NON-EXISTENCE OF STATIONARY TWO-BLACK-HOLE CONFIGURATIONS

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We resume former discussions of the question, whether the spin-spin repulsion and the gravitational attraction of two aligned sub-extremal black holes can balance each other. To answer the question we formulate a boundary value problem for two separate (Killing-) horizons and apply the inverse (scattering) method to solve it. Making use of a universal inequality between angular momentum and horizon area that has to be satisfied by every sub-extremal black hole, we prove the non-existence of the equilibrium situation in question.

1. Introduction

This talk is meant to contribute to the present discussion about the existence or non-existence of stationary equilibrium configurations consisting of separate bodies at rest. In Newtonian theory it is a classical result that there exists no static n-body configuration (with bodies separated by a plane and with $n > 1$). Recently, a similar statement was shown in the context of General relativity: Beig and Schoen[1] were able to prove a non-existence theorem for a reflectionally symmetric *static* n-body configuration. In the meantime, Beig *et al.*[2] have been able to extend this non-existence theorem to symmetric configurations with *anti-aligned* spins.

However, as indicated by post-Newtonian approximations, anti-aligned spins enhance the omnipresent mass attraction. Hence it could be desirable to study examples of *aligned* spins that could generate repulsive effects.

As a characteristic example for such configurations we investigate the possibility of equilibrium between two aligned rotating axisymmetric *black holes*. We will present and review a chain of old and new arguments which finally forbid this equilibrium situation. The following considerations are based on another article.[19]

2. The Double-Kerr-NUT Solution

Interestingly, there exists an exact solution to the Einstein equations that was expected to solve the two-black-hole equilibrium problem[5,8–15] — the *double-Kerr-NUT solution*, first investigated by Kramer and Neugebauer.[11] This solution is a particular case of a more general solution which was constructed by applying an

N-fold *Bäcklund transformation*[a] to an arbitrary seed solution.[16] The double-Kerr-NUT solution can be obtained as the special case of a two-fold ($N = 2$) Bäcklund transformation applied to Minkowski spacetime.[17] Since a single Bäcklund transformation generates the Kerr-NUT solution that contains, by a special choice of its parameters, the Kerr black hole solutions and since Bäcklund transformations act as a "nonlinear superposition principle", the double-Kerr-NUT solution was considered to be a good candidate for the solution of the two-horizon problem.

However, there was no argument requiring that this particular solution be the *only* candidate. The deciding step to remove this objection was to formulate and discuss a boundary value problem for two separate horizons. Using the *inverse scattering method of soliton theory*[b] we were able to show that the solution of this boundary value problem necessarily leads to (a subclass of) the double-Kerr-NUT family of solutions. As a consequence, we can indeed make use of former discussions[14,15] of the double-Kerr-NUT solution. Details of the solution of the boundary value problem can be found in Neugebauer *et al.*[18,19] These articles make use of a particular form of the linear problem[17] (see Eq. (1) of this reference) which is equivalent to the linear problem of Belinski and Zakharov.[3]

3. A Universal Inequality for Sub-extremal Black Holes

In the following we investigate whether the double-Kerr-NUT solution can describe equilibrium between two *sub-extremal* black holes (where we define sub-extremality by existence of trapped surfaces in every sufficiently small interior neighborhood of the event horizon[4]). The possibility of two-black-hole equilibrium configurations involving *degenerate* black holes will be studied in a forthcoming article.[7]

With methods from the calculus of variations it can be shown[6] that every sub-extremal black hole satisfies the inequality

$$8\pi|J| < A \tag{1}$$

between angular momentum J and horizon area A. With the explicit formulae for the quantities A and J for the two gravitational objects (black hole candidates) described by the double-Kerr-NUT solution we can test whether inequality (1) is satisfied for both objects. However, it turns out that for all choices of the free parameters at least one of these two objects violates the inequality. Hence we conclude that *an equilibrium between two sub-extremal black holes is impossible.*

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[a]The Bäcklund transformation is a particular method from soliton theory that creates new solutions to nonlinear equations from a previously known one.

[b]Hereby, an associated linear problem is analyzed, whose integrability conditions are equivalent to the non-linear field equations.

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