the same books* can be accommodated within a relational algebra. This is by no means surprising since any partition is equivalent to some equivalence relation. That the relation denoted by *read the same books* is indeed an equivalence relation on the set of book readers follows from Theorem 2.10.

Our grammar derives the following semantic tree:



(5.18)

That $Id(R,B)$ captures the intended meaning of the verb phrase is shown by being equivalent to the following quantifier logic expression:

$$\{< x, y > |(\forall z)(z \in B \rightarrow (xRz \leftrightarrow yRz))\} \tag{5.19}$$

Our grammar predicts the following inference to be valid:

$$\frac{\begin{array}{l}\textit{John and Mary read the same books}\\ \textit{Bill and Mary read the same books}\end{array}}{\textit{John and Bill read the same books}} \tag{5.20}$$

The proof follows from Theorem 10.

A parallel case is the verb phrase *read different books*. According to our grammar it denotes $Div(R,B)$. That $Div(R,B)$ construes the meaning of this verb phrase is shown by its equivalence to the following quantifier logic expression:

$$\{< x, y > |(\forall z)(z \in B \rightarrow (xRz \leftrightarrow \neg yRz)\} \tag{5.21}$$

There is an interesting duality between universal and negative quantifiers and *same* and *different*:

Theorem 5.1
$UQ + N + TV + DIV + N \leftrightarrow NQ + N + TV + DA + ID + N$

Proof. From Theorem 2.12 we have

1. $X \subseteq Div(R,B) \rightarrow X \cap Id(R,B) = \emptyset$
2. $X \cap Id(R,B) = \emptyset \rightarrow X \subseteq Div(R,B)$

∎

Theorem 5.2
$NQ + N + TV + DIV + N \leftrightarrow UQ + N + TV + DA + ID + N$

[19] Böttner (1992b).

PROOF. Analogous to proof of Theorem 5.1. ∎

Theorem 5.1 and Theorem 5.2 predict the following pairs of sentences to be equivalent:

1.(a) *All students read different novels*
(b) *No students read the same novels*
2.(a) *No students read different novels*
(b) *All students read the same novels*

Our grammar predicts the following argument to be valid:

$$\frac{\textit{John and Mary own all houses together}}{\textit{John and Mary own the same houses}} \tag{5.22}$$

where by *together* we want to explicitly refer to the collective reading of this sentence. From theorem 2.14

$$[\textit{own all houses together}] \subseteq [\textit{own the same houses}] \tag{5.23}$$

follows, from which the assertion follows.

The important point about the validity of (5.23) is that it supports our definition of $Col_u(R, A)$ rather than the standard definition in terms of $Col_e(R, A)$, since (5.23) would not be valid with the weaker definition of this function.

Notice that our analysis of sentences

John and Mary like the same books
John and Mary like different books

preserves the verb phrases *like the same books* and *like different books* as constituents of the natural language sentences. Therefore we think this solution is more natural than the tree arising from the structure proposed by Carnap (1929) who proposed the sentence

$$\begin{array}{c}\textit{Ich habe denselben Lehrer wie du}\\ (\textit{I have the same teacher as you})\end{array} \tag{5.24}$$

to have the logical form

$$\breve{L}``ich = \breve{L}``du \tag{5.25}$$

where L is the denotation for *Lehrer (teacher)*, *ich* and *du* are first and second person singular personal pronouns. Adopting Carnap's solution to our example would return the following root denotation for semantic tree (5.18):

$$B \cap R``\{j\} = B \cap R``\{m\}. \tag{5.26}$$

This structure, however, does not match the syntactic structure of the respective sentence: Assume we let *same* denote the identity relation. We then have to deal with the fact that the constituent *books* occurs once in the syntactic structure but twice in the semantic structure (5.26). The same is true of the constituent *read*. So the processing of the denotation would not be able to account for the subject-predicate structure of the sentence. Moreover, and what is by far worse, Carnap's solution cannot be generalized to

$$\textit{All students read the same books}, \tag{5.27}$$

since its meaning requires a set of equations rather than a single equation:

$$B \cap R``\{s_1\} = ... = B \cap R``\{s_n\}. \tag{5.28}$$

We also think our solution to be more natural than the analysis in terms of polyadic quantifiers proposed in Keenan (1987) for the very same reasons as in the case of Carnap's analysis. Keenan's structure:

$$\text{(ALL,SAME)(STUDENT,READ,BOOKS)} \tag{5.29}$$

where the prefix is called a polyadic quantifier is too discrepant from the syntactic structure and fails to identify the verb phrase constituent as a predicate.

We assume that the verb phrase *read the same books* denotes a binary relation. Support comes from the fact that one can use it in a sentence like

John reads the same books as Mary.

And this sentence is clearly an instance of a relational sentence resembling comparative sentences like

John is as old as Mary.

6 Conclusion

Let us conclude by pointing out two open problems of our analysis that deserve further investigation. One problem has to do with the derivation of collective uses of transitive verbs, the other problem has to do with the extension of proper noun phrases from the combination of just two to more than two.

Our analysis predicts the following argument to be valid:

$$\frac{\textit{John and Mary own a house together}}{\textit{John owns a house}} \tag{6.1}$$

One might object against having this as a valid argument. But considering that collective ownership is

$$\textit{have a share in the possession of} \tag{6.2}$$

rather than

$$\textit{share the possession of ... with} \tag{6.3}$$

this inference may not be too devastating.

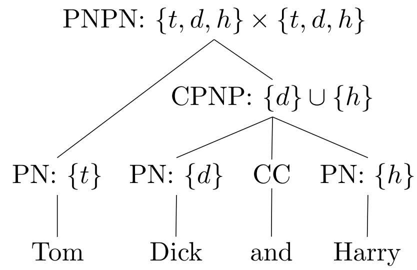
With our grammar so far we are not in a position to derive sentences with more than two proper noun combinations like

$$\textit{Tom, Dick, and Harry are brothers} \tag{6.4}$$

So our grammar is in need for extension. A straightforward way to do this is by adding the rule-function-pair

$$\begin{array}{ll} PPNP & \to PN + CPNP \\ [PPNP] & = ([PN] \cup [CPNP]) \times ([PN] \cup [CPNP]) \end{array} \tag{6.5}$$

It would derive the following semantic tree



(6.6)

The sentence (6.4) would be true iff

$$\{t, d, h\} \times \{t, d, h\} \cap B \neq \emptyset. \tag{6.7}$$

This condition could be fulfilled if there is a pair $< x, y >$ that belongs to both the denotations of *Tom, Dick, and Harry* and *brothers*, i.e. if, for instance, Tom is a brother of Harry. But this is not what (6.4) asserts. What it rather asserts is that any pair of the set of Tom, Dick, and Harry stands in the relation of being a brother of. This rules out either (6.5) or the rule of grammar that accounts for expanding S into $PPNP + ColVP$. Since we think (6.5) to be basically correct let us turn to the second alternative and replace the semantic function associated with the rule expanding S and replace it by

$$[S] = U(-[PPNP] \cup [ColVP]) \tag{6.8}$$

This solution would at least return the desired result for (6.4). But this solution runs into other problems. It would render any argument valid that has the form

$$\frac{X + VP,\ Y \subseteq X}{Y + VP} \tag{6.9}$$

But there are instances for which this form is not valid, like the following:

$$\frac{\textit{Tom, Dick, and Harry inhabit neighboring villages}}{\textit{Tom and Harry inhabit neighboring villages}} \tag{6.10}$$

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