Standard Model Fermions and Infinite-Dimensional R Symmetries

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Following up on our earlier work [K. A. Meissner and H. Nicolai, Phys. Rev. D **91**, 065029 (2015)] where we showed how to amend a scheme originally proposed by M. Gell-Mann to identify the 48 spin-$\frac{1}{2}$ fermions of $N = 8$ supergravity that remain after complete breaking of $N = 8$ supersymmetry with the 3×16 quarks and leptons of the standard model, we further generalize the construction to account for the full $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_w \times \mathrm{U}(1)_Y$ assignments, with an additional family symmetry $\mathrm{SU}(3)_f$. Our proposal relies in an essential way on embedding the SU(8) R symmetry of $N = 8$ supergravity into the (infinite-dimensional) "maximal compact" subgroup $K(E_{10})$ of the conjectured maximal duality symmetry E_{10}. As a by-product, it predicts fractionally charged and possibly strongly interacting massive gravitinos. It also indicates how E_{10} and $K(E_{10})$ can supersede supersymmetry as a guiding principle for unification.

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It is generally believed that neither the particle content nor the dynamics of maximal $N = 8$ supergravity [1,2] can be matched with the standard model (SM) of particle physics. In this Letter we reexamine this widely accepted wisdom on the basis of recent work [3,4], which further develops an old proposal of Gell-Mann's [5] to identify the 48 quarks and leptons of the SM with the 48 spin-$\frac{1}{2}$ fermions that remain after complete breaking of $N = 8$ supersymmetry. As shown in Ref. [3] the mismatch $\pm\frac{1}{6}$ of electric charges in the original Gell-Mann scheme can be rectified by introducing a new vectorlike generator $\mathcal{I}$ not contained in the SU(8) R symmetry group of $N = 8$ supergravity. However, in subsequent work [4] it was shown that this new generator does belong to $K(E_{10})$, the maximal compact subgroup of the hyperbolic Kac-Moody group E_{10} conjectured to be a symmetry underlying M theory [6,7], thus demonstrating the need for new ingredients *beyond* $N = 8$ supergravity. Here we take this construction one step further by exploring how the full $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_w \times \mathrm{U}(1)_Y$ symmetry of the SM might be incorporated into this scheme, together with an additional family symmetry $\mathrm{SU}(3)_f$ that does not commute with the electroweak symmetries.

As the present proposal differs substantially from current paradigms to derive the SM fermions from a Planck scale unified theory of quantum gravity, let us first explain our basic "philosophy." Our considerations here are based on the central conjecture of Ref. [6] according to which the full E_{10} symmetry conjectured to underlie M theory manifests itself in a "near singularity limit." This conjecture itself is based on a BKL-type analysis of cosmological singularities, where the causal decoupling of spatial points entails an effective dimensional reduction to one (time) dimension [8]. In this pregeometric regime, the spatial dependence of the fields is supposed to "deemerge" near the singularity, in the sense that it gets "spread" over the Lie algebra of E_{10}, as outlined in Ref. [6]. We emphasize, however, that explaining the emergence of a space-time based quantum field theory in this scheme, as well as explaining the emergence of space-time symmetries and the distinction between global and local symmetries, is still an unsolved problem.

Subsequent analysis of the fermionic sector of the theory [9] has revealed that the relevant symmetry acting on the fermions is the involutory ("maximal compact") subgroup $K(E_{10})$ of E_{10}, the conjectured R symmetry of M theory. Here as well, one restricts attention in a first step to the fermionic fields at a given spatial point, with the idea that the spatial dependence would emerge in a more complete description based on more faithful realizations of $K(E_{10})$. So far, however, only finite-dimensional *unfaithful* spinorial (double-valued) representations of $K(E_{10})$ are known [9,10], the two simplest of which are in one-to-one correspondence with the spinors of maximal supergravity. The action of $K(E_{10})$ on these representations can be conveniently described in terms of the quotient group $\mathfrak{G}_{\mathcal{R}} = K(E_{10})/\mathfrak{N}_{\mathcal{R}}$, where $\mathfrak{N}_{\mathcal{R}}$ is the normal subgroup associated with the ideal spanned by those $K(E_{10})$ generators that annihilate all elements of the given representation space $\mathcal{R}$. For the Dirac representation corresponding to the supersymmetry parameter we have $\mathfrak{G}_D = \mathrm{Spin}(32)/\mathbb{Z}_2$, while for the Rarita-Schwinger representation describing the physical fermionic degrees of freedom we have $\mathfrak{G}_{RS} = \mathrm{Spin}(288,32)/\mathbb{Z}_2$ [11]; the latter group also contains transformations acting chirally on the $N=8$ fermions. We expect there to exist bigger and less unfaithful representations which can possibly capture more of the spatial dependence [12].

