CMB from EFT

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Abstract: In this work, we study the key role of generic Effective Field Theory (EFT) framework to quantify the correlation functions in a quasi de Sitter background for an arbitrary initial choice of the quantum vacuum state. We perform the computation in unitary gauge in which we apply Stückelberg trick in lowest dimensional EFT operators which are broken under time diffeomorphism. Particularly using this non-linear realization of broken time diffeomorphism and truncating the action by considering the contribution from two derivative terms in the metric we compute the two point and three point correlations from scalar perturbations and two point correlation from tensor perturbations to quantify the quantum fluctuations observed in Cosmic Microwave Background (CMB) map. We also use equilateral limit and squeezed limit configurations for the scalar three point correlations in Fourier space. To give future predictions from EFT setup and to check the consistency of our derived results for correlations, we use the results obtained from all class of canonical single field and general single field $P(X,\phi)$ model. This analysis helps us to fix the coefficients of the relevant operators in EFT in terms of the slow roll parameters and effective sound speed. Finally, using CMB observation from Planck we constrain all of these coefficients of EFT operators for single field slow roll inflationary paradigm.

Keywords: Effective field theories, Cosmology of Theories beyond the SM, De Sitter space.

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1 Introduction

The basic idea of effective field theory (EFT) is very useful in many branches in theoretical physics including particle physics [1, 2], condensed matter physics [3], gravity [4, 5], cosmology [6–25] and hydrodynamics [26, 27]. In a more technical ground EFT framework is an approximated model independent version of the underlying physical theory which is valid up to a specified cut-off scale at high energies, commonly known as UV cut-off scale ($\Lambda_{\rm UV}$), which is in usual practice fixed at the Planck scale M_p. EFT prescription deal with all possible relevant and irrelevant operators allowed by the underlying symmetry in the effective action and all the higher dimensional non renormalizable operators are accordingly suppressed by the UV cut-off scale ($\Lambda_{\rm UV}\sim M_p$). There are two possible approaches exist within the framework of quantum field theory (QFT) using which one can explain the origin of EFT, which are appended below:

1. **Top down approach:** In this case the usual idea is to start with a UV complete fundamental QFT framework which contain all possible degrees of freedom. Further using this setup one can finally derive the EFT of relevant degrees of freedom at low energy scale $\Lambda_s<\Lambda_{\rm UV}\sim M_p$ by doing path integration over all irrelevant field contents [11, 13]. To demonstrate this idea in a more technical ground let us consider a visible sector light scalar field ϕ which has a very small mass $m_\phi<\Lambda_{\rm UV}\sim M_p$ and heavy scalar fields $\Psi_i\forall i=1,2,\cdots,N$ with mass $M_{\Psi_i}>\Lambda_{\rm UV}\sim M_p$, in the hidden sector of the theory. the representative action of the theory is described by the following action [11, 13]:

$$\boxed{S[\phi,\Psi_i,g_{\mu\nu}]=\int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R+\mathcal{L}_{\rm vis}[\phi]+\sum_{i=1}^{N}\mathcal{L}^{(i)}_{\rm hid}[\Psi_i]+\sum_{j=1}^{N}\mathcal{L}^{(j)}_{\rm int}[\phi,\Psi_j]\right]}\,, \qquad (1.1)$$

where $g_{\mu\nu}$ is the classical background metric, $\mathcal{L}_{\rm vis}[\phi]$ is the Lagrangian density of the visible sector light field, $\mathcal{L}^{(i)}_{\rm hid}[\Psi_i]\forall i=1,2,\cdots,N$ is the Lagrangian density of the hidden sector heavy field and $\mathcal{L}^{(j)}_{\rm int}[\phi,\Psi_j]\forall j=1,2,\cdots,N$ is the Lagrangian density of the interaction between hidden sector and visible sector field. Further using Eq. (1.1) one can construct an EFT by performing path integration over the contributions from all hidden sector heavy fields and all possible high frequency contributions as given by:

$$S_{EFT}\left[\phi,g_{\mu\nu}\right]=-i\ln\left[\prod_{j=1}^{N}\int\left[\mathcal{D}\Psi_j\right]S[\phi,\Psi_j,g_{\mu\nu}]\right]=-i\sum_{j=1}^{N}\ln\left[\int\left[\mathcal{D}\Psi_j\right]S[\phi,\Psi_j,g_{\mu\nu}]\right]. \qquad (1.2)$$

Finally one can express the EFT action in terms of the systematic series expansion of visible sector light degrees of freedom and classical gravitational background as [11, 13]:

$$\boxed{S_{EFT}\left[\phi,g_{\mu\nu}\right]=\int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R+\mathcal{L}_{\rm vis}[\phi]+\sum_{\gamma}\sum_{j=1}^{N}\mathcal{C}^{(j)}_{\gamma}(g_c)\frac{\widetilde{\mathcal{O}}^{(j)}_{\gamma}[\phi]}{M_{\Psi_j}^{\Delta_\gamma-4}}\right]}\,, \qquad (1.3)$$

where $\mathcal{C}^{(j)}_{\gamma}(g_c)\forall\gamma,\ \forall j=1,2,\cdots,N$ represent dimensionless coupling constants which depend on the parameter g_c of the UV complete QFT. Also $\widetilde{\mathcal{O}}^{(j)}_{\gamma}[\phi]\forall\gamma,\ \forall j=1,2,\cdots,N$ represent Δ_γ mass dimensional local EFT operators suppressed by the scale $M^{\Delta_\gamma-4}_{\Psi_j}$. In this connection one of the best possible example of UV complete field theoretic setup is string theory from which one can derive an EFT setup at the string scale Λ_s which is identified with M_{Ψ_j} in Eq (1.3).

2. **Bottom up approach:** In this case the usual idea is to start with a low energy model independent effective action allowed by the symmetry requirements. Using such setup the prime job is to find out the appropriate UV complete field theoretic setup allowed by the underlying symmetries [11, 13]. This identification allows us to determine the coefficients of the EFT operators in terms of the model parameters of UV complete field theories. In this paper we follow this approach to write down the most generic EFT framework using which we describe the theory of quantum fluctuations observed in CMB around a quasi de Sitter inflationary background solution of Einstein's equations.

In this paper our prime objective is to compute the expressions for the cosmological two and three point correlation functions in unitary gauge using the well known St$\ddot{u}$ckelberg trick [28, 29] along with the arbitrary choice of initial quantum vacuum state. The working principle of St$\ddot{u}$ckelberg trick in quasi de Sitter background is to break the time diffeormorphism symmetry to generate all the required quantum fluctuations observed in CMB. This is exactly same as applicable in the context of $SU(N)$ non-abelian gauge theory to describe the spontaneous symmetry breaking. In the present context the scalar modes which are appearing from the quantum fluctuation exactly mimic the role of Goldstone mode as appearing in $SU(N)$ non-abelian gauge theory. After breaking the time diffeomorphism in the unitary gauge scalar Goldstone like degrees of freedom are eaten by the metric. In unitary gauge, to write a most generic EFT in terms of operators which breaks time diffeormorphism symmetry, the following contributions play significant role in quasi de Sitter background:

- Polynomial powers of the time fluctuation of the component in the metric, g^{00} such as, $\delta g^{00}=g^{00}+1$,
- Polynomial powers of the time fluctuation in the extrinsic curvature at constant time surfaces, $K_{\mu\nu}$ such as, $\delta K_{\mu\nu}=\left(K_{\mu\nu}-a^2Hh_{\mu\nu}\right)$, where a is the scale factor in quasi de Sitter background.

Construction of EFT action using St$\ddot{u}$ckelberg trick also allows us to characterize all the possible contribution to the model independent simple versions of field theoretic framework based on the models of inflationary paradigm described by single field, where the observables are constrained by CMB observation appearing from Planck data. It is important to note that this idea of constructing EFT action using St$\ddot{u}$ckelberg trick can also be generalized to the EFT framework guided by multiple number of scalar fields as well.

The mian highlighting points of this paper are appended below point-wise:

1. We have presented all the results by restricting up to all possible contributions coming from the two derivative terms in the metric which finally give rise to a consistently truncated EFT action. Consequently, we get consistent predictions for Single Field Slow Roll [30–41] and Generalized Single Field $P(X,\phi)$ models of inflation [42–55]. In earlier works various efforts are made to derive cosmological three point correlation functions by writing a consistent EFT action in the similar theoretical framework. However, the earlier results are

not consistent with the Single Field Slow Roll inflation with effective sound speed $c_S = 1$ as it predicts vanishing three point correlation function for scalar fluctuations. See ref. [6] for more details. The main reason for this inconsistency was ignoring specific contributions from the fluctuation in the EFT action, which give rise to improper truncation.

2. We have computed the analytical expression for the two point and three point correlation function for the scalar fluctuation in quasi de Sitter inflationary background in presence of generalized initial quantum state. Also for the first time we have presented the result for two point correlation function for the tensor fluctuation in this context. To simplify our results we have also presented the results for Bunch Davies vacuum and α, β vacua [1].

3. We have presented the exact analytical expressions for all the coefficients of EFT operators for Single Field Slow Roll and Generalized Single Field $P(X, \phi)$ models of inflation in terms of the time dependent slow roll parameters as well the parameters which characterize the generalized initial quantum state. To give numerical estimates we have further presented the results for Bunch Davies vacuum and α, β vacua.

This paper is organized as follows. In **section 2**, we discuss the overview of the EFT framework under consideration, which includes the construction of the EFT action under broken time diffeomorphism in quasi de Sitter background. In **section 3**, we derive the expression for the two point correlation function from EFT using scalar and tensor mode fluctuation. Further in **section 4** we derive the expression for the scalar three point function from EFT using scalar mode fluctuation in equilateral and squeezed limit configurations. After that in **section 5**, we derive the exact analytical expressions for coefficients of EFT operators for both single field slow roll inflation and generalized single field $P(X, \phi)$ models of inflation. Finally we conclude in **section 6** with some future prospects of the present work.

2 Overview on EFT

2.1 Construction of the generic EFT action

In this section our motivation is to construct the most generic EFT action in the background of quasi de Sitter space. Before going to the further technical details it is important to note that the method of implementing cosmological perturbation using a scalar field is different compared to the generic EFT framework. However the underlying connection can be explained by interpreting the scalar (inflaton) field as a scalar under all space time diffeomorphisms in General Relativity:

$$\boxed{\textbf{Space} - \textbf{time diffeomorphism}: \ x^\mu \Longrightarrow x^\mu + \xi^\mu(t, \mathbf{x}) \quad \forall \ \mu = 0, 1, 2, 3} \,. \tag{2.1}$$

Consequently in the cosmological perturbation the scalar field $\delta\phi$ transform like a scalar under the operation of spatial diffeomorphisms, on the other hand it transforms in non-linear fashion with respect to time diffeomorphisms. The space and time diffeomorphic transformation rules are

[1] In QFT of quasi de Sitter space we deal with a class of non thermal quantum states, characterized by infinite family of two real parameters α and β, commonly known as α, β vacua. It is important to note that α, β quantum states are CP invariant under SO(1,4) de Sitter isometry group. On the other hand we we fix $\beta = 0$ then we get α vacua which is actually CPT invariant under SO(1,4) de Sitter isometry group. Furthermore, if we fix both $\alpha = 0$ and $\beta = 0$ then we get the thermal Bunch Davies vacuum state.

appended bellow:

$$\begin{array}{r}\textbf{Spatial diffeomorphism}: \; t \Longrightarrow t,\; x^i \Longrightarrow x^i + \xi^i(t,\mathbf{x}) \quad \forall\, i=1,2,3 \longrightarrow \delta\phi \Longrightarrow \delta\phi, \\ \textbf{Time diffeomorphism}: \; t \Longrightarrow t + \xi^0(t,\mathbf{x}),\; x^i \Longrightarrow x^i \quad \forall\, i=1,2,3 \longrightarrow \delta\phi \Longrightarrow \delta\phi + \dot{\phi}_0(t)\xi^0(t,\mathbf{x}).\end{array} \tag{2.2}$$

Here $\xi^0(t,\mathbf{x})$ and $\xi^i(t,\mathbf{x}) \forall i = 1,2,3$ are the diffeomorphism parameter. In this context one can choose a specific gauge in which we set the background scalar degrees of freedom as, $\phi(t,\mathbf{x}) = \phi_0(t)$, which is consistent with the requirement that the perturbation in the scalar field vanishes:

$$\boxed{\textbf{Unitary gauge fixing} \quad \Rightarrow \quad \delta\phi(t,\mathbf{x}) = 0}\,, \tag{2.3}$$

In cosmological perturbation theory this is known as unitary gauge in which all degrees of freedom are preserved in the metric of quasi de Sitter space. This phenomenon is analogous to the spontaneous symmetry breaking as appearing in the context of $SU(N)$ gauge theory where the Goldstone mode transform in a non-linear fashion and destroyed by the $SU(N)$ gauge boson in unitary gauge to give a massive spin 1 degrees of freedom after symmetry breaking. In a alternative way one can present the framework of EFT by describing cosmological perturbation theory during inflation where time diffeomorphisms are realized in non-linear fashion.

Now to construct a most general structure of the EFT action suitable for inflationary paradigm we need to follow the step appended below:

1. One must write down the EFT operators that are functions of the metric $g_{\mu\nu}$. Here one of the possibilities is Riemann tensor.

2. Also the EFT operators are invariant under the linearly realized time dependent spatial diffeomorphic transformation:

$$\boxed{\textbf{Spatial diffeomorphism}: \; t \Longrightarrow t, \quad x^i \Longrightarrow x^i + \xi^i(t,\mathbf{x}) \quad \forall\, i=1,2,3}\,. \tag{2.4}$$

   For an example, one can consider an EFT operator constructed by g^{00} or its polynomials without derivatives which transform like a scalar under Eq (2.4).

3. Due to the reduced symmetry of the physical system many more extra contributions are allowed in the EFT action.

4. In the EFT action one can also allow geometrical quantities in a preferred space-time slice. For an example, one can consider the extrinsic curvature $K_{\mu\nu}$ of surfaces at constant time, which transform like a tensor under Eq (2.4).

Consequently the most general EFT action can be written in terms of all possible allowed operators by the space-time diffeomorphism as [6, 19]:

$$\begin{aligned} S = \int d^4x\sqrt{-g}\Bigg[&\frac{M_p^2}{2}R + M_p^2\dot{H}g^{00} - M_p^2\left(3H^2+\dot{H}\right) + \sum_{n=2}^{\infty}\frac{M_n^4(t)}{n!}(\delta g^{00})^n \\ &-\sum_{q=0}^{\infty}\frac{\bar{M}_1^{3-q}(t)}{(q+2)!}\delta g^{00}\left(\delta K^\mu_\mu\right)^{q+1} - \sum_{m=0}^{\infty}\frac{\bar{M}_2^{2-m}(t)}{(m+2)!}\left(\delta K^\mu_\mu\right)^{m+2} - \sum_{m=0}^{\infty}\frac{\bar{M}_3^{2-m}(t)}{(m+2)!}\left[\delta K\right]^{m+2} + \cdots\Bigg]. \end{aligned} \tag{2.5}$$

where the dots stand for higher order fluctuations in the EFT action which contains operators with more derivatives in space-time metric. Here we use the following sets of definitions for extrinsic curvature $K_{\mu\nu}$, unit normal n_μ and induced metric $h_{\mu\nu}$:

$$\begin{aligned} K_{\mu\nu} &= h^\sigma_\mu \nabla_\sigma n_\nu = \frac{\delta^0_\mu \partial_\nu g^{00} + \delta^0_\nu \partial_\mu g^{00}}{2(-g^{00})^{3/2}} + \frac{\delta^0_\mu \delta^0_\nu g^{0\sigma}\partial_\sigma g^{00}}{2(-g^{00})^{5/2}} - \frac{g^{0\rho}\left(\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}\right)}{2(-g^{00})^{1/2}}, \\ h_{\mu\nu} &= g_{\mu\nu} + n_\mu n_\nu, \qquad n_\mu = \frac{\partial_\mu t}{\sqrt{-g^{\mu\nu}\partial_\mu t \partial_\nu t}} = \frac{\delta^0_\mu}{\sqrt{-g^{00}}}. \end{aligned} \tag{2.6}$$

Here $\delta K_{\mu\nu}$ represents the variation of the extrinsic curvature of constant time surfaces with respect to the unperturbed background FLRW metric in quasi de Sitter space-time:

$$\delta g^{00} = g^{00} + 1, \qquad \delta K_{\mu\nu} = K_{\mu\nu} - a^2 H h_{\mu\nu}. \tag{2.7}$$

Additionally, we have used a shorthand notation $[\delta K]$ to define the following tensor contraction rule useful to quantify the EFT action [19]:

$$[\delta K]^{m+2} = \delta K^{\mu_1}_{\mu_2} \delta K^{\mu_2}_{\mu_3} \delta K^{\mu_3}_{\mu_4} \cdots \delta K^{\mu_{m+1}}_{\mu_{m+2}} \delta K^{\mu_{m+2}}_{\mu_1}. \tag{2.8}$$

Before going to the further details let us first point out the few important characteristics of the EFT action which are appended bellow:

- In the EFT action the operators $M_p^2 \dot{H} g^{00}$ and $M_p^2\left(3H^2 + \dot{H}\right)$ are completely specified by the Hubble parameter $H(t)$ which is the solution of Friedman's Eqns in unperturbed background.
- Rest of the contributions in EFT action captures the effect of quantum fluctuations, which are characterized by the perturbation around the background FLRW solution of all UV complete theories of inflation.
- The coefficients of the operators appearing in the EFT action are in general time dependent.

Now as we are interested to compute the two and three point correlation function, we have restricted to the following truncated EFT action [6, 19]:

$$\boxed{\begin{aligned} S = \int d^4x \sqrt{-g}\Bigg[&\frac{M_p^2}{2}R + M_p^2 \dot{H} g^{00} - M_p^2\left(3H^2 + \dot{H}\right) + \frac{M_2^4(t)}{2!}\left(g^{00}+1\right)^2 + \frac{M_3^4(t)}{3!}\left(g^{00}+1\right)^3 \\ &- \frac{\bar{M}_1^3(t)}{2}\left(g^{00}+1\right)\delta K^\mu_\mu - \frac{\bar{M}_2^2(t)}{2}(\delta K^\mu_\mu)^2 - \frac{\bar{M}_3^2(t)}{2}\delta K^\mu_\nu \delta K^\nu_\mu\Bigg].\end{aligned}} \tag{2.9}$$

where we have considered the terms in two derivatives in the metric [2].

[2]As we are dealing with EFT, in principle one can consider opeators which includes higher derivatives in the metric i.e. $\left(g^{00}+1\right)^2 \delta K^2$, $\delta K^2 \delta K^\nu_\mu \delta K^\mu_\nu$, δK^3, $\delta K \delta N^2$ (here $\delta N = N - 1$, where N is the lapse function in ADM formalism. See ref. [56] for more details.) etc contributions. But since we have considered the terms two derivative in the metric we have truncated the EFT action in the form presencted in Eq. (2.9) and the form of the EFT action is exactly similar with ref. [6]. In this paper our prime objective is to concentrate only on the leading order tree level contributions and for this reason we have not considered any subleading suppressed contributions or any other contributions which are coming from the quantum loop corrections. Additionally, we have also neglected the term like $\left(g^{00}+1\right)^2 \delta K$ in the EFT action as this term is suppressed by the contribution $H^2\epsilon << 1$ in the decoupling limit and also the higher derivatives of the Goldsotone mode π after implementing the symmetry breaking through Stückelberg trick.

2.2 EFT as a theory of Goldstone Boson

2.2.1 Stückelberg trick I: An example from $SU(N)$ gauge theory with massive gauge boson in flat background

In the unitary gauge the EFT action consist of graviton mode two helicities and scalar mode respectively. In this context first we apply a broken time diffeomorphic transformation on Goldstone boson. As a result $SU(N)$ gauge symmetry [6, 57] is non-linearly realized in the framework of EFT. This mechanism is commonly known as Stückelberg trick. Let us mention two crucial roles of Stückelberg trick in gauge theory:

1. Using this trick in $SU(N)$ gauge theory [6, 57] one can study the physical implications from longitudinal components of a massive gauge boson degrees of freedom.

2. It is expected that in the weak coupling limit the contribution from the mixing terms are very small and consequently Goldstone modes decouple from the theory.

To give a specific example of Stückelberg trick we consider $SU(N)$ gauge theory characterized by a non-abelian gauge field A^a_μ in the background of Minkowski flat space-time. In unitary gauge this theory is described by the following action:

$$\boxed{S = \int d^4x \left[-\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{m^2}{2}\mathrm{Tr}(A_\mu A^\mu)\right]}, \tag{2.10}$$

where $A_\mu = A^a_\mu T_a$ and $F^a_{\mu\nu} = \partial_{[\mu}A^a_{\nu]}$. Here the label $a = 1, 2, \cdots, N$ for $SU(N)$ gauge theory. Also T_a are the generators of non-abelian gauge group which satisfy the following properties:

$$\left[T^a, T^b\right] = if^{abc}T_c, \qquad \mathrm{Tr}(T^a) = 0, \qquad \mathrm{Tr}(T^aT^b) = \frac{\delta^{ab}}{2}. \tag{2.11}$$

Here $f^{abc}\ \forall a, b, c = 1, 2, \cdots, N$ are the structure constants of the non-abelian $SU(N)$ gauge theory.

It is important to mention that, in this context the $SU(N)$ gauge transformation on the non-abelian gauge field can be written as:

$$A_\mu \Longrightarrow \tilde{A}_\mu = \frac{i}{g}UD_\mu U^\dagger, \qquad \text{with} \quad D_\mu = \partial_\mu - igA_\mu \tag{2.12}$$

where D_μ is the covariant derivative. Here g is the gauge coupling parameter for $SU(N)$ non-abelian gauge theory. Under this gauge transformation each of the terms in the action stated in Eq (2.10) transform as:

$$\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \Longrightarrow \mathrm{Tr}(\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}) = \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}), \tag{2.13}$$

$$\frac{m^2}{2}\mathrm{Tr}(A_\mu A^\mu) \Longrightarrow \frac{m^2}{2}\mathrm{Tr}(\tilde{A}_\mu\tilde{A}^\mu) = \frac{m^2}{2g}\mathrm{Tr}[(D_\mu U^\dagger)(D^\mu U)], \tag{2.14}$$

where U is the unitary operator in $SU(N)$ non-abelian gauge theory.

Consequently after doing $SU(N)$ gauge transformation action can be expressed as:

$$\boxed{S \Longrightarrow \tilde{S} = S + \underbrace{\int d^4x \left[\frac{m^2}{2}\mathrm{Tr}(A_\mu A^\mu) - \frac{m^2}{2g}\mathrm{Tr}[(D_\mu U^\dagger)(D^\mu U)]\right]}_{\textbf{Additional part which breaks SU(N) gauge symmetry}}}. \tag{2.15}$$

where $\underbrace{}$ term signifies the gauge symmetry breaking contribution in the unitary gauge.

Further it is important to note that the $SU(N)$ gauge symmetry can be restored by defining the previously mentioned unitary operator in a following fashion:

$$U = \exp\left[iT^a\pi^a(t,\mathbf{x})\right], \tag{2.16}$$

where one can identify the $\pi^a\ \forall\ a = 1, 2, \cdots, N$ s with the Goldstone modes, which transform in a linear fashion under the action of the following gauge transformation:

$$U \Longrightarrow \tilde{U} = \exp\left[iT^a\tilde{\pi}^a(t,\mathbf{x})\right] = \Sigma(t,\mathbf{x})\exp\left[iT^a\pi^a(t,\mathbf{x})\right] = \underbrace{\Sigma(t,\mathbf{x})}_{Local\ operator} U. \tag{2.17}$$

For the sake of simplicity one can rescale the Goldstone modes by absorbing the mass of the $SU(N)$ gauge field m and the $SU(N)$ gauge coupling parameter g by introducing the following canonical normalization as given by:

$$\boxed{\text{Canonical normalization :} \qquad \pi_c = \frac{m}{g}\pi}\,. \tag{2.18}$$

Consequently, the action in terms of canonically normalized field π_c can be written after $SU(N)$ gauge transformation as:

$$\boxed{\begin{aligned} S \Longrightarrow \tilde{S} = S + \int d^4x \Bigg[&\frac{m^2}{2}\text{Tr}(A_\mu A^\mu) - \underbrace{\frac{1}{2}\text{Tr}[(\partial_\mu\pi_c)(\partial^\mu\pi_c)]}_{\textbf{Kinetic term of Goldstone}} \\ &\underbrace{-\frac{2g^2}{m}\text{Tr}(A_\mu\partial^\mu\pi_c) + \frac{g^2}{2}\text{Tr}(A_\mu A^\mu\pi_c^2) + ig\text{Tr}(\pi_c A_\mu\partial^\mu\pi_c)}_{\textbf{Mixing terms after canonical normalization}}\Bigg]. \end{aligned}} \tag{2.19}$$

It is important to note the important facts from Eq (2.19) which are appended below:

- The last two terms in Eq (2.19) are the mixing terms between the transverse component of the $SU(N)$ gauge field, Goldstone boson and its kinetic term respectively.
- Here one can neglect all such mixing contributions at the energy scale $E_{mix} >> m$. Consequently, two sectors decouple from each other as they are weakly coupled in the energy scale $E_{mix} >> m$ and the Eq (2.19) takes the following form:

$$\boxed{S \Longrightarrow \tilde{S} = S + \int d^4x \left[\frac{m^2}{2}\text{Tr}(A_\mu A^\mu) - \frac{1}{2}\text{Tr}[(\partial_\mu\pi_c)(\partial^\mu\pi_c)]\right]}\,. \tag{2.20}$$

2.2.2 Stückelberg trick II: Broken time diffeomorphism in quasi de Sitter background

Here one need to perform a time diffeomorphism with a local parameter $\xi^0(t,\mathbf{x})$, which is interpreted as a Goldstone field $\pi(t,\mathbf{x})$. These Goldstone modes shifts under the application of time diffeomorphism, as given by:

$$\boxed{\textbf{Time diffeomorphism :}\ t \Longrightarrow t + \xi^0(t,\mathbf{x}),\ x^i \Longrightarrow x^i \quad \forall\ i = 1,2,3 \longrightarrow \pi(t,\mathbf{x}) \rightarrow \pi(t,\mathbf{x}) - \xi^0(t,\mathbf{x}).} \tag{2.21}$$

The π is the Goldstone mode which describes the scalar perturbations around the background FLRW metric. The effective action in the unitary gauge can be reproduced by gauge fixing the time diffeomorphism as:

$$\boxed{\textbf{Unitary gauge fixing} \quad \Rightarrow \quad \pi(t,\mathbf{x}) = 0 \quad \Rightarrow \quad \tilde{\pi}(t,\mathbf{x}) = -\xi^0(t,\mathbf{x})}\,. \tag{2.22}$$

To construct the EFT action it is important to write down the transformation property of each operators under the application of broken time diffeomorphism, which are given by:

1. **Rule for metric:** Under broken time diffeomorphism contravariant and covariant metric transform as:

$$\begin{aligned}\textbf{Contravariant metric}: \quad g^{00} &\Longrightarrow (1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi})g^{0i}\partial_i\pi + g^{ij}\partial_i\pi\partial_j\pi, \\ g^{0i} &\Longrightarrow (1+\dot{\pi})g^{0i} + g^{ij}\partial_j\pi, \\ g^{ij} &\Longrightarrow g^{ij}.\end{aligned} \tag{2.23}$$

$$\begin{aligned}\textbf{Covariant metric}: \quad g_{00} &\Longrightarrow (1+\dot{\pi})^2 g_{00}, \\ g_{0i} &\Longrightarrow (1+\dot{\pi})g_{0i} + g_{00}\dot{\pi}\partial_i\pi, \\ g_{ij} &\Longrightarrow g_{ij} + g_{0j}\partial_i\pi + g_{i0}\partial_j\pi.\end{aligned} \tag{2.24}$$

2. **Rule for Ricci scalar and Ricci tensor:** Under broken time diffeomorphism Ricci scalar and the spatial component of the Ricci tensor on 3-hypersurface transform as:

$$\begin{aligned}\textbf{Ricci scalar}: \quad {}^{(3)}R &\Longrightarrow {}^{(3)}R + \frac{4}{a^2}H(\partial^2\pi), \\ \textbf{Spatial Ricci tensor}: \quad {}^{(3)}R_{ij} &\Longrightarrow {}^{(3)}R_{ij} + H(\partial_i\partial_j\pi + \delta_{ij}\partial^2\pi).\end{aligned} \tag{2.25}$$

3. **Rule for extrinsic curvature:** Under broken time diffeomorphism trace and the spatial, time and mixed component of the extrinsic curvature transform as:

$$\begin{aligned}\textbf{Trace}: \quad \delta K &\Longrightarrow \delta K - 3\pi\dot{H} - \frac{1}{a^2}(\partial^2\pi), \\ \textbf{Spatial extrinsic curvature}: \quad \delta K_{ij} &\Longrightarrow \delta K_{ij} - \pi\dot{H}h_{ij} - \partial_i\partial_j\pi \\ \textbf{Temporal extrinsic curvature}: \quad \delta K^0_0 &\Longrightarrow \delta K^0_0, \\ \textbf{Mixed extrinsic curvature}: \quad \delta K^0_i &\Longrightarrow \delta K^0_i, \\ \textbf{Mixed extrinsic curvature}: \quad \delta K^i_0 &\Longrightarrow \delta K^i_0 + 2Hg^{ij}\partial_j\pi.\end{aligned} \tag{2.26}$$

4. **Rule for time dependent EFT coefficients:** Under broken time diffeomorphism time dependent EFT coefficients transform after canonical normalization $\pi_c = F^2(t)\pi$ as:

$$\boxed{\begin{aligned}\textbf{EFT coefficient}: \quad F(t) \Longrightarrow F(t+\pi) &= \left[\sum_{n=0}^{\infty} \frac{\pi^n}{n!}\frac{d^n}{dt^n}\right] F(t) \\ &= \left[\sum_{n=0}^{\infty} \underbrace{\frac{\pi_c^n}{n! F^{2n}}}_{\textbf{Suppression}} \frac{d^n}{dt^n}\right] F(t) \approx F(t)\ .\end{aligned}} \tag{2.27}$$

Here $F(t)$ corresponds to all EFT coefficients mention in the EFT action.

5. **Rule for Hubble parameter:** Under broken time diffeomorphism, time dependent EFT coefficients transform after using the following canonical normalization:

$$\boxed{\textbf{Canonical normalization}: \quad \pi_c = F^2(t)\pi}\ , \tag{2.28}$$

as given by:

$$\boxed{\begin{aligned}\textbf{Hubble parameter}: \quad H(t) \Longrightarrow H(t+\pi) &= \left[\sum_{n=0}^{\infty} \frac{\pi^n}{n!}\frac{d^n}{dt^n}\right] H(t) \\ &= \left[1 \underbrace{- \pi H(t)\epsilon - \frac{\pi^2 H(t)}{2}\left(\dot{\epsilon} - 2\epsilon^2\right) + \cdots}_{\textbf{Correction terms}}\right] H(t)\ .\end{aligned}} \tag{2.29}$$

Here $\epsilon = -\dot{H}/H^2$ is the slow-roll parameter.

Now to construct the EFT action we need to also understand the behaviour of all the operators appearing in the weak coupling regime of EFT. In this regime one can neglect the mixing contributions between the gravity and Goldstone modes. To demonstrate this explicitly let us start with the EFT operator:

$$\mathcal{O}_1(t) = -\dot{H} M_p^2 g^{00}. \tag{2.30}$$

Under broken time diffeomorphism, the operator $\mathcal{O}(t)$ transform as:

$$\boxed{\mathcal{O}_1(t) \Longrightarrow \left[1 + \frac{\pi}{\epsilon}\left(\dot{\epsilon} - 2H\epsilon^2\right) + \cdots\right]\left[(1+\dot{\pi})^2\mathcal{O}_1(t) - \dot{H} M_p^2\left(2(1+\dot{\pi})\partial_i\pi g^{0i} + g^{ij}\partial_i\pi\partial_j\pi\right)\right]}\ . \tag{2.31}$$

For further simplification the temporal component of the metric g^{00} can be written as, $g^{00} = \bar{g}^{00} + \delta g^{00}$, where the background metric is given by, $\bar{g}^{00} = -1$ and the metric fluctuation is characterized by δg^{00} [6, 19]. Using this in Eq (2.31) and considering only the first term in Eq (2.31) we get a kinetic term, $M_p^2 \dot{H}\dot{\pi}^2\bar{g}^{00}$ and a mixing contribution, $M_p^2\dot{H}\dot{\pi}\delta g^{00}$ respectively. Further we use a canonical normalized metric fluctuation from the mixing contribution as given by:

$$\boxed{\textbf{Canonical normalization}: \quad \delta g_c^{00} = M_p \delta g^{00}}\ , \tag{2.32}$$

in terms of which one can write, $M_p^2\dot{H}\dot{\pi}\delta g^{00} = \sqrt{\dot{H}}\dot{\pi}_c\delta g_c^{00}$. Consequently, at above the energy scale $E_{mix} = \sqrt{\dot{H}}$, we can neglect this mixing term in the weak coupling regime.

