COUPLING THE SO(2) SUPERGRAVITY THROUGH DIMENSIONAL REDUCTION

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We construct the pure supergravity theory in five dimensions which after dimensional reduction gives SO(2) supergravity coupled to an SO(2) vector matter multiplet.

Already some years ago, the pure extended SO(2) supergravity theory was constructed by different methods [1]. Also, the coupling of this theory to SO(2) matter was found by application of the Noether method [2] and by using U(1) invariance [3]. The subsequent discovery of the auxiliary fields and a corresponding tensor calculus for $N = 2$ [4] made it possible to recover the results of ref. [2] in a more systematic fashion. Similarly, the superspace approach should lead to a full understanding of the coupling between matter and gravity [5]. However, although these methods, in principle, enable us to write down even more general models involving SO(2) multiplets, this has, to date, not been done, presumably because of the large number of auxiliary fields that have to be introduced [4].

In this paper we intend to construct an explicit lagrangian coupling SO(2) gravity to the SO(2) vector multiplet (which contains one vector, two Majorana spinors (= one Dirac spinor) and two scalars). Our method of construction is that of Cremmer et al. [6]. We first derive the action and transformation laws in five dimensions and then dimensionally reduce [6] the theory to four dimensions, thereby obtaining the desired lagrangian. In many respects this theory can be used as a toy model for SO(8) supergravity to gain familiarity with it.

This method has the advantage of being rather straightforward and we expect that our results can be rederived by use of the above-mentioned methods and are analogous to ref. [3]. As in $D = 11$, the field content of the five-dimensional theory turns out to be surprisingly simple: a fünfbein, which upon dimensional reduction yields a vierbein e_μ^α, a vector B_μ^5 and a scalar [‡1] $e_{\dot{5}}{}^5 \equiv \exp P$; a five-dimensional vector A_M subject to the gauge transformation

$$\delta A_M = \partial_M \Lambda\,, \tag{1}$$

which gives another four-dimensional vector A_μ and a scalar $A_{\dot 5}$; and finally, a *Dirac* spinor ψ_M which is decomposed into a Dirac gravitino ψ_μ and an ordinary Dirac spinor $\psi_{\dot 5}$. It is easy to see that the physical degrees of freedom are just those of one SO(2) gravity and one SO(2) vector multiplet.

Our conventions are the same as in ref. [6]: as the metric, we take (+– – – –), and by

$$\Gamma^{A_1 \dots A_k} \equiv \Gamma^{[A_1} \cdots \Gamma^{A_k]} \tag{2}$$

we denote the fully antisymmetrized product of the 4 × 4 Dirac matrices $\Gamma^{A_1}, \dots, \Gamma^{A_k}$. In particular,

$$\Gamma^5 = -\Gamma_5 \equiv \mathrm{i}\gamma^5 = \mathrm{i}\gamma_5 \tag{3}$$

is anti-hermitean. Tangent space indices are from the beginning of the alphabet while curved ones are from the

[‡1] We use the notation $\dot 5$ for the curved internal index and 5 for the flat one.

middle.

The lagrangian we find reads

$$\mathcal{L} = -\tfrac{1}{4}eR(\omega) - \tfrac{1}{2}\mathrm{i}e[\bar\psi_M\Gamma^{MNP}\mathrm{D}_N((3\omega-\hat\omega)/2)\,\psi_P + \bar\psi_P\overleftarrow{\mathrm{D}}_N((3\omega-\hat\omega)/2)\,\Gamma^{MNP}\psi_M] - \tfrac{1}{4}eF_{MN}F^{MN}$$
$$-\tfrac{1}{8}\sqrt{3}\,\mathrm{i}e\,\bar\psi_M X^{MNPQ}\psi_N(F_{PQ}+\hat F_{PQ}) - (6\sqrt{3})^{-1}\epsilon^{MNPQR}F_{MN}F_{PQ}A_R\,. \tag{4}$$

The quartic terms have been absorbed into the supercovariant fields $\hat\omega, \hat F$ to be defined below. We have used the following definitions:

$$\mathrm{D}_N\psi_P = \partial_N\psi_P + \tfrac{1}{4}\omega_{NAB}\Gamma^{AB}\psi_P\,, \qquad \omega_{MAB} = \omega^0{}_{MAB}(e) + \kappa_{MAB}\,, \tag{5}$$

with the contorsion tensor

$$\kappa_{MAB} = -\tfrac{1}{2}\mathrm{i}\bar\psi_Q\Gamma^{QR}{}_{MAB}\psi_R + \tfrac{1}{2}\mathrm{i}[(\bar\psi_M\Gamma_B\psi_A - \bar\psi_A\Gamma_B\psi_M) + (\bar\psi_B\Gamma_M\psi_A - \bar\psi_A\Gamma_M\psi_B) - (\bar\psi_M\Gamma_A\psi_B - \bar\psi_B\Gamma_A\psi_M)] \tag{6}$$