Although the present scheme necessarily transcends $N=8$ supergravity, this theory nevertheless continues to play a crucial role in "guiding" our proposal, in that we insist on remaining compatible with the original scheme of Ref. [5], and with the known vacuum structure of gauged maximal supergravities with stationary points preserving a residual $\mathrm{SU}(3)\times\mathrm{U}(1)$ symmetry [13–16]. The group $\mathrm{SU}(3)_c\times U(1)_{em}$ is believed to be the gauge symmetry that survives to the lowest energies in the SM, but a naive identification of the supergravity SU(3) with the color group $\mathrm{SU}(3)_c$ does not work, as is immediately obvious from Eq. (6). For this reason, M. Gell-Mann introduced an additional family symmetry $\mathrm{SU}(3)_f$ that acts between the three particle families (generations) and proposed to identify the residual SU(3) of supergravity with the *diagonal subgroup* of color and family [5]. This scheme "almost" works in the sense that, after the removal of eight Goldstinos there is complete agreement of the SU(3) assignments, but there remains a systematic mismatch in that the U(1) charges of the supergravity fermions are systemically off by $\pm\frac{1}{6}$ from the electric charges of the quarks and leptons. As noticed in Ref. [3] this mismatch can be remedied by means of the extra generator (8) which "deforms" the $\mathrm{SU}(3)\times\mathrm{U}(1)$ group. Although this deformation is no longer contained in SU(8), it *is* contained in $K(E_{10})$ [4]. One notable feature here is that even though accompanied by a vast enlargement of the R symmetry, our proposal makes do with the original 56 spin-$\frac{1}{2}$ fermions and eight gravitinos, whereas more conventional schemes would require correspondingly larger multiplets for larger groups.

Accordingly, we now focus on the fermionic sector of $N=8$ supergravity, which consists of eight gravitinos ψ_μ^i transforming in the **8**, and a tri-spinor of spin-$\frac{1}{2}$ fermions χ^{ijk} transforming in the **56** of SU(8), where χ^{ijk} is fully antisymmetric in the SU(8) indices i, j, k. We adopt the conventions and notations of Ref. [2], where complex conjugation raises (or lowers) indices, such that, for instance, $\chi^{ijk}=(\chi_{ijk})^*$, and where upper (lower) position of the SU(8) indices indicates positive (negative) chirality. Hence the chiral SU(8) transformations act as $\chi^{ijk}\to U^i_l U^j_m U^k_n \chi^{lmn}, \chi_{ijk}\to U^l_i U^m_j U^n_k \chi_{lmn}$ with $U\in\mathrm{SU}(8)$, $U_i^j\equiv(U^i_j)^*$ and $U^i_k U^k_j=\delta^i_j$.

As already pointed out, a special role is played by the subgroup $\mathrm{SU}(3)\times\mathrm{U}(1)$, which is contained in the vector-like gauge group $\mathrm{SO}(8)\subset\mathrm{SU}(8)$ via the real embedding $\mathrm{SU}(3)\times\mathrm{U}(1)\subset\mathrm{SO}(6)\times\mathrm{SO}(2)\subset\mathrm{SO}(8)$ [5,14]. To study the relevant decompositions we introduce boldface indices $\mathbf{1},...,\mathbf{4}$, and barred indices $\bar{\mathbf{1}},\ldots,\bar{\mathbf{4}}$ for the conjugate representations as in Refs. [3,14], but now with unit normalization so that $v^{\mathbf{1}}\equiv 2^{-1/2}(v^1+iv^2)$, $v^{\mathbf{2}}\equiv 2^{-1/2}(v^3+iv^4)$, etc. and $v^{\bar{\mathbf{1}}}\equiv 2^{-1/2}(v^1-iv^2)$, $v^{\bar{\mathbf{2}}}\equiv 2^{-1/2}(v^3-iv^4)$, etc.