One can also consider mixing contributions $M_p^2\dot{H}\dot{\pi}^2\delta g^{00}$ and $\pi M_p^2\ddot{H}\dot{\pi}\bar{g}^{00}$, which can be recast after canonical normalization as, $M_p^2\dot{H}\dot{\pi}^2\delta g^{00} = \dot{\pi}_c^2\delta g_c^{00}/M_p$ and $\pi M_p^2\ddot{H}\dot{\pi}\bar{g}^{00} = \ddot{H}\pi_c\dot{\pi}_c\bar{g}^{00}/\dot{H}$ with $\ddot{H}/\dot{H} << 1$. Here all higher order terms in $\dot{\pi}$ will lead to additional Planck-suppression after canonical normalization. Consequently, we can neglect the contribution from $M_p^2\dot{H}\dot{\pi}\delta g^{00}$ term at the scale $E > E_{mix}$. Finally, in the weak coupling regime one can recast Eq (2.31) as:

$$\boxed{\mathcal{O}_1(t) \Longrightarrow \mathcal{O}_1(t)\left[\dot{\pi}^2 - \frac{1}{a^2}(\partial_i\pi)^2\right]}\,. \tag{2.33}$$

2.2.3 The Goldstone action from EFT

Finally in the weak coupling limit (or decoupling limit) we get the following simplified EFT action:

$$S_{EFT} = S_g + S_\pi, \tag{2.34}$$

where the gravitational part and the Goldstone action is given by:

$$S_g = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - M_p^2\left(3H^2 + \dot{H}\right)\right], \tag{2.35}$$

$$S_\pi = S_\pi^{(2)} + S_\pi^{(3)} + \cdots, \tag{2.36}$$

where the second and third order Goldstone action can be written as:

$$\boxed{\begin{aligned} S_\pi^{(2)} &= \int d^4x\ a^3\left[-M_p^2\dot{H}\left(\dot{\pi}^2 - \frac{1}{a^2}(\partial_i\pi)^2\right) + 2M_2^4\dot{\pi}^2\right. \\ &\left. + \frac{1}{2}\left(\bar{M}_3^2 + 3\bar{M}_2^2\right)H^2(1-\epsilon)\frac{(\partial_i\pi)^2}{a^2} - \left(\bar{M}_3^2 + 3\bar{M}_2^2\right)H^2\frac{(\partial_i\pi)^2}{a^2} - \bar{M}_1^3\dot{\pi}\frac{1}{a^2}(\partial_i^2\pi)\right]\end{aligned}}\,. \tag{2.37}$$

$$\boxed{\begin{aligned} S_\pi^{(3)} &= \int d^4x\ a^3\left[\left(2M_2^4 - \frac{4}{3}M_3^4\right)\dot{\pi}^3 - 2M_2^4\dot{\pi}\frac{1}{a^2}(\partial_i\pi)^2\right. \\ &- \bar{M}_3^2\pi\dot{H}\frac{1}{a^2}\partial_i^2\pi - 3\bar{M}_2^2\dot{H}\pi\frac{1}{a^2}(\partial_i^2\pi) + \frac{3}{2}\bar{M}_1^3\pi\dot{H}\frac{1}{a^2}(\partial_i\pi)^2 \\ &\left. - \frac{3}{2}\bar{M}_1^3\dot{H}\pi\dot{\pi}^2 - \bar{M}_1^3\dot{\pi}\frac{1}{a^2}(\partial_i\pi)^2\right]\end{aligned}}\,. \tag{2.38}$$

Here we introduce EFT sound speed c_S as:

$$c_S \equiv \frac{1}{\sqrt{1 - \frac{2M_2^4}{\dot{H}M_p^2}}}. \tag{2.39}$$

Here if we set $M_2 = 0$ or equivalently if we say that $\frac{M_2^4}{2!}(g^{00}+1)^2$ term is absent in the effective Lagrangian then Eq (2.39) suggests that in that case sound speed $c_S = 1$, which is true for single field canonical slow roll inflation. Next using Eq (2.39) and applying integration by parts in the

Goldstone part of the Lagrangian we get [3]:

$$\boxed{S_\pi^{(2)} = \int d^4x\ a^3 \left(-\frac{M_p^2\dot{H}}{c_S^2}\right)\left[\dot{\pi}^2 - c_S^2\left(1 - \frac{\bar{M}_1^3 H}{M_p^2\dot{H}} - \left[\bar{M}_3^2 + 3\bar{M}_2^2\right]\frac{H^2(1+\epsilon)}{2M_p^2\dot{H}}\right)\frac{1}{a^2}(\partial_i\pi)^2\right]}\ . \quad (2.44)$$

$$\boxed{\begin{aligned} S_\pi^{(3)} = \int d^4x\ a^3 \Bigg[&\left\{\left(1 - \frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3}M_3^4\right\}\dot{\pi}^3 \\ &- \left\{\left(1 - \frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H\right\}\frac{1}{a^2}\dot{\pi}(\partial_i\pi)^2 \\ &- \frac{9}{2}\bar{M}_1^3 H^2\pi\dot{\pi}^2 + \frac{3}{2}\bar{M}_1^3 H\frac{1}{a^2}\pi\frac{d}{dt}(\partial_i\pi)^2\Bigg]\ .\end{aligned}} \quad (2.45)$$

In the present context metric fluctuation of the spatial components are given by:

$$g_{ij} = a^2(t)\left[(1 + 2\zeta(t,\mathbf{x}))\,\delta_{ij} + \gamma_{ij}\right] \quad \forall \quad i = 1,2,3, \quad (2.46)$$

where $a(t)$ is the scale factor in FLRW quasi de Sitter background space-time. Also $\zeta(t,\mathbf{x})$ is known as curvature perturbation which signifies scalar fluctuation. On the other hand, tensor fluctuations are identified with γ_{ij}, which is spin-2, transverse and traceless rank 2 tensor. Here under the broken time diffeomorphism the scale factor $a(t)$ transforms in the following fashion:

$$\boxed{a(t) \Longrightarrow a(t - \pi(t,\mathbf{x})) = a(t) - H\pi(t,\mathbf{x})a(t) + \cdots \approx a(t)\left(1 - H\pi(t,\mathbf{x})\right)}\ . \quad (2.47)$$

Further using Eq (2.46) and Eq (2.47), we get:

$$a^2(t)\left(1 - H\pi(t,\mathbf{x})\right)^2 \approx a^2(t)\left(1 - 2H\pi(t,\mathbf{x})\right) = a^2(t)\left(1 + 2\zeta(t,\mathbf{x})\right). \quad (2.48)$$

This implies that the curvature perturbation $\zeta(t,\mathbf{x})$ can be written in terms of Goldstone modes $\pi(t,\mathbf{x})$ in the following way [4]:

$$\boxed{\textbf{Quantum fluctuation in terms of Goldstone mode:} \quad \zeta(t,\mathbf{x}) = -H\pi(t,\mathbf{x})}\ . \quad (2.50)$$

[3] Let us concentrate on the following contribution in the second and third order peturbed EFT action, which can be written after integration by parts as:

$$\begin{aligned} S_\pi^2 \supset -\int d^3x\ dt\ a^3\ \bar{M}_1^3\ \frac{\dot{\pi}}{a^2}\ (\partial_i^2\pi) &= \int d^3x\ dt\ a^3\ \frac{\bar{M}_1^3}{a^2}\left[-\partial_i\left(\dot{\pi}\partial_i\pi\right) + \frac{1}{2}\frac{d}{dt}(\partial_i\pi)^2\right] \\ &= \int d^3x\ dt\ a^3\ \frac{\bar{M}_1^3}{2}\left[\frac{d}{dt}\left(\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{H}{a^2}(\partial_i\pi)^2\right] \\ &= -\int d^3x\ dt\ a^3\ \frac{\bar{M}_1^3}{2}\ H\ \frac{(\partial_i\pi)^2}{a^2}. \end{aligned} \quad (2.40)$$

$$S_\pi^3 \supset -\int d^3x\ dt\ a^3\ \bar{M}_1^3\ \frac{3}{2}\ \dot{H}\ \pi\ \dot{\pi}^2 = \int d^3x\ dt\ a^3\left[\frac{3}{2}\bar{M}_1^3 H\dot{\pi}^3 - \frac{9}{2}H^2\bar{M}_1^3\pi\dot{\pi}^2\right]. \quad (2.41)$$

$$S_\pi^3 \supset \int d^3x\ dt\ a^3\ \bar{M}_1^3\ \frac{3}{2}\ \dot{H}\ \pi\ \frac{(\partial_i\pi)^2}{a^2} = -\int d^3x\ dt\ a^3\left[\frac{3}{2}\bar{M}_1^3\frac{H\pi}{a^2}(\partial_i\pi)^2 + \frac{\dot{\pi}}{a^2}\frac{3}{2}\bar{M}_1^3(\partial_i\pi)^2\right]. \quad (2.42)$$

$$S_\pi^3 \supset -3\int d^3x\ dt\ a^3\ \bar{M}_2^2\ \dot{H}\ \pi\ \frac{\pi}{a^2}\ (\partial_i^2\pi) = \int d^3x\ dt\ a^3\ 3\bar{M}_2^2\left[\frac{H\pi}{a^2}(\partial_i\pi)^2 + \frac{\dot{\pi}}{a^2}(\partial_i\pi)^2\right]. \quad (2.43)$$

[4] Here we have considered the linear relation between the curvature perturbation (ζ) and the Goldstone mode (π). In this context one can consider the following non-linear relation to compute the three point correlation function from the present setup:

$$\zeta(t,\mathbf{x}) = -H\pi(t,\mathbf{x}) - \frac{(\epsilon - \eta)}{2}H^2\pi^2(t,\mathbf{x}) + \cdots, \quad (2.49)$$

Further using Eq (2.50), the effective action for the Goldstone part of the Lagrangian can be recast in terms of curvature perturbation $\zeta(t,\mathbf{x})$ as:

$$\boxed{S^{(2)}_{\zeta} \approx \int d^4x\; a^3\;\left(\frac{M_p^2\epsilon}{c_S^2}\right)\left[\dot{\zeta}^2 - c_S^2\left(1-\frac{\bar{M}_1^3 H}{M_p^2\dot{H}} - \left[\bar{M}_3^2+3\bar{M}_2^2\right]\frac{H^2(1+\epsilon)}{2M_p^2\dot{H}}\right)\frac{1}{a^2}(\partial_i\zeta)^2\right]}\,. \tag{2.51}$$

$$\boxed{\begin{aligned} S^{(3)}_{\zeta} \approx \int d^4x\;\frac{a^3}{H^3}\Bigg[&-\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3 H-\frac{4}{3}M_3^4\right\}\dot{\zeta}^3\\ &+\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3 H\right\}\frac{1}{a^2}\dot{\zeta}(\partial_i\zeta)^2\\ &+\frac{9}{2}\bar{M}_1^3H^2\zeta\dot{\zeta}^2-\frac{3}{2}\bar{M}_1^3H\frac{1}{a^2}\zeta\frac{d}{dt}\left(\partial_i\zeta\right)^2\Bigg]\,.\end{aligned}} \tag{2.52}$$

For further simplification we introduce few new parameters which are appended bellow [5]:

- First of all we define an effective sound speed $\tilde{c}_S$, which can be expressed in terms of the usual EFT sound speed c_S as [6]:

$$\tilde{c}_S = c_S\sqrt{1-\frac{\bar{M}_1^3 H}{M_p^2\dot{H}} - \left[\bar{M}_3^2+3\bar{M}_2^2\right]\frac{H^2(1+\epsilon)}{2M_p^2\dot{H}}}. \tag{2.53}$$

  Since the following approximations:

$$\left|\frac{\bar{M}_1^3 H}{M_p^2\dot{H}}\right| << 1, \qquad \left|\left[\bar{M}_3^2+3\bar{M}_2^2\right]\frac{H^2(1+\epsilon)}{2M_p^2\dot{H}}\right| << 1, \tag{2.54}$$

  are valid in the present context of discussion, one can recast the effective sound speed in the following simplified form as:

$$\tilde{c}_S \approx c_S\left\{1+\frac{1}{2\epsilon H M_p^2}\left[\bar{M}_1^3+\left(\bar{M}_3^2+3\bar{M}_2^2\right)\frac{H(1+\epsilon)}{2}\right]\right\}. \tag{2.55}$$

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where the slow-roll parameters are given by, $\epsilon=-\dot{H}/H^2$ and $\eta=\epsilon-\frac{1}{2}\frac{d\ln\epsilon}{d\mathcal{N}}$. Here $\mathcal{N}=\int H\,dt$, represents the number of e-foldings. However the contribution from such non-linear term is extremely small and proportional to subleading terms ϵ^2, η^2 and $\epsilon\eta$ in the expression for the three point function and the associated bispectrum. From the observational perspective such contributions also not so important and can be treated as very very small correction to the leading order result computed in this paper.

[5]Here we have used few choices for the simplifications of the further computation of the two and three point correlation function in the EFT coefficients which are partly motivated from the ref. [58]. Also it is important to note that, since we are restricted our computation up to tree level and and not considering any quantum effects through loop correction, we have discussed the radiative stablity or naturalness of these choices under quantum corrections.

[6]Here it is important to point out that, in the case when $M_2=0$ we have the EFT sound speed $c_S=1$ exactly, which is true for all canonical slow-roll models of inflation driven by a single field. But since here the EFT coefficients are sufficitly small $\bar{M}_i\forall i=1,2,3(\sim\mathcal{O}(10^{-2}-10^{-3}))$ it is expected that $\tilde{c}_S\approx c_S$ and for the situation $c_S=1$ one can approximately fix $\tilde{c}_S\approx 1$. So for canonical slow-roll model one can easily approimate the redefined sound speed $\tilde{c}_S$ with the ususal EFT sound speed c_S without loosing any generality. But such small EFT coefficients $\bar{M}_i\forall i=1,2,3(\sim\mathcal{O}(10^{-2}-10^{-3}))$ play significant role in the computation of the three point function and the associated bispectrum as in the absence of these coefficients the amplitude of the bispectrum f_{NL} is zero. This also implies that for canonical slow-roll model of single field inflation the amount of non-Gaussinity is not very large and this completely consistent with the previous finding that in that case the amplitude of the bispectrum $f_{NL}\propto\epsilon$ (where ϵ is the slow-roll parameter), at the leading order of the computation. See ref. [30] for details.

- Secondly, we introduce the following connecting relationship between M_3 and M_2 given by:

$$M_3^4 c_S^2 = -\tilde{c}_3 M_2^4. \tag{2.56}$$

When $M_2 = 0$ then from Eq (2.39) we can see that the sound speed $c_S = 1$ and Eq (2.56) also implies that $M_3 = 0$ in that case.

- Next we define the following connecting relationship between M_3 and $\bar{M}_1$ given by:

$$M_3^4 \tilde{c}_4 = -H\bar{M}_1^3 \tilde{c}_3. \tag{2.57}$$

When $M_2 = 0$ then from Eq (2.39) we can see that the sound speed $c_S = 1$ (which is actually the result for single field canonical slow-roll models of inflation) and Eq (2.56) and Eq (2.57) also implies the following possibilities:

  1. $M_3 = 0$, $\bar{M}_1 \neq 0$ and $\frac{\tilde{c}_3}{\tilde{c}_4} \rightarrow 0$. We will look into this possibility in detail during our computation for $c_S = 1$ case as this will finally give rise to non vanishing three point function (non-gaussianity).
  2. $M_3 = 0$, $\bar{M}_1 = 0$ and $\frac{\tilde{c}_3}{\tilde{c}_4} \neq 0$. We don't consider this possibility for $c_S = 1$ case because for this case third ($S^{(3)}_{\zeta}$) action for curvature perturbation vanishes, which will give rise zero three point function (non-gaussianity).

- For further simplification one can also assume that:

$$\bar{M}_3^2 + 3\bar{M}_2^2 = \frac{\bar{M}_1^3}{H\tilde{c}_5} \tag{2.58}$$

so that one can write:

$$\frac{1}{\epsilon H M_p^2}\left[\bar{M}_1^3 + \left(\bar{M}_3^2 + 3\bar{M}_2^2\right)\frac{H(1+\epsilon)}{2}\right] = \frac{\bar{M}_1^3}{\epsilon H M_p^2}\left[1 + \frac{(1+\epsilon)}{2\tilde{c}_5}\right]. \tag{2.59}$$

For $c_S = 1$ this implies the following two possibilities:

  1. $\bar{M}_1 \neq 0$ and $\tilde{c}_5 = -\frac{1}{2}(1+\epsilon)$. We will look into this possibility in detail during our computation for $c_S = 1$ case as this will finally give rise to non vanishing three point function (non-gaussianity).
  2. $\bar{M}_1 = 0$. We don't consider this possibility for $c_S = 1$ case because for this case third ($S^{(3)}_{\zeta}$) action for curvature perturbation vanishes, which will give rise zero three point function (non-gaussianity).

Consequently, the effective sound speed can be recast as:

$$\tilde{c}_S = c_S\sqrt{1 + \frac{\Delta\bar{M}_1^3}{2\epsilon H M_p^2}} \approx c_S\left\{1 + \frac{\Delta\bar{M}_1^3}{4\epsilon H M_p^2}\right\} \tag{2.60}$$

where Δ is defined as, $\Delta = 2 + \frac{1+\epsilon}{\tilde{c}_5}$. Here $\Delta = 0$ for $\tilde{c}_5 = -\frac{1}{2}(1+\epsilon)$ when $c_S = 1$. Consequently we have $\tilde{c}_S = c_S = 1$ in that case.

- For further simplification one can also assume that:

$$\bar{M}_3^2 \approx \bar{M}_2^2 = \frac{\bar{M}_1^3}{4H\tilde{c}_5}. \tag{2.61}$$

Here $c_S = 1$ this implies the following two possibilities:

1. $\bar{M}_3^2 \approx \bar{M}_2^2 \neq 0, \bar{M}_1 \neq 0$ and $\tilde{c}_5 = -\frac{1}{2}(1+\epsilon)$ as mentioned earlier. We will look into this possibility in detail during our computation for $c_S = 1$ case as this will finally give rise to non vanishing non-gaussianity.
2. $\bar{M}_3^2 \approx \bar{M}_2^2 = 0, \bar{M}_1 = 0$. As mentioned earlier here we don't consider this possibility for $c_S = 1$ case because for this case second $(S_\zeta^{(2)})$ and third order $(S_\zeta^{(3)})$ action for curvature perturbation vanishes, which will give rise zero non-gaussianity.

- Next we define the following connecting relationship between M_4 and M_3 given by:

$$M_4^4 \tilde{c}_6 = M_3^4 \tilde{c}_4 = -H\bar{M}_1^3 \tilde{c}_3. \tag{2.62}$$

When $M_2 = 0$ then from Eq (2.39) we can see that the sound speed $c_S = 1$ and Eq (2.56) and Eq (2.62) also implies the following possibilities:

1. $M_4 \neq 0$, $M_3 = 0$, $\bar{M}_1 \neq 0$ and $\frac{\tilde{c}_3}{\tilde{c}_4} \to 0$. We will look into this possibility in detail during our computation for $c_S = 1$ case as this will finally give rise to non vanishing three point function (non-gaussianity).
2. $M_4 = 0$, $M_3 = 0$, $\bar{M}_1 = 0$ and $\frac{\tilde{c}_3}{\tilde{c}_4} \neq 0$. We don't consider this possibility for $c_S = 1$ case because for this case third $(S_\zeta^{(3)})$ order action for curvature perturbation vanishes, which will give rise zero three point function (non-gaussianity).

Further using all such new defined parameters the EFT action for Goldstone boson can be recast as [7]:

For $\mathbf{c_S = 1}$:

$$S_\zeta^{(2)} \approx \int d^4x\, a^3\, M_p^2 \epsilon \left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i \zeta)^2\right]. \tag{2.63}$$

$$\begin{aligned} S_\zeta^{(3)} \approx \int d^4x\, \frac{a^3}{H^3} \Bigg[-\left\{\frac{3}{2}\bar{M}_1^3 H\right\} \dot{\zeta}^3 + \left\{\frac{3}{2}\bar{M}_1^3 H\right\} \frac{1}{a^2} \dot{\zeta} (\partial_i \zeta)^2 \\ + \frac{9}{2}\bar{M}_1^3 H^2 \zeta \dot{\zeta}^2 - \frac{3}{2}\bar{M}_1^3 H \frac{1}{a^2} \zeta \frac{d}{dt}(\partial_i \zeta)^2 \Bigg]. \end{aligned} \tag{2.64}$$

For $\mathbf{c_S < 1}$:

$$S_\zeta^{(2)} \approx \int d^4x\, a^3 \left(\frac{M_p^2 \epsilon}{c_S^2}\right) \left[\dot{\zeta}^2 - \tilde{c}_S^2 \frac{1}{a^2} (\partial_i \zeta)^2\right]. \tag{2.65}$$

$$\begin{aligned} S_\zeta^{(3)} \approx \int d^4x\, a^3 \frac{\epsilon M_p^2}{H} \left(1 - \frac{1}{c_S^2}\right) \Bigg[\left\{1 + \frac{3\tilde{c}_4}{4c_S^2} + \frac{2\tilde{c}_3}{3c_S^2}\right\} \dot{\zeta}^3 - \left\{1 + \frac{3\tilde{c}_4}{4c_S^2}\right\} \frac{1}{a^2} \dot{\zeta} (\partial_i \zeta)^2 \\ - \frac{9H\tilde{c}_4}{4c_S^2} \zeta \dot{\zeta}^2 + \frac{3\tilde{c}_4}{4c_S^2} \frac{1}{a^2} \zeta \frac{d}{dt} (\partial_i \zeta)^2 \Bigg]. \end{aligned} \tag{2.66}$$

[7]Here it is important to note that, for the case $c_S = 1$ we have written an approximated form of the second and third order action by assuming that $\tilde{c}_S \approx c_S \sim 1$, which is true for all canonical slow-roll models of inflation driven by a single field. Here the EFT coefficients are sufficitly small $\bar{M}_i \forall i = 1, 2, 3 (\sim \mathcal{O}(10^{-2} - 10^{-3}))$ for which it is expected that $\tilde{c}_S \approx c_S$ and for the situation $c_S = 1$ one can approximately fix $\tilde{c}_S \approx 1$.

3 Two point correlation function from EFT

3.1 For scalar modes

3.1.1 Mode equation and solution for scalar perturbation

Here we compute the two point correlation from scalar perturbation. For this purpose we consider the second order perturbed action as given by [8]:

$$S^{(2)}_{\zeta} \approx \int d^4x\ a^3 \left(\frac{M_p^2\epsilon}{c_S^2}\right)\left[\dot{\zeta}^2 - c_S^2\left(1 - \frac{\bar{M}_1^3 H}{M_p^2 \dot{H}} - [\bar{M}_3^2 + 3\bar{M}_2^2]\frac{H^2(1+\epsilon)}{2M_p^2\dot{H}}\right)\frac{1}{a^2}(\partial_i\zeta)^2\right], \quad (3.1)$$

which can be recast for $c_S = 1$ and $c_S < 1$ case as:

$$\boxed{\underline{\textbf{For } c_S = 1:} \quad S^{(2)}_{\zeta} \approx \int d^4x\ a^3\ M_p^2\epsilon\left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i\zeta)^2\right]}, \quad (3.2)$$

$$\boxed{\underline{\textbf{For } c_S < 1:} \quad S^{(2)}_{\zeta} \approx \int d^4x\ a^3 \left(\frac{M_p^2\epsilon}{c_S^2}\right)\left[\dot{\zeta}^2 - \tilde{c}_S^2\frac{1}{a^2}(\partial_i\zeta)^2\right]}, \quad (3.3)$$

where the effective sound speed $\tilde{c}_S$ is defined earlier.

Next we define Mukhanov-Sasaki variable variable $v(\eta, \mathbf{x})$ which is defined as:

$$\boxed{\textbf{Mukhanov - Sasaki variable:} \quad v(\eta, \mathbf{x}) = z\ \zeta(\eta, \mathbf{x})\ M_p = -z\ H\ \pi(\eta, \mathbf{x})\ M_p}. \quad (3.4)$$

In general the parameter z is defined for the present EFT setup as, $z = \frac{a\sqrt{2\epsilon}}{\tilde{c}_S}$. Now in terms of $v(\eta, \mathbf{x})$ the second order action for the curvature perturbation can be recast as:

$$\boxed{S^{(2)}_{\zeta} \approx \int d^3x\ d\eta\ \left[v'^2 - \tilde{c}_S^2(\partial_i v)^2\frac{1}{a^2}(\partial_i\zeta)^2 - m_{eff}^2(\eta)v^2\right]}, \quad (3.5)$$

where the effective mass parameter $m_{eff}(\eta)$ is defined as, $m^2_{eff}(\eta) = -\frac{1}{z}\frac{d^2 z}{d\eta^2}$. Here η is the conformal time which can be expressed in terms of physical time t as, $\eta = \int \frac{dt}{a(t)}$. The conformal time described here is negative and lying within $-\infty < \eta < 0$. During inflation the scale factor and the parameter z can be expressed in terms of the conformal time η as:

$$a(\eta) = \begin{cases} -\frac{1}{H\eta} & \textbf{for dS} \\ -\frac{1}{H\eta}(1+\epsilon) & \textbf{for qdS}. \end{cases} \quad (3.6)$$

and

$$z = \frac{a\sqrt{2\epsilon}}{c_S} = \begin{cases} -\frac{1}{H\eta}\frac{\sqrt{2\epsilon}}{c_S} & \textbf{for dS} \\ -\frac{1}{H\eta}\frac{\sqrt{2\epsilon}}{c_S}(1+\epsilon) & \textbf{for qdS}. \end{cases} \quad (3.7)$$

Additionally it is important to note that for de Sitter and quasi de Sitter case the relation between conformal time η and physical time t can be expressed as, $t = -\frac{1}{H}\ln(-H\eta)$. Within this setup inflation ends when the conformal time $\eta \sim 0$.

[8] See also ref. [30] and [43], where similar computation have performed for canonical single field slow roll and generalized slow roll models of inflation in presence of Bunch-Davies vacuum state.

Now further doing the Fourier transform:

$$v(\eta, \boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^3}\, v_{\mathbf{k}}(\eta)\, e^{i\mathbf{k}.\mathbf{x}} \tag{3.8}$$

one can write down the equation of motion for scalar fluctuation as:

$$\boxed{\textbf{Mukhanov – Sasaki Eqn for scalar mode :} \qquad v''_{\mathbf{k}} + \left(\tilde{c}_S^2 k^2 + m^2_{eff}(\eta)\right) v_{\mathbf{k}} = 0}\,. \tag{3.9}$$

Here it is important to note that for de Sitter and quasi de Sitter case the effective mass parameter can be expressed as:

$$m^2_{eff}(\eta) = \begin{cases} -\dfrac{2}{\eta^2} & \textbf{for} \quad \textbf{dS} \\ -\dfrac{\left(\nu^2 - \frac{1}{4}\right)}{\eta^2} & \textbf{for} \quad \textbf{qdS}. \end{cases} \tag{3.10}$$

Here in the de Sitter and quasi de Sitter case the parameter ν can be written as:

$$\nu = \begin{cases} \dfrac{3}{2} & \textbf{for} \quad \textbf{dS} \\ \dfrac{3}{2} + 3\epsilon - \eta + \dfrac{s}{2} & \textbf{for} \quad \textbf{qdS}, \end{cases} \tag{3.11}$$

where ϵ, η and s are the slow-roll parameter defined as:

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = 2\epsilon - \frac{\dot{\epsilon}}{2H\epsilon}, \qquad s = \frac{\dot{c_S}}{Hc_S}. \tag{3.12}$$

In the slow-roll regime of inflation $\epsilon << 1$ and $|\eta| << 1$ and at the end of inflation sow-roll condition breaks when any of the criteria satisfy, (1) $\epsilon = 1$ or $|\eta| = 1$, (2) $\epsilon = 1 = |\eta|$.

The general solution for $v_{\mathbf{k}}(\eta)$ thus can be written as:

$$\boxed{v_{\mathbf{k}}(\eta) = \begin{cases} \sqrt{-\eta}\left[C_1 H^{(1)}_{\frac{3}{2}}\left(-k\tilde{c}_S\eta\right) + C_2 H^{(2)}_{\frac{3}{2}}\left(-k\tilde{c}_S\eta\right)\right] & \textbf{for} \quad \textbf{dS} \\ \sqrt{-\eta}\left[C_1 H^{(1)}_{\nu}\left(-k\tilde{c}_S\eta\right) + C_2 H^{(2)}_{\nu}\left(-k\tilde{c}_S\eta\right)\right] & \textbf{for} \quad \textbf{qdS}. \end{cases}} \tag{3.13}$$

Here C_1 and C_2 are the arbitrary integration constants and the numerical values depend on the choice of the initial vacuum. In the present context we consider the following choice of the vacuum for the computation:

1. **Bunch Davies vacuum:** In this case we choose, $C_1 = 1, C_2 = 0$.
2. **α, β vacuum:** In this case we choose $C_1 = \cosh\alpha, C_2 = e^{i\beta}\sinh\alpha$. Here β is a phase factor.

For the most general solution as stated in Eq (3.13) one can consider the limiting physical situations, as given by, I. **Superhorizon regime:** $k\tilde{c}_S\eta << -1$, II. **Horizon crossing:** $k\tilde{c}_S\eta = -1$, III. **Subhorizon regime:** $k\tilde{c}_S\eta >> -1$.

Finally, considering the behaviour of the mode function in the subhorizon regime and superhorizon regime one can write the expression in de Sitter and quasi de Sitter case as:

$$v_{\mathbf{k}}(\eta) = \begin{cases} \dfrac{1}{i\eta}\dfrac{1}{\sqrt{2}\left(k\tilde{c}_S\right)^{\frac{3}{2}}}\left[C_1 e^{-ik\tilde{c}_S\eta}\left(1 + ik\tilde{c}_S\eta\right)e^{-i\pi} - C_2 e^{ikc_S\eta}\left(1 - ik\tilde{c}_S\eta\right)e^{i\pi}\right] & \textbf{for} \quad \textbf{dS} \\ 2^{\nu-\frac{3}{2}}\dfrac{1}{i\eta}\dfrac{1}{\sqrt{2}\left(k\tilde{c}_S\right)^{\frac{3}{2}}}\left(-k\tilde{c}_S\eta\right)^{\frac{3}{2}-\nu}\left|\dfrac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|\left[C_1 e^{-ik\tilde{c}_S\eta}\left(1 + ik\tilde{c}_S\eta\right)e^{-\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)} \right. & \\ \left. \qquad - C_2 e^{ikc_S\eta}\left(1 - ik\tilde{c}_S\eta\right)e^{\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)}\right] & \textbf{for} \quad \textbf{qdS}. \end{cases}$$

Further using Eq (3.14) one can write down the expression for the curvature perturbation $\zeta(\eta, \mathbf{k}) = \frac{v_{\mathbf{k}}(\eta)}{z\; M_p}$ as:

$$\zeta(\eta,\mathbf{k}) = \begin{cases} \frac{iH\tilde{c}_S}{2\;M_p\sqrt{\epsilon}\,(k\tilde{c}_S)^{\frac{3}{2}}}\left[C_1 e^{-ik\tilde{c}_S\eta}\left(1+ik\tilde{c}_S\eta\right)e^{-i\pi} - C_2 e^{ikc_S\eta}\left(1-ik\tilde{c}_S\eta\right)e^{i\pi}\right] & \textbf{for}\;\; \textbf{dS} \\ 2^{\nu-\frac{3}{2}}\frac{iH\tilde{c}_S}{2\;M_p\sqrt{\epsilon}(1+\epsilon)(\tilde{c}_S k)^{\frac{3}{2}}}(-k\tilde{c}_S\eta)^{\frac{3}{2}-\nu}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|\left[C_1 e^{-ik\tilde{c}_S\eta}\left(1+ik\tilde{c}_S\eta\right)e^{-\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)}\right. & \\ \qquad\left. - C_2 e^{ikc_S\eta}\left(1-ik\tilde{c}_S\eta\right)e^{\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)}\right] & \textbf{for}\;\; \textbf{qdS}. \end{cases}$$

One can further compute the two point function for scalar fluctuation as:

$$\boxed{\langle\zeta(\eta,\mathbf{k})\zeta(\eta,\mathbf{q})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{q})P_\zeta(k,\eta)}\;, \tag{3.14}$$

where $P_\zeta(k, \eta)$ is the power spectrum at time η for scalar fluctuations and in the present context it is defined as:

$$P_\zeta(k,\eta) = \frac{|v_{\mathbf{k}}(\eta)|^2}{z^2 M_p^2} = \begin{cases} \frac{H^2}{4\;M_p^2\epsilon\tilde{c}_S}\frac{1}{k^3}\left|C_1 e^{-ik\tilde{c}_S\eta}\left(1+ik\tilde{c}_S\eta\right)e^{-i\pi} - C_2 e^{ikc_S\eta}\left(1-ik\tilde{c}_S\eta\right)e^{i\pi}\right|^2 & \textbf{for}\;\; \textbf{dS} \\ 2^{2\nu-3}\frac{H^2}{4\;M_p^2\epsilon(1+\epsilon)^2\tilde{c}_S}\frac{1}{k^3}(-k\tilde{c}_S\eta)^{3-2\nu}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \\ \left|C_1 e^{-ik\tilde{c}_S\eta}\left(1+ik\tilde{c}_S\eta\right)e^{-\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)} - C_2 e^{ikc_S\eta}\left(1-ik\tilde{c}_S\eta\right)e^{\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)}\right|^2 & \textbf{for}\;\; \textbf{qdS}. \end{cases} \tag{3.15}$$

3.1.2 Primordial power spectrum for scalar perturbation

Finally at the horizon crossing one can write further the two point correlation function as [9]:

$$\boxed{\langle\zeta(\mathbf{k})\zeta(\mathbf{q})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{q})P_\zeta(k)}\;, \tag{3.16}$$

where $P_\zeta(k)$ is the power spectrum at time η for scalar fluctuations and it is defined as:

$$P_\zeta(k) = \left[\frac{|v_{\mathbf{k}}(\eta)|^2}{z^2 M_p^2}\right]_{|k\tilde{c}_S\eta|=1} = P_\zeta(k_*)\frac{1}{k^3} = \begin{cases} \frac{H^2}{4\;M_p^2\epsilon\tilde{c}_S}\frac{1}{k^3}\left[|C_1|^2+|C_2|^2-(C_1^*C_2+C_1C_2^*)\right] & \textbf{for}\;\; \textbf{dS} \\ 2^{2\nu-3}\frac{H^2}{4\;M_p^2\epsilon(1+\epsilon)^2\tilde{c}_S}\frac{1}{k^3}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[|C_1|^2+|C_2|^2\right. & \\ \qquad\left. -\left(C_1^*C_2 e^{i\pi\left(\nu+\frac{1}{2}\right)} + C_1C_2^* e^{-i\pi\left(\nu+\frac{1}{2}\right)}\right)\right] & \textbf{for}\;\; \textbf{qdS}, \end{cases} \tag{3.17}$$

where $P_\zeta(k_*)$ is power spectrum for scalar fluctuation at the pivot scale $k = k_*$.For simplicity one can keep $k^3/2\pi^2$ dependence outside and further define amplitude of the power spectrum $\Delta_\zeta(k_*)$ at the pivit scale $k = k_*$ as:

$$\Delta_\zeta(k_*) = \frac{k^3}{2\pi^2}P_\zeta(k) = \frac{1}{2\pi^2}P_\zeta(k_*) = \begin{cases} \frac{H^2}{8\pi^2\;M_p^2\epsilon\tilde{c}_S}\left[|C_1|^2+|C_2|^2-(C_1^*C_2+C_1C_2^*)\right] & \textbf{for}\;\; \textbf{dS} \\ 2^{2\nu-3}\frac{H^2}{8\pi^2\;M_p^2\epsilon(1+\epsilon)^2\tilde{c}_S}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[|C_1|^2+|C_2|^2\right. & \\ \qquad\left. -\left(C_1^*C_2 e^{i\pi\left(\nu+\frac{1}{2}\right)} + C_1C_2^* e^{-i\pi\left(\nu+\frac{1}{2}\right)}\right)\right] & \textbf{for}\;\; \textbf{qdS}. \end{cases} \tag{3.18}$$

For **Bunch Davies** and α, β vacua power spectrum can be written as:

[9] See also ref. [30] and [43], where similar computation have performed for canonical single field slow roll and generalized slow roll models of inflation in presence of Bunch-Davies vacuum state.

- **For Bunch Davies vacuum :**
  In this case by setting $C_1 = 1$ and $C_2 = 0$ we get the following expression for the power spectrum:

$$P_\zeta(k) = \begin{cases} \frac{H^2}{4\ M_p^2 \epsilon \tilde{c}_S} \frac{1}{k^3} & \textbf{for}\ \ \textbf{dS} \\ 2^{2\nu-3} \frac{H^2}{4\ M_p^2 \epsilon (1+\epsilon)^2 \tilde{c}_S} \frac{1}{k^3} \left| \frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)} \right|^2 & \textbf{for}\ \ \textbf{qdS}. \end{cases} \tag{3.19}$$

  Also the power spectrum $\Delta_\zeta(k_*)$ at the pivit scale $k = k_*$ as:

$$\Delta_\zeta(k_*) = \begin{cases} \frac{H^2}{8\pi^2\ M_p^2 \epsilon \tilde{c}_S} & \textbf{for}\ \ \textbf{dS} \\ 2^{2\nu-3} \frac{H^2}{8\pi^2\ M_p^2 \epsilon (1+\epsilon)^2 \tilde{c}_S} \left| \frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)} \right|^2 & \textbf{for}\ \ \textbf{qdS}. \end{cases} \tag{3.20}$$

- **For α, β vacuum :**
  In this case by setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get the following expression for the power spectrum:

$$P_\zeta(k) = \begin{cases} \frac{H^2}{4\ M_p^2 \epsilon \tilde{c}_S} \frac{1}{k^3} \left[\cosh 2\alpha - \sinh 2\alpha \cos\beta\right] & \textbf{for}\ \ \textbf{dS} \\ 2^{2\nu-3} \frac{H^2}{4\ M_p^2 \epsilon (1+\epsilon)^2 \tilde{c}_S} \frac{1}{k^3} \left| \frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)} \right|^2 & \\ \left[\cosh 2\alpha - \sinh 2\alpha \cos\left(\pi\left(\nu + \frac{1}{2}\right) + \beta\right)\right] & \textbf{for}\ \ \textbf{qdS}. \end{cases} \tag{3.21}$$

  Also the power spectrum $\Delta_\zeta(k_*)$ at the pivit scale $k = k_*$ as:

$$\Delta_\zeta(k_*) = \begin{cases} \frac{H^2}{8\pi^2\ M_p^2 \epsilon \tilde{c}_S} \left[\cosh 2\alpha - \sinh 2\alpha \cos\beta\right] & \textbf{for}\ \ \textbf{dS} \\ 2^{2\nu-3} \frac{H^2}{8\pi^2\ M_p^2 \epsilon (1+\epsilon)^2 \tilde{c}_S} \left| \frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)} \right|^2 & \\ \left[\cosh 2\alpha - \sinh 2\alpha \cos\left(\pi\left(\nu + \frac{1}{2}\right) + \beta\right)\right] & \textbf{for}\ \ \textbf{qdS}. \end{cases} \tag{3.22}$$

Finally at the horizon crossing we get the following expression for the spectral tilt for scalar fluctuation at the pivot scale $k = k_*$ as:

$$n_\zeta(k_*) - 1 = \left[\frac{d\ln\Delta_\zeta(k)}{d\ln k}\right]_{|k\tilde{c}_S\eta|=1} = 2\eta - 4\epsilon - \tilde{s}, \tag{3.23}$$

where $\tilde{s}$ is defined as, $\tilde{s} = \frac{\dot{\tilde{c}}_S}{H\tilde{c}_S}$.