Similar rules apply to the fermions; for instance,

$$\varphi^{\mathbf{12\bar{4}}}\equiv\frac{1}{2\sqrt{2}}(\chi^{137}+i\chi^{237}+i\chi^{147}-i\chi^{138}-\chi^{247}+\chi^{238}+\chi^{148}+i\chi^{248}), \qquad (1)$$

and so on. It is important here that the spinor $\varphi^{\bar{\mathbf{1}}\bar{\mathbf{2}}\mathbf{4}}$ with barred and unbarred indices interchanged is *not* the complex conjugate of $\varphi^{\mathbf{12\bar{4}}}$ because the spinor components χ^{ijk} are themselves complex. Writing Eq. (1) in the schematic form $\varphi=S\circ\chi$ it is easy to see that S decomposes into 12 blocks of 2-by-2 matrices and four blocks of 8-by-8 matrices; see Ref. [17] for explicit formulas.

For the further analysis we single out the first three indices with labels $\mathbf{a},\mathbf{b},\ldots=\mathbf{1},\mathbf{2},\mathbf{3}$, on which the SU(3) subgroup acts. There is furthermore a two-parameter family of U(1) subgroups embedded as follows into SO(8)

$$Y(\alpha,\beta)=\begin{pmatrix} 0 & -\alpha & 0 & 0 & 0 & 0 & 0 & 0\\ \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -\alpha & 0 & 0 & 0 & 0\\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & -\alpha & 0 & 0\\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta\\ 0 & 0 & 0 & 0 & 0 & 0 & \beta & 0 \end{pmatrix}. \qquad (2)$$

This matrix commutes with $\mathrm{U}(3)\times\mathrm{U}(1)\subset\mathrm{SO}(8)$ for all α, β, and thus defines an $\mathrm{SU}(3)\times\mathrm{U}(1)$ subgroup of SO(8) for each choice of α and β. As shown in Ref. [5] we must take

$$\alpha=\frac{1}{6},\qquad \beta=\frac{1}{2}, \qquad (3)$$

in order to match the $N=8$ fermions with those of the standard model (and incidentally also in accord with the structure of $N=2$ AdS supermultiplets [14]). With this choice one easily reads off the $\mathrm{SU}(3)\times\mathrm{U}(1)$ assignments for the gravitinos

$$\psi_\mu^{\mathbf{a}}\in\left(\mathbf{3},\frac{1}{6}\right),\qquad \psi_\mu^{\bar{\mathbf{a}}}\in\left(\bar{\mathbf{3}},-\frac{1}{6}\right),$$
$$\psi_\mu^{\mathbf{4}}\in\left(\mathbf{1},\frac{1}{2}\right),\qquad \psi_\mu^{\bar{\mathbf{4}}}\in\left(\mathbf{1},-\frac{1}{2}\right). \qquad (4)$$

The 56 spin-$\frac{1}{2}$ fermions are split into $6+2$ Goldstinos

$$\varphi^{\mathbf{a}} \equiv \varphi^{\mathbf{a}4\bar{4}} \in \left(\mathbf{3}, \frac{1}{6}\right), \qquad \varphi^{\bar{\mathbf{a}}} \equiv \varphi^{\bar{\mathbf{a}}4\bar{4}} \in \left(\bar{\mathbf{3}}, -\frac{1}{6}\right),$$
$$\varphi^{4} \equiv \varphi^{\mathbf{abc}} \in \left(\mathbf{1}, \frac{1}{2}\right), \qquad \varphi^{\bar{4}} \equiv \varphi^{\bar{\mathbf{a}}\bar{\mathbf{b}}\bar{\mathbf{c}}} \in \left(\mathbf{1}, -\frac{1}{2}\right), \tag{5}$$

and the remaining 48 spin-$\frac{1}{2}$ fermions:

$$\varphi^{\mathbf{ab}4} \in \left(\bar{\mathbf{3}}, \frac{5}{6}\right), \quad \varphi^{\mathbf{ab}\bar{4}} \in \left(\bar{\mathbf{3}}, -\frac{1}{6}\right),$$
$$\varphi^{\bar{\mathbf{a}}\bar{\mathbf{b}}4} \in \left(\mathbf{3}, \frac{1}{6}\right), \quad \varphi^{\bar{\mathbf{a}}\bar{\mathbf{b}}\bar{4}} \in \left(\mathbf{3}, -\frac{5}{6}\right),$$
$$\varphi^{\mathbf{ab}\bar{\mathbf{c}}} \in \left(\mathbf{3}, \frac{1}{6}\right) \oplus \left(\bar{\mathbf{6}}, \frac{1}{6}\right), \quad \varphi^{\bar{\mathbf{a}}\bar{\mathbf{b}}\mathbf{c}} \in \left(\bar{\mathbf{3}}, -\frac{1}{6}\right) \oplus \left(\mathbf{6}, -\frac{1}{6}\right),$$
$$\varphi^{\mathbf{a}\bar{\mathbf{b}}4} \in \left(\mathbf{8}, \frac{1}{2}\right) \oplus \left(\mathbf{1}, \frac{1}{2}\right), \quad \varphi^{\mathbf{a}\bar{\mathbf{b}}\bar{4}} \in \left(\mathbf{8}, -\frac{1}{2}\right) \oplus \left(\mathbf{1}, -\frac{1}{2}\right). \tag{6}$$

To be sure, in order to properly identify the Goldstinos, we must allow for a possible mixing between the representations with the same $\mathrm{SU}(3) \times \mathrm{U}(1)$ content, as is also obvious from the mass eigenstates at the $\mathrm{SU}(3) \times \mathrm{U}(1)$ stationary point [14]. However, the precise form of the mixing depends on the dynamics which is expected to deviate from $N = 8$ supergravity and which at this point is unknown. It is an essential assumption here that even with such mixing we can bring the fermions to the form with the labeling exhibited above, possibly by means of a chiral redefinition of the fermions. We then assume that all supersymmetries are broken at a high scale, such that all gravitinos acquire a very large mass.

Now, as first pointed out in Ref. [5], with the above choice of Goldstinos the three generations of SM fermions can be matched with the remaining 48 spin-$\frac{1}{2}$ fermions from Eq. (6), provided one identifies the supergravity SU(3) with the diagonal subgroup of color $\mathrm{SU}(3)_c$ and a new family symmetry $\mathrm{SU}(3)_f$, viz.,

$$\mathrm{SU}(3) \equiv [\mathrm{SU}(3)_c \times \mathrm{SU}(3)_f]_{\mathrm{diag}}, \tag{7}$$

and furthermore allows for a spurion charge $\pm\frac{1}{6}$ to correct the U(1) charges in Eq. (6) so as to recover the known electric charges of quarks and leptons. More specifically, the assignments are as follows: [5]

$$
\begin{array}{llr}
(u,c,t)_L & \mathbf{3}_c \times \bar{\mathbf{3}}_f \to \mathbf{8} \oplus \mathbf{1} & \frac{2}{3} = \frac{1}{2} + q \\
(\bar{u},\bar{c},\bar{t})_L & \bar{\mathbf{3}}_c \times \mathbf{3}_f \to \mathbf{8} \oplus \mathbf{1} & -\frac{2}{3} = -\frac{1}{2} - q \\
(d,s,b)_L & \mathbf{3}_c \times \mathbf{3}_f \to \mathbf{6} \oplus \bar{\mathbf{3}} & -\frac{1}{3} = -\frac{1}{6} - q \\
(\bar{d},\bar{s},\bar{b})_L & \bar{\mathbf{3}}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{6}} \oplus \mathbf{3} & \frac{1}{3} = \frac{1}{6} + q \\
(\nu_e,\nu_\mu,\nu_\tau)_L & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}} & 0 = -\frac{1}{6} + q \\
(\bar{\nu}_e,\bar{\nu}_\mu,\bar{\nu}_\tau)_L & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3} & 0 = \frac{1}{6} - q \\
(e^-,\mu^-,\tau^-)_L & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3} & -1 = -\frac{5}{6} - q \\
(e^+,\mu^+,\tau^+)_L & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}} & 1 = \frac{5}{6} + q,
\end{array}
$$

where we made use of the fact that right-chiral particles can be equivalently described by their left-chiral antiparticles, and where the spurion shift is $q = \frac{1}{6}$ with alternating signs between triplets and antitriplets of $\mathrm{SU}(3)_f$.