3.2 For tensor modes

3.2.1 Mode equation and solution for tensor perturbation

Here we compute the two point correlation from tensor perturbation. For this purpose we consider the second order perturbed action as given by [10]:

$$\boxed{S_\gamma^{(2)} \approx \int d^4x\ a^3\ \frac{M_p^2}{8}\left[\left(1 - \frac{\bar{M}_3^2}{M_p^2}\right)\dot{\gamma}_{ij}\dot{\gamma}_{ij} - \frac{1}{a^2}(\partial_m\gamma_{ij})^2\right] = \int d^3x\ d\eta\ a^2\ \frac{M_p^2}{8}\left[\left(1 - \frac{\bar{M}_3^2}{M_p^2}\right)\gamma_{ij}'^2 - (\partial_m\gamma_{ij})^2\right]}. \tag{3.24}$$

[10]See also ref. [30] and [43], where similar computation have performed for canonical single field slow roll and generalized slow roll models of inflation in presence of Bunch-Davies vacuum state.

In Fourier space one can write $\gamma_{ij}(\eta, \mathbf{x})$ as:

$$\gamma_{ij}(\eta, \mathbf{x}) = \sum_{\lambda=\times,+} \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \epsilon^{\lambda}_{ij}(k)\ \gamma_{\lambda}(\eta, \mathbf{k})\ e^{i\mathbf{k}.\mathbf{x}}, \tag{3.25}$$

where the rank-2 polarization tensor ϵ^{λ}_{ij} satisfies the properties, $\epsilon^{\lambda}_{ii} = k^i \epsilon^{\lambda}_{ij} = 0$, $\sum_{i,j} \epsilon^{\lambda}_{ij}\epsilon^{\lambda'}_{ij} = 2\delta_{\lambda\lambda'}$. Similar like scalar fluctuation here we also define a new variable $u_{\lambda}(\eta, \mathbf{k})$ in Fourier space as:

$$u_{\lambda}(\eta, \mathbf{k}) = \frac{a}{\sqrt{2}}\ M_p\ \gamma_{\lambda}(\eta, \mathbf{k}) = \begin{cases} -\dfrac{1}{\sqrt{2}H\eta}\ M_p\ \gamma_{\lambda}(\eta, \mathbf{k}) & \textbf{for} \ \ \textbf{dS} \\ -\dfrac{1}{\sqrt{2}H\eta}(1+\epsilon)\ \ M_p\ \gamma_{\lambda}(\eta, \mathbf{k}) & \textbf{for} \ \ \textbf{qdS}. \end{cases} \tag{3.26}$$

Using $u_{\lambda}(\eta, \mathbf{k})$ one can further write Eq (3.24) as:

$$\boxed{S^{(2)}_{\gamma} \approx \int d^3x\ d\eta\ a^2\ \frac{M_p^2}{4}\left[\left(1 - \frac{\bar{M}_3^2}{M_p^2}\right) u^{'2}_{\lambda}(\eta, \mathbf{k}) - \left(k^2 - \frac{a^{''}}{a}\right)(u_{\lambda}(\eta, \mathbf{k}))^2\right]}\ . \tag{3.27}$$

From this action one can find out the mode equation for tensor fluctuation as:

$$\boxed{\textbf{Mukhanov}-\textbf{Sasaki Eqn for tensor mode :} \qquad u^{''}_{\lambda}(\eta, \mathbf{k}) + \frac{\left(k^2 - \frac{a^{''}}{a}\right)}{\left(1 - \frac{\bar{M}_3^2}{M_p^2}\right)} u_{\lambda}(\eta, \mathbf{k}) = 0}\ . \tag{3.28}$$

Further we introduce a new parameter c_T defined as:

$$c_T = \frac{1}{\sqrt{1 - \frac{\bar{M}_3^2}{M_p^2}}}. \tag{3.29}$$

The general solution for the mode equation for graviton fluctuation can finally written as:

$$\boxed{u_{\lambda}(\eta, \mathbf{k}) = \begin{cases} \sqrt{-\eta}\left[D_1 H^{(1)}_{\frac{1}{2}\sqrt{1+8c_T^2}}(-kc_T\eta) + D_2 H^{(2)}_{\frac{1}{2}\sqrt{1+8c_T^2}}(-kc_T\eta)\right] & \textbf{for} \ \ \textbf{dS} \\ \sqrt{-\eta}\left[D_1 H^{(1)}_{\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}}(-kc_T\eta) + D_2 H^{(2)}_{\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}}(-kc_T\eta)\right] & \textbf{for} \ \ \textbf{qdS}. \end{cases}} \tag{3.30}$$

Here D_1 and D_2 are the arbitrary integration constants and the numerical values depend on the choice of the initial vacuum. In the present context we consider the following choice of the vacuum for the computation:

1. **Bunch Davies vacuum:** In this case we choose, $D_1 = 1, \quad D_2 = 0$.

2. **α, β vacuum:** In this case we choose $D_1 = \cosh\alpha, \quad D_2 = e^{i\beta}\sinh\alpha$. Here β is a phase factor.

For the most general solution as stated in Eq (3.30) one can consider the limiting physical situations, as given by, I. **Superhorizon regime:** $|kc_T\eta| << 1$, II. **Horizon crossing:** $|kc_T\eta| = 1$, III. **Subhorizon regime:** $|kc_T\eta| >> 1$.

Finally, considering the behaviour of the mode function in the subhorizon regime and superhorizon regime we get:

$$u_\lambda(\eta,\mathbf{k})=\begin{cases}2^{\frac{1}{2}\sqrt{1+8c_T^2}-\frac{3}{2}}\frac{1}{i\eta}\frac{1}{\sqrt{2}\,(kc_T)^{\frac{3}{2}}}(-kc_T\eta)^{\frac{3}{2}-\frac{1}{2}\sqrt{1+8c_T^2}}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right| \\ \left[D_1e^{-ikc_T\eta}\left(1+ikc_T\eta\right)e^{-\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}-D_2e^{ikc_T\eta}\left(1-ikc_T\eta\right)e^{\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}\right] & \textbf{for dS} \\ 2^{\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-\frac{3}{2}}\frac{1}{i\eta}\frac{1}{\sqrt{2}\,(kc_T)^{\frac{3}{2}}}(-kc_T\eta)^{\frac{3}{2}-\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right| \\ \left[D_1e^{-ikc_T\eta}\left(1+ikc_T\eta\right)e^{-\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}-D_2e^{ikc_T\eta}\left(1-ikc_T\eta\right)e^{\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right] & \textbf{for qdS}.\end{cases}\tag{3.31}$$

Further using Eq (3.14) one can write down the expression for the curvature perturbation $\zeta(\eta,\mathbf{k})$ as:

$$h_\lambda(\eta,\mathbf{k})=\frac{u_\lambda(\eta,\mathbf{k})}{a\;M_p}=\begin{cases}2^{\frac{1}{2}\sqrt{1+8c_T^2}-\frac{3}{2}}\frac{iH}{M_p}\frac{1}{(kc_T)^{\frac{3}{2}}}(-kc_T\eta)^{\frac{3}{2}-\frac{1}{2}\sqrt{1+8c_T^2}}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right| \\ \left[D_1e^{-ikc_T\eta}\left(1+ikc_T\eta\right)e^{-\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}-D_2e^{ikc_T\eta}\left(1-ikc_T\eta\right)e^{\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}\right] & \textbf{for dS} \\ 2^{\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-\frac{3}{2}}\frac{iH}{M_p\,(1+\epsilon)}\frac{1}{(kc_T)^{\frac{3}{2}}}(-kc_T\eta)^{\frac{3}{2}-\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}} \\ \left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|\left[D_1e^{-ikc_T\eta}\left(1+ikc_T\eta\right)e^{-\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right. \\ \left.\qquad-D_2e^{ikc_T\eta}\left(1-ikc_T\eta\right)e^{\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right] & \textbf{for qdS}.\end{cases}\tag{3.32}$$

3.2.2 Primordial power spectrum for tensor perturbation

One can further compute the two point function for tensor fluctuation as:

$$\boxed{\langle h(\eta,\mathbf{k})h(\eta,\mathbf{q})\rangle=\sum_{\lambda,\lambda'}\langle h_\lambda(\eta,\mathbf{k})h_{\lambda'}(\eta,\mathbf{q})\rangle=(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{q})P_h(k,\eta)}\,,\tag{3.33}$$

where $P_h(k,\eta)$ is the power spectrum at time η for tensor fluctuations and in the present context it is defined as:

$$P_h(k,\eta)=\frac{4|h_\lambda(\eta,\mathbf{k})|^2}{a^2M_p^2}=\begin{cases}2^{\sqrt{1+8c_T^2}-3}\frac{4H^2}{M_p^2}\frac{1}{(kc_T)^3}(-kc_T\eta)^{3-\sqrt{1+8c_T^2}}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 \\ \left|D_1e^{-ikc_T\eta}\left(1+ikc_T\eta\right)e^{-\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}-D_2e^{ikc_T\eta}\left(1-ikc_T\eta\right)e^{\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}\right|^2 & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{4H^2}{M_p^2\,(1+\epsilon)^2}\frac{1}{(kc_T)^3}(-kc_T\eta)^{3-\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}} \\ \left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left|D_1e^{-ikc_T\eta}\left(1+ikc_T\eta\right)e^{-\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right. \\ \left.\quad-D_2e^{ikc_T\eta}\left(1-ikc_T\eta\right)e^{\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right|^2 & \textbf{for qdS}.\end{cases}\tag{3.34}$$

Finally at the horizon crossing we get the following two point correlation function for tensor perturbation as:

$$\boxed{\langle h(\mathbf{k})h(\mathbf{q})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{q})P_h(k)}\,, \tag{3.35}$$

where $P_h(k)$ is known as the power spectrum at the horizon crossing for tensor fluctuations and in the present context it is defined as:

$$P_h(k) = P_h(k_*)\frac{1}{k^3} = \begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{4H^2}{M_p^2c_T^3}\frac{1}{k^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[|D_1|^2+|D_2|^2\right. \\ \quad \left.-\left(D_1^*D_2e^{i\pi\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}+D_1D_2^*e^{-i\pi\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}\right)\right] & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{4H^2}{M_p^2\left(1+\epsilon\right)^2c_T^3}\frac{1}{k^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[|D_1|^2+|D_2|^2\right. \\ \quad \left.-\left(D_1^*D_2e^{i\pi\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}+D_1D_2^*e^{-i\pi\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right)\right] & \textbf{for qdS.} \end{cases} \tag{3.36}$$

where $P_h(k_*)$ is power spectrum for tensor fluctuation at the pivot scale $k = k_*$. For simplicity one can keep $k^3/2\pi^2$ dependence outside and further define amplitude of the power spectrum $\Delta_h(k_*)$ at the pivit scale $k = k_*$ as:

$$\Delta_h(k_*) = \frac{k^3}{2\pi^2}P_h(k) = \frac{1}{2\pi^2}P_h(k_*)$$
$$= \begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{2H^2}{\pi^2M_p^2c_T^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[|D_1|^2+|D_2|^2\right. \\ \quad \left.-\left(C_1^*C_2e^{i\pi\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}+D_1D_2^*e^{-i\pi\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}\right)\right] & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{2H^2}{\pi^2M_p^2\left(1+\epsilon\right)^2c_T^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[|D_1|^2+|D_2|^2\right. \\ \quad \left.-\left(D_1^*D_2e^{i\pi\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}+D_1D_2^*e^{-i\pi\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right)\right] & \textbf{for qdS.} \end{cases} \tag{3.37}$$

For **Bunch Davies** and α, β vacua we get:

- **For Bunch Davies vacuum :**
  In this case by setting $D_1 = 1$ and $D_2 = 0$ we get the following expression for the power spectrum:

$$P_h(k) = \begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{4H^2}{M_p^2c_T^3}\frac{1}{k^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{4H^2}{M_p^2\left(1+\epsilon\right)^2c_T^3}\frac{1}{k^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS.} \end{cases} \tag{3.38}$$

  Also the power spectrum $\Delta_\zeta(k_*)$ at the pivit scale $k = k_*$ as:

$$\Delta_h(k_*) = \begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{2H^2}{\pi^2M_p^2c_T^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{2H^2}{\pi^2M_p^2\left(1+\epsilon\right)^2c_T^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS.} \end{cases} \tag{3.39}$$

- **For α,β vacuum :**
In this case by setting $D_1=\cosh\alpha$ and $D_2=e^{i\beta}\sinh\alpha$ we get the following expression for the power spectrum:

$$P_h(k)=\begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{4H^2}{M_p^2c_T^3}\frac{1}{k^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 \\ \left[\cosh 2\alpha-\sinh 2\alpha\cos\left(\pi\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{4H^2}{M_p^2\left(1+\epsilon\right)^2c_T^3}\frac{1}{k^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 \\ \left[\cosh 2\alpha-\sinh 2\alpha\cos\left(\pi\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for qdS}. \end{cases} \tag{3.40}$$

Also the power spectrum $\Delta_\zeta(k_*)$ at the pivit scale $k=k_*$ as:

$$\Delta_h(k_*)=\begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{2H^2}{\pi^2M_p^2c_T^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 \\ \left[\cosh 2\alpha-\sinh 2\alpha\cos\left(\pi\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{2H^2}{\pi^2M_p^2\left(1+\epsilon\right)^2c_T^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 \\ \left[\cosh 2\alpha-\sinh 2\alpha\cos\left(\pi\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for qdS}. \end{cases} \tag{3.41}$$

Now let us consider a special case for tensor fluctuation where $c_T=1$ and it implies the following two possibilities:

1. $\bar{M}_3=0$. But for this case as we have assumed earlier $\bar{M}_3^2\approx\bar{M}_3^2=\bar{M}_1^3/4H\tilde{c}_5$, then $\bar{M}_1=0$ which is not our matter of interest in this work as this leads to zero three point function for scalar fluctuation. But if we assume that $\bar{M}_3^2\neq\bar{M}_3^2$ but $\bar{M}_3^2=\bar{M}_1^3/4H\tilde{c}_5$ then by setting $\bar{M}_3=0$ one can get $\bar{M}_1\neq 0$, which is necessarily required for non-vanishing three point function for scalar fluctuation.

2. $\bar{M}_3<<M_p$. In this case if we assume $\bar{M}_3^2\approx\bar{M}_3^2=\bar{M}_1^3/4H\tilde{c}_5$, then $\bar{M}_1^3/4H\tilde{c}_5M_p^2<<1$ and $\bar{M}_2<<M_p$. This is perfectly ok of generating non-vanishing three point function for scalar fluctuation.

If we set $c_T=1$ then for **Bunch Davies** and α,β vacua power spectrum can be recast into the following simplified form:

- **For Bunch Davies vacuum :**
In this case by setting $D_1=1$ and $D_2=0$ we get the following expression for the power spectrum:

$$P_h(k)=\begin{cases} \frac{4H^2}{M_p^2}\frac{1}{k^3} & \textbf{for dS} \\ 2^{2\nu-3}\frac{4H^2}{M_p^2\left(1+\epsilon\right)^2}\frac{1}{k^3}\left|\frac{\Gamma\left(\nu\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases} \tag{3.42}$$

Also the power spectrum $\Delta_\zeta(k_*)$ at the pivit scale $k = k_*$ as:

$$\Delta_h(k_*) = \begin{cases} \dfrac{2H^2}{\pi^2 M_p^2} & \textbf{for dS} \\ 2^{\nu-3}\dfrac{2H^2}{\pi^2 M_p^2 (1+\epsilon)^2}\left|\dfrac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases} \tag{3.43}$$

- **For α, β vacuum :**
  In this case by setting $D_1 = \cosh\alpha$ and $D_2 = e^{i\beta}\sinh\alpha$ we get the following expression for the power spectrum:

$$P_h(k) = \begin{cases} \dfrac{4H^2}{M_p^2}\dfrac{1}{k^3}\left[\cosh 2\alpha - \sinh 2\alpha\cos\beta\right] & \textbf{for dS} \\ 2^{2\nu-3}\dfrac{4H^2}{M_p^2(1+\epsilon)^2}\dfrac{1}{k^3}\left|\dfrac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[\cosh 2\alpha - \sinh 2\alpha\cos\left(\pi\left(\nu+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for qdS}. \end{cases} \tag{3.44}$$

Also the power spectrum $\Delta_\zeta(k_*)$ at the pivit scale $k = k_*$ as:

$$\Delta_h(k_*) = \begin{cases} \dfrac{2H^2}{\pi^2 M_p^2}\left[\cosh 2\alpha - \sinh 2\alpha\cos\beta\right] & \textbf{for dS} \\ 2^{2\nu-3}\dfrac{2H^2}{\pi^2 M_p^2(1+\epsilon)^2}\left|\dfrac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \\ \left[\cosh 2\alpha - \sinh 2\alpha\cos\left(\pi\left(\nu+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for qdS}. \end{cases} \tag{3.45}$$

4 Scalar Three point correlation function from EFT

4.1 Basic setup

Here we compute the three point correlation function for perturbations from scalar modes. For this purpose we consider the third order perturbed action for the scalar modes as given by [11]:

$$\boxed{\begin{aligned} S^{(3)}_\zeta \approx \int d^4x\,\frac{a^3}{H^3}\Bigg[&-\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3 H-\frac{4}{3}M_3^4\right\}\dot{\zeta}^3 \\ &+\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3 H\right\}\frac{1}{a^2}\dot{\zeta}(\partial_i\zeta)^2 \\ &+\frac{9}{2}\bar{M}_1^3 H^2\zeta\dot{\zeta}^2-\frac{3}{2}\bar{M}_1^3 H\frac{1}{a^2}\zeta\frac{d}{dt}(\partial_i\zeta)^2\Bigg], \end{aligned}} \tag{4.1}$$

[11] Here it is important to note that the **red** colored terms are the new contribution in the EFT action considered in this paper, which are not present in ref. [6]. From the EFT action itself it is clear that for effective sound speed $c_S = 1$ three point correlation function and the associated bispectrum vanishes if we don't contribution these **red** colored terms. This is obviously true if we fix $c_S = 1$ in the result obtained in ref. [6]. On the other hand if we consider these **red** colored terms then the result is consistent with ref. [30] with $c_S = 1$ and with ref. [43] with $c_S \neq 1$. This implies that $c_S = 1$ is not fully radiatively stable in single field slow roll inflation. However, if we include the effects produced by quantum correction through loop effects, then a small deviation in the effective sound speed $1 - c_S \sim \epsilon\,(H/M_p)^2$ can be produced. See ref. [6] where this fact is clearly pointed. But for inflation we know that in the inflationary regime the slow roll parameter $\epsilon < 1$ and the scale of inflation is $H/M_p << 1$, which imply this deviation is also very small and not very interesting for our purpose studied in this paper.

which can be recast for $c_S = 1$ and $c_S < 1$ case as:

For $c_S = 1$:

$$S^{(3)}_{\zeta} \approx \int d^4x \, \frac{a^3}{H^3}\left[-\left\{\frac{3}{2}\bar{M}^3_1 H\right\}\dot{\zeta}^3 + \left\{\frac{3}{2}\bar{M}^3_1 H\right\}\frac{1}{a^2}\dot{\zeta}(\partial_i\zeta)^2 + \frac{9}{2}\bar{M}^3_1 H^2\zeta\dot{\zeta}^2 - \frac{3}{2}\bar{M}^3_1 H\frac{1}{a^2}\zeta\frac{d}{dt}\left(\partial_i\zeta\right)^2\right], \tag{4.2}$$

For $c_S < 1$:

$$S^{(3)}_{\zeta} \approx \int d^4x \, a^3\,\frac{\epsilon M^2_p}{H}\left(1-\frac{1}{c^2_S}\right)\left[\left\{1+\frac{3\tilde{c}_4}{4c^2_S}+\frac{2\tilde{c}_3}{3c^2_S}\right\}\dot{\zeta}^3 - \left\{1+\frac{3\tilde{c}_4}{4c^2_S}\right\}\frac{1}{a^2}\dot{\zeta}(\partial_i\zeta)^2 - \frac{9H\tilde{c}_4}{4c^2_S}\zeta\dot{\zeta}^2 + \frac{3\tilde{c}_4}{4c^2_S}\frac{1}{a^2}\zeta\frac{d}{dt}\left(\partial_i\zeta\right)^2\right], \tag{4.3}$$

To extract further informations from third order action, first of all one needs to start with the Fourier transform of the curvature perturbation $\zeta(\eta, \mathbf{x})$ defined as:

$$\zeta(\eta, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\zeta_{\mathbf{k}}(\eta)\exp(i\mathbf{k}.\mathbf{x}), \tag{4.4}$$

where $\zeta_{\mathbf{k}}(\eta)$ is the time dependent part of the curvature fluctuation after Fourier transform and can be expressed in terms of the normalized time dependent scalar mode function $v_{\mathbf{k}}(\eta)$ as:

$$\hat{\zeta}(\eta, \mathbf{k}) = \frac{v_{\mathbf{k}}(\eta)}{zM_p} = \frac{\zeta\left(\eta, \mathbf{k}\right)a\left(\mathbf{k}\right) + \zeta^*\left(\eta, -\mathbf{k}\right)a^{\dagger}\left(-\mathbf{k}\right)}{zM_p} \tag{4.5}$$

where z is explicitly defined earlier and $a(\mathbf{k}), a^{\dagger}(\mathbf{k})$ are the creation and annihilation operator satisfies the following commutation relations:

$$\left[a(\mathbf{k}), a^{\dagger}(-\mathbf{k}^{'})\right] = (2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}^{'}), \quad \left[a(\mathbf{k}), a(\mathbf{k}^{'})\right] = 0, \quad \left[a^{\dagger}(\mathbf{k}), a^{\dagger}(\mathbf{k}^{'})\right] = 0. \tag{4.6}$$

4.2 Computation of scalar three point function in interaction picture

Presently our prime objective is to compute the three point function of the curvature fluctuation in momentum space from S^2_{ζ} with respect to the arbitrary choice of vacuum, which leads to important result in the context of primordial cosmology. Further using the interaction picture the three point function of the curvature fluctuation in momentum space can be expressed as:

$$\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle = -i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\; a(\eta)\;\langle 0|\left[\hat{\zeta}(\eta_f, \mathbf{k_1})\hat{\zeta}(\eta_f, \mathbf{k_2})\hat{\zeta}(\eta_f, \mathbf{k_3}), H_{int}\left(\eta\right)\right]|0\rangle, \tag{4.7}$$

where $a(\eta)$ is the scale factor defined in the earlier section in terms of Hubble parameter H and conformal time scale η. Here $|0\rangle$ represents any arbitrary vacuum state and for discussion we will only derive the results for Bunch-Davies vacuum and α,β vacuum. In the interaction picture the Hamiltonian can written as [12]:

$$\boxed{\begin{aligned} H_{int}(\eta) = -\frac{1}{H^3}\left[-\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3H-\frac{4}{3}M_3^4\right\}\dot{\zeta}^3\right. \\ +\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3H\right\}\frac{1}{a^2}\dot{\zeta}(\partial_i\zeta)^2 \\ \left.+\frac{9}{2}\bar{M}_1^3H^2\zeta\dot{\zeta}^2-\frac{3}{2}\bar{M}_1^3H\frac{1}{a^2}\zeta\frac{d}{dt}(\partial_i\zeta)^2\right].\end{aligned}} \tag{4.8}$$

which gives the primary information to compute the explicit expression for the three point function in the present context. After substituting the interaction Hamiltonian we finally get the following expression for the three point function for the scalar fluctuation:

$$\begin{aligned}\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle = -i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\ a(\eta)\ \int\frac{d^3x}{H^3}\int\int\int\frac{d^3k_4}{(2\pi)^3}\frac{d^3k_5}{(2\pi)^3}\frac{d^3k_6}{(2\pi)^3}\ e^{i(\mathbf{k_4}+\mathbf{k_5}+\mathbf{k_6}).\mathbf{x}} \\ \left\{\alpha_1\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}^{'}(\eta,\mathbf{k_4})\hat{\zeta}^{'}(\eta,\mathbf{k_5})\hat{\zeta}^{'}(\eta,\mathbf{k_6})\right]|0\rangle\right. \\ -\alpha_2(\mathbf{k_5}.\mathbf{k_6})\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}^{'}(\eta,\mathbf{k_4})\hat{\zeta}(\eta,\mathbf{k_5})\hat{\zeta}(\eta,\mathbf{k_6})\right]|0\rangle \\ +\alpha_3\ a(\eta)\ \langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}(\eta,\mathbf{k_4})\hat{\zeta}^{'}(\eta,\mathbf{k_5})\hat{\zeta}^{'}(\eta,\mathbf{k_6})\right]|0\rangle \\ -\alpha_4(\mathbf{k_5}.\mathbf{k_6})\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}(\eta,\mathbf{k_4})\hat{\zeta}^{'}(\eta,\mathbf{k_5})\hat{\zeta}(\eta,\mathbf{k_6})\right]|0\rangle \\ \left.-\alpha_5(\mathbf{k_5}.\mathbf{k_6})\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}(\eta,\mathbf{k_4})\hat{\zeta}(\eta,\mathbf{k_5})\hat{\zeta}^{'}(\eta,\mathbf{k_6})\right]|0\rangle\right\},\end{aligned} \tag{4.9}$$

where the coefficients $\alpha_j\forall j=1,2,3,4,5$ are defined as [13]:

$$\alpha_1 = \left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3H-\frac{4}{3}M_3^4\right\}, \tag{4.10}$$

$$\alpha_2 = -\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3H\right\}, \tag{4.11}$$

$$\alpha_3 = -\frac{9}{2}\bar{M}_1^3H^2, \tag{4.12}$$

$$\alpha_4 = \frac{3}{2}\bar{M}_1^3H, \tag{4.13}$$

$$\alpha_5 = \frac{3}{2}\bar{M}_1^3H. \tag{4.14}$$

Now let us evaluate the co-efficients of $\alpha_1,\alpha_2,\alpha_3,\alpha_4$ in the present context using Wick's theorem:

[12] See also ref. [30] and [43], where similar computation have performed for canonical single field slow roll and generalized slow roll models of inflation in presence of Bunch-Davies vacuum state.

[13] Here it is clearly observed that for canonical single field slow-roll model, which is described by $c_S=1$ we have $M_3=0$ and other EFT coeffecients are sufficiently small, $\bar{M}_i\forall i=1,2,3(\sim\mathcal{O}()10^{-3}-10^{-2})$. This directly implies that the contribution in the three point function and in the associated bispectrum is very small and also consistent with the previous result as obtained in ref. [30]. Additionally it is important to mention that, in momentum space the bispectrum containts additional terms in presence of any arbitrary choice of the quantum vacuum initial state. Also, if we compare with the ref. [6]

1. **Coefficient of α_1**:

$$\begin{aligned}
&\langle 0| \left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}^{'}(\eta,\mathbf{k_4})\hat{\zeta}^{'}(\eta,\mathbf{k_5})\hat{\zeta}^{'}(\eta,\mathbf{k_6})\right]|0\rangle \\
&= \langle 0|a(\mathbf{k_1})a(\mathbf{k_2})a(\mathbf{k_3})a^{\dagger}(-\mathbf{k_4})a^{\dagger}(-\mathbf{k_5})a^{\dagger}(-\mathbf{k_6})|0\rangle \\
&\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^{'*}(\eta_f,-\mathbf{k_4})\bar{v}^{'*}(\eta_f,-\mathbf{k_5})\bar{v}^{'*}(\eta_f,-\mathbf{k_6}) \\
&+\langle 0|a(\mathbf{k_4})a(\mathbf{k_5})a(\mathbf{k_6})a^{\dagger}(-\mathbf{k_1})a^{\dagger}(-\mathbf{k_2})a^{\dagger}(-\mathbf{k_3})|0\rangle \\
&\bar{v}^{*}(\eta_f,-\mathbf{k_1})\bar{v}^{*}(\eta_f,-\mathbf{k_2})\bar{v}^{*}(\eta_f,-\mathbf{k_3})\bar{v}^{'}(\eta,\mathbf{k_4})\bar{v}^{'}(\eta,\mathbf{k_5})\bar{v}^{'}(\eta,\mathbf{k_6}).
\end{aligned} \tag{4.15}$$

2. **Coefficient of α_2**:

$$\begin{aligned}
&(\mathbf{k_5}.\mathbf{k_6})\langle 0| \left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}^{'}(\eta,\mathbf{k_4})\hat{\zeta}(\eta,\mathbf{k_5})\hat{\zeta}(\eta,\mathbf{k_6})\right]|0\rangle \\
= \quad &(\mathbf{k_5}.\mathbf{k_6})\langle 0|a(\mathbf{k_1})a(\mathbf{k_2})a(\mathbf{k_3})a^{\dagger}(-\mathbf{k_4})a^{\dagger}(-\mathbf{k_5})a^{\dagger}(-\mathbf{k_6})|0\rangle \\
&\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^{'*}(\eta_f,-\mathbf{k_4})\bar{v}^{*}(\eta_f,-\mathbf{k_5})\bar{v}^{*}(\eta_f,-\mathbf{k_6}) \\
&+(\mathbf{k_5}.\mathbf{k_6})\langle 0|a(\mathbf{k_4})a(\mathbf{k_5})a(\mathbf{k_6})a^{\dagger}(-\mathbf{k_1})a^{\dagger}(-\mathbf{k_2})a^{\dagger}(-\mathbf{k_3})|0\rangle \\
&\bar{v}^{*}(\eta_f,-\mathbf{k_1})\bar{v}^{*}(\eta_f,-\mathbf{k_2})\bar{v}^{*}(\eta_f,-\mathbf{k_3})\bar{v}^{'}(\eta_f,\mathbf{k_4})\bar{v}(\eta_f,\mathbf{k_5})\bar{v}(\eta_f,\mathbf{k_6}).
\end{aligned} \tag{4.16}$$

3. **Coefficient of α_3**:

$$\begin{aligned}
&\langle 0| \left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}(\eta,\mathbf{k_4})\hat{\zeta}^{'}(\eta,\mathbf{k_5})\hat{\zeta}^{'}(\eta,\mathbf{k_6})\right]|0\rangle \\
= \quad &\langle 0|a(\mathbf{k_1})a(\mathbf{k_2})a(\mathbf{k_3})a^{\dagger}(-\mathbf{k_4})a^{\dagger}(-\mathbf{k_5})a^{\dagger}(-\mathbf{k_6})|0\rangle \\
&\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^{*}(\eta_f,-\mathbf{k_4})\bar{v}^{'*}(\eta_f,-\mathbf{k_5})\bar{v}^{'*}(\eta_f,-\mathbf{k_6}) \\
&+\langle 0|a(\mathbf{k_4})a(\mathbf{k_5})a(\mathbf{k_6})a^{\dagger}(-\mathbf{k_1})a^{\dagger}(-\mathbf{k_2})a^{\dagger}(-\mathbf{k_3})|0\rangle \\
&\bar{v}^{*}(\eta_f,-\mathbf{k_1})\bar{v}^{*}(\eta_f,-\mathbf{k_2})\bar{v}^{*}(\eta_f,-\mathbf{k_3})\bar{v}(\eta_f,\mathbf{k_4})\bar{v}^{'}(\eta_f,\mathbf{k_5})\bar{v}^{'}(\eta_f,\mathbf{k_6}).
\end{aligned} \tag{4.17}$$

4. **Coefficient of α_4**:

$$\begin{aligned}
&(\mathbf{k_5}.\mathbf{k_6})\langle 0| \left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}(\eta,\mathbf{k_4})\hat{\zeta}^{'}(\eta,\mathbf{k_5})\hat{\zeta}(\eta,\mathbf{k_6})\right]|0\rangle \\
= \quad &(\mathbf{k_5}.\mathbf{k_6})\langle 0|a(\mathbf{k_1})a(\mathbf{k_2})a(\mathbf{k_3})a^{\dagger}(-\mathbf{k_4})a^{\dagger}(-\mathbf{k_5})a^{\dagger}(-\mathbf{k_6})|0\rangle \\
&\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^{*}(\eta_f,-\mathbf{k_4})\bar{v}^{'*}(\eta_f,-\mathbf{k_5})\bar{v}^{*}(\eta_f,-\mathbf{k_6}) \\
&+(\mathbf{k_5}.\mathbf{k_6})\langle 0|a(\mathbf{k_4})a(\mathbf{k_5})a(\mathbf{k_6})a^{\dagger}(-\mathbf{k_1})a^{\dagger}(-\mathbf{k_2})a^{\dagger}(-\mathbf{k_3})|0\rangle \\
&\bar{v}^{*}(\eta_f,-\mathbf{k_1})\bar{v}^{*}(\eta_f,-\mathbf{k_2})\bar{v}^{*}(\eta_f,-\mathbf{k_3})\bar{v}(\eta_f,\mathbf{k_4})\bar{v}^{'}(\eta_f,\mathbf{k_5})\bar{v}(\eta_f,\mathbf{k_6}).
\end{aligned} \tag{4.18}$$

5. **Coefficient of α_5**:

$$\begin{aligned}
&(\mathbf{k_5}.\mathbf{k_6})\langle 0| \left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}(\eta,\mathbf{k_4})\hat{\zeta}(\eta,\mathbf{k_5})\hat{\zeta}^{'}(\eta,\mathbf{k_6})\right]|0\rangle \\
= \quad &\langle 0|a(\mathbf{k_1})a(\mathbf{k_2})a(\mathbf{k_3})a^{\dagger}(-\mathbf{k_4})a^{\dagger}(-\mathbf{k_5})a^{\dagger}(-\mathbf{k_6})|0\rangle \\
&\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^{*}(\eta_f,-\mathbf{k_4})\bar{v}^{*}(\eta_f,-\mathbf{k_5})\bar{v}^{'*}(\eta_f,-\mathbf{k_6}) \\
&+\langle 0|a(\mathbf{k_4})a(\mathbf{k_5})a(\mathbf{k_6})a^{\dagger}(-\mathbf{k_1})a^{\dagger}(-\mathbf{k_2})a^{\dagger}(-\mathbf{k_3})|0\rangle \\
&\bar{v}^{*}(\eta_f,-\mathbf{k_1})\bar{v}^{*}(\eta_f,-\mathbf{k_2})\bar{v}^{*}(\eta_f,-\mathbf{k_3})\bar{v}(\eta_f,\mathbf{k_4})\bar{v}(\eta_f,\mathbf{k_5})\bar{v}^{'}(\eta_f,\mathbf{k_6}).
\end{aligned} \tag{4.19}$$

where we define $\bar{v}$ as:

$$\bar{v}(\eta, \mathbf{k}) = \frac{v(\eta, \mathbf{k})}{zM_p}. \quad (4.20)$$

Further we also use the following result to simplify the the co-efficients of $\alpha_1, \alpha_2, \alpha_3, \alpha_4$:

$$\begin{aligned}
&\langle 0|a(\mathbf{k_1})a(\mathbf{k_2})a(\mathbf{k_3})a^\dagger(-\mathbf{k_4})a^\dagger(-\mathbf{k_5})a^\dagger(-\mathbf{k_6})|0\rangle \\
&= \langle 0|a(\mathbf{k_4})a(\mathbf{k_5})a(\mathbf{k_6})a^\dagger(-\mathbf{k_1})a^\dagger(-\mathbf{k_2})a^\dagger(-\mathbf{k_3})|0\rangle \\
&= (2\pi)^9 \left\{\delta^{(3)}(\mathbf{k_4}+\mathbf{k_1})\left[\delta^{(3)}(\mathbf{k_5}+\mathbf{k_2})\delta^{(3)}(\mathbf{k_6}+\mathbf{k_3}) + \delta^{(3)}(\mathbf{k_5}+\mathbf{k_3})\delta^{(3)}(\mathbf{k_6}+\mathbf{k_2})\right]\right. \\
&\quad +\delta^{(3)}(\mathbf{k_4}+\mathbf{k_2})\left[\delta^{(3)}(\mathbf{k_5}+\mathbf{k_1})\delta^{(3)}(\mathbf{k_6}+\mathbf{k_3}) + \delta^{(3)}(\mathbf{k_5}+\mathbf{k_3})\delta^{(3)}(\mathbf{k_6}+\mathbf{k_1})\right] \\
&\quad \left.+\delta^{(3)}(\mathbf{k_4}+\mathbf{k_3})\left[\delta^{(3)}(\mathbf{k_5}+\mathbf{k_1})\delta^{(3)}(\mathbf{k_6}+\mathbf{k_3}) + \delta^{(3)}(\mathbf{k_5}+\mathbf{k_2})\delta^{(3)}(\mathbf{k_6}+\mathbf{k_1})\right]\right\}. \quad (4.21)
\end{aligned}$$

Finally one can write the following expresion for the three point function for the scalar fluctuation [14]:

$$\boxed{\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})B_{EFT}(k_1,k_2,k_3)}\,. \quad (4.22)$$

where $B_{EFT}(k_1, k_2, k_3)$ is the bispectrum for scalar fluctuation. In the present computation one can further write down the expression for the bispectrum as:

$$\boxed{B_{EFT}(k_1,k_2,k_3) = \sum_{j=1}^{5}\alpha_j\Theta_j(k_1,k_2,k_3)}\,, \quad (4.23)$$

where $\Theta_j(k_1, k_2, k_3)\forall j = 1, 2, 3, 4, 5$ is defined in the next subsections. Here it is important to note that we have derived the expression for the three point function and the associated bispectrum for effective sound speed $c_S = 1$ and $c_S < 1$ with a choice of general quantum vacuum state.

4.2.1 Coefficient of α_1

Here we can write the function $\Theta_1(k_1, k_2, k_3)$ as:

$$\begin{aligned}
\Theta_1(k_1,k_2,k_3) = 6i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\, \frac{a(\eta)}{H^3}&\left[\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^{'*}(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_2})\bar{v}^{'*}(\eta,\mathbf{k_3})\right. \\
&\left.+\bar{v}^{*}(\eta_f,\mathbf{k_1})\bar{v}^{*}(\eta_f,\mathbf{k_2})\bar{v}^{*}(\eta_f,\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_3})\right]. \quad (4.24)
\end{aligned}$$

[14]See also ref. [30] and [43], where similar computation have performed for canonical single field slow roll and generalized slow roll models of inflation in presence of Bunch-Davies vacuum state.

Further using the integrals from the Appendix we finally get the following simplified expression for the three point function for the scalar fluctuations [15]:

$$\Theta_1(k_1,k_2,k_3) = \frac{3H^2}{16\epsilon^3 M_p^6}\frac{1}{k_1k_2k_3}\left[\left\{\frac{1}{K^3}\left[(C_1-C_2)^3\left(C_1^{*3}+C_2^{*3}\right)+(C_1^*-C_2^*)^3\left(C_1^3+C_2^3\right)\right]\right.\right.$$
$$\left.\left.+\left[(C_1-C_2)^3 C_1^*C_2^*\left(C_1^*-C_2^*\right)+(C_1^*-C_2^*)^3 C_1C_2\left(C_1-C_2\right)\right]\sum_{i=1}^{3}\frac{1}{(2k_i-K)^3}\right\}\right], \tag{4.26}$$

Finally for Bunch Davies and α, β vacuum we get the following contribution in the three point function for scalar fluctuations:

- **For Bunch Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$\Theta_1(k_1,k_2,k_3) = \frac{6H^2}{16\epsilon^3 M_p^6}\frac{1}{k_1k_2k_3}\frac{1}{K^3}, \tag{4.27}$$

- **For α, β vacuum:**
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\ \sinh\alpha$ we get:

$$\Theta_1(k_1,k_2,k_3) = \frac{3H^2}{16\epsilon^3 M_p^6}\frac{1}{k_1k_2k_3}\left\{\frac{1}{K^3}\left[\left(\cosh\alpha - e^{i\beta}\sinh\alpha\right)^3\left(\cosh^3\alpha + e^{-3i\beta}\sinh^3\alpha\right)\right.\right.$$
$$\left.+\left(\cosh\alpha - e^{-i\beta}\sinh\alpha\right)^3\left(\cosh^3\alpha + e^{3i\beta}\sinh^3\alpha\right)\right]$$
$$+\frac{1}{2}\left[\left(\cosh\alpha - e^{i\beta}\sinh\alpha\right)^3 e^{-i\beta}\sinh 2\alpha\left(\cosh\alpha - e^{-i\beta}\sinh\alpha\right)\right.$$
$$\left.\left.+\left(\cosh\alpha - e^{-i\beta}\sinh\alpha\right)^3 e^{i\beta}\sinh 2\alpha\left(\cosh\alpha - e^{i\beta}\sinh\alpha\right)\right]\sum_{i=1}^{3}\frac{1}{(2k_i-K)^3}\right\}. \tag{4.28}$$

[15] Here ot is important to point out that in de-Sitter space if we consider the Bunch Davies vacuum state then here only the term with $1/K^3$ will appear explicitly in the expression for the three point function and in the associated bispectrum. On the other hand if we consider all other non-trivial quantum vacuum state in our computation then the rest of the contribution will explicitly appear. From the perspective of observation this is obviously an important information as for the non trivial quantum vacuum state we get additional contribution in the bispectrum which may enhance the amplitude of the non-Gaussianity in squeezed limiting configuration. Additionally, it is important to mention that in quasi de Sitter case we get extra contributions $1/\tilde{c}_S^{6\nu-9}$ and $1/(1+\epsilon)^5$. Also the factor $1/(k_1k_2k_3)$ will be replaced by $1/(k_1k_2k_3)^{2(\nu-1)}$. Consequently, in quasi de Sitter case this contribution in the bispectrum can be recast as:

$$\Theta_1(k_1,k_2,k_3) = \frac{3H^2}{16\epsilon^3 M_p^6\tilde{c}_S^{6\nu-9}(1+\epsilon)^5}\frac{1}{(k_1k_2k_3)^{2(\nu-1)}}\left[\left\{\frac{1}{K^3}\left[(C_1-C_2)^3\left(C_1^{*3}+C_2^{*3}\right)+(C_1^*-C_2^*)^3\left(C_1^3+C_2^3\right)\right]\right.\right.$$
$$\left.\left.+\left[(C_1-C_2)^3 C_1^*C_2^*\left(C_1^*-C_2^*\right)+(C_1^*-C_2^*)^3 C_1C_2\left(C_1-C_2\right)\right]\sum_{i=1}^{3}\frac{1}{(2k_i-K)^3}\right\}\right], \tag{4.25}$$

4.2.2 Coefficient of α_2

Here we can write the function $\Theta_2(k_1,k_2,k_3)$ as:

$$\begin{aligned}\Theta_2(k_1,k_2,k_3) = i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\, \frac{a(\eta)}{H^3}\Big\{&2(\mathbf{k_2}.\mathbf{k_3})\Big[\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^{'*}(\eta,\mathbf{k_1})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^*(\eta,\mathbf{k_3})\\ &+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_1})\bar{v}(\eta,-\mathbf{k_2})\bar{v}(\eta,-\mathbf{k_3})\Big]\\ &+2(\mathbf{k_3}.\mathbf{k_1})\Big[\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^{'*}(\eta,\mathbf{k_2})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^*(\eta,\mathbf{k_3})\\ &+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_2})\bar{v}(\eta,-\mathbf{k_1})\bar{v}(\eta,-\mathbf{k_3})\Big]\\ &+2(\mathbf{k_1}.\mathbf{k_2})\Big[\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^{'*}(\eta,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^*(\eta,\mathbf{k_2})\\ &+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}(\eta,-\mathbf{k_2})\Big]\Big\}.\end{aligned} \tag{4.29}$$

Using the results derived in Appendix we finally get the following simplified expression for the three point function for the scalar fluctuations [16]:

$$\boxed{\begin{aligned}\Theta_2(k_1,k_2,k_3) = \frac{H^2}{32\epsilon^3 M_p^6 \tilde{c}_S^2}\frac{1}{(k_1k_2k_3)^3}\Big[&k_1^2(\mathbf{k_2}.\mathbf{k_3})G_1(k_1,k_2,k_3)\\ &+k_2^2(\mathbf{k_1}.\mathbf{k_3})G_2(k_1,k_2,k_3)+k_3^2(\mathbf{k_1}.\mathbf{k_2})G_3(k_1,k_2,k_3)\Big],\end{aligned}} \tag{4.31}$$

[16]Here ot is important to point out that in de-Sitter space if we consider the Bunch Davies vacuum state then here only the term with $1/K^3$ will appear explicitly in the expression for the three point function and in the associated bispectrum. On the other hand if we consider all other non-trivial quantum vacuum state in our computation then the rest of the contribution will explicitly appear. From the perspective of observation this is obviously an important information as for the non trivial quantum vacuum state we get additional contribution in the bispectrum which may enhance the amplitude of the non-Gaussianity in squeezed limiting configuration. Additionally, it is important to mention that in quasi de Sitter case we get extra contributions $1/\tilde{c}_S^{6\nu-7}$ and $1/(1+\epsilon)^5$. Also the factor $1/(k_1k_2k_3)^3$ will be replaced by $1/(k_1k_2k_3)^{2\nu}$. Consequently, in quasi de Sitter case this contribution in the bispectrum can be recast as:

$$\begin{aligned}\Theta_2(k_1,k_2,k_3) = \frac{H^2}{32\epsilon^3 M_p^6 \tilde{c}_S^{6\nu-7}(1+\epsilon)^5}\frac{1}{(k_1k_2k_3)^{2\nu}}\Big[&k_1^2(\mathbf{k_2}.\mathbf{k_3})G_1(k_1,k_2,k_3)\\ &+k_2^2(\mathbf{k_1}.\mathbf{k_3})G_2(k_1,k_2,k_3)+k_3^2(\mathbf{k_1}.\mathbf{k_2})G_3(k_1,k_2,k_3)\Big],\end{aligned} \tag{4.30}$$

where the momentum dependent functions $G_1(k_1,k_2,k_3)$, $G_2(k_1,k_2,k_3)$ and $G_3(k_1,k_2,k_3)$ are defined as:

$$\begin{aligned}
G_1(k_1,k_2,k_3) &= \frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
&+\left\{\frac{1}{(2k_1-K)^3}\left[K^2+2k_2k_3+K(K-5k_1)-2(K-k_1)k_1+4k_1^2\right]\right. \\
&+\frac{1}{(2k_2-K)^3}\left[K^2-4k_2k_3+K(k_3-5k_2)+6k_2^2\right] \\
&\left.+\frac{1}{(2k_1-K)^3}\left[K^2-4k_2k_3+K(k_2-5k_3)+6k_3^2\right]\right\} \\
&\quad\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (4.32)
\end{aligned}$$

$$\begin{aligned}
G_2(k_1,k_2,k_3) &= \frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
&+\left\{\frac{1}{(2k_2-K)^3}\left[K^2+2k_1k_3+K(K-5k_2)-2(K-k_2)k_2+4k_2^2\right]\right. \\
&+\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_3+K(k_3-5k_1)+6k_1^2\right] \\
&\left.+\frac{1}{(2k_3-K)^3}\left[K^2-4k_1k_3+K(k_1-5k_3)+6k_3^2\right]\right\} \\
&\quad\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (4.33)
\end{aligned}$$

$$\begin{aligned}
G_3(k_1,k_2,k_3) &= \frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
&+\left\{\frac{1}{(2k_3-K)^3}\left[K^2+2k_1k_2+K(K-5k_3)-2(K-k_3)k_3+4k_3^2\right]\right. \\
&+\frac{1}{(2k_2-K)^3}\left[K^2-4k_1k_2+K(k_1-5k_2)+6k_2^2\right] \\
&\left.+\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_2+K(k_2-5k_1)+6k_1^2\right]\right\} \\
&\quad\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (4.34)
\end{aligned}$$

Here $\sum_{i=1}^{3}\mathbf{k_i}=\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}=0$. Consequently one can write:

$$\mathbf{k_1}.\mathbf{k_2}=\frac{1}{2}\left(k_3^2-k_2^2-k_1^2\right),\quad \mathbf{k_1}.\mathbf{k_3}=\frac{1}{2}\left(k_2^2-k_1^2-k_3^2\right),\quad \mathbf{k_2}.\mathbf{k_3}=\frac{1}{2}\left(k_1^2-k_2^2-k_3^2\right), \quad (4.35)$$

and using these results one can further recast the three point function for the scalar fluctuation as:

$$\boxed{\begin{aligned}\Theta_2(k_1,k_2,k_3) &= \frac{H^2}{64\epsilon^3M_p^6\tilde{c}_S^2}\frac{1}{(k_1k_2k_3)^3}\left[k_1^2\left(k_1^2-k_2^2-k_3^2\right)G_1(k_1,k_2,k_3)\right. \\ &\left.+k_2^2\left(k_2^2-k_1^2-k_3^2\right)G_2(k_1,k_2,k_3)+k_3^2\left(k_3^2-k_2^2-k_1^2\right)G_3(k_1,k_2,k_3)\right].\end{aligned}} \quad (4.36)$$

Finally for Bunch Davies and α,β vacuum we get the following contribution in the three point function for scalar fluctuations:

- **For Bunch Davies vacuum:**
After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$G_1(k_1,k_2,k_3) = \frac{2}{K^3}\left[K^2 + 2k_2k_3 + K(K-k_1)\right]. \tag{4.37}$$

$$G_2(k_1,k_2,k_3) = \frac{2}{K^3}\left[K^2 + 2k_1k_3 + K(K-k_2)\right]. \tag{4.38}$$

$$G_3(k_1,k_2,k_3) = \frac{2}{K^3}\left[K^2 + 2k_1k_2 + K(K-k_3)\right]. \tag{4.39}$$

Consequently we get:

$$\begin{aligned}\Theta_2(k_1,k_2,k_3) = \frac{H^2}{32\epsilon^3 M_p^6 \tilde{c}_S^2}\frac{1}{(k_1k_2k_3)^3}\frac{1}{K^3}\Big[k_1^2\left(k_1^2-k_2^2-k_3^2\right)\left[K^2+2k_2k_3+K(K-k_1)\right] \\ + k_2^2\left(k_2^2-k_1^2-k_3^2\right)\left[K^2+2k_1k_3+K(K-k_2)\right] \\ + k_3^2\left(k_3^2-k_2^2-k_1^2\right)\left[K^2+2k_1k_2+K(K-k_3)\right]\Big].\end{aligned} \tag{4.40}$$

- **For α, β vacuum:**
After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\,\sinh\alpha$ we get:

$$\begin{aligned}G_1(k_1,k_2,k_3) =& \frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]J_1(\alpha,\beta) \\ &+\left\{\frac{1}{(2k_1-K)^3}\left[K^2+2k_2k_3+K(K-5k_1)-2(K-k_1)k_1+4k_1^2\right]\right. \\ &+\frac{1}{(2k_2-K)^3}\left[K^2-4k_2k_3+K(k_3-5k_2)+6k_2^2\right] \\ &\left.+\frac{1}{(2k_1-K)^3}\left[K^2-4k_2k_3+K(k_2-5k_3)+6k_3^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{4.41}$$

$$\begin{aligned}G_2(k_1,k_2,k_3) =& \frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right]J_1(\alpha,\beta) \\ &+\left\{\frac{1}{(2k_2-K)^3}\left[K^2+2k_1k_3+K(K-5k_2)-2(K-k_2)k_2+4k_2^2\right]\right. \\ &+\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_3+K(k_3-5k_1)+6k_1^2\right] \\ &\left.+\frac{1}{(2k_3-K)^3}\left[K^2-4k_1k_3+K(k_1-5k_3)+6k_3^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{4.42}$$

$$\begin{aligned}G_3(k_1,k_2,k_3) =& \frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]J_1(\alpha,\beta) \\ &+\left\{\frac{1}{(2k_3-K)^3}\left[K^2+2k_1k_2+K(K-5k_3)-2(K-k_3)k_3+4k_3^2\right]\right. \\ &+\frac{1}{(2k_2-K)^3}\left[K^2-4k_1k_2+K(k_1-5k_2)+6k_2^2\right] \\ &\left.+\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_2+K(k_2-5k_1)+6k_1^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{4.43}$$

where $J_1(\alpha,\beta)$ and $J_2(\alpha,\beta)$ are defined as:

$$\begin{aligned}J_1(\alpha,\beta) =& \left[\left(\cosh\alpha - e^{i\beta}\sinh\alpha\right)^3\left(\cosh^3\alpha + e^{-3i\beta}\sinh^3\alpha\right)\right. \\ &\left.+\left(\cosh\alpha - e^{-i\beta}\sinh\alpha\right)^3\left(\cosh^3\alpha + e^{3i\beta}\sinh^3\alpha\right)\right],\end{aligned} \tag{4.44}$$

$$J_2(\alpha,\beta) = \frac{1}{2}\left[\left(\cosh\alpha - e^{i\beta}\sinh\alpha\right)^3 e^{-i\beta}\sinh 2\alpha\left(\cosh\alpha - e^{-i\beta}\sinh\alpha\right)\right.$$
$$\left. + \left(\cosh\alpha - e^{-i\beta}\sinh\alpha\right)^3 e^{i\beta}\sinh 2\alpha\left(\cosh\alpha - e^{i\beta}\sinh\alpha\right)\right]. \quad (4.45)$$

Consequently the three point function for the scalar fluctuation can also be written after substituting all the momentum dependent functions $G_1(k_1,k_2,k_3)$, $G_2(k_1,k_2,k_3)$ and $G_3(k_1,k_2,k_3)$ for α,β vacuum.

4.2.3 Coefficient of α_3

Here we can write the function $\Theta_3(k_1,k_2,k_3)$ as:

$$\begin{aligned}\Theta_3(k_1,k_2,k_3) = 2i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\, \frac{a^2(\eta)}{H^3}\Big\{&\Big[\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_2})\bar{v}^{'*}(\eta,\mathbf{k_3})\\ &+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_3})\Big]\\ &+\Big[\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^{'*}(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_3})\\ &+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_3})\Big]\\ &+\Big[\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^{'*}(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_2})\\ &+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_2})\Big]\Big\}.\end{aligned} \quad (4.46)$$

Using the results obtained in the Appendix we finally get the following simplified expression for the three point function for the scalar fluctuations [17]:

$$\boxed{\begin{aligned}\Theta_3(k_1,k_2,k_3) = \frac{H}{32\epsilon^3 M_p^6}\frac{1}{(k_1k_2k_3)^3}&\left[(k_2k_3)^2 M_1(k_1,k_2,k_3)\right.\\ &\left.+(k_1k_3)^2 M_2(k_1,k_2,k_3)+(k_1k_2)^2 M_3(k_1,k_2,k_3)\right],\end{aligned}} \quad (4.48)$$

[17]Here ot is important to point out that in de-Sitter space if we consider the Bunch Davies vacuum state then here only the term with $1/K^2$ will appear explicitly in the expression for the three point function and in the associated bispectrum. On the other hand if we consider all other non-trivial quantum vacuum state in our computation then the rest of the contribution will explicitly appear. From the perspective of observation this is obviously an important information as for the non trivial quantum vacuum state we get additional contribution in the bispectrum which may enhance the amplitude of the non-Gaussianity in squeezed limiting configuration. Additionally, it is important to mention that in quasi de Sitter case we get extra contributions $1/\tilde{c}_S^{6\nu-9}$ and $1/(1+\epsilon)^3$. Also the factor $1/(k_1k_2k_3)^3$ will be replaced by $1/(k_1k_2k_3)^{2\nu}$. Consequently, in quasi de Sitter case this contribution in the bispectrum can be recast as:

$$\begin{aligned}\Theta_3(k_1,k_2,k_3) = \frac{H}{32\epsilon^3 M_p^6\tilde{c}_S^{6\nu-9}(1+\epsilon)^3}\frac{1}{(k_1k_2k_3)^{2\nu}}&\left[(k_2k_3)^2 M_1(k_1,k_2,k_3)\right.\\ &\left.+(k_1k_3)^2 M_2(k_1,k_2,k_3)+(k_1k_2)^2 M_3(k_1,k_2,k_3)\right],\end{aligned} \quad (4.47)$$

where the momentum dependent functions $M_1(k_1,k_2,k_3)$, $M_2(k_1,k_2,k_3)$ and $M_3(k_1,k_2,k_3)$ are defined as:

$$\begin{aligned}
M_1(k_1,k_2,k_3) &= \frac{1}{K^2}(K+k_1)\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
&+\left\{\frac{(K-3k_1)}{(2k_1-K)^2}+\frac{(K+k_1-2k_2)}{(2k_2-K)^2}+\frac{(K+k_1-2k_3)}{(2k_3-K)^2}\right\} \\
&\quad \left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (4.49)\\
M_2(k_1,k_2,k_3) &= \frac{1}{K^2}(K+k_2)\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
&+\left\{\frac{(K-3k_2)}{(2k_2-K)^2}+\frac{(K+k_2-2k_1)}{(2k_1-K)^2}+\frac{(K+k_2-2k_3)}{(2k_3-K)^2}\right\} \\
&\quad \left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (4.50)\\
M_3(k_1,k_2,k_3) &= \frac{1}{K^2}(K+k_3)\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
&+\left\{\frac{(K-3k_3)}{(2k_3-K)^2}+\frac{(K+k_3-2k_2)}{(2k_2-K)^2}+\frac{(K+k_3-2k_1)}{(2k_1-K)^2}\right\} \\
&\quad \left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (4.51)
\end{aligned}$$

Finally for Bunch Davies and α,β vacuum we get the following contribution in the three point function for scalar fluctuations:

- **For Bunch Davies vacuum:**
  After setting $C_1=1$ and $C_2=0$ we get:

$$M_1(k_1,k_2,k_3) = \frac{2}{K^2}(K+k_1). \qquad (4.52)$$

$$M_2(k_1,k_2,k_3) = \frac{2}{K^2}(K+k_2). \qquad (4.53)$$

$$M_3(k_1,k_2,k_3) = \frac{2}{K^2}(K+k_3). \qquad (4.54)$$

  Consequently we get the following contribution:

$$\Theta_3(k_1,k_2,k_3) = \frac{H}{16\epsilon^3M_p^6}\frac{1}{(k_1k_2k_3)^3}\frac{1}{K^2}\left[(k_2k_3)^2(K+k_1)]+(k_1k_3)^2(K+k_2)+(k_1k_2)^2(K+k_3)\right], \qquad (4.55)$$

- **For α,β vacuum:**
  After setting $C_1=\cosh\alpha$ and $C_2=e^{i\beta}\ \sinh\alpha$ we get:

$$M_1(k_1,k_2,k_3) = \frac{(K+k_1)J_1(\alpha,\beta)}{K^2}+\left\{\frac{(K-3k_1)}{(2k_1-K)^2}+\frac{(K+k_1-2k_2)}{(2k_2-K)^2}+\frac{(K+k_1-2k_3)}{(2k_3-K)^2}\right\}J_2(\alpha,\beta). \quad (4.56)$$

$$M_2(k_1,k_2,k_3) = \frac{(K+k_2)J_1(\alpha,\beta)}{K^2}+\left\{\frac{(K-3k_2)}{(2k_2-K)^2}+\frac{(K+k_2-2k_1)}{(2k_1-K)^2}+\frac{(K+k_2-2k_3)}{(2k_3-K)^2}\right\}J_2(\alpha,\beta). \quad (4.57)$$

$$M_3(k_1,k_2,k_3) = \frac{(K+k_3)J_1(\alpha,\beta)}{K^2}+\left\{\frac{(K-3k_3)}{(2k_3-K)^2}+\frac{(K+k_3-2k_2)}{(2k_2-K)^2}+\frac{(K+k_3-2k_1)}{(2k_1-K)^2}\right\}J_2(\alpha,\beta). \quad (4.58)$$

  where $J_1(\alpha,\beta)$ and $J_2(\alpha,\beta)$ are defined earlier. Consequently the three point function for the scalar fluctuation can also be written after substituting all the momentum dependent functions $M_1(k_1,k_2,k_3)$, $M_2(k_1,k_2,k_3)$ and $M_3(k_1,k_2,k_3)$ for α,β vacuum.

4.2.4 Coefficient of α_4

Here we can write the function $\Theta_4(k_1,k_2,k_3)$ as:

$$\Theta_4(k_1,k_2,k_3) = i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\;\frac{a(\eta)}{H^3}\left\{(\mathbf{k_2}.\mathbf{k_3})X_1(k_1,k_2,k_3)+(\mathbf{k_1}.\mathbf{k_3})X_2(k_1,k_2,k_3)+(\mathbf{k_1}.\mathbf{k_2})X_3(k_1,k_2,k_3)\right\}, \tag{4.59}$$

where the momentum dependent functions $X_1(k_1,k_2,k_3)$, $X_2(k_1,k_2,k_3)$ and $X_3(k_1,k_2,k_3)$ can be expressewd in terms of the various combinations of the scalar mode functions as:

$$\begin{aligned}
X_1(k_1,k_2,k_3) &= \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_2})\bar{v}^*(\eta,\mathbf{k_3})\\
&+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_2})\bar{v}(\eta,-\mathbf{k_3})\\
&+\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\\
&+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_2}), \qquad (4.60)\\
X_2(k_1,k_2,k_3) &= \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^{'*}(\eta,\mathbf{k_1})\bar{v}^*(\eta,\mathbf{k_3})\\
&+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_1})\bar{v}(\eta,-\mathbf{k_3})\\
&+\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^{'*}(\eta,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\\
&+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1}), \qquad (4.61)\\
X_3(k_1,k_2,k_3) &= \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^{'*}(\eta,\mathbf{k_1})\bar{v}^*(\eta,\mathbf{k_2})\\
&+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_1})\bar{v}(\eta,-\mathbf{k_2})\\
&+(\mathbf{k_1}.\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^{'*}(\eta,\mathbf{k_2})\bar{v}^*(\eta,\mathbf{k_1})\\
&+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_1}), \qquad (4.62)
\end{aligned}$$

Using the results obtained in the Appendix we finally get the following simplified expression for the three point function for the scalar fluctuations [18]:

$$\boxed{\begin{aligned}
\Theta_4(k_1,k_2,k_3) &= -\frac{H^2}{64\tilde{c}_S^2\epsilon^3M_p^6}\frac{1}{(k_1k_2k_3)^3}\left[k_2^2(\mathbf{k_2}.\mathbf{k_3})\mathcal{F}_1(k_1,k_2,k_3)+k_3^2(\mathbf{k_2}.\mathbf{k_3})\mathcal{F}_2(k_1,k_2,k_3)\right.\\
&+k_1^2(\mathbf{k_1}.\mathbf{k_3})\mathcal{F}_3(k_1,k_2,k_3)+k_3^2(\mathbf{k_1}.\mathbf{k_3})\mathcal{F}_4(k_1,k_2,k_3)\\
&\left.+k_1^2(\mathbf{k_1}.\mathbf{k_2})\mathcal{F}_5(k_1,k_2,k_3)+k_2^2(\mathbf{k_1}.\mathbf{k_2})\mathcal{F}_6(k_1,k_2,k_3)\right],
\end{aligned}} \tag{4.64}$$

[18] Here ot is important to point out that in de-Sitter space if we consider the Bunch Davies vacuum state then here only the term with $1/K^3$ will appear explicitly in the expression for the three point function and in the associated bispectrum. On the other hand if we consider all other non-trivial quantum vacuum state in our computation then the rest of the contribution will explicitly appear. From the perspective of observation this is obviously an important information as for the non trivial quantum vacuum state we get additional contribution in the bispectrum which may enhance the amplitude of the non-Gaussianity in squeezed limiting configuration. Additionally, it is important to mention that in quasi de Sitter case we get an extra contribution $1/(1+\epsilon)^5$. Also the factor $1/(k_1k_2k_3)^3$ will be replaced by $1/(k_1k_2k_3)^{2\nu}$ and $1/\tilde{c}_S^2$ is replaced by $1/\tilde{c}_S^{6\nu-7}$. Consequently, in quasi de Sitter case this contribution in the bispectrum can be recast as:

$$\begin{aligned}
\Theta_4(k_1,k_2,k_3) &= -\frac{H^2}{64\tilde{c}_S^{6\nu-7}\epsilon^3M_p^6(1+\epsilon)^5}\frac{1}{(k_1k_2k_3)^{2\nu}}\left[k_2^2(\mathbf{k_2}.\mathbf{k_3})\mathcal{F}_1(k_1,k_2,k_3)+k_3^2(\mathbf{k_2}.\mathbf{k_3})\mathcal{F}_2(k_1,k_2,k_3)\right.\\
&\left.+k_1^2(\mathbf{k_1}.\mathbf{k_3})\mathcal{F}_3(k_1,k_2,k_3)+k_3^2(\mathbf{k_1}.\mathbf{k_3})\mathcal{F}_4(k_1,k_2,k_3)+k_1^2(\mathbf{k_1}.\mathbf{k_2})\mathcal{F}_5(k_1,k_2,k_3)+k_2^2(\mathbf{k_1}.\mathbf{k_2})\mathcal{F}_6(k_1,k_2,k_3)\right],
\end{aligned} \tag{4.63}$$

where the momentum dependent functions $\mathcal{F}_i(k_1,k_2,k_3)\forall i=1,2,\cdots,6$ are defined as:

$$\begin{aligned}
\mathcal{F}_1(k_1,k_2,k_3) = & \frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
& +\left\{\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_3+K(k_3-5k_1)+6k_1^2\right]\right. \\
& +\frac{1}{(2k_2-K)^3}\left[(K-2k_2)(K-2k_2+k_1)+(K+2k_1-2k_2)k_3\right] \\
& \left.+\frac{1}{(2k_3-K)^3}\left[K^2-4k_1k_3+K(k_1-5k_3)+6k_3^2\right]\right\} \\
& \left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (4.65)
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_2(k_1,k_2,k_3) = & \frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
& +\left\{\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_2+K(k_2-5k_1)+6k_1^2\right]\right. \\
& +\frac{1}{(2k_3-K)^3}\left[(K-2k_3)(K-2k_3+k_1)+(K+2k_1-2k_3)k_2\right] \\
& \left.+\frac{1}{(2k_2-K)^3}\left[K^2-4k_1k_2+K(k_1-5k_2)+6k_2^2\right]\right\} \\
& \left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (4.66)
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_3(k_1,k_2,k_3) = & \frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
& +\left\{\frac{1}{(2k_2-K)^3}\left[K^2-4k_2k_3+K(k_3-5k_2)+6k_2^2\right]\right. \\
& +\frac{1}{(2k_1-K)^3}\left[(K-2k_1)(K-2k_1+k_2)+(K+2k_2-2k_1)k_2\right] \\
& \left.+\frac{1}{(2k_3-K)^3}\left[K^2-4k_2k_3+K(k_2-5k_3)+6k_3^2\right]\right\} \\
& \left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (4.67)
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_4(k_1,k_2,k_3) = & \frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
& +\left\{\frac{1}{(2k_2-K)^3}\left[K^2-4k_1k_2+K(k_1-5k_2)+6k_2^2\right]\right. \\
& +\frac{1}{(2k_3-K)^3}\left[(K-2k_3)(K-2k_3+k_2)+(K+2k_2-2k_3)k_2\right] \\
& \left.+\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_2+K(k_2-5k_1)+6k_1^2\right]\right\} \\
& \left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (4.68)
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_5(k_1,k_2,k_3) = & \frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
& +\left\{\frac{1}{(2k_3-K)^3}\left[K^2-4k_2k_3+K(k_2-5k_3)+6k_3^2\right]\right. \\
& +\frac{1}{(2k_1-K)^3}\left[(K-2k_1)(K-2k_1+k_2)+(K+2k_2-2k_1)k_3\right] \\
& \left.+\frac{1}{(2k_2-K)^3}\left[K^2-4k_2k_3+K(k_3-5k_2)+6k_2^2\right]\right\} \\
& \left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (4.69)
\end{aligned}$$

$$\begin{aligned}\mathcal{F}_6(k_1,k_2,k_3) =& \frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right]\\ &+\left\{\frac{1}{(2k_3-K)^3}\left[K^2-4k_1k_3+K(k_1-5k_3)+6k_3^2\right]\right.\\ &+\frac{1}{(2k_2-K)^3}\left[(K-2k_2)(K-2k_2+k_3)+(K+2k_3-2k_2)k_1\right]\\ &\left.+\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_3+K(k_3-5k_1)+6k_1^2\right]\right\}\\ &\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \end{aligned} \tag{4.70}$$

Further after simplification one can recast the three point function for the scalar fluctuation as:

$$\begin{aligned}\Theta_4(k_1,k_2,k_3) =& -\frac{H^2}{128\tilde{c}_S^2\epsilon^3M_p^6}\frac{1}{(k_1k_2k_3)^3}\left[k_2^2\left(k_1^2-k_2^2-k_3^2\right)\mathcal{F}_1(k_1,k_2,k_3)+k_3^2\left(k_1^2-k_2^2-k_3^2\right)\mathcal{F}_2(k_1,k_2,k_3)\right.\\ &+k_1^2\left(k_2^2-k_1^2-k_3^2\right)\mathcal{F}_3(k_1,k_2,k_3)+k_3^2\left(k_2^2-k_1^2-k_3^2\right)\mathcal{F}_4(k_1,k_2,k_3)\\ &\left.+k_1^2\left(k_3^2-k_2^2-k_1^2\right)\mathcal{F}_5(k_1,k_2,k_3)+k_2^2\left(k_3^2-k_2^2-k_1^2\right)\mathcal{F}_6(k_1,k_2,k_3)\right], \end{aligned} \tag{4.71}$$

Finally for Bunch Davies and α,β vacuum we get the following contribution in the three point function for scalar fluctuations:

- **For Bunch Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$\mathcal{F}_1(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right]. \tag{4.72}$$
$$\mathcal{F}_2(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]. \tag{4.73}$$
$$\mathcal{F}_3(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]. \tag{4.74}$$
$$\mathcal{F}_4(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]. \tag{4.75}$$
$$\mathcal{F}_5(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]. \tag{4.76}$$
$$\mathcal{F}_6(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right]. \tag{4.77}$$

  Consequently the three point function for the scalar fluctuation can be expressed as:

$$\begin{aligned}\Theta_4(k_1,k_2,k_3) =& -\frac{H^2}{128\tilde{c}_S^2\epsilon^3M_p^6}\frac{1}{(k_1k_2k_3)^3}\left[k_2^2\left(k_1^2-k_2^2-k_3^2\right)\left[K^2+2k_1k_3+K(K-k_2)\right]\right.\\ &+k_3^2\left(k_1^2-k_2^2-k_3^2\right)\left[K^2+2k_1k_2+K(K-k_3)\right]\\ &+k_1^2\left(k_2^2-k_1^2-k_3^2\right)\left[K^2+2k_2k_3+K(K-k_1)\right]\\ &+k_3^2\left(k_2^2-k_1^2-k_3^2\right)\left[K^2+2k_1k_2+K(K-k_3)\right]\\ &+k_1^2\left(k_3^2-k_2^2-k_1^2\right)\left[K^2+2k_2k_3+K(K-k_1)\right]\\ &\left.+k_2^2\left(k_3^2-k_2^2-k_1^2\right)\left[K^2+2k_1k_3+K(K-k_2)\right]\right], \end{aligned} \tag{4.78}$$

- **For α, β vacuum:**
After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\ \sinh\alpha$ we get:

$$\begin{aligned}\mathcal{F}_1(k_1,k_2,k_3) = &\frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right]J_1(\alpha,\beta)\\&+\left\{\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_3+K(k_3-5k_1)+6k_1^2\right]\right.\\&+\frac{1}{(2k_2-K)^3}\left[(K-2k_2)(K-2k_2+k_1)+(K+2k_1-2k_2)k_3\right]\\&\left.+\frac{1}{(2k_3-K)^3}\left[K^2-4k_1k_3+K(k_1-5k_3)+6k_3^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{4.79}$$

$$\begin{aligned}\mathcal{F}_2(k_1,k_2,k_3) = &\frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]J_1(\alpha,\beta)\\&+\left\{\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_2+K(k_2-5k_1)+6k_1^2\right]\right.\\&+\frac{1}{(2k_3-K)^3}\left[(K-2k_3)(K-2k_3+k_1)+(K+2k_1-2k_3)k_2\right]\\&\left.+\frac{1}{(2k_2-K)^3}\left[K^2-4k_1k_2+K(k_1-5k_2)+6k_2^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{4.80}$$

$$\begin{aligned}\mathcal{F}_3(k_1,k_2,k_3) = &\frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]J_1(\alpha,\beta)\\&+\left\{\frac{1}{(2k_2-K)^3}\left[K^2-4k_2k_3+K(k_3-5k_2)+6k_2^2\right]\right.\\&+\frac{1}{(2k_1-K)^3}\left[(K-2k_1)(K-2k_1+k_2)+(K+2k_2-2k_1)k_2\right]\\&\left.+\frac{1}{(2k_3-K)^3}\left[K^2-4k_2k_3+K(k_2-5k_3)+6k_3^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{4.81}$$

$$\begin{aligned}\mathcal{F}_4(k_1,k_2,k_3) = &\frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]J_1(\alpha,\beta)\\&+\left\{\frac{1}{(2k_2-K)^3}\left[K^2-4k_1k_2+K(k_1-5k_2)+6k_2^2\right]\right.\\&+\frac{1}{(2k_3-K)^3}\left[(K-2k_3)(K-2k_3+k_2)+(K+2k_2-2k_3)k_2\right]\\&\left.+\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_2+K(k_2-5k_1)+6k_1^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{4.82}$$

$$\begin{aligned}\mathcal{F}_5(k_1,k_2,k_3) = &\frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]J_1(\alpha,\beta)\\&+\left\{\frac{1}{(2k_3-K)^3}\left[K^2-4k_2k_3+K(k_2-5k_3)+6k_3^2\right]\right.\\&+\frac{1}{(2k_1-K)^3}\left[(K-2k_1)(K-2k_1+k_2)+(K+2k_2-2k_1)k_3\right]\\&\left.+\frac{1}{(2k_2-K)^3}\left[K^2-4k_2k_3+K(k_3-5k_2)+6k_2^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{4.83}$$

$$\mathcal{F}_6(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right] J_1(\alpha,\beta)$$
$$+\left\{\frac{1}{(2k_3-K)^3}\left[K^2-4k_1k_3+K(k_1-5k_3)+6k_3^2\right]\right.$$
$$+\frac{1}{(2k_2-K)^3}\left[(K-2k_2)(K-2k_2+k_3)+(K+2k_3-2k_2)k_1\right]$$
$$\left.+\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_3+K(k_3-5k_1)+6k_1^2\right]\right\} J_2(\alpha,\beta). \quad (4.84)$$

where $J_1(\alpha,\beta)$ and $J_2(\alpha,\beta)$ are defined earlier. Consequently the three point function for the scalar fluctuation can also be written after substituting all the momentum dependent functions $\mathcal{F}_i(k_1,k_2,k_3)\forall i=1,2,\cdots,6$ for α,β vacuum.