The main advance reported in Ref. [3] was to show that with the representation (6) of the $N = 8$ spin-$\frac{1}{2}$ fermions the spurion shift can be accounted for by means of the $\mathrm{U}(1)_q$ transformations $\exp(\frac{1}{6}\omega\mathcal{I})$, where

$$\mathcal{I} := \frac{1}{2}(T \otimes \mathbf{1} \otimes \mathbf{1} + \mathbf{1} \otimes T \otimes \mathbf{1} + \mathbf{1} \otimes \mathbf{1} \otimes T + T \otimes T \otimes T), \tag{8}$$

with $T \equiv Y(1,1)$ [cf. (2)]. Because T acts as $+i$ (or $-i$) on an unbarred (barred) index it is easy to see that $\mathcal{I}$ gives $(+i)$ on φ's with no or only one barred index, and $(-i)$ on φ's with two or three barred indices. Importantly, the term with $T \otimes T \otimes T$, and hence the generator $\mathcal{I}$ are *not* elements of SU(8). However, in Ref. [4] it was shown that this new generator does belong to $K(E_{10})$, and that furthermore the action of the generator $T \otimes T \otimes T$ can be extended to the gravitinos, with $\mathcal{I} \circ \psi = T \circ \psi$. Therefore the action of $\frac{1}{6}\mathcal{I}$ adds $\pm\frac{1}{6}$ to the U(1) charges of the gravitinos in Eq. (4). Consequently, $\psi_\mu^{\mathbf{a}}$ has electric charge $\frac{1}{3}$, and ψ_μ^{4} has electric charge $\frac{2}{3}$, consistent with the assignments (5); $\psi_\mu^{\bar{\mathbf{a}}}$ and $\psi_\mu^{\bar{4}}$ have the opposite electric charges.

While the new generator $\mathcal{I}$ commutes with the original $\mathrm{SU}(3) \times \mathrm{U}(1)$, and, therefore, merely deforms it but does not enlarge it, this is not so with the remaining generators of SU(8). As shown in Ref. [11], repeated commutation of $\mathcal{I}$ with the elements of SU(8) generates a much bigger group acting on χ^{ijk}, namely, SU(56). Because $\mathcal{I} \in K(E_{10})$, the latter group is also contained in $K(E_{10})$, and should, consequently, be viewed as a subgroup of the quotient group $\mathfrak{G}_{RS} = \mathrm{Spin}(288,32)/\mathbb{Z}_2$. The fact that this SU(56) group acts chirally motivates us to look for a realization of the full SM symmetries within $K(E_{10})$.

To this aim we now "take the tri-spinor apart" into its $8+48$ components, and write out the correspondence more explicitly for the left-chiral particles

$$\begin{aligned} U_L^{\mathrm{a}\bar{\mathfrak{a}}} &\equiv (u^{\mathrm{a}}, c^{\mathrm{a}}, t^{\mathrm{a}})_L \equiv (\varphi^{\mathrm{a}\bar{\mathbf{1}}\mathbf{4}}, \varphi^{\mathrm{a}\bar{\mathbf{2}}\mathbf{4}}, \varphi^{\mathrm{a}\bar{\mathbf{3}}\mathbf{4}}), \\ D_L^{\mathrm{a}\mathfrak{a}} &\equiv (d^{\mathrm{a}}, s^{\mathrm{a}}, b^{\mathrm{a}})_L \equiv (\varphi^{\mathrm{a}\bar{\mathbf{2}}\bar{\mathbf{3}}}, \varphi^{\mathrm{a}\bar{\mathbf{3}}\bar{\mathbf{1}}}, \varphi^{\mathrm{a}\bar{\mathbf{1}}\bar{\mathbf{2}}}), \\ N_L^{\bar{\mathfrak{a}}} &\equiv (\nu_e, \nu_\mu, \nu_\tau)_L \equiv (\varphi^{\mathbf{23}\bar{\mathbf{4}}}, \varphi^{\mathbf{31}\bar{\mathbf{4}}}, \varphi^{\mathbf{12}\bar{\mathbf{4}}}), \\ E_L^{\mathfrak{a}} &\equiv (e^-, \mu^-, \tau^-)_L \equiv (\varphi^{\bar{\mathbf{2}}\bar{\mathbf{3}}\bar{\mathbf{4}}}, \varphi^{\bar{\mathbf{3}}\bar{\mathbf{1}}\bar{\mathbf{4}}}, \varphi^{\bar{\mathbf{1}}\bar{\mathbf{2}}\bar{\mathbf{4}}}), \end{aligned} \tag{9}$$

and for the left-chiral antiparticles,

$$\begin{aligned} \bar{U}_L^{\bar{\mathrm{a}}\mathfrak{a}} &\equiv (\bar{u}^{\bar{\mathrm{a}}}, \bar{c}^{\bar{\mathrm{a}}}, \bar{t}^{\bar{\mathrm{a}}})_L \equiv (\varphi^{\bar{\mathrm{a}}\mathbf{1}\bar{\mathbf{4}}}, \varphi^{\bar{\mathrm{a}}\mathbf{2}\bar{\mathbf{4}}}, \varphi^{\bar{\mathrm{a}}\mathbf{3}\bar{\mathbf{4}}}), \\ \bar{D}_L^{\bar{\mathrm{a}}\bar{\mathfrak{a}}} &\equiv (\bar{d}^{\bar{\mathrm{a}}}, \bar{s}^{\bar{\mathrm{a}}}, \bar{b}^{\bar{\mathrm{a}}})_L \equiv (\varphi^{\bar{\mathrm{a}}\mathbf{23}}, \varphi^{\bar{\mathrm{a}}\mathbf{31}}, \varphi^{\bar{\mathrm{a}}\mathbf{12}}), \\ \bar{N}_L^{\mathfrak{a}} &\equiv (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)_L \equiv (\varphi^{\bar{\mathbf{2}}\bar{\mathbf{3}}\mathbf{4}}, \varphi^{\bar{\mathbf{3}}\bar{\mathbf{1}}\mathbf{4}}, \varphi^{\bar{\mathbf{1}}\bar{\mathbf{2}}\mathbf{4}}), \\ \bar{E}_L^{\bar{\mathfrak{a}}} &\equiv (e^+, \mu^+, \tau^+)_L \equiv (\varphi^{\mathbf{234}}, \varphi^{\mathbf{314}}, \varphi^{\mathbf{124}}). \end{aligned} \tag{10}$$

We emphasize again that the spinors (10) are not simply complex conjugates of (9), but independent fields (as would be completely obvious if we had written them as right-chiral particles). Thus, we now have color indices $\mathrm{a}, \mathrm{b}, \ldots \equiv \mathbf{1}, \mathbf{2}, \mathbf{3}$ and family indices $\mathfrak{a}, \mathfrak{b}, \ldots \equiv \mathbf{1}, \mathbf{2}, \mathbf{3}$ (or $([\bar{\mathbf{2}}\,\bar{\mathbf{3}}], [\bar{\mathbf{3}}\,\bar{\mathbf{1}}], [\bar{\mathbf{1}}\,\bar{\mathbf{2}}])$), thereby disentangling the diagonal SU(3) subgroup into its factors $\mathrm{SU}(3)_c$ and $\mathrm{SU}(3)_f$. Accordingly, we stipulate that $\mathrm{SU}(3)_c$ acts on indices $\mathrm{a}, \mathrm{b}, \ldots$ as $D^{\mathrm{a}\mathfrak{a}} \to i\lambda^{\mathrm{a}}_{\mathrm{b}} D^{\mathrm{b}\mathfrak{a}}$, $U^{\mathrm{a}\bar{\mathfrak{a}}} \to i\lambda^{\mathrm{a}}_{\mathrm{b}} U^{\mathrm{b}\bar{\mathfrak{a}}}$, where λ is any of the (Hermitean) Gell-Mann matrices. Similarly, the $\mathrm{SU}(3)_f$ acts on the indices $\mathfrak{a}, \mathfrak{b}, \ldots$, such that for instance $D^{\mathrm{a}\mathfrak{a}} \to i\lambda^{\mathfrak{a}}_{\mathfrak{b}} D^{\mathrm{a}\mathfrak{b}}$ and $U^{\mathrm{a}\bar{\mathfrak{a}}} \to -i(\lambda^*)^{\bar{\mathfrak{a}}}_{\bar{\mathfrak{b}}} U^{\mathrm{a}\bar{\mathfrak{b}}}$. The indices $\mathbf{4}$ and $\bar{\mathbf{4}}$ are unaffected by these SU(3) actions.