4.2.5 Coefficient of α_5

$$\Theta_5(k_1,k_2,k_3) = i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\,\frac{a(\eta)}{H^3}\left\{(\mathbf{k_2}.\mathbf{k_3})Y_1(k_1,k_2,k_3)+(\mathbf{k_1}.\mathbf{k_3})Y_2(k_1,k_2,k_3)\right.$$
$$\left.+(\mathbf{k_1}.\mathbf{k_2})Y_3(k_1,k_2,k_3)\right\}, \quad (4.85)$$

where the momentum dependent functions $Y_1(k_1,k_2,k_3)$, $Y_2(k_1,k_2,k_3)$ and $Y_3(k_1,k_2,k_3)$ can be expressewd in terms of the various combinations of the scalar mode functions as:

$$Y_1(k_1,k_2,k_3) = \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^{*'}(\eta,\mathbf{k_3})$$
$$+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_3})$$
$$+\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^{*'}(\eta,\mathbf{k_2})$$
$$+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}(\eta,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_2}), \quad (4.86)$$
$$Y_2(k_1,k_2,k_3) = \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^{*'}(\eta,\mathbf{k_3})$$
$$+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_2})\bar{v}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_3})$$
$$+\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^{*'}(\eta,\mathbf{k_1})$$
$$+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_2})\bar{v}(\eta,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_1}), \quad (4.87)$$
$$Y_3(k_1,k_2,k_3) = \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_2})$$
$$+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_2})$$
$$+\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^{*'}(\eta,\mathbf{k_1})$$
$$+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_1}), \quad (4.88)$$

Here we get the following contribution in the three point function for scalar fluctuations [19]:

$$\Theta_5(k_1,k_2,k_3) = \Theta_4(k_1,k_2,k_3), \quad (4.90)$$

where $\Theta_4(k_1,k_2,k_3)$ is defined earlier. Here the result is exactly same as derived for the coefficient α_4.

[19] Here ot is important to point out that in de-Sitter space if we consider the Bunch Davies vacuum state then here only the term with $1/K^3$ will appear explicitly in the expression for the three point function and in the associated bispectrum. On the other hand if we consider all other non-trivial quantum vacuum state in our computation then the rest of the contribution will explicitly appear. From the perspective of observation this is obviously an important information as for the non trivial quantum vacuum state we get additional contribution in the bispectrum which may enhance the amplitude of the non-Gaussianity in squeezed limiting configuration. Additionally, it is important to mention that in quasi de Sitter case we get an extra contribution $1/(1+\epsilon)^5$. Also the factor $1/(k_1k_2k_3)^3$ will be replaced by $1/(k_1k_2k_3)^{2\nu}$ and $1/\tilde{c}_S^2$ is replaced by $1/\tilde{c}_S^{6\nu-7}$. Consequently, in quasi de Sitter case this contribution in

4.3 Limiting configurations of scalar bispectrum

To analyze the features of the bispectrum computed from the present setup here we further consider the following two configurations:

4.3.1 Equilateral limit configuration

Equilateral limit configuration is characterized by the condition, $k_1 = k_2 = k_3 = k$, where $k_i = |\mathbf{k_i}| \forall i = 1, 2, 3$. Consequently we have, $K = 3k$.

For this case, the bispectrum can be written as:

$$\boxed{B_{EFT}(k,k,k) = \sum_{j=1}^{5} \alpha_j \Theta_j(k,k,k)}\,, \tag{4.91}$$

where $\alpha_j \forall j = 1, 2, \cdots, 5$ are defined earlier and $\Theta_j(k,k,k) \forall j = 1, 2, \cdots, 5$ are given by:

$$\Theta_1(k,k,k) = \frac{3H^2}{16\epsilon^3 M_p^6} \frac{1}{k^6} \left[\frac{1}{27} U_1 - 3U_2\right], \tag{4.92}$$

$$\Theta_2(k,k,k) = -\frac{3H^2}{64\epsilon^3 M_p^6 \tilde{c}_S^2} \frac{1}{k^6} \left[\frac{17}{27} U_1 - 3U_2\right], \tag{4.93}$$

$$\Theta_3(k,k,k) = \frac{3H}{32\epsilon^3 M_p^6} \frac{1}{k^6} \left[\frac{10}{9} U_1 - \frac{22}{49} U_2\right], \tag{4.94}$$

$$\Theta_4(k,k,k) = \frac{3H^2}{64\tilde{c}_S^2 \epsilon^3 M_p^6} \frac{1}{k^6} \left[\frac{17}{27} U_1 - 3U_2\right] = \Theta_5(k,k,k), \tag{4.95}$$

where U_1 and U_2 are defined as:

$$\begin{aligned} U_1 &= \left[(C_1 - C_2)^3 \left(C_1^{*3} + C_2^{*3}\right) + (C_1^* - C_2^*)^3 \left(C_1^3 + C_2^3\right)\right], \\ U_2 &= \left[(C_1 - C_2)^3 C_1^* C_2^* \left(C_1^* - C_2^*\right) + (C_1^* - C_2^*)^3 C_1 C_2 \left(C_1 - C_2\right)\right]. \end{aligned} \tag{4.96}$$

Further substituting the explicit expressions for $\alpha_j \forall j = 1, 2, \cdots, 5$ and $\Theta_j(k,k,k) \forall j = 1, 2, \cdots, 5$ we get the following expression for the bispectrum for scalar fluctuations:

$$\boxed{B_{EFT}(k,k,k) = \frac{3H^2}{16\epsilon^3 \tilde{c}_S M_p^6} \frac{1}{k^6} \sum_{p=1}^{2} f_p U_p}\,, \tag{4.97}$$

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the bispectrum can be recast as:

$$\begin{aligned} \Theta_5(k_1,k_2,k_3) = \Theta_4(k_1,k_2,k_3) = &-\frac{H^2}{64\tilde{c}_S^{6\nu-7}\epsilon^3 M_p^6 (1+\epsilon)^5} \frac{1}{(k_1k_2k_3)^{2\nu}} \left[k_2^2(\mathbf{k_2}.\mathbf{k_3})\mathcal{F}_1(k_1,k_2,k_3) + k_3^2(\mathbf{k_2}.\mathbf{k_3})\mathcal{F}_2(k_1,k_2,k_3)\right. \\ &+ k_1^2(\mathbf{k_1}.\mathbf{k_3})\mathcal{F}_3(k_1,k_2,k_3) + k_3^2(\mathbf{k_1}.\mathbf{k_3})\mathcal{F}_4(k_1,k_2,k_3) + k_1^2(\mathbf{k_1}.\mathbf{k_2})\mathcal{F}_5(k_1,k_2,k_3) + \left. k_2^2(\mathbf{k_1}.\mathbf{k_2})\mathcal{F}_6(k_1,k_2,k_3)\right], \end{aligned} \tag{4.89}$$

where $f_p \forall p = 1,2$ are defined as:

$$f_1 = \frac{\tilde{c}_s}{27}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3}M_3^4\right\} + \frac{17}{108\tilde{c}_s}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H\right\} - \frac{5}{2}\bar{M}_1^3 H\tilde{c}_s + \frac{17}{36\tilde{c}_s}\bar{M}_1^3 H, \quad (4.98)$$

$$f_2 = -3\tilde{c}_s\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3}M_3^4\right\} - \frac{3}{4\tilde{c}_s}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H\right\} + \frac{99}{98}\bar{M}_1^3 H\tilde{c}_s - \frac{9}{4\tilde{c}_s}\bar{M}_1^3 H. \quad (4.99)$$

- **For Bunch Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$ we get:
  $$U_1 = 2, \quad U_2 = 0. \quad (4.100)$$
  Consequently we get the following expression for the bispectrum for scalar fluctuations:
  $$B_{EFT}(k,k,k) = \frac{H^2}{4\epsilon\tilde{c}_S M_p^2}\frac{1}{M_p^4\epsilon^2}\frac{1}{k^6}\left[\frac{\tilde{c}_s}{18}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3}M_3^4\right\} + \frac{17}{72\tilde{c}_s}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H\right\} - \frac{15}{4}\bar{M}_1^3 H\tilde{c}_s + \frac{17}{24\tilde{c}_s}\bar{M}_1^3 H\right]. \quad (4.101)$$
  For $\tilde{c}_S = 1 = c_S$ case we know that $M_2 = 0$ and $M_3 = 0$ which we have already shown earlier. As a result the bispectrum for scalar fluctuation can be expressed in the following simplified form:
  $$B_{EFT}(k,k,k) = -\frac{H^2}{4\epsilon M_p^2}\frac{1}{M_p^4\epsilon^2}\frac{1}{k^6}\frac{125}{48}\bar{M}_1^3 H. \quad (4.102)$$
  For $\tilde{c}_S < 1$ and $c_S < 1$ case one can also recast the bispectrum for scalar fluctuations in the following simplified form:
  $$B_{EFT}(k,k,k) = \frac{H^2}{4\epsilon\tilde{c}_S M_p^2}\frac{1}{M_p^4\epsilon^2}\frac{1}{k^6}\bar{M}_1^3 H\left[\frac{\tilde{c}_S}{18}\left\{\frac{3}{2} + \frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4} + \frac{2c_S^2}{\tilde{c}_4}\right\} + \frac{17}{72\tilde{c}_S}\left\{\frac{2c_S^2}{\tilde{c}_4} + \frac{3}{2}\right\} - \frac{15}{4}\tilde{c}_S + \frac{17}{24\tilde{c}_S}\right]. \quad (4.103)$$

- **For α, β vacuum:**
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\ \sinh\alpha$ we get:
  $$U_1 = J_1(\alpha,\beta), \quad U_2 = J_2(\alpha,\beta), \quad (4.104)$$
  where $J_1(\alpha,\beta)$ and $J_2(\alpha,\beta)$ are defined earlier.
  Consequently we get the following expression for the bispectrum for scalar fluctuations:
  $$B_{EFT}(k,k,k) = \frac{H^2}{4\epsilon\tilde{c}_S M_p^2}\frac{1}{M_p^4\epsilon^2}\frac{1}{k^6}\left[\left(\frac{\tilde{c}_s}{36}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3}M_3^4\right\} + \frac{17}{144\tilde{c}_s}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H\right\} - \frac{15}{8}\bar{M}_1^3 H\tilde{c}_s + \frac{17}{48\tilde{c}_s}\bar{M}_1^3 H\right) J_1(\alpha,\beta) + \left(-\frac{9}{4}\tilde{c}_s\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3}M_3^4\right\} - \frac{9}{16\tilde{c}_s}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H\right\} + \frac{297}{392}\bar{M}_1^3 H\tilde{c}_s - \frac{27}{16\tilde{c}_s}\bar{M}_1^3 H\right) J_2(\alpha,\beta)\right]. \quad (4.105)$$

For $\tilde{c}_S = 1 = c_S$ case we know that $M_2 = 0$ and $M_3 = 0$ which we have already shown earlier. As a result the bispectrum for scalar fluctuation can be expressed in the following simplified form:

$$B(k,k,k) = -\frac{H^2}{4\epsilon M_p^2}\frac{1}{M_p^4\epsilon^2}\frac{1}{k^6}\bar{M}_1^3 H\left[\frac{125}{96}J_1(\alpha,\beta) + \frac{8073}{1568}J_2(\alpha,\beta)\right]. \quad (4.106)$$

For $\tilde{c}_S < 1$ and $c_S < 1$ case one can also recast the bispectrum for scalar fluctuations in the following simplified form:

$$\begin{aligned} B_{EFT}(k,k,k) = \frac{H^2}{4\epsilon\tilde{c}_S M_p^2}\frac{1}{M_p^4\epsilon^2}\frac{1}{k^6}\bar{M}_1^3 H &\left[\left(\frac{\tilde{c}_S}{36}\left\{\frac{3}{2} + \frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4} + \frac{2c_S^2}{\tilde{c}_4}\right\} + \frac{17}{144\tilde{c}_S}\left\{\frac{2c_S^2}{\tilde{c}_4} + \frac{3}{2}\right\}\right.\right. \\ &- \left.\frac{15}{8}\tilde{c}_S + \frac{17}{48\tilde{c}_S}\right) J_1(\alpha,\beta) + \left(-\frac{9}{4}\tilde{c}_S\left\{\frac{3}{2} + \frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4} + \frac{2c_S^2}{\tilde{c}_4}\right\} - \frac{9}{16\tilde{c}_S}\left\{\frac{2c_S^2}{\tilde{c}_4} + \frac{3}{2}\right\}\right. \\ &\left.\left. + \frac{297}{392}\tilde{c}_S - \frac{27}{16\tilde{c}_S}\right) J_2(\alpha,\beta)\right]. \end{aligned} \quad (4.107)$$

4.3.2 Squeezed limit configuration

Squeezed limit configuration is characterized by the condition, $k_1 \approx k_2 (= k_L) >> k_3 (= k_S)$, where $k_i = |\mathbf{k_i}| \forall i = 1, 2, 3$. Also k_L and k_S characterize long and short mode momentum respectively. Consequently we have, $K = 2k_L + k_S$. For this case, the bispectrum can be written as:

$$\boxed{B_{EFT}(k_L, k_L, k_S) = \sum_{j=1}^{5}\alpha_j\Theta_j(k_L, k_L, k_S)}, \quad (4.108)$$

where $\alpha_j \forall j = 1, 2, \cdots, 5$ are defined earlier and $\Theta_j(k_L, k_L, k_S) \forall j = 1, 2, \cdots, 5$ are given by:

$$\Theta_1(k_L, k_L, k_S) \approx \frac{3H^2}{128\epsilon^3 M_p^6}\frac{1}{k_L^5 k_S}\left[U_1 - 16U_2\left(\frac{k_L}{k_S}\right)^3\right], \quad (4.109)$$

$$\begin{aligned} \Theta_2(k_L, k_L, k_S) = -\frac{H^2}{64\epsilon^3 M_p^6\tilde{c}_S^2}\frac{1}{k_L^5 k_S}&\left\{\frac{3}{4}\left[U_1 - 3U_2\right] + \frac{3}{4}\left[U_1 - U_2\left(1 + \frac{8}{3}\left(\frac{k_L}{k_S}\right)^2\right)\right]\right. \\ &\left. + \frac{5}{4}\left(2 - \left(\frac{k_S}{k_L}\right)^2\right)\left[U_1 - U_2\left(1 - \frac{8}{5}\left(\frac{k_L}{k_S}\right)^3\right)\right]\right\}, \end{aligned} \quad (4.110)$$

$$\Theta_3(k_L, k_L, k_S) = \frac{H}{64\epsilon^3 M_p^6}\frac{1}{k_L^5 k_S}\left\{3\left[U_1 - U_2\right] + \left(\frac{k_L}{k_S}\right)^2\left[U_1 - \left(1 + 8\left(\frac{k_L}{k_S}\right)\right)U_2\right]\right\}, \quad (4.111)$$

$$\begin{aligned} \Theta_4(k_L, k_L, k_S) = -\frac{H^2}{64\tilde{c}_S^2\epsilon^3 M_p^6}&\left\{\frac{3}{4}\left(\frac{1}{k_L^5 k_S} + \frac{k_S}{k_L^7}\right)\left[U_1 - U_2\left(1 + \frac{4}{3}\left(\frac{k_L}{k_S}\right)^2\right)\right]\right. \\ &+ \frac{5}{4}\left(\frac{1}{k_L^5 k_S} + \frac{k_S}{k_L^7}\right)\left[U_1 - U_2\left(1 - \frac{16}{5}\left(\frac{k_L}{k_S}\right)^2\right)\right] \\ &\left. + \frac{3}{2k_L^3 k_S^3}\left(2 - \left(\frac{k_S}{k_L}\right)^2\right)\left[U_1 - U_2\left(1 + \frac{8}{3}\left(\frac{k_L}{k_S}\right) + \frac{4}{3}\left(\frac{k_L}{k_S}\right)^2\right)\right]\right\} \\ = \Theta_5(k_L, k_L, k_S), & \end{aligned} \quad (4.112)$$

where U_1 and U_2 are defined earlier.

Further substituting the explicit expressions for $\alpha_j \forall j = 1, 2, \cdots, 5$ and $\Theta_j(k_L, k_L, k_S) \forall j = 1, 2, \cdots, 5$ we get the following expression for the bispectrum for scalar fluctuations:

$$B_{EFT}(k_L, k_L, k_S) = \frac{H^2}{64\epsilon^3 \tilde{c}_S M_p^6} \sum_{p=1}^{2} g_p(k_L, k_S) U_p, \tag{4.113}$$

where $g_p(k_L, k_S) \forall p = 1, 2$ are defined as:

$$\begin{aligned} g_1(k_L, k_S) = & \frac{3\tilde{c}_S}{2k_L^5 k_S} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2} \bar{M}_1^3 H - \frac{4}{3} M_3^4 \right\} \\ & + \frac{1}{\tilde{c}_S k_L^5 k_S} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2} \bar{M}_1^3 H \right\} \left(2 - \frac{5}{4} \left(\frac{k_S}{k_L}\right)^2\right) \\ & - \frac{9}{2} \bar{M}_1^3 H \frac{\tilde{c}_S}{k_L^5 k_S} \left(3 + \left(\frac{k_L}{k_S}\right)^2\right) + \frac{3}{\tilde{c}_S} \bar{M}_1^3 H \left\{ 2 \left(\frac{1}{k_L^5 k_S} + \frac{k_S}{k_L^7}\right) + \frac{3}{2k_L^3 k_S^3} \left(2 - \left(\frac{k_S}{k_L}\right)^2\right) \right\}, \end{aligned} \tag{4.114}$$

$$\begin{aligned} g_2(k_L, k_S) = & \frac{24\tilde{c}_S}{k_L^5 k_S} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2} \bar{M}_1^3 H - \frac{4}{3} M_3^4 \right\} \left(\frac{k_L}{k_S}\right)^2 \\ & + \frac{1}{\tilde{c}_S k_L^5 k_S} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2} \bar{M}_1^3 H \right\} \left\{ 3 + 2 \left(\frac{k_S}{k_L}\right)^2 \right. \\ & \left. + \frac{5}{4} \left(2 - \frac{5}{4} \left(\frac{k_S}{k_L}\right)^2\right) \left(1 - \frac{8}{5} \left(\frac{k_S}{k_L}\right)^3\right) \right\} - \frac{9}{2} \bar{M}_1^3 H \frac{\tilde{c}_S}{k_L^5 k_S} \left\{ 3 + \left(\frac{k_L}{k_S}\right)^2 \left(1 + 8 \left(\frac{k_L}{k_S}\right)\right) \right\} \\ & + \frac{3}{\tilde{c}_S} \bar{M}_1^3 H \left\{ 2 \left(\frac{1}{k_L^5 k_S} + \frac{k_S}{k_L^7}\right) + \frac{3}{2k_L^3 k_S^3} \left(2 - \left(\frac{k_S}{k_L}\right)^2\right) \right\}. \end{aligned} \tag{4.115}$$

- **For Bunch Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$U_1 = 2, \quad U_2 = 0. \tag{4.116}$$

Consequently we get the following expression for the bispectrum for scalar fluctuations:

$$\begin{aligned} B_{EFT}(k_L, k_L, k_S) = & \frac{H^2}{4\epsilon \tilde{c}_S M_p^2} \frac{1}{8M_p^4 \epsilon^2} \left[\frac{3\tilde{c}_S}{2k_L^5 k_S} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2} \bar{M}_1^3 H - \frac{4}{3} M_3^4 \right\} \right. \\ & + \frac{1}{\tilde{c}_S k_L^5 k_S} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2} \bar{M}_1^3 H \right\} \left(2 - \frac{5}{4} \left(\frac{k_S}{k_L}\right)^2\right) \\ & - \frac{9}{2} \bar{M}_1^3 H \frac{\tilde{c}_S}{k_L^5 k_S} \left(3 + \left(\frac{k_L}{k_S}\right)^2\right) \\ & \left. + \frac{3}{\tilde{c}_S} \bar{M}_1^3 H \left\{ 2 \left(\frac{1}{k_L^5 k_S} + \frac{k_S}{k_L^7}\right) + \frac{3}{2k_L^3 k_S^3} \left(2 - \left(\frac{k_S}{k_L}\right)^2\right) \right\} \right]. \end{aligned} \tag{4.117}$$

For $\tilde{c}_S = 1 = c_S$ case we know that $M_2 = 0$ and $M_3 = 0$ which we have already shown earlier. As a result the bispectrum for scalar fluctuation can be expressed in the following simplified form:

$$B_{EFT}(k_L,k_L,k_S) = \frac{H^2}{4\epsilon M_p^2}\frac{1}{8M_p^4\epsilon^2}\bar{M}_1^3 H\left[\frac{9}{4k_L^5k_S}+\frac{3}{2k_L^5k_S}\left(2-\frac{5}{4}\left(\frac{k_S}{k_L}\right)^2\right)\right. \tag{4.118}$$
$$\left.-\frac{9}{2}\frac{1}{k_L^5k_S}\left(3+\left(\frac{k_L}{k_S}\right)^2\right)+3\left\{2\left(\frac{1}{k_L^5k_S}+\frac{k_S}{k_L^7}\right)+\frac{3}{2k_L^3k_S^3}\left(2-\left(\frac{k_S}{k_L}\right)^2\right)\right\}\right].$$

For $\tilde{c}_S < 1$ and $c_S < 1$ case one can also recast the bispectrum for scalar fluctuations in the following simplified form:

$$\begin{aligned} B_{EFT}(k_L,k_L,k_S) = \frac{H^2}{4\epsilon\tilde{c}_S M_p^2}\frac{1}{8M_p^4\epsilon^2}\bar{M}_1^3 H&\left[\frac{3\tilde{c}_S}{2k_L^5k_S}\left\{\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right\}\right.\\ &+\frac{1}{\tilde{c}_S k_L^5k_S}\left\{\frac{2c_S^2}{\tilde{c}_4}+\frac{3}{2}\right\}\left(2-\frac{5}{4}\left(\frac{k_S}{k_L}\right)^2\right)\\ &-\frac{9}{2}\frac{\tilde{c}_S}{k_L^5k_S}\left(3+\left(\frac{k_L}{k_S}\right)^2\right)\\ &\left.+\frac{3}{\tilde{c}_S}\left\{2\left(\frac{1}{k_L^5k_S}+\frac{k_S}{k_L^7}\right)+\frac{3}{2k_L^3k_S^3}\left(2-\left(\frac{k_S}{k_L}\right)^2\right)\right\}\right]. \end{aligned} \tag{4.119}$$

- **For α,β vacuum:**
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\ \sinh\alpha$ we get:

$$U_1 = J_1(\alpha,\beta), \quad U_2 = J_2(\alpha,\beta), \tag{4.120}$$

where $J_1(\alpha,\beta)$ and $J_2(\alpha,\beta)$ are defined earlier.

Consequently we get the following expression for the bispectrum for scalar fluctuations:

$$B_{EFT}(k_L,k_L,k_S) = \frac{H^2}{4\epsilon\tilde{c}_S M_p^2}\frac{1}{16M_p^4\epsilon^2}\left[g_1(k_L,k_S)J_1(\alpha,\beta)+g_2(k_L,k_S)J_2(\alpha,\beta)\right]. \tag{4.121}$$

For $\tilde{c}_S = 1 = c_S$ case we know that $M_2 = 0$ and $M_3 = 0$ which we have already shown earlier. As a result the factors $g_1(k_L,k_S)$ and $g_2(k_L,k_S)$ appearing in the expression for bispectrum for scalar fluctuation can be expressed in the following simplified form:

$$\begin{aligned} g_1(k_L,k_S) = \frac{9}{4k_L^5k_S}\bar{M}_1^3H &+ \frac{3}{2k_L^5k_S}\bar{M}_1^3H\left(2-\frac{5}{4}\left(\frac{k_S}{k_L}\right)^2\right)\\ &-\frac{9}{2}\bar{M}_1^3H\frac{1}{k_L^5k_S}\left(3+\left(\frac{k_L}{k_S}\right)^2\right)\\ &+3\bar{M}_1^3H\left\{2\left(\frac{1}{k_L^5k_S}+\frac{k_S}{k_L^7}\right)+\frac{3}{2k_L^3k_S^3}\left(2-\left(\frac{k_S}{k_L}\right)^2\right)\right\}, \end{aligned} \tag{4.122}$$

$$g_2(k_L,k_S) = \frac{36}{k_L^5 k_S}\bar{M}_1^3 H\left(\frac{k_L}{k_S}\right)^2 + \frac{3}{2k_L^5 k_S}\bar{M}_1^3 H\left\{3+2\left(\frac{k_S}{k_L}\right)^2 + \frac{5}{4}\left(2-\frac{5}{4}\left(\frac{k_S}{k_L}\right)^2\right)\left(1-\frac{8}{5}\left(\frac{k_S}{k_L}\right)^3\right)\right\} - \frac{9}{2}\bar{M}_1^3 H\frac{1}{k_L^5 k_S}\left\{3+\left(\frac{k_L}{k_S}\right)^2\left(1+8\left(\frac{k_L}{k_S}\right)\right)\right\} + 3\bar{M}_1^3 H\left\{2\left(\frac{1}{k_L^5 k_S}+\frac{k_S}{k_L^7}\right) + +\frac{3}{2k_L^3 k_S^3}\left(2-\left(\frac{k_S}{k_L}\right)^2\right)\right\}. \quad (4.123)$$

For $\tilde{c}_S < 1$ and $c_S < 1$ case one can also recast the factors $g_1(k_L,k_S)$ and $g_2(k_L,k_S)$ as appearing in the expression for bispectrum for scalar fluctuations in the following simplified form:

$$g_1(k_L,k_S) = \frac{3\tilde{c}_S}{2k_L^5 k_S}\left\{\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right\} + \frac{1}{\tilde{c}_S k_L^5 k_S}\left\{\frac{2c_S^2}{\tilde{c}_4}+\frac{3}{2}\right\}\left(2-\frac{5}{4}\left(\frac{k_S}{k_L}\right)^2\right) - \frac{9}{2}\bar{M}_1^3 H\frac{\tilde{c}_S}{k_L^5 k_S}\left(3+\left(\frac{k_L}{k_S}\right)^2\right) + \frac{3}{\tilde{c}_S}\bar{M}_1^3 H\left\{2\left(\frac{1}{k_L^5 k_S}+\frac{k_S}{k_L^7}\right)+\frac{3}{2k_L^3 k_S^3}\left(2-\left(\frac{k_S}{k_L}\right)^2\right)\right\}, \quad (4.124)$$

$$g_2(k_L,k_S) = \frac{24\tilde{c}_S}{k_L^5 k_S}\left\{\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right\}\left(\frac{k_L}{k_S}\right)^2 + \frac{1}{\tilde{c}_S k_L^5 k_S}\left\{\frac{2c_S^2}{\tilde{c}_4}+\frac{3}{2}\right\}\left\{3+2\left(\frac{k_S}{k_L}\right)^2+\frac{5}{4}\left(2-\frac{5}{4}\left(\frac{k_S}{k_L}\right)^2\right)\left(1-\frac{8}{5}\left(\frac{k_S}{k_L}\right)^3\right)\right\} - \frac{9}{2}\bar{M}_1^3 H\frac{\tilde{c}_S}{k_L^5 k_S}\left\{3+\left(\frac{k_L}{k_S}\right)^2\left(1+8\left(\frac{k_L}{k_S}\right)\right)\right\} + \frac{3}{\tilde{c}_S}\bar{M}_1^3 H\left\{2\left(\frac{1}{k_L^5 k_S}+\frac{k_S}{k_L^7}\right)++\frac{3}{2k_L^3 k_S^3}\left(2-\left(\frac{k_S}{k_L}\right)^2\right)\right\}. \quad (4.125)$$

5 Determination of EFT coefficients and future predictions

In this section we compute the exact analytical expression for the EFT coefficients for two specific cases- 1. Canonical single field slow roll inflation and 2. General single field $P(X,\phi)$ models of inflation, where $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ is the kinetic term. To determine the EFT coefficients for canonical single field slow roll model or from general single field $P(X,\phi)$ model of inflation we will follow the following strategy:

1. First of all, we will start with the general expression for the three point function and the bispectrum for scalar perturbations with arbitrary choice of quantum vacuum. Then we take the Bunch-Davies and α,β vacuum to match with the standard results of scalar three point function.

2. Next we take the equilateral limit and squeezed limit configuration of the bispectrum obtained from single fleid slow roll model and general single field $P(X, \phi)$ model.

3. Further we equate the equilateral limit and squeezed limit configuration of the bispectrum computed from the EFT of inflation with the single field slow roll or from general single field $P(X, \phi)$ model.

4. Finally, for sound speed $c_S = 1$ and $c_S < 1$ we get the analytical expressions for all the EFT coefficients for canonical single field slow roll models or from generalized single field $P(X, \phi)$ models of inflation.

5.1 For canonical Single Field Slow Roll inflation

Here our prime objective is to derive the EFT coefficients by computing the most general expression for the three point function for scalar fluctuations from the canonical single field slow roll model of inflation for arbitrary vacuum. Then we give specific example for Bunch-Davies and α, β vacuum for completeness.

5.1.1 Basic setup

Let us start with the action for single scalar field (inflaton) which has canonical kinetic term as given by:

$$\boxed{\textbf{Canonical model}: \; S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + X - V(\phi)\right]}\,, \tag{5.1}$$

where $V(\phi)$ is the potential which satisfies slow-roll condition for inflation.

It is important to mention here that perturbations to the homogeneous situation discussed above are introduced in the ADM formalism ehere the metric takes the form [30]:

$$\boxed{\textbf{ADM metric}: \; ds^2 = -N^2dt^2 + g_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)}\,, \tag{5.2}$$

where g_{ij} is the metric on the spatial three surface characterized by t, lapse N and shift N_i. Here we choose synchronous gauge to maintain diffeomorphism invariance of the theory where the gauge fixing conditions are given by:

$$\boxed{\textbf{Synchronous gauge}: \quad N = 1, \quad N^i = 0}\,, \tag{5.3}$$

and the perturbed metric is given by:

$$\boxed{g_{ij} = a^2(t)\left[(1 + 2\zeta(t, \mathbf{x}))\delta_{ij} + \gamma_{ij}\right], \quad \gamma_{ii} = 0}\,, \tag{5.4}$$

where $\zeta(t, \mathbf{x})$ and γ_{ij} are defined earlier. Here it is important to note that, the structure of g_{ij} is exactly same that we have mentioned in case of EFT framework discussed in this paper. Note that in the context of ADM formalism one can treat the scalar field ϕ, induced metric g_{ij} as the dynamical variables. On the other hand, N and N^i mimics the role of Lagrange multipliers in ADM formalism. Consequently, one needs to impose the equations of motion of N, N^i as additional constraints in the synchronous gauge where the gauge condition as stated in Eq. (5.3) holds good perfectly. More precisely, in this context the equations of motion of N and N^i correspond to time and spatial reparametrization invariance.

Further using the ADM metric as stated in Eq (5.2), the action for the single scalar field Eq (5.1) can be recast as [30]:

$$\boxed{S=\int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}N\,{}^{(3)}R-NV+\frac{1}{2N}\left(E_{ij}E^{ij}-E^2\right)+\frac{1}{2N}\left(\dot{\phi}-N^i\partial_i\phi\right)^2-Ng^{ij}\partial_i\phi\partial_j\phi\right]}\,,\quad(5.5)$$

where ${}^{(3)}R$ is the Ricci scalar curvature of the spatial slice. Also here E_{ij} and E is defined as [30]:

$$E_{ij}:=\frac{1}{2}\left(\dot{g}_{ij}-\nabla_iN_j-\nabla_jN_i\right)=NK_{ij},\quad(5.6)$$

$$E:=E^i_i=g_{ij}E^{ij}=g_{ij}g^{im}g^{jn}E_{mn}=Ng_{ij}g^{im}g^{jn}K_{mn}.\quad(5.7)$$

Here the covarinat derivative ∇_i, is taken with respect to the 3-metric g_{ij}. Also in this context the extrinsic curvature K_{ij} is defined as [30]:

$$K_{ij}=\frac{1}{N}E_{ij}=\frac{1}{2N}\left(\dot{g}_{ij}-\nabla_iN_j-\nabla_jN_i\right).\quad(5.8)$$

Additionally we choose the following two gauges:

$$\boxed{\underline{\textbf{Gauge I}}:\quad\delta\phi(t,\mathbf{x})=0,\quad\zeta(t,\mathbf{x})\neq0,\quad\partial_i\gamma_{ij}=0,\quad\gamma_{ii}=0}\,.\quad(5.9)$$

$$\boxed{\underline{\textbf{Gauge II}}:\quad\delta\phi(t,\mathbf{x})\neq0,\quad\zeta(t,\mathbf{x})=0,\quad\partial_i\gamma_{ij}=0,\quad\gamma_{ii}=0}\,.\quad(5.10)$$

For our present computations, we will work in **Gauge I** as this is exactly same as the unitary gauge that we have used in the context of EFT framework. Also the tensor perturbation γ_{ij} is exactly same for the unitary gauge that we have used for EFT setup.

5.1.2 Scalar three point function for Single Field Slow Roll inflation

Before computing the three point function for scalar mode fluctuation here it is important to note that the two point function for Single Field Slow Roll inflation is exactly same with the results obtained for EFT of inflation with sound speed $c_S=1$ and $\tilde{c}_S=1$, which can be obtained by setting the EFT coefficients, $M_2=0$, $M_3=0$, $\bar{M}_1\neq0$, $M_4\neq0$, $\bar{M}_2\neq0$, $\bar{M}_3\neq0$, $\tilde{c}_5=-\frac{1}{2}(1+\epsilon)$ [20]. Using three point function we can able to fix all of these coefficients.

[20]In case of Single Field Slow Roll inflation amplitude of power spectrum and spectral tilt for scalar fluctuation can be written at the horizon crossing $|k\eta|=1$ as:

$$\underline{\textbf{For Bunch – Davies vacuum :}}\quad\Delta_\zeta(k_*)=\begin{cases}\dfrac{V(\phi_*)}{24\pi^2\,M_p^4\epsilon_V} & \textbf{for dS}\\[2ex] 2^{6\epsilon_V-2\eta_V}\dfrac{V(\phi_*)}{24\pi^2\,M_p^4\epsilon_V(1+\epsilon_V)^2}\left|\dfrac{\Gamma(\frac{3}{2}+4\epsilon_V-\eta_V)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS}.\end{cases}$$

$$\underline{\textbf{For }\alpha,\beta\textbf{ vacuum :}}\quad\Delta_\zeta(k_*)=\begin{cases}\dfrac{V(\phi_*)}{24\pi^2\,M_p^4\epsilon_V}\left[\cosh2\alpha-\sinh2\alpha\cos\beta\right] & \textbf{for dS}\\[2ex] 2^{6\epsilon_V-2\eta_V}\dfrac{V(\phi_*)}{24\pi^2\,M_p^4\epsilon_V(1+\epsilon_V)^2}\left|\dfrac{\Gamma(\frac{3}{2}+4\epsilon_V-\eta_V)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \\[2ex] \left[\cosh2\alpha-\sinh2\alpha\cos\left(\pi\left(2+4\epsilon_V-\eta_V\right)+\beta\right)\right] & \textbf{for qdS}.\end{cases}$$

and

$$n_\zeta(k_*)-1=2\eta_V-6\epsilon_V.\quad(5.11)$$

We here now proceed to calculate the three point function for the scalar fluctuation $\zeta(t, \mathbf{x})$ in the interacting picture with arbitrary vacuum. Then we cite results for Bunch-Davies and α, β vacuum. For Single Field Slow Roll inflation the third order term in the action Eq. (5.5) is given by [30]:

$$\boxed{S^{(3)}_{\zeta} = \int d^4x \, \left[a^3\epsilon^2\tilde{\zeta}\dot{\tilde{\zeta}}^2 + a\epsilon^2\tilde{\zeta}(\partial\tilde{\zeta})^2 - 2a^3\epsilon\dot{\tilde{\zeta}}\partial_i\tilde{\zeta}\partial_i(\epsilon\partial^{-2}\dot{\tilde{\zeta}})\right]}\,, \tag{5.12}$$

which is derived from Eq (5.5) and here after neglecting all the contribution from the terms which are subleading in the slow-roll parameters. Additionally here we use the following field redefinition:

$$\zeta = \tilde{\zeta} + \left\{\epsilon - \frac{\eta}{2}\right\}\tilde{\zeta}^2, \tag{5.13}$$

where ϵ, η, δ and s are slow-roll parameters which are defined in the context of Single Field Slow Roll inflation as:

$$\epsilon \sim \frac{1}{2M_p^2}\frac{\dot{\phi}^2}{H^2}, \quad \eta \sim \epsilon - \delta, \quad \delta = \frac{\ddot{\phi}}{H\dot{\phi}}, \quad s = 0. \tag{5.14}$$

Here one can also express the slow-roll parameters ϵ and η in terms of the slowly varying potential $V(\phi)$ as, $\epsilon \sim \epsilon_V$, $\eta \sim \eta_V - \epsilon_V$, $\delta \sim 2\epsilon_V - \eta_V$. where the new slow-roll parameter ϵ_V and η_V are defined as, $\epsilon_V = \frac{M_p^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2, \quad \eta_V = M_p^2\left(\frac{V''(\phi)}{V(\phi)}\right)$. Here $'$ represents $d/d\phi$.

Now it is important to note that, in the present context of discussion we are interested in the three point function for the scalar fluctuation field ζ, not for the redefined scalar field fluctuation $\tilde{\zeta}$ and for this reason one can write down the exact connection between the three point fulction for the scalar fluction field ζ and redefined scalar fluctuation field $\tilde{\zeta}$ in position space as:

$$\begin{aligned}\langle\zeta(\mathbf{x_1})\zeta(\mathbf{x_2})\zeta(\mathbf{x_3})\rangle &= \langle\tilde{\zeta}(\mathbf{x_1})\tilde{\zeta}(\mathbf{x_2})\tilde{\zeta}(\mathbf{x_3})\rangle + (2\epsilon - \eta)\left[\langle\zeta(\mathbf{x_1})\zeta(\mathbf{x_2})\rangle\langle\zeta(\mathbf{x_1})\zeta(\mathbf{x_3})\rangle\right. \\ &\quad \left.+ \langle\zeta(\mathbf{x_2})\zeta(\mathbf{x_1})\rangle\langle\zeta(\mathbf{x_2})\zeta(\mathbf{x_3})\rangle + \langle\zeta(\mathbf{x_3})\zeta(\mathbf{x_1})\rangle\langle\zeta(\mathbf{x_3})\zeta(\mathbf{x_2})\rangle\right].\end{aligned} \tag{5.15}$$

After taking the Fourier transform of the scalar fluction field ζ and redefined scalar fluctuation field $\tilde{\zeta}$ one can express connection between three point function in momentum space and this is our main point of interest also.

The interaction Hamiltonian for the redefined scalar fluctuation $\tilde{\zeta}$ can be expressed as:

$$H_{int} = \int d^3x \, \left[a\ \epsilon^2\tilde{\zeta}\tilde{\zeta}'^2 + a\ \epsilon^2\tilde{\zeta}(\partial\tilde{\zeta})^2 - 2a\epsilon\tilde{\zeta}'\partial_i\tilde{\zeta}\partial_i(\epsilon\partial^{-2}\tilde{\zeta}')\right]. \tag{5.16}$$

Further following the in-in formalism in interaction picture the expression for the three point function for the redefined scalar fluctuation $\tilde{\zeta}$ and then transforming the final result in terms of the scalar fluctuation ζ in momentum one can write the following expression:

$$\boxed{\begin{aligned}\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle &= -i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\ a(\eta)\ \langle 0|\left[\zeta(\eta_f,\mathbf{k_1})\zeta(\eta_f,\mathbf{k_2})\zeta(\eta_f,\mathbf{k_3}), H_{int}(\eta)\right]|0\rangle \\ &= (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})B_{SFSR}(k_1,k_2,k_3)\ ,\end{aligned}} \tag{5.17}$$

where $B_{SFSR}(k_1, k_2, k_3)$ represents the bispectrum of scalar fluctuation ζ, which is computed from Single Field Slow Roll inflation. Here the final expression for the bispectrum of scalar fluctuation for arbitray vacuum is given by:

$$\boxed{\begin{aligned} B_{SFSR}(k_1,k_2,k_3) = & \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[2(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2\sum_{i=1}^{3}k_i^3\right.\\ & +\epsilon\left(|C_1|^2-|C_2|^2\right)^2\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ & +\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2\right.\\ & \left.\left.+8\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\sum_{m=1}^{3}\frac{1}{K-2k_m}\right)\right].\end{aligned}} \tag{5.18}$$

For Bunch Davies and α,β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch Davies vacuum**:
  After setting $C_1 = 1$ and $C_2 = 0$ we get [30]:

$$\begin{aligned} B_{SFSR}(k_1,k_2,k_3) = & \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[2(2\epsilon-\eta)\sum_{i=1}^{3}k_i^3\right.\\ & \left.+\epsilon\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\right].\end{aligned} \tag{5.19}$$

- **For α,β vacuum**:
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get [32]:

$$\begin{aligned} B_{SFSR}(k_1,k_2,k_3) = & \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[2(2\epsilon-\eta)\cosh^2 2\alpha\sum_{i=1}^{3}k_i^3\right.\\ & +\epsilon\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ & \left.+\epsilon\sinh^2 2\alpha\cos^2\beta\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+8\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\sum_{m=1}^{3}\frac{1}{K-2k_m}\right)\right].\end{aligned} \tag{5.20}$$

Further we consider equilateral limit and squeezed limit in which we finally get:

1. Equilateral limit configuration:
   Here the bispectrum for scalar perturbations in presence of arbitray quantum vacuum can be expressed as:

$$\boxed{\begin{aligned} B_{SFSR}(k,k,k) = & \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2\right.\\ & \left.+27\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\right].\end{aligned}} \tag{5.21}$$

Now for Bunch Davies and α, β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$\boxed{B_{SFSR}(k,k,k) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[23\epsilon - 6\eta\right]}\,. \tag{5.22}$$

- **For α, β vacuum:**
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get:

$$\boxed{B_{SFSR}(k,k,k) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[6(2\epsilon-\eta)\cosh^2 2\alpha + 11\epsilon + 27\epsilon\sinh^2 2\alpha\cos^2\beta\right]}\,. \tag{5.23}$$

2. Squeezed limit configuration:
   Here the bispectrum for scalar perturbations in presence of arbitray quantum vacuum can be expressed as:

$$\boxed{B_{SFSR}(k_L,k_L,k_S) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k_L^3 k_S^3}\sum_{j=-1}^{3} a_j\left(\frac{k_S}{k_L}\right)^j}\,, \tag{5.24}$$

where the expansion coefficients $a_j \forall j = -1, \cdots, 3$ for arbitrary vacuum are defined as:

$$\begin{aligned}
a_{-1} &= 16\epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2\,,\\
a_0 &= 4(2\epsilon-\eta)\left(|C_1|^2 + |C_2|^2\right)^2 + 4\epsilon\left(|C_1|^2 - |C_2|^2\right)^2 + 4\epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2\,,\\
a_1 &= 34\epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2\,,\quad a_2 = 10\epsilon\left(|C_1|^2 - |C_2|^2\right)^2 + 10\epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2\,,\\
a_3 &= 2(2\epsilon-\eta)\left(|C_1|^2 + |C_2|^2\right)^2 - 5\epsilon\left(|C_1|^2 - |C_2|^2\right)^2 - \epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2\,.
\end{aligned}$$

Now for Bunch Davies and α, β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$, we get the following expression for the expansion coefficients $a_j \forall j = -1, \cdots, 3$:

$$a_{-1} = 0,\quad a_0 = 4(3\epsilon-\eta),\quad a_1 = 0,\quad a_2 = 10\epsilon,\quad a_3 = -(\epsilon+2\eta). \tag{5.25}$$

  Consequently the bispectrum can be recast as:

$$\boxed{B_{SFSR}(k_L,k_L,k_S) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k_L^3 k_S^3}\left[4(3\epsilon-\eta) + 10\epsilon\left(\frac{k_S}{k_L}\right)^2 - (\epsilon+2\eta)\left(\frac{k_S}{k_L}\right)^3\right]}\,. \tag{5.26}$$

- **For α, β vacuum:**
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$, we get the following expression for the expansion coefficients $a_j \forall j = -1, \cdots, 3$:

$$\begin{aligned}
a_{-1} &= 16\epsilon\sinh^2 2\alpha\cos^2\beta,\quad a_0 = 4(2\epsilon-\eta)\cosh^2 2\alpha + 4\epsilon + 4\epsilon\sinh^2 2\alpha\cos^2\beta,\\
a_1 &= 34\epsilon\sinh^2 2\alpha\cos^2\beta,\quad a_2 = 10\epsilon + 10\epsilon\sinh^2 2\alpha\cos^2\beta,\\
a_3 &= 2(2\epsilon-\eta)\cosh^2 2\alpha - 5\epsilon - \epsilon\sinh^2 2\alpha\cos^2\beta.
\end{aligned}$$

Consequently the bispectrum can be recast as:

$$B_{SFSR}(k_L, k_L, k_S) = \frac{H^4}{32\epsilon^2 M_p^4} \frac{1}{k_L^3 k_S^3} \left[16\epsilon \sinh^2 2\alpha \cos^2\beta \left(\frac{k_S}{k_L}\right)^{-1} + \left(4(2\epsilon-\eta)\cosh^2 2\alpha + 4\epsilon + 4\epsilon \sinh^2 2\alpha \cos^2\beta\right) + 34\epsilon \sinh^2 2\alpha \cos^2\beta \left(\frac{k_S}{k_L}\right) + \left(10\epsilon + 10\epsilon \sinh^2 2\alpha \cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^2 + \left(2(2\epsilon-\eta)\cosh^2 2\alpha - 5\epsilon - \epsilon \sinh^2 2\alpha \cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^3 \right]. \quad (5.27)$$

5.1.3 Expression for EFT coefficients for Single Field Slow Roll inflation

Here our prime objective is to derive the analytical expressions for EFT coefficients for Single Field Slow Roll inflation. To serve this purpose we start with a claim that the three point function and the associated bispectrum for the scalar fluctuations computed from Single Field Slow Roll inflation is exactly same as that we have computed from EFT setup for consistent UV completion. Here we use the equilateral limit and squeezed limit configurations to extract the analytical expression for the EFT coefficients. In the two limiting cases the results are following:

1. Equilateral limit configuration:
For this case with arbitrary vacuum one can write:

$$B_{EFT}(k,k,k) = B_{SFSR}(k,k,k), \quad (5.28)$$

which implies that:

$$\begin{aligned}
\bar{M}_1 &= \left\{ \frac{HM_p^2\epsilon\left[6(\eta-2\epsilon)\left(|C_1|^2+|C_2|^2\right)^2 - 11\epsilon\left(|C_1|^2-|C_2|^2\right)^2 - 27\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\right]}{\left[\frac{125}{12}U_1+\frac{8073}{196}U_2\right]} \right\}^{\frac{1}{3}}, \\
\bar{M}_2 \approx \bar{M}_3 &= \sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}} = \left\{ \frac{2M_p^2\epsilon\left[6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2 + 11\epsilon\left(|C_1|^2-|C_2|^2\right)^2 + 27\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\right]}{(1+\epsilon)\left[\frac{125}{3}U_1+\frac{8073}{49}U_2\right]} \right\}^{\frac{1}{2}}, \\
\tilde{c}_5 &= -\frac{1}{2}\left(1+\epsilon\right), \quad M_2 = 0, \quad M_3 = 0, \\
M_4 &= \left(-\frac{\tilde{c}_3}{\tilde{c}_6} H\bar{M}_1^3\right)^{\frac{1}{4}} = \left\{ \frac{\tilde{c}_3 H^2 M_p^2\epsilon\left[6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2 + 11\epsilon\left(|C_1|^2-|C_2|^2\right)^2 + 27\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\right]}{\tilde{c}_6\left[\frac{125}{12}U_1+\frac{8073}{196}U_2\right]} \right\}^{\frac{1}{4}}.
\end{aligned} \quad (5.29)$$

where for arbitrary vacuum U_1 and U_2 are defined as:

$$U_1 = \left[(C_1 - C_2)^3 \left(C_1^{*3} + C_2^{*3}\right) + \left(C_1^* - C_2^*\right)^3 \left(C_1^3 + C_2^3\right)\right], \quad (5.30)$$

$$U_2 = \left[(C_1 - C_2)^3 C_1^* C_2^* \left(C_1^* - C_2^*\right) + \left(C_1^* - C_2^*\right)^3 C_1 C_2 \left(C_1 - C_2\right)\right]. \quad (5.31)$$

To constraint all these coefficients of EFT operators using CMB observation from Planck TT+low P data we use [31]:

$$\epsilon < 0.012 \;\; (95\% \text{ CL}), \; \eta = -0.0080^{+0.0088}_{-0.0146} \;\; (68\% \text{ CL}), \; c_S = 1 \;\; (95\% \text{ CL}),$$
$$H = H_{inf} \leq 1.09 \times 10^{-4}\, M_p \sqrt{\epsilon\, c_S}, \quad (5.32)$$

where $M_p = 2.43\times 10^{18}$ GeV is the reduced Planck mass. Now for Bunch Davies and α,β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch Davies vacuum**:
  After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$\bar{M}_1 = \left\{\frac{6}{125}HM_p^2\epsilon\left[6\eta-23\epsilon\right]\right\}^{\frac{1}{3}}, \quad \bar{M}_2\approx\bar{M}_3=\sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}}=\left\{\frac{3}{125(1+\epsilon)}M_p^2\epsilon\left[23\epsilon-6\eta\right]\right\}^{\frac{1}{2}},$$
$$\tilde{c}_5=-\frac{1}{2}\left(1+\epsilon\right), \quad M_2=0,\quad M_3=0,\quad M_4=\left(-\frac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}}=\left\{\frac{6\tilde{c}_3}{125\tilde{c}_6}H^2M_p^2\epsilon\left[23\epsilon-6\eta\right]\right\}^{\frac{1}{4}}. \quad (5.33)$$

  Further using the constraint stated in Eq (5.32) we finally get the following constraints on the coefficients of EFT operators:

$$1.23\times10^{-3}\ M_p<|\bar{M}_1|<1.41\times10^{-3}\ M_p,\quad 8.79\times10^{-3}\ M_p<|\bar{M}_2|\approx|\bar{M}_3|<1.08\times10^{-2}\ M_p,$$
$$M_2=0,\quad M_3=0,\quad 3.86\times10^{-4}\ M_p<M_4\times(-\tilde{c}_6/\tilde{c}_3)^{1/4}<4.29\times10^{-4}\ M_p. \quad (5.34)$$

- **For α,β vacuum**:
  After setting $C_1=\cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get:

$$\begin{aligned}
\bar{M}_1 &= \left\{\frac{HM_p^2\epsilon\left[6(\eta-2\epsilon)\cosh^2 2\alpha-11\epsilon-27\epsilon\sinh^2 2\alpha\cos^2\beta\right]}{\left[\frac{125}{12}J_1(\alpha,\beta)+\frac{8073}{196}J_2(\alpha,\beta)\right]}\right\}^{\frac{1}{3}},\\
\bar{M}_2 &\approx \bar{M}_3=\sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}}=\left\{\frac{2M_p^2\epsilon\left[6(2\epsilon-\eta)\cosh^2 2\alpha+11\epsilon+27\epsilon\sinh^2 2\alpha\cos^2\beta\right]}{(1+\epsilon)\left[\frac{125}{3}J_1(\alpha,\beta)+\frac{8073}{49}J_2(\alpha,\beta)\right]}\right\}^{\frac{1}{2}},\\
\tilde{c}_5 &= -\frac{1}{2}\left(1+\epsilon\right),\quad M_2=0,\quad M_3=0,\\
M_4 &= \left(-\frac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}}=\left\{\frac{\tilde{c}_3H^2M_p^2\epsilon\left[6(2\epsilon-\eta)\cosh^2 2\alpha+11\epsilon+27\epsilon\sinh^2 2\alpha\cos^2\beta\right]}{\tilde{c}_6\left[\frac{125}{12}J_1(\alpha,\beta)+\frac{8073}{196}J_2(\alpha,\beta)\right]}\right\}^{\frac{1}{4}}.
\end{aligned} \quad (5.35)$$

  Further using the contraint stated in Eq (5.32) we finally get the following constraints on the coefficients of EFT operators for a given value of the parameters α and β (say for $\alpha=0.1$ and $\beta=0.1$):

$$9.1\times10^{-4}\ M_p<|\bar{M}_1|<1.1\times10^{-3}\ M_p,\quad 1.11\times10^{-2}\ M_p<|\bar{M}_2|\approx|\bar{M}_3|<1.5\times10^{-2}\ M_p,$$
$$M_2=0,\quad M_3=0,\quad 3.06\times10^{-4}\ M_p<M_4\times(-\tilde{c}_6/\tilde{c}_3)^{1/4}<3.54\times10^{-4}\ M_p. \quad (5.36)$$

2. Squeezed limit configuration:
   For this case with arbitrary vacuum one can write:

$$B_{EFT}(k_L,k_L,k_S)=B_{SFSR}(k_L,k_L,k_S), \quad (5.37)$$

which implies that:

$$\bar{M}_1=\left\{2HM_p^2\epsilon\frac{\sum_{j=-1}^{3}a_j\left(\frac{k_S}{k_L}\right)^j}{\sum_{m=-1}^{3}b_m\left(\frac{k_S}{k_L}\right)^m}\right\}^{\frac{1}{3}}, \quad \bar{M}_2\approx\bar{M}_3=\sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}}=\left\{-\frac{M_p^2\epsilon}{(1+\epsilon)}\frac{\sum_{j=-1}^{3}a_j\left(\frac{k_S}{k_L}\right)^j}{\sum_{m=-1}^{3}b_m\left(\frac{k_S}{k_L}\right)^m}\right\}^{\frac{1}{2}},$$
$$\tilde{c}_5=-\frac{1}{2}\left(1+\epsilon\right),\quad M_2=0,\quad M_3=0,\ M_4=\left(-\frac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}}=\left\{\frac{2H^2M_p^2\epsilon\ \tilde{c}_3}{\tilde{c}_6}\frac{\sum_{j=-1}^{3}a_j\left(\frac{k_S}{k_L}\right)^j}{\sum_{m=-1}^{3}b_m\left(\frac{k_S}{k_L}\right)^m}\right\}^{\frac{1}{4}}. \quad (5.38)$$

where the expansion coefficients $a_j \forall j = -1, \cdots, 3$ are defined earlier and here the coefficients $b_m \forall m = -1, \cdots, 3$ for arbitrary vacuum are defined as:

$$b_{-1} = -36U_2, \quad b_0 = \frac{9}{2}\left(U_1 + 9U_2\right), \quad b_1 = 0, \quad b_2 = \left(\frac{27}{4}U_1 + \frac{17}{2}U_2\right), \quad b_3 = 0\,, \tag{5.39}$$

where U_1 and U_2 are already defined earlier.

Now for Bunch Davies and α, β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$, we get $U_1 = 2$ and $U_2 = 0$. Consequently the expansion coefficients can be recast as:

$$a_{-1} = 0, \quad a_0 = 4(3\epsilon - \eta), \quad a_1 = 0, \quad a_2 = 10\epsilon, \quad a_3 = -(\epsilon + 2\eta)\,, \tag{5.40}$$

  and

$$b_{-1} = 0, \quad b_0 = 9, \quad b_1 = 0, \quad b_2 = \frac{27}{2}, \quad b_3 = 0\,, \tag{5.41}$$

  Finally the EFT coefficients for scalar fluctuation can be written as:

$$\boxed{\begin{aligned}
\bar{M}_1 &= \left\{\frac{2HM_p^2\epsilon\left[4(3\epsilon-\eta)+10\epsilon\left(\frac{k_S}{k_L}\right)^2-(\epsilon+2\eta)\left(\frac{k_S}{k_L}\right)^3\right]}{\left[18-\frac{27}{2}\left(\frac{k_S}{k_L}\right)^2\right]}\right\}^{\frac{1}{3}}, \\
\bar{M}_2 &\approx \bar{M}_3 = \sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}} = \left\{\frac{M_p^2\epsilon\left[4(\eta-3\epsilon)-10\epsilon\left(\frac{k_S}{k_L}\right)^2+(\epsilon+2\eta)\left(\frac{k_S}{k_L}\right)^3\right]}{(1+\epsilon)\left[18-\frac{27}{2}\left(\frac{k_S}{k_L}\right)^2\right]}\right\}^{\frac{1}{2}}, \\
\tilde{c}_5 &= -\frac{1}{2}\left(1+\epsilon\right), \quad M_2 = 0, \quad M_3 = 0, \\
M_4 &= \left(-\frac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}} = \left\{\frac{2H^2M_p^2\epsilon\;\tilde{c}_3}{\tilde{c}_6}\frac{\left[4(\eta-3\epsilon)-10\epsilon\left(\frac{k_S}{k_L}\right)^2+(\epsilon+2\eta)\left(\frac{k_S}{k_L}\right)^3\right]}{\left[18-\frac{27}{2}\left(\frac{k_S}{k_L}\right)^2\right]}\right\}^{\frac{1}{4}}.
\end{aligned}} \tag{5.42}$$

  Further using the contraint stated in Eq (5.32) we finally get the following constraints on the coefficients of EFT operators for a given value of the parameter k_S/k_L (say for $k_S/k_L = 0.1$):

$$\begin{aligned}
&1.22 \times 10^{-3}\ M_p < |\bar{M}_1| < 1.56 \times 10^{-3}\ M_p, \quad 8.67 \times 10^{-3}\ M_p < |\bar{M}_2| \approx |\bar{M}_3| < 1.25 \times 10^{-2}\ M_p, \\
&M_2 = 0, \quad M_3 = 0, \quad 3.75 \times 10^{-4}\ M_p < M_4 \times (-\tilde{c}_6/\tilde{c}_3)^{1/4} < 4.51 \times 10^{-4}\ M_p.
\end{aligned} \tag{5.43}$$

- **For α, β vacuum:**
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$, we get $U_1 = J_1(\alpha, \beta)$ and $U_2 = J_2(\alpha, \beta)$. Consequently the expansion coefficients can be recast as:

$$\begin{aligned}
a_{-1} &= 16\epsilon\sinh^2 2\alpha\cos^2\beta, \quad a_0 = 4(2\epsilon-\eta)\cosh^2 2\alpha + 4\epsilon + 4\epsilon\sinh^2 2\alpha\cos^2\beta, \\
a_1 &= 34\epsilon\sinh^2 2\alpha\cos^2\beta, \quad a_2 = 10\epsilon + 10\epsilon\sinh^2 2\alpha\cos^2\beta, \\
a_3 &= 2(2\epsilon-\eta)\cosh^2 2\alpha - 5\epsilon - \epsilon\sinh^2 2\alpha\cos^2\beta.
\end{aligned} \tag{5.44}$$

and

$$b_{-1} = -36J_2(\alpha,\beta),\ b_0 = \frac{9}{2}\left(J_1(\alpha,\beta)+9J_2(\alpha,\beta)\right),$$
$$b_2 = \left(\frac{27}{4}J_1(\alpha,\beta)+\frac{17}{2}J_2(\alpha,\beta)\right),\quad b_3 = 0 = b_1. \tag{5.45}$$

Finally the EFT coefficients for scalar fluctuation can be written as:

$$\begin{aligned}
\bar{M}_1 =& \left\{2HM_p^2\epsilon\left[-36J_2(\alpha,\beta)\left(\frac{k_S}{k_L}\right)^{-1}+9\left(J_1(\alpha,\beta)+\frac{J_2(\alpha,\beta)}{2}\right)\right.\right.\\
&\left.-\frac{3}{4}\left(9J_1(\alpha,\beta)-7J_2(\alpha,\beta)\right)\left(\frac{k_S}{k_L}\right)^2\right]^{-1}\\
&\left[16\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)^{-1}+\left(4(2\epsilon-\eta)\cosh^2 2\alpha+4\epsilon+4\epsilon\sinh^2 2\alpha\cos^2\beta\right)\right.\\
&+34\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)+\left(10\epsilon+10\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^2\\
&\left.\left.+\left(2(2\epsilon-\eta)\cosh^2 2\alpha-5\epsilon-\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^3\right]\right\}^{\frac{1}{3}},\\
\bar{M}_2 \approx \bar{M}_3 =& \sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}} = \left\{\frac{M_p^2\epsilon}{(1+\epsilon)}\left[36J_2(\alpha,\beta)\left(\frac{k_S}{k_L}\right)^{-1}-9\left(J_1(\alpha,\beta)+\frac{J_2(\alpha,\beta)}{2}\right)\right.\right.\\
&\left.+\frac{3}{4}\left(9J_1(\alpha,\beta)-7J_2(\alpha,\beta)\right)\left(\frac{k_S}{k_L}\right)^2\right]^{-1}\\
&\left[16\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)^{-1}+\left(4(2\epsilon-\eta)\cosh^2 2\alpha+4\epsilon+4\epsilon\sinh^2 2\alpha\cos^2\beta\right)\right.\\
&+34\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)+\left(10\epsilon+10\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^2\\
&\left.\left.+\left(2(2\epsilon-\eta)\cosh^2 2\alpha-5\epsilon-\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^3\right]\right\}^{\frac{1}{2}},\\
\tilde{c}_5 =& -\frac{1}{2}(1+\epsilon),\quad M_2=0,\quad M_3=0,\\
M_4 =& \left(-\frac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}} = \left\{\frac{2H^2M_p^2\epsilon\ \tilde{c}_3}{\tilde{c}_6}\left[36J_2(\alpha,\beta)\left(\frac{k_S}{k_L}\right)^{-1}-9\left(J_1(\alpha,\beta)+\frac{J_2(\alpha,\beta)}{2}\right)\right.\right.\\
&\left.+\frac{3}{4}\left(9J_1(\alpha,\beta)-7J_2(\alpha,\beta)\right)\left(\frac{k_S}{k_L}\right)^2\right]^{-1}\\
&\left[16\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)^{-1}+\left(4(2\epsilon-\eta)\cosh^2 2\alpha+4\epsilon+4\epsilon\sinh^2 2\alpha\cos^2\beta\right)\right.\\
&+34\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)+\left(10\epsilon+10\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^2\\
&\left.\left.+\left(2(2\epsilon-\eta)\cosh^2 2\alpha-5\epsilon-\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^3\right]\right\}^{\frac{1}{4}}.
\end{aligned} \tag{5.46}$$

Further using the contraint stated in Eq (5.32) we finally get the following constraints on the coefficients of EFT operators for a given value of the parameters α, β and k_S/k_L (say for $\alpha = 0.1$, $\beta = 0.1$ and $k_S/k_L = 0.1$):

$$\begin{aligned}
&6.05\times10^{-4}\ M_p < |\bar{M}_1| < 7.15\times10^{-4}\ M_p,\quad 3.03\times10^{-3}\ M_p < |\bar{M}_2| \approx |\bar{M}_3| < 3.89\times10^{-3}\ M_p,\\
&M_2 = 0,\quad M_3 = 0,\quad 2.22\times10^{-3}\ M_p < M_4\times(-\tilde{c}_6/\tilde{c}_3)^{1/4} < 2.51\times10^{-3}\ M_p.
\end{aligned} \tag{5.47}$$

5.2 For General Single Field $P(X, \phi)$ inflation

Here our prime objective is to derive the EFT coefficients by computing the most general expression for the three point function for scalar fluctuations from the General Single Field $P(X, \phi)$ model of inflation for arbitrary vacuum. Then we give specific example for Bunch-Davies and α, β vacuum for completeness.

5.2.1 Basic setup

Let us start with the action for single scalar field (inflaton) which is described by the general function $P(X, \phi)$, contains non canonical kinetic term in general and it is minimally coupled to the gravity [32, 43]:

$$\boxed{S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + P(X,\phi)\right]}\,. \tag{5.48}$$

In case of general structure of $P(X, \phi)$ the pressure p, the energy density ρ and effective speed of sound parameter c_S can be written as [43]:

$$p = P(X,\phi), \quad \rho = 2XP_{,X}(X,\phi) - P(X,\phi), \quad c_S = \sqrt{\frac{P_{,X}(X,\phi)}{P_{,X}(X,\phi) + 2XP_{,XX}(X,\phi)}}. \tag{5.49}$$

In case of general $P(X, \phi)$ theory the slow roll parameters can be expressed as [43]:

$$\begin{aligned}
\epsilon &= \frac{XP_X(X,\phi)}{H^2M_p^2}, \quad \eta = \epsilon - \frac{\delta}{P_{,X}(X,\phi)}\left[P_{,X}(X,\phi) + XP_{,XX}(X,\phi)\right], \\
s &= \frac{2X\delta}{P_{,X}(X,\phi)}\frac{\left[XP^2_{,XX}(X,\phi) - P_{,X}(X,\phi)P_{,XX}(X,\phi) - XP_{,X}(X,\phi)P_{,XXX}(X,\phi)\right]}{\left[P_{,X}(X,\phi) + XP_{,XX}(X,\phi)\right]}.
\end{aligned} \tag{5.50}$$

In case of **Single Field Slow Roll inflation** we have:

$$P(X,\phi) = X - V(\phi), \tag{5.51}$$

where $V(\phi)$ is the single field slowly varying potential. For this case if we compute the effective sound speed then it turns out to be $c_S = 1$, which is consistent with our result obtained in the previous section. Also if we compute the expressions for the slow roll parameters ϵ, η, δ and s the results are also perfectly matches with the results obtained in Eq (5.14).

Similarly in case **DBI inflationary model** one can identify the function $P(X, \phi)$ as [44]:

$$P(X,\phi) = -\frac{1}{f(\phi)}\sqrt{1 - 2Xf(\phi)} + \frac{1}{f(\phi)} - V(\phi), \tag{5.52}$$

where the inflaton ϕ is identified to be the position of a D3 barne which is moving in warped throat geometry and $f(\phi)$ characterize the warp factor [21]. For the effective potential $V(\phi)$ one can consider following mathematical structures of the potentials in the UV and IR regime [44]:

- **UV regime**: In this case the inflaton moves from the UV regime of the warped geometric space to the IR regime under the influence of the effective potential, $V(\phi) \simeq \frac{1}{2}m^2\phi^2$, where the inflaton mass satisfy the constraint $m >> M_p\sqrt{\lambda}$. In this specific situation the inflaton starts rolling very far away from the origin of the effective potential and then rolls down in a relativistic fashion to the minimum of potential situated at the origin.