For the electroweak $\mathrm{SU}(2)_w$ we treat $(U, D)_L$ and $(N, E)_L$ as doublets, viz.

$$\begin{pmatrix} U^{\mathrm{a}\bar{\mathfrak{a}}} \\ D^{\mathrm{a}\mathfrak{a}} \end{pmatrix} \to i\boldsymbol{\omega} \cdot \boldsymbol{\tau} \begin{pmatrix} U^{\mathrm{a}\bar{\mathfrak{a}}} \\ D^{\mathrm{a}\mathfrak{a}} \end{pmatrix}, \qquad \begin{pmatrix} N^{\bar{\mathfrak{a}}} \\ E^{\mathfrak{a}} \end{pmatrix} \to i\boldsymbol{\omega} \cdot \boldsymbol{\tau} \begin{pmatrix} N^{\bar{\mathfrak{a}}} \\ E^{\mathfrak{a}} \end{pmatrix}, \tag{11}$$

with the Pauli isospin matrices $\boldsymbol{\tau} \equiv (\tau^1, \tau^2, \tau^3)$ and isospin parameters $\boldsymbol{\omega}$. By contrast, the left-chiral antiparticles $(\bar{U}, \bar{D}, \bar{N}, \bar{E})$ are treated as $\mathrm{SU}(2)_w$ *singlets*, and this is perfectly consistent because these spinors are independent, as we already said. There is no need to discuss $\mathrm{U}(1)_Y$ separately, as the associated hypercharges can be recovered from the standard formula $Q = T^3 + Y$. It is an unaccustomed feature that the electroweak $\mathrm{SU}(2)_w$ does *not* commute with $\mathrm{SU}(3)_f$, as the upper and lower components of the electroweak doublets are assigned to opposite representations of $\mathrm{SU}(3)_f$, such that the $\mathrm{SU}(3)_f$ is realized via the block matrix

$$\begin{pmatrix} (\mathcal{U}^*)^{\bar{\mathfrak{a}}}_{\bar{\mathfrak{b}}} & 0 \\ 0 & \mathcal{U}^{\mathfrak{a}}_{\mathfrak{b}} \end{pmatrix}, \qquad \mathcal{U} \in \mathrm{SU}(3)_f \tag{12}$$

in the electroweak isospin space. This prescription is reminiscent of old attempts to merge the spin rotation group SU(2) with the flavor symmetry SU(3) into SU(6), but the groups $\mathrm{SU}(2)_w$ and $\mathrm{SU}(3)_f$ are expected to be simultaneously realized as noncommuting symmetries only in the phase with unbroken $K(E_{10})$, which does not admit a local realization within space-time based quantum field theory (this may explain why attempts at a group theoretical family unification have not met with success so far). We recall that the action of E_{10} on the bosonic fields likewise does not admit a local realization [6].

While obviously motivated by the known symmetry assignments of quarks and leptons the above construction does not tell us how these symmetries act on the gravitinos and Goldstinos, and more specifically, whether the SU(3) acting on them should be identified with $\mathrm{SU}(3)_c$ or $\mathrm{SU}(3)_f$. It is clear that for consistency this action must be the same on (4) and (5). It then appears reasonable to demand the absence of gauge anomalies for quarks and leptons to persist with these fields. The simplest choice ensuring this is to assign (4) and (5) to transform as $\mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{1} \oplus \bar{\mathbf{1}}$ of $\mathrm{SU}(3)_c$, but as singlets under electroweak $\mathrm{SU}(2)_w$ and $\mathrm{SU}(3)_f$. With this choice we are led to predict fractionally charged and strongly interacting massive gravitinos, implying a novel menagerie of exotic dark matter candidates, possibly even fractionally charged massive bound states of gravitinos and quarks. Otherwise, the gravitinos would have to be assigned to chiral representations of $\mathrm{SU}(2)_w$ (in which case the largest anomaly-free subgroup acting chirally on the gravitinos is indeed $\mathrm{SU}(2) \times \mathrm{U}(1)$ [18]) and could acquire masses only jointly with electroweak symmetry breaking.