[21] For AdS like throat geometry, $f(\phi) \approx \frac{\lambda}{\phi^4}$, where λ is the parameter in string theory which depends on the flux number.

- **IR regime**: In this case the inflaton started moving from the IR regime of the warped space geometry to the UV regime under the influence of the effective potential, $V(\phi) \simeq V_0 - \frac{1}{2}m^2\phi^2$, where the inflaton mass is comparable to the scale of inflation, as given by, $m \approx H$. In this specific situation, the inflaton starts rolling down near the origin of the effective potential and rolls down in a relativistic fashion away from it.

In case of DBI model the pressure p and the energy density ρ can be written as [44]:

$$p = \frac{1}{f(\phi)}(1 - c_S) - V(\phi), \quad \rho = \frac{1}{f(\phi)}\left(\frac{1}{c_S} - 1\right) + V(\phi), \quad c_S = \sqrt{1 - 2Xf(\phi)} = \sqrt{1 - \dot{\phi}^2 f(\phi)}, \quad (5.53)$$

where $X = \dot{\phi}^2/2$. In this context the slow roll parameter [44]:

$$\epsilon = \frac{3\dot{\phi}^2}{2\left[c_S V(\phi) + \frac{1}{f(\phi)}(1 - c_S)\right]} \approx \frac{3}{2\left[1 + c_S f(\phi)V(\phi)\right]}. \quad (5.54)$$

is not small and as a result the effective sound speed is very small, $c_S << 1$. Consequently the inflaon speed during inflation is given by the expression, $\dot{\phi} = \pm\frac{1}{\sqrt{f(\phi)}}$. Additionally it is important to note that in the context of DBI inflation the other slow roll parameters η and s can be computed as:

$$\eta \approx \frac{\left[3\sqrt{1 + c_S f(\phi)V(\phi)} + \frac{\sqrt{3f(\phi)c_S}}{2} M_p\dot{\phi}c_S\left\{f(\phi)V'(\phi) + V(\phi)f'(\phi) - \frac{1}{2c_S^2}\left(2\ddot{\phi}f(\phi) + \dot{\phi}^2 f'(\phi)\right)\right\}\right]}{\left[1 + c_S f(\phi)V(\phi)\right]^{\frac{3}{2}}},$$

$$s = -\frac{\sqrt{3f(\phi)c_S}}{2c_S^2} M_p\dot{\phi}\left[2\ddot{\phi}f(\phi) + \dot{\phi}^2 f'(\phi)\right]. \quad (5.55)$$

In the slow roll regime to validate slow-roll approximation along with $c_S << 1$ we need to satisfy the constraint condition for DBI inflation, $2c_S f(\phi)V(\phi) >> 1$.

5.2.2 Scalar three point function for General Single Field $P(X, \phi)$ inflation

Before computing the three point function for scalar mode fluctuation here it is important to note that the two point function for General Single Field $P(X, \phi)$ inflation is exactly same with the results obtained for EFT of inflation with sound speed $c_S << 1$ and $\tilde{c}_S << 1$, which can be obtained by setting the EFT coefficients, $M_2 \neq 0$, $M_3 \neq 0$, $\bar{M}_1 \neq 0$, $M_4 \neq 0$, $\bar{M}_2 \neq 0$, $\bar{M}_3 \neq 0$, $\tilde{c}_5 \neq -\frac{1}{2}(1 + \epsilon)$ [22]. Using three point function we can able to fix all of these coefficients.

Now here before going to the details of the computation for three point function just using the knowledge of two two point function we can easily identify the exact analytical expression for the EFT coeficient M_2. For this we need to identify the effective sound speed computed from General Single Field $P(X, \phi)$ inflation with the result obtained for the proposed EFT setup. Consequently

[22] In case of General Single Field $P(X, \phi)$ inflation amplitude of power spectrum and spectral tilt for scalar fluctuation can be written at the horizon crossing $|k\tilde{c}_S\eta| = 1$ as:

$$\text{For Bunch – Davies vacuum :} \quad \Delta_\zeta(k_*) = \begin{cases} \dfrac{2X_* P_{,X}(X_*, \phi_*) - P(X_*, \phi_*)}{24\pi^2 M_p^4 \tilde{c}_S \epsilon} & \textbf{for dS} \\ 2^{3\epsilon - \eta + \frac{s}{2}} \dfrac{2X_* P_{,X}(X_*, \phi_*) - P(X_*, \phi_*)}{24\pi^2 M_p^4 \epsilon (1+\epsilon)^2} \left|\dfrac{\Gamma(\frac{3}{2} + 3\epsilon - \eta + \frac{s}{2})}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases}$$

we get:

$$M_2=\left(-\frac{XP_{,XX}(X,\phi)}{P_{,X}(X,\phi)}\dot{H}M_p^2\right)^{\frac{1}{4}}=\begin{cases}0 & \textbf{for Single Field Slow Roll}\\ \left[\left(\frac{\dot{\phi}^2 f(\phi)}{\dot{\phi}^2 f(\phi)-1}\right)\frac{\dot{H}M_p^2}{2}\right]^{\frac{1}{4}} & \textbf{for DBI}.\end{cases} \tag{5.57}$$

We here now proceed to calculate the three point function for the scalar fluctuation $\zeta(t,\mathbf{x})$ in the interacting picture with arbitrary vacuum in case of General Single Field $P(X,\phi)$ inflation. Then we cite results for Bunch-Davies and α,β vacuum and give a specific example for DBI model of inflation.

Here we introduce two new parameters [43]:

$$\Sigma_1(X,\phi) = XP_{,X}(X,\phi)+2X^2P_{,XX}(X,\phi)=\frac{\epsilon H^2M_p^2}{c_S^2}, \tag{5.58}$$

$$\Sigma_2(X,\phi) = X^2P_{,XX}(X,\phi)+\frac{2}{3}X^3P_{,XXX}(X,\phi). \tag{5.59}$$

which will appear in the expression for three point function for the scalar fluctuation. For **Single Field Slow Roll inflation** and **DBI inflation** we get the following expressions for these parameters [43]:

$$\Sigma_1(X,\phi)=\begin{cases}X=\epsilon H^2M_p^2 & \textbf{for Single Field Slow Roll}\\ \frac{X}{(1-2Xf(\phi))^{\frac{3}{2}}}=\frac{\epsilon H^2M_p^2}{c_S^2} & \textbf{for DBI}.\end{cases} \tag{5.60}$$

$$\Sigma_2(X,\phi)=\begin{cases}0 & \textbf{for Single Field Slow Roll}\\ \frac{X^2f(\phi)}{(1-2Xf(\phi))^{\frac{5}{2}}} & \textbf{for DBI}.\end{cases} \tag{5.61}$$

For General Single Field $P(X,\phi)$ inflation the third order term in the action Eq. (5.48) is given by [43]:

$$\boxed{\begin{aligned}S^{(3)}_{\zeta} = \int d^4x\,\Bigg[&-a^3\left\{\Sigma_1(X,\phi)\left(1-\frac{1}{c_S^2}\right)+2\Sigma_2(X,\phi)\right\}\frac{\dot{\tilde{\zeta}}^3}{H^3}+\frac{a^3\epsilon\left(\epsilon-3+3c_S^2\right)}{c_S^4}\tilde{\zeta}\dot{\tilde{\zeta}}^2\\ &+\frac{a\epsilon\left(\epsilon-2s+1-c_S^2\right)}{c_S^2}\tilde{\zeta}(\partial\tilde{\zeta})^2-2a^3\epsilon\dot{\tilde{\zeta}}\partial_i\tilde{\zeta}\partial_i\left(\frac{\epsilon}{c_S^2}\partial^{-2}\dot{\tilde{\zeta}}\right)\Bigg],\end{aligned}} \tag{5.62}$$

$$\textbf{For } \alpha,\beta \textbf{ vacuum :}\quad \Delta_\zeta(k_*)=\begin{cases}\frac{2X_*P_{,X}(X_*,\phi_*)-P(X_*,\phi_*)}{24\pi^2\;M_p^4\tilde{c}_S\epsilon}\left[\cosh 2\alpha-\sinh 2\alpha\cos\beta\right] & \textbf{for dS}\\ 2^{6\epsilon-2\eta+s}\frac{2X_*P_{,X}(X_*,\phi_*)-P(X_*,\phi_*)}{24\pi^2\;M_p^4\epsilon(1+\epsilon)^2}\left|\frac{\Gamma\left(\frac{3}{2}+3\epsilon-\eta+\frac{s}{2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \\ \left[\cosh 2\alpha-\sinh 2\alpha\cos\left(\pi\left(2+3\epsilon-\eta+\frac{s}{2}\right)+\beta\right)\right] & \textbf{for qdS}.\end{cases}$$

and

$$n_\zeta(k_*)-1=2\eta-6\epsilon-s. \tag{5.56}$$

which is derived from Eq (5.5) and here after neglecting all the contribution from the terms which are sub-leading in the slow-roll parameters. Additionally here we use the following field redefinition:

$$\zeta = \tilde{\zeta} + \frac{1}{c_S^2}\left\{\epsilon - \frac{\eta}{2}\right\}\tilde{\zeta}^2, \tag{5.63}$$

where ϵ, η, δ and s are already defined earlier for General Single Field $P(X,\phi)$ inflation.

Now it is important to note that, in the present context of discussion we are interested in the three point function for the scalar fluctuation field ζ, not for the redefined scalar field fluctuation $\tilde{\zeta}$ and for this reason one can write down the exact connection between the three point fulction for the scalar fluction field ζ and redefined scalar fluctuation field $\tilde{\zeta}$ in position space as:

$$\begin{aligned}\langle\zeta(\mathbf{x_1})\zeta(\mathbf{x_2})\zeta(\mathbf{x_3})\rangle &= \langle\tilde{\zeta}(\mathbf{x_1})\tilde{\zeta}(\mathbf{x_2})\tilde{\zeta}(\mathbf{x_3}) + \frac{(2\epsilon-\eta)}{c_S^2}\left[\langle\zeta(\mathbf{x_1})\zeta(\mathbf{x_2})\rangle\langle\zeta(\mathbf{x_1})\zeta(\mathbf{x_3})\rangle\right.\\ &\left.+\langle\zeta(\mathbf{x_2})\zeta(\mathbf{x_1})\rangle\langle\zeta(\mathbf{x_2})\zeta(\mathbf{x_3})\rangle + \langle\zeta(\mathbf{x_3})\zeta(\mathbf{x_1})\rangle\langle\zeta(\mathbf{x_3})\zeta(\mathbf{x_2})\rangle\right].\end{aligned} \tag{5.64}$$

After taking the Fourier transform of the scalar fluctuation field ζ and redefined scalar fluctuation field $\tilde{\zeta}$ one can express connection between three point function in momentum space and this is our main point of interest also.

The interaction Hamiltonian for the redefined scalar fluctuation $\tilde{\zeta}$ can be expressed as:

$$\begin{aligned}H_{int} = \int d^3x \Bigg[&-\left\{\Sigma_1(X,\phi)\left(1-\frac{1}{c_S^2}\right)+2\Sigma_2(X,\phi)\right\}\frac{\tilde{\zeta}'^3}{H^3} + \frac{a\,\epsilon\left(\epsilon-3+3c_S^2\right)}{c_S^4}\tilde{\zeta}\tilde{\zeta}'^2\\ &+\frac{a\,\epsilon\left(\epsilon-2s+1-c_S^2\right)}{c_S^2}\tilde{\zeta}(\partial\tilde{\zeta})^2 - 2a\tilde{\zeta}'\partial_i\tilde{\zeta}\partial_i\left(\frac{\epsilon}{c_S^2}\partial^{-2}\tilde{\zeta}'\right)\Bigg].\end{aligned} \tag{5.65}$$

Further following the in-in formalism in interaction picture the expression for the three point function for the redefined scalar fluctuation $\tilde{\zeta}$ and then transforming the final result in terms of the scalar fluctuation ζ in momentum one can write the following expression:

$$\boxed{\begin{aligned}\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle &= -i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\; a(\eta)\;\langle 0|\left[\zeta(\eta_f,\mathbf{k_1})\zeta(\eta_f,\mathbf{k_2})\zeta(\eta_f,\mathbf{k_3}),H_{int}(\eta)\right]|0\rangle\\ &= (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})B_{GSF}(k_1,k_2,k_3),\end{aligned}} \tag{5.66}$$

where $B_{GSF}(k_1,k_2,k_3)$ represents the bispectrum of scalar fluctuation ζ, which is computed from General Single Field $P(X,\phi)$ inflation. Here the final expression for the bispectrum of scalar fluctuation for arbitray vacuum is given by:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[\frac{3}{2}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\left(|C_1|^2+|C_2|^2\right)^2\frac{(k_1k_2k_3)^2}{K^3}\right.\\ &+\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2\left(\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &+\frac{s}{c_S^2}\left(|C_1|^2+|C_2|^2\right)^2\left(-2\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &+2(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2\sum_{i=1}^{3}k_i^3\\ &+\epsilon\left(|C_1|^2-|C_2|^2\right)^2\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &+\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\left(-\textstyle\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2\right.\\ &\left.\left.\qquad+8\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\sum_{m=1}^{3}\frac{1}{K-2k_m}\right)+\mathcal{O}(\epsilon)\right], \end{aligned} \tag{5.67}$$

where $\mathcal{O}(\epsilon)$ characterizes the sub-leading corrections in the three point function for the scalar fluctuation computed from General Single Field $P(X,\phi)$ inflation.

Now further we consider a very specific class of models, where the following constraint condition [23] $P_{,X\phi}(X,\phi)=0$ perfectly holds good. In this case one can write down the following simplified expression for the bispectrum of scalar fluctuation for arbitray vacuum as:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\left(|C_1|^2+|C_2|^2\right)^2\frac{(k_1k_2k_3)^2}{K^3}\right.\\ &+\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2\left(\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &+2(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2\sum_{i=1}^{3}k_i^3\\ &+\epsilon\left(|C_1|^2-|C_2|^2\right)^2\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &\left.+\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+8\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\sum_{m=1}^{3}\frac{1}{K-2k_m}\right)\right], \end{aligned} \tag{5.68}$$

[23] Strictly speaking, DBI model is one of the exceptions where this condition is not applicable. On the other hand, in case of Single Field Slow Roll inflation this condition is applicable. But in that case one can set $c_S=1$ and get back all the results derived in the earlier section. Additionally it is important to mention that, here we consider those models also where $c_S << 1$ alongwith this given constraint.

where the new parameter ϵ_X is defined as [24]:

$$\epsilon_X = -\frac{\dot{X}H_{,X}}{H^2}. \tag{5.70}$$

For Bunch Davies and α,β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch Davies vacuum**:
  After setting $C_1 = 1$ and $C_2 = 0$ we get [43]:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) =& \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(_1k_2k_3)^3}\left[\frac{3}{2}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\frac{(k_1k_2k_3)^2}{K^3}\right.\\ &+\left(\frac{1}{c_S^2}-1\right)\left(\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &+\frac{s}{c_S^2}\left(-2\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &+2(2\epsilon-\eta)\sum_{i=1}^{3}k_i^3\\ &\left.+\epsilon\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\right]. \end{aligned} \tag{5.71}$$

  Further for restricted class of General Single Field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi) = 0$, one can further write down the following expression for the bispectrum:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) =& \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\frac{(k_1k_2k_3)^2}{K^3}\right.\\ &+\left(\frac{1}{c_S^2}-1\right)\left(\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &+2(2\epsilon-\eta)\sum_{i=1}^{3}k_i^3\\ &\left.+\epsilon\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\right], \end{aligned} \tag{5.72}$$

- **For α,β vacuum**:

[24] For Single Field Slow Roll inflation the newly introduced parameter ϵ_X is computed as:

$$\epsilon_X = \epsilon(\eta-\epsilon) \approx \epsilon_V(\eta_V - 2\epsilon_V). \tag{5.69}$$

After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get [32]:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) =& \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[\frac{3}{2}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\cosh^2 2\alpha\frac{(k_1k_2k_3)^2}{K^3}\right.\\ &+\left(\frac{1}{c_S^2}-1\right)\cosh^2 2\alpha\left(\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &+\frac{s}{c_S^2}\cosh^2 2\alpha\left(-2\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &+2(2\epsilon-\eta)\cosh^2 2\alpha\sum_{i=1}^{3}k_i^3+\epsilon\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &\left.+\epsilon\sinh^2 2\alpha\cos^2\beta\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+8\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\sum_{m=1}^{3}\frac{1}{K-2k_m}\right)\right]. \end{aligned} \tag{5.73}$$

Further for restricted class of General Single Field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, one can further write down the following expression for the bispectrum:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) =& \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\cosh^2 2\alpha\frac{(k_1k_2k_3)^2}{K^3}\right.\\ &+\left(\frac{1}{c_S^2}-1\right)\cosh^2 2\alpha\left(\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &+2(2\epsilon-\eta)\cosh^2 2\alpha\sum_{i=1}^{3}k_i^3+\epsilon\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &\left.+\epsilon\sinh^2 2\alpha\cos^2\beta\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+8\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\sum_{m=1}^{3}\frac{1}{K-2k_m}\right)\right], \end{aligned} \tag{5.74}$$

Further we consider equilateral limit and squeezed limit in which we finally get:

1. Equilateral limit configuration:
   Here the bispectrum for scalar perturbations in presence of arbitray quantum vacuum can be expressed as:

$$\boxed{\begin{aligned} B_{GSF}(k,k,k) =& \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\left(|C_1|^2+|C_2|^2\right)^2\right.\\ &-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2-\frac{34}{3}\frac{s}{c_S^2}\left(|C_1|^2+|C_2|^2\right)^2\\ &\left.+6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2+27\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\right]. \end{aligned}} \tag{5.75}$$

   Further for restricted class of General Single Field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, one can further write down the following expression for the bispectrum:

$$B_{GSF}(k_1,k_2,k_3) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\left(|C_1|^2+|C_2|^2\right)^2-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2 \right.$$
$$\left. +6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2+27\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\right], \quad (5.76)$$

Now for Bunch Davies and α,β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch Davies vacuum**:
  After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$B_{GSF}(k,k,k) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)-\frac{34}{3}\frac{s}{c_S^2}+23\epsilon-6\eta\right]. \quad (5.77)$$

  Further for restricted class of General Single Field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, we get:

$$B_{GSF}(k_1,k_2,k_3) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)+23\epsilon-6\eta\right], \quad (5.78)$$

- **For α,β vacuum**:
  After setting $C_1=\cosh\alpha$ and $C_2=e^{i\beta}\sinh\alpha$ we get:

$$\begin{aligned} B_{GSF}(k,k,k) = & \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\cosh^2 2\alpha\right. \\ & -\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\cosh^2 2\alpha-\frac{34}{3}\frac{s}{c_S^2}\cosh^2 2\alpha \\ & \left.+6(2\epsilon-\eta)\cosh^2 2\alpha+11\epsilon+27\epsilon\sinh^2 2\alpha\cos^2\beta\right]. \end{aligned} \quad (5.79)$$

  Further for restricted class of General Single Field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, we get:

$$\begin{aligned} B_{GSF}(k,k,k) = & \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\cosh^2 2\alpha-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\cosh^2 2\alpha\right. \\ & \left.+6(2\epsilon-\eta)\cosh^2 2\alpha+11\epsilon+27\epsilon\sinh^2 2\alpha\cos^2\beta\right]. \end{aligned} \quad (5.80)$$

2. Squeezed limit configuration:
   Here the bispectrum for scalar perturbations in presence of arbitray quantum vacuum can be expressed as:

$$B_{GSF}(k_L,k_L,k_S) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k_L^3 k_S^3}\sum_{j=-1}^{3} t_j\left(\frac{k_S}{k_L}\right)^j, \quad (5.81)$$

where the expansion coefficients $t_j \forall j = -1, \cdots, 3$ for arbitrary vacuum are defined as:

$$\begin{aligned}
t_{-1} &= 16\epsilon \left(C_1^* C_2 + C_1 C_2^*\right)^2, \\
t_0 &= \left(4(2\epsilon - \eta) - \frac{6s}{c_S^2}\right)\left(|C_1|^2 + |C_2|^2\right)^2 + 4\epsilon\left(|C_1|^2 - |C_2|^2\right)^2 + 4\epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2, \\
t_1 &= 34\epsilon \left(C_1^* C_2 + C_1 C_2^*\right)^2, \\
t_2 &= \left\{\frac{3}{16}\left(\frac{1}{c_S^2} - 1 - \frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right) - 6\left(\frac{1}{c_S^2} - 1\right) - \frac{6s}{c_S^2}\right\}\left(|C_1|^2 + |C_2|^2\right)^2 \\
&\qquad + 10\epsilon\left(|C_1|^2 - |C_2|^2\right)^2 + 10\epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2, \\
t_3 &= \left\{2(2\epsilon - \eta) - \frac{9}{32}\left(\frac{1}{c_S^2} - 1 - \frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right) + 5\left(\frac{1}{c_S^2} - 1\right) + \frac{2s}{c_S^2}\right\}\left(|C_1|^2 + |C_2|^2\right)^2 \\
&\qquad - 5\epsilon\left(|C_1|^2 - |C_2|^2\right)^2 - \epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2.
\end{aligned} \tag{5.82}$$

Further for restricted class of General Single Field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi) = 0$, we get:

$$\begin{aligned}
t_{-1} &= 16\epsilon \left(C_1^* C_2 + C_1 C_2^*\right)^2, \\
t_0 &= 4(2\epsilon - \eta)\left(|C_1|^2 + |C_2|^2\right)^2 + 4\epsilon\left(|C_1|^2 - |C_2|^2\right)^2 + 4\epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2, \\
t_1 &= 34\epsilon \left(C_1^* C_2 + C_1 C_2^*\right)^2, \\
t_2 &= \left\{\frac{1}{8}\left(\frac{1}{c_S^2} - 1 - \frac{s\epsilon}{3\epsilon_X}\right) - 6\left(\frac{1}{c_S^2} - 1\right)\right\}\left(|C_1|^2 + |C_2|^2\right)^2 \\
&\qquad + 10\epsilon\left(|C_1|^2 - |C_2|^2\right)^2 + 10\epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2, \\
t_3 &= \left\{2(2\epsilon - \eta) - \frac{3}{16}\left(\frac{1}{c_S^2} - 1 - \frac{s\epsilon}{3\epsilon_X}\right) + 5\left(\frac{1}{c_S^2} - 1\right)\right\}\left(|C_1|^2 + |C_2|^2\right)^2 \\
&\qquad - 5\epsilon\left(|C_1|^2 - |C_2|^2\right)^2 - \epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2.
\end{aligned} \tag{5.83}$$

Now for Bunch Davies and α, β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$, we get the following expression for the expansion coefficients $t_j \forall j = -1, \cdots, 3$:

$$\begin{aligned}
t_{-1} &= 0, \\
t_0 &= 4(3\epsilon - \eta) - \frac{6s}{c_S^2}, \\
t_1 &= 0, \\
t_2 &= 10\epsilon + \left\{\frac{3}{16}\left(\frac{1}{c_S^2} - 1 - \frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right) - 6\left(\frac{1}{c_S^2} - 1\right) - \frac{6s}{c_S^2}\right\}, \\
t_3 &= -(\epsilon + 2\eta) + \left\{5\left(\frac{1}{c_S^2} - 1\right) + \frac{2s}{c_S^2} - \frac{9}{32}\left(\frac{1}{c_S^2} - 1 - \frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\right\}.
\end{aligned} \tag{5.84}$$

Consequently the bispectrum can be recast as:

$$\begin{aligned}B_{GSF}(k_L,k_L,k_S) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k_L^3 k_S^3}\left[\left\{4(3\epsilon-\eta)-\frac{6s}{c_S^2}\right\}\right.\\&+\left(10\epsilon+\left\{\frac{3}{16}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-6\left(\frac{1}{c_S^2}-1\right)-\frac{6s}{c_S^2}\right\}\right)\left(\frac{k_S}{k_L}\right)^2\\&+\left(-(\epsilon+2\eta)+\left\{5\left(\frac{1}{c_S^2}-1\right)+\frac{2s}{c_S^2}\right.\right.\\&\left.\left.\left.-\frac{9}{32}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\right\}\right)\left(\frac{k_S}{k_L}\right)^3\right].\end{aligned} \tag{5.85}$$

Further for restricted class of General Single Field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, we get:

$$\begin{aligned}t_{-1} &= 0,\\ t_0 &= 4(3\epsilon-\eta),\\ t_1 &= 0,\\ t_2 &= \left\{\frac{1}{8}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-6\left(\frac{1}{c_S^2}-1\right)\right\}+10\epsilon,\\ t_3 &= \left\{5\left(\frac{1}{c_S^2}-1\right)-\frac{3}{16}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\right\}-(\epsilon+2\eta)\end{aligned} \tag{5.86}$$

for such case bispectrum is given by:

$$\begin{aligned}B_{GSF}(k_L,k_L,k_S) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k_L^3 k_S^3}\left[4(3\epsilon-\eta)\right.\\&+\left(\left\{\frac{1}{8}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-6\left(\frac{1}{c_S^2}-1\right)\right\}+10\epsilon\right)\left(\frac{k_S}{k_L}\right)^2\\&\left.+\left(-(\epsilon+2\eta)+\left\{5\left(\frac{1}{c_S^2}-1\right)-\frac{3}{16}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\right\}\right)\left(\frac{k_S}{k_L}\right)^3\right].\end{aligned} \tag{5.87}$$

- **For α,β vacuum**:
  After setting $C_1=\cosh\alpha$ and $C_2=e^{i\beta}\sinh\alpha$, we get the following expression for the expansion coefficients $a_j \forall j=-1,\cdots,3$:

$$\begin{aligned}t_{-1} &= 16\epsilon\sinh^2 2\alpha\cos^2\beta,\\ t_0 &= \left(4(2\epsilon-\eta)-\frac{6s}{c_S^2}\right)\cosh^2 2\alpha+4\epsilon+4\epsilon\sinh^2 2\alpha\cos^2\beta,\\ t_1 &= 34\epsilon\sinh^2 2\alpha\cos^2\beta,\\ t_2 &= \left\{\frac{3}{16}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-6\left(\frac{1}{c_S^2}-1\right)-\frac{6s}{c_S^2}\right\}\cosh^2 2\alpha\\&\quad+10\epsilon+10\epsilon\sinh^2 2\alpha\cos^2\beta,\\ t_3 &= \left\{2(2\epsilon-\eta)-\frac{9}{32}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)+5\left(\frac{1}{c_S^2}-1\right)+\frac{2s}{c_S^2}\right\}\cosh^2 2\alpha\\&\quad-5\epsilon-\epsilon\sinh^2 2\alpha\cos^2\beta.\end{aligned} \tag{5.88}$$

Consequently the bispectrum can be recast as:

$$
\begin{aligned}
B_{GSF}(k_L,k_L,k_S) =& \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k_L^3 k_S^3}\left[16\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)^{-1}\right.\\
&+\left(\left(4(2\epsilon-\eta)-\frac{6s}{c_S^2}\right)\cosh^2 2\alpha+4\epsilon+4\epsilon\sinh^2 2\alpha\cos^2\beta\right)\\
&\quad+34\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)\\
&+\left(\left\{\frac{3}{16}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-6\left(\frac{1}{c_S^2}-1\right)-\frac{6s}{c_S^2}\right\}\cosh^2 2\alpha\right.\\
&\quad\left.+10\epsilon+10\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^2\\
&+\left(\left\{2(2\epsilon-\eta)-\frac{9}{32}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)+5\left(\frac{1}{c_S^2}-1\right)+\frac{2s}{c_S^2}\right\}\cosh^2 2\alpha\right.\\
&\quad\left.\left.-5\epsilon-\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^3\right].
\end{aligned}
\tag{5.89}
$$

Further for restricted class of General Single Field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, we get:

$$
\begin{aligned}
t_{-1} &= 16\epsilon\sinh^2 2\alpha\cos^2\beta,\\
t_0 &= 4(2\epsilon-\eta)\cosh^2 2\alpha+4\epsilon+4\epsilon\sinh^2 2\alpha\cos^2\beta,\\
t_1 &= 34\epsilon\sinh^2 2\alpha\cos^2\beta,\\
t_2 &= \left\{\frac{1}{8}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-6\left(\frac{1}{c_S^2}-1\right)\right\}\cosh^2 2\alpha\\
&\qquad+10\epsilon+10\epsilon\sinh^2 2\alpha\cos^2\beta,\\
t_3 &= \left\{2(2\epsilon-\eta)-\frac{3}{16}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)+5\left(\frac{1}{c_S^2}-1\right)\right\}\cosh^2 2\alpha\\
&\qquad-5\epsilon-\epsilon\sinh^2 2\alpha\cos^2\beta.
\end{aligned}
\tag{5.90}
$$

Consequently the bispectrum can be recast as:

$$
\begin{aligned}
B_{GSF}(k_L,k_L,k_S) =& \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k_L^3 k_S^3}\left[16\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)^{-1}\right.\\
&+\left(4(2\epsilon-\eta)\cosh^2 2\alpha+4\epsilon+4\epsilon\sinh^2 2\alpha\cos^2\beta\right)\\
&\quad+34\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)\\
&+\left(\left\{\frac{1}{8}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-6\left(\frac{1}{c_S^2}-1\right)\right\}\cosh^2 2\alpha\right.\\
&\quad\left.+10\epsilon+10\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^2\\
&+\left(\left\{2(2\epsilon-\eta)-\frac{3}{16}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)+5\left(\frac{1}{c_S^2}-1\right)\right\}\cosh^2 2\alpha\right.\\
&\quad\left.\left.-5\epsilon-\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^3\right].
\end{aligned}
\tag{5.91}
$$

5.2.3 Expression for EFT coefficients for General Single Field $P(X,\phi)$ inflation

Here our prime objective is to derive the analytical expressions for EFT coefficients for General Single Field $P(X,\phi)$ inflation. To serve this purpose here we start with a claim that the three point function and the associated bispectrum for the scalar fluctuations computed from General Single Field $P(X,\phi)$ inflation is exactly same as that we have computed from EFT setup. Here we use the equilateral limit and squeezed limit configurations to extract the analytical expression for the EFT coefficients. In the two limiting cases the results are following:

1. Equilateral limit configuration:
   For this case with arbitrary vacuum one can write:

$$B_{EFT}(k,k,k) = B_{GSF}(k,k,k), \tag{5.92}$$

which implies that [25]:

$$\begin{aligned}
\bar{M}_1 &= \left\{\frac{\hat{A}}{2\hat{B}H}\left[-1+\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right\}^{\frac{1}{3}}, \quad \bar{M}_2 \approx \bar{M}_3 = \sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}} = \sqrt{\frac{\hat{A}}{8\hat{B}H^2\tilde{c}_5}\left[-1+\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]},\\
M_2 &= \left(-\frac{XP_{,XX}(X,\phi)}{P_{,X}(X,\phi)}\dot{H}M_p^2\right)^{\frac{1}{4}},\\
M_3 &= \left\{-\frac{\hat{A}}{2\hat{B}}\frac{\tilde{c}_3}{\tilde{c}_4}\left[-1+\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right\}^{\frac{1}{4}},\\
M_4 &= \left(-\frac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}} = \left(-\frac{\hat{A}}{2\hat{B}}\frac{\tilde{c}_3}{\tilde{c}_6}\left[-1+\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right)^{\frac{1}{4}}.
\end{aligned} \tag{5.94}$$

where the factors $\hat{A}$, $\hat{B}$ and $\hat{C}$ are defined as:

$$\begin{aligned}
\hat{A} &= \left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{U_1}{27}-3U_2\right]-\frac{5}{2}U_1+\frac{99}{98}U_2\\
&\quad -\frac{\Delta}{4}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\left(|C_1|^2+|C_2|^2\right)^2\right.\\
&\quad -\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2-\frac{34}{3}\frac{s}{c_S^2}\left(|C_1|^2+|C_2|^2\right)^2\\
&\quad \left.+6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2+27\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\right],\\
\hat{B} &= \left\{\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{U_1}{27}-3U_2\right]-\frac{5}{2}U_1+\frac{99}{98}U_2\right\}\frac{\Delta c_S^2}{2\epsilon H^2M_p^2},\\
\hat{C} &= \frac{H^2M_p^2\epsilon c_S^2}{2}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\left(|C_1|^2+|C_2|^2\right)^2\right.\\
&\quad -\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2-\frac{34}{3}\frac{s}{c_S^2}\left(|C_1|^2+|C_2|^2\right)^2\\
&\quad \left.+6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2+27\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\right].
\end{aligned} \tag{5.95}$$

[25] Here we also get another solution:

$$\bar{M}_1 = \left\{\frac{A}{2BH}\left[-1-\sqrt{1+\frac{4BC}{A^2}}\right]\right\}^{\frac{1}{3}}, \tag{5.93}$$

which is redundant in the present context as this solution is not consistent with the $c_S = 1$ and $\tilde{c}_S = 1$ limit result as computed in the earlier section for Single Field Slow Roll inflation.

Further for restricted class of General Single Field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, we get the following expression for the factors $\hat{A}$, $\hat{B}$ and $\hat{C}$ as:

$$\begin{aligned}
\hat{A} &= \left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{U_1}{27}-3U_2\right]-\frac{5}{2}U_1+\frac{99}{98}U_2 \\
&-\frac{\Delta}{4}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\left(|C_1|^2+|C_2|^2\right)^2-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2\right. \\
&\left.+6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2+27\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\right], \\
\hat{B} &= \left\{\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{U_1}{27}-3U_2\right]-\frac{5}{2}U_1+\frac{99}{98}U_2\right\}\frac{\Delta c_S^2}{2\epsilon H^2 M_p^2}, \qquad (5.96) \\
\hat{C} &= \frac{H^2M_p^2\epsilon c_S^2}{2}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\left(|C_1|^2+|C_2|^2\right)^2-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2\right. \\
&\left.+6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2+27\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\right].
\end{aligned}$$

where for arbitrary vacucum U_1 and U_2 are defined as:

$$U_1 = \left[(C_1-C_2)^3\left(C_1^{*3}+C_2^{*3}\right)+\left(C_1^*-C_2^*\right)^3\left(C_1^3+C_2^3\right)\right], \qquad (5.97)$$

$$U_2 = \left[(C_1-C_2)^3 C_1^*C_2^*\left(C_1^*-C_2^*\right)+\left(C_1^*-C_2^*\right)^3 C_1C_2\left(C_1-C_2\right)\right]. \qquad (5.98)$$

If we take $c_S=1$ and $\tilde{c}_S=1$ then we get then we get back all the results obtained for Single Field Slow Roll inflation in the previous section.

Now for Bunch Davies and α,β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch Davies vacuum:**
  After setting $C_1=1$ and $C_2=0$ we get the following expression for the factors $\hat{A}$, $\hat{B}$ and $\hat{C}$ as:

$$\begin{aligned}
\hat{A} &= \frac{2}{27}\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)-5-\frac{\Delta}{4}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\right. \\
&\left.-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)-\frac{34}{3}\frac{s}{c_S^2}+(23\epsilon-6\eta)\right], \\
\hat{B} &= \left\{\frac{2}{27}\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)-5\right\}\frac{\Delta c_S^2}{2\epsilon H^2M_p^2}, \qquad (5.99) \\
\hat{C} &= \frac{H^2M_p^2\epsilon c_S^2}{2}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)-\frac{34}{3}\frac{s}{c_S^2}+(23\epsilon-6\eta)\right].
\end{aligned}$$

  Further for restricted class of General Single Field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, we get the following expression for the factors A, B and C as:

$$\begin{aligned}
\hat{A} &= \frac{2}{27}\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)-5-\frac{\Delta}{4}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)+(23\epsilon-6\eta)\right], \\
\hat{B} &= \left\{\frac{2}{27}\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)-5\right\}\frac{\Delta c_S^2}{2\epsilon H^2M_p^2}, \qquad (5.100) \\
\hat{C} &= \frac{H^2M_p^2\epsilon c_S^2}{2}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)+(23\epsilon-6\eta)\right].
\end{aligned}$$

- **For α,β vacuum**:
After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get the following expression for the factors $\hat{A}$, $\hat{B}$ and $\hat{C}$ as:

$$\begin{aligned}
\hat{A} &= \left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{J_1(\alpha,\beta)}{27}-3J_2(\alpha,\beta)\right]-\frac{5}{2}J_1(\alpha,\beta)+\frac{99}{98}J_2(\alpha,\beta)\\
&\quad -\frac{\Delta}{4}\left[\left\{\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)-\frac{34}{3}\frac{s}{c_S^2}\right\}\cosh^2 2\alpha\right.\\
&\quad \left.+6(2\epsilon-\eta)\cosh^2 2\alpha+11\epsilon+27\epsilon\sinh^2\alpha\cos^2\beta\right],\\
\hat{B} &= \left\{\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{J_1(\alpha,\beta)}{27}-3J_2(\alpha,\beta)\right]-\frac{5}{2}J_1(\alpha,\beta)+\frac{99}{98}J_2(\alpha,\beta)\right\}\frac{\Delta c_S^2}{2\epsilon H^2 M_p^2}, \qquad (5.101)\\
\hat{C} &= \frac{H^2M_p^2\epsilon c_S^2}{2}\left[\left\{\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)-\frac{34}{3}\frac{s}{c_S^2}\right\}\cosh^2 2\alpha\right.\\
&\quad \left.+6(2\epsilon-\eta)\cosh^2 2\alpha+11\epsilon+27\epsilon\sinh^2\alpha\cos^2\beta\right].
\end{aligned}$$

Further for restricted class of General Single Field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, we get the following expression for the factors $\hat{A}$, $\hat{B}$ and $\hat{C}$ as:

$$\begin{aligned}
\hat{A} &= \left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{J_1(\alpha,\beta)}{27}-3J_2(\alpha,\beta)\right]-\frac{5}{2}J_1(\alpha,\beta)+\frac{99}{98}J_2(\alpha,\beta)\\
&\quad -\frac{\Delta}{4}\left[\left\{\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)+6(2\epsilon-\eta)\right\}\cosh^2 2\alpha\right.\\
&\quad \left.+11\epsilon+27\epsilon\sinh^2\alpha\cos^2\beta\right],\\
\hat{B} &= \left\{\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{J_1(\alpha,\beta)}{27}-3J_2(\alpha,\beta)\right]-\frac{5}{2}J_1(\alpha,\beta)+\frac{99}{98}J_2(\alpha,\beta)\right\}\frac{\Delta c_S^2}{2\epsilon H^2 M_p^2}, \qquad (5.102)\\
\hat{C} &= \frac{H^2M_p^2\epsilon c_S^2}{2}\left[\left\{\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)+6(2\epsilon-\eta)\right\}\cosh^2 2\alpha\right.\\
&\quad \left.+11\epsilon+27\epsilon\sinh^2\alpha\cos^2\beta\right].
\end{aligned}$$

2. Squeezed limit configuration:
For this case with arbitray vacuum one can write:

$$B_{EFT}(k_L,k_L,k_S) = B_{GSF}(k_L,k_L,k_S), \qquad (5.103)$$

which implies that:

$$\boxed{\begin{aligned}
&\bar{M}_1 = \left\{\frac{\hat{A}}{2\hat{B}H}\left[-1+\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right\}^{\frac{1}{3}}, \quad \bar{M}_2\approx\bar{M}_3=\sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}}=\sqrt{\frac{\hat{A}}{8\hat{B}H^2\tilde{c}_5}\left[-1+\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]},\\
&M_2 = \left(-\frac{XP_{,XX}(X,\phi)}{P_{,X}(X,\phi)}\dot{H}M_p^2\right)^{\frac{1}{4}}, \quad M_3=\left\{-\frac{\hat{A}}{2\hat{B}}\frac{\tilde{c}_3}{\tilde{c}_4}\left[-1+\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right\}^{\frac{1}{4}},\\
&M_4 = \left(-\frac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}}=\left(-\frac{\hat{A}}{2\hat{B}}\frac{\tilde{c}_3}{\tilde{c}_6}\left[-1+\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right)^{\frac{1}{4}}.
\end{aligned}} \qquad (5.104)$$

where the factors $\hat{A}$, $\hat{B}$ and $\hat{C}$ are defined as [26]:

$$\hat{A} = \hat{P}_1 + \hat{P}_2 - \sum_{j=-1}^{3} t_j \left(\frac{k_S}{k_L}\right)^j, \quad \hat{B} = \frac{\hat{P}_1 \Delta}{2\epsilon H^2 M_p^2}, \quad \hat{C} = 2\epsilon H^2 M_p^2 \sum_{j=-1}^{3} t_j \left(\frac{k_S}{k_L}\right)^j, \quad (5.106)$$

where the expansion coefficients $t_j \forall j = -1, \cdots, 3$ are defined earlier for general $P(X,\phi)$ model and also for restricted class of model where $P_{,X\phi}(X,\phi) = 0$ constarint satisfies.

Here the factors $\hat{P}_1$ and $\hat{P}_2$ are defined as:

$$\hat{P}_1 = \sum_{m=-1}^{3} \hat{e}_m \left(\frac{k_S}{k_L}\right)^m, \qquad \hat{P}_2 = \sum_{m=-1}^{3} \hat{h}_m \left(\frac{k_S}{k_L}\right)^m, \quad (5.107)$$

where the expansion coefficients $\hat{e}_m \forall m = -1, \cdots, 3$ and $\hat{h}_m \forall m = -1, \cdots, 3$ for arbitrary vacuum are defined as:

$$\begin{aligned} \hat{e}_{-1} &= -36U_2, \quad \hat{e}_0 = \left[-\frac{9}{2}U_1 + \left(24\left\{\frac{3}{2} + \frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4} + \frac{2c_S^2}{\tilde{c}_4}\right\} - \frac{9}{2}\right)U_2\right], \\ \hat{e}_1 &= 0, \quad \hat{e}_2 = \left[-\frac{27}{2}U_2 + \left(\frac{3}{2}\left\{\frac{3}{2} + \frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4} + \frac{2c_S^2}{\tilde{c}_4}\right\} - \frac{27}{2}\right)U_1\right], \quad \hat{e}_3 = 0, \end{aligned} \quad (5.108)$$

and

$$\begin{aligned} \hat{h}_{-1} &= 0, \quad \hat{h}_0 = \frac{9}{c_S^2}\left(U_1 + U_2\right), \quad \hat{h}_1 = 0, \\ \hat{h}_2 &= \left[\left(15 + 2\left\{\frac{3}{2} + \frac{2c_S^2}{\tilde{c}_4}\right\}\right)U_1 + \left(\frac{45}{2} + 3\left\{\frac{3}{2} + \frac{2c_S^2}{\tilde{c}_4}\right\}\right)U_2\right], \quad \hat{h}_3 = 0, \end{aligned} \quad (5.109)$$

where U_1 and U_2 are already defined earlier.

Now for Bunch Davies and α, β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$, we get $U_1 = 2$ and $U_2 = 0$. Consequently the expansion coefficients can be recast as:

$$\hat{e}_{-1} = 0, \quad \hat{e}_0 = -9, \quad \hat{e}_1 = 0, \quad \hat{e}_2 = -27, \quad \hat{e}_3 = 0, \quad (5.110)$$

  and

$$\hat{h}_{-1} = 0, \quad \hat{h}_0 = \frac{18}{c_S^2}, \quad \hat{h}_1 = 0, \quad \hat{h}_2 = \left(30 + 4\left\{\frac{3}{2} + \frac{2c_S^2}{\tilde{c}_4}\right\}\right), \quad \hat{h}_3 = 0, \quad (5.111)$$

[26] Here we also get another solution:

$$\bar{M}_1 = \left\{\frac{\hat{A}}{2\hat{B}H}\left[-1 - \sqrt{1 + \frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right\}^{\frac{1}{3}}, \quad (5.105)$$

which is redundant in the present context as this solution is not consistent with the $c_S = 1$ and $\tilde{c}_S = 1$ limit result as computed in the earlier section for General Single Field $P(X,\phi)$ inflation.

- **For α,β vacuum**:
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$, we get $U_1 = J_1(\alpha,\beta)$ and $U_2 = J_2(\alpha,\beta)$. Consequently the expansion coefficients can be recast as:

$$\hat{e}_{-1} = -36J_2(\alpha,\beta), \quad \hat{e}_0 = \left[-\frac{9}{2}J_1(\alpha,\beta) + \left(24\left\{\frac{3}{2} + \frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4} + \frac{2c_S^2}{\tilde{c}_4}\right\} - \frac{9}{2}\right)J_2(\alpha,\beta)\right],$$
$$\hat{e}_1 = 0, \quad \hat{e}_2 = \left[-\frac{27}{2}J_2(\alpha,\beta) + \left(\frac{3}{2}\left\{\frac{3}{2} + \frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4} + \frac{2c_S^2}{\tilde{c}_4}\right\} - \frac{27}{2}\right)J_1(\alpha,\beta)\right], \quad \hat{e}_3 = 0, \tag{5.112}$$

and

$$\hat{h}_{-1} = 0, \quad \hat{h}_0 = \frac{9}{c_S^2}\left(U_1 + J_2(\alpha,\beta)U_2\right), \quad \hat{h}_1 = 0,$$
$$\hat{h}_2 = \left[\left(15 + 2\left\{\frac{3}{2} + \frac{2c_S^2}{\tilde{c}_4}\right\}\right)J_1(\alpha,\beta) + \left(\frac{45}{2} + 3\left\{\frac{3}{2} + \frac{2c_S^2}{\tilde{c}_4}\right\}\right)J_2(\alpha,\beta)\right], \quad \hat{h}_3 = 0. \tag{5.113}$$

6 Conclusion

To summarize, in this paper, we have addressed the following issues:

- We have derived the analytical expressions for the two point correlation function for scalar and tensor fluctuations and three point correlation function for scalar fluctuations from EFT framework in quasi de Sitter background in a model independent way. For this computation we use an arbitrary quantum state as initial choice of vacuum. Such a choice finally give rise to the most general expressions for the two point and three point correlation functions for primordial fluctuation in EFT. Further we have simplified our results by considering Bunch Davies vacuum and α,β vacuum state.

- During our computation we have truncated the EFT action by considering the all possible two derivative terms in the metric. This allows us to derive correct expressions for the two point and three point correlation functions for EFT which are consistent with both single field slow roll model and generalized non-canonical $P(X,\phi)$ single field models minimally coupled with gravity [27].

- Further we have derived the analytical expressions for the coefficients of all relevant EFT operators for single field slow roll model and generalized non-canonical $P(X,\phi)$ single field models. We have derived the results in terms of slow roll parameters, effective sound speed parameter and the constants which are fixed by the choice of arbitrary initial vacuum state. Next, we have simplified our results also presented the results by considering Bunch Davies vacuum and α,β vacuum state.

- Finally using the CMB observation from Planck we constrain all of these EFT coefficients for various single field slow roll models and generalized non-canonical $P(X,\phi)$ models of inflation.

The future directions of this paper are appended below point-wise:

[27] This is really an important outcome as the earlier derived results for the three point function for EFT in quasi de Sitter background was not consistent with the known result for single field slow roll model, where effective sound speed is fixed at $\tilde{c}_S = 1$.

- One can further carry forward this work to compute four point scalar correlation function from EFT framework using arbitrary initial choice of the quantum vacuum state. The present work can also be extended for the computation of the three point correlation from tensor fluctuation, other three point cross correlations between scalar and tensor mode fluctuation in the context of EFT with arbitrary initial vacuum.

- In the present EFT framework we have not considered the effects of any additional heavy fields ($m >> H$) in the effective action. One can redo the analysis with such additional effects in the EFT framework to study the quantum entanglement, cosmological decoherence and Bell's inequality violation in the context of primordial cosmology. Once acan also futher generalize this compuatation for any arbitrary spin fields which are consistent with the unitarity bound.

- The analyticity property of response functions and scattering amplitudes in QFT implies significant connection between observables in IR regime and the underlying dynamics valid in the short-distance scale. Such analytic property is directly connected to the causality and unitarity of the QFT under consideration. Following this idea one can also study the analyticity property in the present version of EFT or including the effective of massive fields ($m >> H$) in the effective action.

- There are other open issues as well which one can study within the framework of EFT:

  1. Role of out of time ordered correlations from open quantum system [59–61].
  2. EFT framework in a quantum dissipative systems and its application to cosmology [62–64].
  3. Thermalization, quantum critical quench and its application to the phenomena of reheating in early universe cosmology [66].

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Appendix

A Brief overview on Schwinger-Keldysh (In-In) formalism

To compute the any n-point correlation function in quasi de Sitter space we use Schwinger-Keldysh (In-In) formalism. In this framework the expectation value of a product of operators $\mathcal{O}(t)$ at time

t can be written as:

$$\boxed{\langle\mathcal{O}(t)\rangle=\left\langle\left(T\;\exp\left[-i\int_{-\infty}^{t}H_{int}(t^{'})dt^{'}\right]\right)^{\dagger}\mathcal{O}(t)\left(T\;\exp\left[-i\int_{-\infty}^{t}H_{int}(t^{''})dt^{''}\right]\right)\right\rangle}\,,\qquad (A.1)$$

where it is important to note that all the fields appearing in the right hand side belong to the Heisenberg picture. Here correlation function is computed with respect to the initial quantum vacuum state $|in\rangle$, which in general can be any arbitrary vacuum state. In cosmological literature concept of Bunch Davies and α,β vacuum are commonly used. To mention the mathematical structure of the quantm vacuum state $|in\rangle$ we first consider an arbitrary state $|\Omega(t)\rangle$, which can be expanded in terms of the eigen basis state $|m\rangle$ of the free Hamiltonian as:

$$|\Omega(t)\rangle=\sum|m\rangle\langle m|\Omega(t)\rangle. \qquad (A.2)$$

Further the time evolved quantum state from time $t=t_1$ to $t=t_2$ can be written as:

$$|\Omega(t_2)\rangle=T\;\exp\left[-i\int_{t_1}^{t_2}H_{int}(t^{'})dt^{'}\right]|\Omega(t_1)\rangle=\underbrace{|0\rangle\langle 0|\Omega\rangle}_{\textbf{Free part}}+\underbrace{\sum_{m=1}^{\infty}\exp\left[iE_m(t_2-t_1)\right]|m\rangle\langle m|\Omega(t_1)\rangle}_{\textbf{Interacting part}}.\ (A.3)$$

It is clearly observed that we have expressed any arbitrary quantum state in terms of the free part and the interacting part of the theory. Further for further computation we set $t_2=-\infty(1-i\epsilon)$ which clearly projects all excited quantum states. Using this we have the following connecting relation between the interacting vacuum and the free vacuum state, as given by:

$$|\Omega(-\infty(1-i\epsilon))\rangle\equiv|0\rangle\langle 0|\Omega\rangle. \qquad (A.4)$$

Finally, at any arbitrary time the interacting vacuum can be written as:

$$|in\rangle=T\;\exp\left[-i\int_{-\infty(1-i\epsilon)}^{t}H_{int}(t^{'})dt^{'}\right]|\Omega(-\infty(1-i\epsilon))\rangle=T\;\exp\left[-i\int_{-\infty(1-i\epsilon)}^{t}H_{int}(t^{'})dt^{'}\right]|0\rangle\langle 0|\Omega\rangle. \quad (A.5)$$

For our computation initially we have written the expression which is valid for any arbitrary choice of quantum vacuum state. But for simplicity further we consider two specific choice of vacuum state- Bunch-Davies vacuum and α,β vacuum, which are commonly used in cosmological physics. Now in this context the total Hamiltonian of the theory can be written in terms of the free and interacting part as, $H=H_0+H_{int}$, where interaction Hamiltonian is described by H_{int} and the free field Hamiltonian is described by H_0.

In the context of cosmological perturbation theory one can follow the same formalism where one usually start with the Einstein-Hilbert gravity action with the any matter content in the effective action. For this purpose one uses the well known ADM formalism to derive an action which contains only dynamical degrees of freedom. From this action one need to perform the following steps:

- First of all one need to construct the canonically conjugate momenta and the Hamiltonian for the system.
- Then need to separate out the quadratic part from the higher order contributions in the Hamiltonian.

Now in this context let us consider a part of the effective action which contains the third order contribution and all other higher order contribution in cosmological perturbation theory, represented by L_{int}. In this case the usual expression for the interaction Hamiltonian is given by, $H_{int} = -L_{int}$. Further to make a direct connection to the in-out formalism in QFT used in the computation of S-matrix, one can further insert complete sets of states labeled by α and β in Eq (A.1) and finally get:

$$\langle \mathcal{O}(t)\rangle = \int d\alpha \int d\beta \langle 0| \left(T\ \exp\left[-i\int_{-\infty}^{t} H_{int}(t^{'})dt^{'}\right]\right)^{\dagger} |\alpha\rangle \overbrace{\langle\alpha|\mathcal{O}(t)|\beta\rangle}^{=\mathcal{O}_{\alpha\beta}(t)}\langle\beta| \left(T\ \exp\left[-i\int_{-\infty}^{t} H_{int}(t^{''})dt^{''}\right]\right)|0\rangle, \tag{A.6}$$

Here the in-in quantum correlation is interpreted as the product of the vacuum transition amplitudes and in the matrix element $\langle\alpha|\mathcal{O}(t)|\beta\rangle \equiv \mathcal{O}_{\alpha\beta}(t)$, where one needs to sum over all possible quantum out states. Further to compute the quantum correlations using Schwinger-Keldysh (In-In) formalism one needs to consider the following steps:

- First of all one needs to define the time integration in the time evolution operator $U(t)$ to go over a contour in the complex plane i.e.

$$U(t) = T\ \exp\left[-i\int_{-\infty}^{t} H_{int}(t^{'})dt^{'}\right] \Rightarrow T\ \exp\left[-i\int_{-\infty(1+i\epsilon)}^{t} H_{int}(t^{'})dt^{'}\right], \tag{A.7}$$

  where we have redefined the time interval by including small imaginary contribution as given by $t \rightarrow t(1 \pm i\epsilon)$. With this specific choice one can finally write the following expression for the n-point correlation function:

$$\boxed{\langle \mathcal{O}(t)\rangle = \left\langle \left(T\ \exp\left[-i\int_{-\infty(1-i\epsilon)}^{t} H_{int}(t^{'})dt^{'}\right]\right)^{\dagger} \mathcal{O}(t) \left(T\ \exp\left[-i\int_{-\infty(1+i\epsilon)}^{t} H_{int}(t^{''})dt^{''}\right]\right)\right\rangle}\ . \tag{A.8}$$

  Here it is important to note that complex conjugation of the time evolution operator $U(t)$ signifies the fact that the time-ordered contour does not at all coincide with the time-backward contour.

- Next we analytically continue the expression for the interaction Hamiltonian as appearing in the time evolution operator $U(t)$ i.e. $H_{ini}(t) \rightarrow H_{int}(t(1 \pm i\epsilon))$.

- Next we consider the following Dyson Swinger series:

$$T\ \exp\left[-i\int_{-\infty(1+i\epsilon)}^{t} H_{int}(t^{'})dt^{'}\right] = 1 + \sum_{N=1}^{\infty}\frac{(-i)^N}{N!}\prod_{i=1}^{N}\int_{-\infty(1+i\epsilon)}^{t_i} dt_i\ H_{int}(t_i), \tag{A.9}$$

  using which finally we get the following simplified expression for the n-point correlation function:

$$\boxed{\langle \mathcal{O}(t)\rangle = \sum_{N=0}^{\infty}\frac{(-i)^N}{N!}\prod_{i=1}^{N}\int_{-\infty(1+i\epsilon)}^{t_i} dt_i\ \langle 0|\left[H_{int}(t_i), \mathcal{O}(t)\right]|0\rangle = \sum_{n=0}^{\infty}\langle\mathcal{O}(t)\rangle^{(n)}}\ . \tag{A.10}$$

  where $|0\rangle$ is the initial quantum vacuum state under consideration. Here expanding in the powers of interacting Hamiltonian $H_{int}(t)$ we finally get:

1. Zeroth order term $\langle \mathcal{O}(t)\rangle^{(0)}$ in Dyson Swinger series:
   Here the zeroth order term in Dyson Swinger series can be expressed as:

$$\langle \mathcal{O}(t)\rangle^{(0)} = \langle 0|\mathcal{O}(t)|0\rangle. \tag{A.11}$$

2. First order term $\langle \mathcal{O}(t)\rangle^{(1)}$ in Dyson Swinger series:
   Here the first order term in Dyson Swinger series can be expressed as:

$$\langle \mathcal{O}(t)\rangle^{(1)} = 2\mathrm{Re}\left[-i\int_{-\infty(1+i\epsilon)}^{t} dt^{'}\,\langle 0|\mathcal{O}(t)H_{int}(t^{'})|0\rangle\right]. \tag{A.12}$$

3. Second order term $\langle \mathcal{O}(t)\rangle^{(2)}$ in Dyson Swinger series:
   Here the second order term in Dyson Swinger series can be expressed as:

$$\begin{aligned}\langle \mathcal{O}(t)\rangle^{(2)} = -2\mathrm{Re}\Bigg[&\int_{-\infty(1+i\epsilon)}^{t_1} dt_1 \int_{-\infty(1+i\epsilon)}^{t_2} dt_2\langle 0|\mathcal{O}(t)H_{int}(t_1)H_{int}(t_2)|0\rangle\Bigg]\\ &+\int_{-\infty(1+i\epsilon)}^{t_1} dt_1 \int_{-\infty(1+i\epsilon)}^{t_2} dt_2\langle 0|H_{int}(t_1)\mathcal{O}(t)H_{int}(t_2)|0\rangle.\end{aligned} \tag{A.13}$$

   Following this trick one can easily write down the expression for any n-point correlation function of the given operator $\mathcal{O}(t)$.

B Choice of initial quantum vacuum state

In general one can consider an arbitrary initial quantum vacuum state which is specified by the two sets of constants (C_1, C_2) and (D_1, D_2) as appearing in solution of the scalar and tensor mode fluctuation. In general in this context a quantum state is described by this two number as $|C_1, C_2\rangle$ and $|D_1, D_2\rangle$ and defined as, $C(\mathbf{k})|C_1, C_2\rangle = 0\;\forall\,\mathbf{k}, \quad D(\mathbf{k})|D_1, D_2\rangle = 0\;\forall\,\mathbf{k}$, where $C(\mathbf{k})$ and $D(\mathbf{k})$ are the annihilation operators for scalar and tensor mode fluctuations as appearing in cosmological perturbation theory.

In general ground one can write down the most general state $|C_1, C_2\rangle$ in terms of the well known Bunch-Davies vacuum state as:

$$\begin{aligned}|C_1, C_2\rangle &= \prod_{\mathbf{k}} \frac{1}{\sqrt{|C_1|}} \exp\left[\frac{C_2^*}{2C_1^*}\,C^{\dagger}(\mathbf{k})C^{\dagger}(-\mathbf{k})\right]|0\rangle\\ &= \frac{1}{\mathcal{N}_C}\;\exp\left[\frac{C_2^*}{2C_1^*}\sum_{\mathbf{k}}\;C^{\dagger}(\mathbf{k})C^{\dagger}(-\mathbf{k})\right]|0\rangle = \frac{1}{\mathcal{N}_C}\;\exp\left[\frac{C_2^*}{2C_1^*}\int\frac{d^3k}{(2\pi)^3}\;C^{\dagger}(\mathbf{k})C^{\dagger}(-\mathbf{k})\right]|0\rangle,\end{aligned} \tag{B.1}$$

where $\mathcal{N}_C = \sqrt{|C_1|}$ are the overall normalization constant for scalar and tensor mode fluctuations. For the tensor modes the calclation is similar.

Here it is important to mention that the quantum vacuum state $|C_1, C_2\rangle$ satisfy the following constraint equation:

$$\begin{aligned}\hat{\mathbf{P}}_C|C_1, C_2\rangle &= \int\frac{d^3p}{(2\pi)^3}\,\mathbf{p}\;C^{\dagger}(\mathbf{p})C(\mathbf{p})|C_1, C_2\rangle = \prod_{\mathbf{k}}\int\frac{d^3p}{(2\pi)^3}\;\frac{\mathbf{p}\;C^{\dagger}(\mathbf{p})C(\mathbf{p})}{\sqrt{|C_1|}}\exp\left[\frac{C_2^*}{2C_1^*}\;C^{\dagger}(\mathbf{k})C^{\dagger}(-\mathbf{k})\right]|0\rangle\\ &= \int\frac{d^3p}{(2\pi)^3}\;\frac{\mathbf{p}\;C^{\dagger}(\mathbf{p})C(\mathbf{p})}{\sqrt{|C_1|}}\exp\left[\frac{C_2^*}{2C_1^*}\;\sum_{\mathbf{k}}C^{\dagger}(\mathbf{k})C^{\dagger}(-\mathbf{k})\right]|0\rangle\\ &= \int\frac{d^3p}{(2\pi)^3}\;\frac{\mathbf{p}\;C^{\dagger}(\mathbf{p})C(\mathbf{p})}{\sqrt{|C_1|}}\exp\left[\frac{C_2^*}{2C_1^*}\;\int\frac{d^3k}{(2\pi)^3}C^{\dagger}(\mathbf{k})C^{\dagger}(-\mathbf{k})\right]|0\rangle = 0,\end{aligned} \tag{B.2}$$

which is true for the quantum vacuum state $|D_1, D_2\rangle$ for tensor modes also. Since scalar modes are exactly similar to tensor modes we will not speak about tensor modes in the next part.

Additionally, here it is important to note that, the annihilation and creation operators for Bunch-Davies vacuum $(a(\mathbf{k}), a^\dagger(\mathbf{k}))$ and the arbitrary quantum vacumm $|C_1, C_2\rangle$ state $(C(\mathbf{k}), C^\dagger(\mathbf{k}))$ are connected via the following sets of Bogoliubov transformations:

$$C(\mathbf{k}) = C_1^*\ a(\mathbf{k}) - C_2^*\ a^\dagger(-\mathbf{k}), \tag{B.3}$$

$$a(\mathbf{k}) = C_1\ C(\mathbf{k}) + C_2^*\ C^\dagger(-\mathbf{k}). \tag{B.4}$$

1. Bunch-Davies vacuum:
   Bunch-Davies vacuum is specified by fixing the coefficients to, $C_1 = 1 = D_1, \quad C_2 = 0 = D_2$, in the solution of the scalar and tensor mode fluctuation as derived earlier. In this case the quantum vacuum state $|0\rangle$ is defined as the state that gets annihilated by the annihilation operator, as given by, $a(\mathbf{k})|0\rangle = 0\ \forall\ \mathbf{k}$. Here the creation and annihilation operators $a(\mathbf{k})$ and $a^\dagger(\mathbf{k})$ satisfy the following canonical commutation relations:

$$\left[a(\mathbf{k}), a(\mathbf{k}^{'})\right] = 0, \quad \left[a^\dagger(\mathbf{k}), a^\dagger(\mathbf{k}^{'})\right] = 0, \quad \left[a(\mathbf{k}), a^\dagger(\mathbf{k}^{'})\right] = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}^{'}). \tag{B.5}$$

2. α, β vacuum:
   α, β vacuum is specified by fixing the coefficients to, $C_1 = \cosh\alpha = D_1, \quad C_2 = e^{i\beta}\sinh\alpha = D_2$, in the solution of the scalar and tensor mode fluctuation as derived earlier. In this case the quantum vacuum state $|\alpha,\beta\rangle$ is defined as the state that gets annihilated by the annihilation operator, as given by, $b(\mathbf{k})|\alpha,\beta\rangle = 0\ \forall\ \mathbf{k}$. Here the creation and annihilation operators $b(\mathbf{k})$ and $b^\dagger(\mathbf{k})$ satisfy the following canonical commutation relations:

$$\left[b(\mathbf{k}), b(\mathbf{k}^{'})\right] = 0, \quad \left[b^\dagger(\mathbf{k}), b^\dagger(\mathbf{k}^{'})\right] = 0, \quad \left[b(\mathbf{k}), b^\dagger(\mathbf{k}^{'})\right] = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}^{'}). \tag{B.6}$$

   Here one can write the Bunch Davies vacuum state $|0\rangle$ as a special class of $|\alpha,\beta\rangle$ vacuum state. Also using Bogoliubov transformation one can write down $|\alpha,\beta\rangle$ vacuum state in terms of the Bunch Davies vacuum state $|0\rangle$, as given by:

$$\begin{aligned}
|\alpha,\beta\rangle &= \prod_{\mathbf{k}} \frac{1}{\sqrt{|\cosh\alpha|}} \exp\left[-\frac{i}{2}e^{-i\beta}\tanh\alpha\ a^\dagger(\mathbf{k})a^\dagger(-\mathbf{k})\right]|0\rangle \\
&= \frac{1}{\mathcal{N}}\ \exp\left[-\frac{i}{2}e^{-i\beta}\tanh\alpha \sum_{\mathbf{k}}\ a^\dagger(\mathbf{k})a^\dagger(-\mathbf{k})\right]|0\rangle \\
&= \frac{1}{\mathcal{N}}\ \exp\left[-\frac{i}{2}e^{-i\beta}\tanh\alpha \int \frac{d^3k}{(2\pi)^3}\ a^\dagger(\mathbf{k})a^\dagger(-\mathbf{k})\right]|0\rangle,
\end{aligned} \tag{B.7}$$

   where $\mathcal{N} = \sqrt{|\cosh\alpha|}$ is the overall normalization constant. Here it is important to mention that the $|\alpha,\beta\rangle$ vacuum state satisfy the following constraint equation:

$$\begin{aligned}
\hat{\mathbf{P}}|\alpha,\beta\rangle &= \int \frac{d^3p}{(2\pi)^3}\ \mathbf{p}\ a^\dagger(\mathbf{p})a(\mathbf{p})|\alpha,\beta\rangle \\
&= \prod_{\mathbf{k}} \int \frac{d^3p}{(2\pi)^3}\ \frac{\mathbf{p}\ a^\dagger(\mathbf{p})a(\mathbf{p})}{\sqrt{|\cosh\alpha|}} \exp\left[-\frac{i}{2}e^{-i\beta}\tanh\alpha\ a^\dagger(\mathbf{k})a^\dagger(-\mathbf{k})\right]|0\rangle \\
&= \int \frac{d^3p}{(2\pi)^3}\ \frac{\mathbf{p}\ a^\dagger(\mathbf{p})a(\mathbf{p})}{\sqrt{|\cosh\alpha|}} \exp\left[-\frac{i}{2}e^{-i\beta}\tanh\alpha\ \sum_{\mathbf{k}} a^\dagger(\mathbf{k})a^\dagger(-\mathbf{k})\right]|0\rangle \\
&= \int \frac{d^3p}{(2\pi)^3}\ \frac{\mathbf{p}\ a^\dagger(\mathbf{p})a(\mathbf{p})}{\sqrt{|\cosh\alpha|}} \exp\left[-\frac{i}{2}e^{-i\beta}\tanh\alpha\ \int \frac{d^3k}{(2\pi)^3} a^\dagger(\mathbf{k})a^\dagger(-\mathbf{k})\right]|0\rangle = 0.
\end{aligned} \tag{B.8}$$

Additionally, here it is important to note that, the creation and annihilation operators for Bunch-Davies vacuum and $|\alpha,\beta\rangle$ vacuum state are connected via the following sets of Bogoliubov transformations:

$$b(\mathbf{k}) = \cosh\alpha\; a(\mathbf{k}) + i\; e^{-i\beta}\sinh\alpha\; a^{\dagger}(-\mathbf{k}), \tag{B.9}$$
$$a(\mathbf{k}) = \cosh\alpha\; b(\mathbf{k}) - i\; e^{-i\beta}\sinh\alpha\; b^{\dagger}(-\mathbf{k}). \tag{B.10}$$

C Useful integrals as appearing in scalar three point function

All the useful integrals appearing in the scalar three point function are appended bellow:

1. $$\int_{-\infty}^{0} d\eta\; \eta^2\; e^{\pm iK\tilde{c}_S\eta} = \mp\frac{2}{iK^3\tilde{c}_S^3}, \tag{C.1}$$

2. $$\int_{-\infty}^{0} d\eta\; \eta^2\; e^{\mp i(2k_a-K)\tilde{c}_S\eta} = \pm\frac{2}{i(2k_a-K)^3\tilde{c}_S^3}, \tag{C.2}$$

3. $$\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\; (1\mp ik_b\tilde{c}_S\eta)(1\mp ik_c\tilde{c}_S\eta)\; e^{\pm iK\tilde{c}_S\eta} = \frac{1}{iK^3\tilde{c}_S}\left[K^2+2k_bk_c+K(K-k_a)\right], \tag{C.3}$$

4. $$\int_{-\infty}^{0} d\eta\; (1-ik_b\tilde{c}_S\eta)(1-ik_c\tilde{c}_S\eta)\; e^{i(K-2k_a)\tilde{c}_S\eta} = -\int_{-\infty}^{0} d\eta\; (1+ik_b\tilde{c}_S\eta)(1+ik_c\tilde{c}_S\eta)\; e^{-i(K-2k_a)\tilde{c}_S\eta}$$
$$= -\frac{1}{i(2k_a-K)^3\tilde{c}_S}\left[K^2+2k_bk_c+K(K-5k_a)-2(K-k_a)k_a+4k_a^2\right], \tag{C.4}$$

5. $$\int_{-\infty}^{0} d\eta\; (1-ik_b\tilde{c}_S\eta)(1\mp ik_c\tilde{c}_S\eta)\; e^{i(K-2k_b)\tilde{c}_S\eta} = -\int_{-\infty}^{0} d\eta\; (1\mp ik_b\tilde{c}_S\eta)(1\pm ik_c\tilde{c}_S\eta)\; e^{\mp i(K-2k_b)\tilde{c}_S\eta}$$
$$= -\frac{1}{i(2k_b-K)^3\tilde{c}_S}\left[K^2-4k_bk_c+K(k_c-5k_b)+6k_b^2\right], \tag{C.5}$$

6. $$\int_{-\infty}^{0} d\eta\; (1\mp ik_a\tilde{c}_S\eta)\; e^{\pm iK\tilde{c}_S\eta} = \pm\frac{1}{iK^2\tilde{c}_S}(K+k_a), \tag{C.6}$$

7. $$\int_{-\infty}^{0} d\eta\; (1\pm ik_a\tilde{c}_S\eta)\; e^{\pm i(K-2k_a)\tilde{c}_S\eta} = \pm\frac{(K-3k_a)}{i(2k_a-K)^2\tilde{c}_S}, \tag{C.7}$$

8. $$\int_{-\infty}^{0} d\eta\; (1\mp ik_a\tilde{c}_S\eta)\; e^{\pm i(K-2k_b)\tilde{c}_S\eta} = \pm\frac{(K+k_a-2k_b)}{i(2k_b-K)^2\tilde{c}_S}, \tag{C.8}$$

9. $$\int_{-\infty}^{0} d\eta\; (1-ik_a\tilde{c}_S\eta)(1-ik_c\tilde{c}_S\eta)\; e^{i(K-2k_b)\tilde{c}_S\eta}$$
$$= -\frac{1}{i(2k_b-K)^3\tilde{c}_S}\left[(K-2k_b)(K+k_a-2k_b)+(K+2k_a-2k_b)k_c\right]. \tag{C.9}$$

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