Having realized the action of all SM symmetries on all fermions, we can now express these actions directly in terms of the original tri-spinor χ^{ijk}, that is, in the form

$$\chi^{ijk} \to M^{ijk}_{lmn}(G)\chi^{lmn}, \tag{13}$$

with

$$M(G) = S^{-1} \circ G \circ S \in \mathrm{SU}(56), \tag{14}$$

where $G \in \mathrm{SU}(3)_c \times \mathrm{SU}(2)_w \times \mathrm{U}(1)_Y$ or $\in \mathrm{SU}(3)_f$. The matrix $M(G)$ is real [that is, $M \in \mathrm{SO}(56)$] for the vectorlike rotations in $\mathrm{SU}(3)_c$, $\mathrm{SU}(3)_f$, and $\mathrm{U}(1)_{em}$, and complex (but still unitary) for chiral electroweak rotations; more generally, transformations that do not mix barred and unbarred indices are vectorlike, whereas transformations that do mix them are chiral. As there is no space here to present a complete list of 56-by-56 matrices we have collected explicit formulas in Ref. [17].

Obviously, $K(E_{10})$ is humongously larger than any symmetry so far considered for fermion unification, and it is therefore all the more remarkable that it admits a realization on precisely 48 spin-$\frac{1}{2}$ fermions and eight massive gravitinos. This enlargement beyond SU(8) hinges crucially on the presence of timelike imaginary roots in E_{10} (which have no analog in finite dimensional Lie groups), and thus directly on the hyperbolicity of E_{10} [11]. However, like for the bosonic sector [6], these considerations are limited, at least for the time being, to one given spatial point, and therefore the main challenge remains to incorporate the spatial dependence, and to explain how $K(E_{10})$ "unfolds" in terms of bigger and less unfaithful representations to give rise to emergent space-time and gauge symmetries. A realization of the present scheme would evidently require part of $K(E_{10})$ to become dynamical (e.g., with composite vector bosons $W^{\pm}$ and Z), in a partial realization of a conjecture already made in Ref. [1] for the R symmetry SU(8), but now for a much bigger group, and with a very different mechanism for extracting space-time fields. Similarly, one may ask why the theory should pick a vacuum where $K(E_{10})$ is broken to the SM symmetries in the way described above; we note, however, that the necessity of canceling SM gauge anomalies provides an extremely constraining criterion for vacuum selection and symmetry breaking. Finally, there is the question how supersymmetry is ultimately disposed of: is it actually there and only broken by some clever mechanism, or is it altogether absent even at the most fundamental level? Indeed, there is evidence that (conventional) supersymmetry is incompatible with $K(E_{10})$ precisely because of the presence of imaginary roots [19]. This could imply that supersymmetry in particle physics may remain a chimera, and even in the context of maximal $N = 8$ supergravity not more than a secondary manifestation of a deeper and more powerful underlying duality symmetry.

While much effort will be required to validate the present proposal (assuming there is any truth in it), it offers three main advantages in comparison with other schemes for fermion unification: (i) it can explain the SM fermion spectrum as is, without $N = 1$ superpartners or other exotica, (ii) it confirms, and is compatible *only* with the existence of three generations of SM fermions, and (iii) it can be directly and immediately falsified by the detection of any new fundamental spin-$\frac{1}{2}$ fermion, be it via the discovery of low energy $N = 1$ supersymmetry or of new sterile fermions. Our proposal underlines the potential importance of the infinite-dimensional exceptional algebras E_{10} and $K(E_{10})$, and shows how these so far elusive structures might supplant supersymmetry as a guiding principle for unification, and how they might become essential for making contact with SM physics.

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