The field strength tensor is

$$F_{MN} = \partial_M A_N - \partial_N A_M\,, \tag{7}$$

and we have also introduced the notation

$$X^{MNPQ} = \Gamma^{MNPQ} + g^{MP}g^{NQ} - g^{MQ}g^{NP}\,. \tag{8}$$

The lagrangian (4) is invariant under the following supersymmetry transformations:

$$\delta e_M{}^A = -\mathrm{i}\bar\epsilon\Gamma^A\psi_M + \mathrm{i}\bar\psi_M\Gamma^A\epsilon\,, \qquad \delta\psi_M = \hat{\mathrm{D}}_M(\hat\omega)\,\epsilon \equiv \mathrm{D}_M(\hat\omega)\,\epsilon + (4\sqrt{3})^{-1}(\Gamma_M{}^{PQ} - 4\delta_M^P\Gamma^Q)\,\epsilon\hat F_{PQ}\,,$$
$$\delta A_M = \tfrac{1}{2}\mathrm{i}\sqrt{3}(\bar\epsilon\psi_M - \bar\psi_M\epsilon)\,. \tag{9}$$

We are using second order formalism throughout, so ω_{MAB} is obtained from its equation of motion. Its supercovariant extension is given by

$$\hat\omega_{MAB} \equiv \omega_{MAB} + \tfrac{1}{2}\mathrm{i}\bar\psi_Q\Gamma^{QR}{}_{MAB}\psi_R\,. \tag{10}$$

The supercovariant field strength is

$$\hat F_{MN} = F_{MN} + \tfrac{1}{2}\mathrm{i}\sqrt{3}(\bar\psi_M\psi_N - \bar\psi_N\psi_M)\,, \tag{11}$$

and one readily verifies that

$$\delta\hat F_{AB} = -\mathrm{i}\sqrt{3}\,e_{[A}{}^M e_{B]}{}^N(\bar\epsilon\hat{\mathrm{D}}_M\psi_N - \hat{\mathrm{D}}_N\bar\psi_M\epsilon)\,, \tag{12}$$

where $\hat{\mathrm{D}}_M\psi_N$ is defined analogously to $\delta\psi_M = \hat{\mathrm{D}}_M\epsilon$. To obtain the action and transformation laws we have mimicked the procedure of ref. [6]: the coefficients in the transformation rules, as well as the form of the $\bar\psi X\psi F$ interaction, are determined by requiring the closure of the algebra on the Bose fields and the cancellation of $\bar\epsilon\psi F$ terms. To obtain a full cancellation of all terms of type $\bar\epsilon\psi F^2$ we are forced to add a term proportional to

$$\epsilon^{MNPQR}F_{MN}F_{PQ}A_R\,, \tag{13}$$

again in complete analogy with the eleven-dimensional theory [6]. The quartic terms are then fixed so as to reproduce the supercovariant equation of motion

$$\Gamma^{MNP}\hat{\mathrm{D}}_N(\hat\omega)\,\psi_P = 0\,. \tag{14}$$

This is a test of consistency and involves the following Fierz rearrangement identity:

$$g^{R[N}\Gamma^{MPQ]}\psi_N\cdot\bar\psi_P\Gamma_R\psi_Q=\tfrac14(\Gamma^{MNPQ}\psi_N\cdot\bar\psi_P\psi_Q-\psi_N\cdot\bar\psi_P\Gamma^{MNPQ}\psi_Q)\,. \tag{15}$$

The next step in our construction is the dimensional reduction which allows us to make some contact with the real world and to identify the physical fields of the model. In the simplest scheme, one assumes independence of the fifth coordinate and, again, our derivation is analogous to the one that has been carried out for the $N = 8$ theory by Cremmer and Julia [7].

Using the non-diagonal part of local SO(1,4), we gauge away the field $e_{\dot 5}^{\alpha}$, so the fünfbein assumes the form

$$e_M{}^A=\begin{pmatrix} e_\mu{}^\alpha & B_\mu{}^5\\ 0 & e_{\dot 5}{}^5=\exp P\end{pmatrix},\qquad e_A{}^M=\begin{pmatrix} e_\alpha{}^\mu & -B_\alpha{}^{\dot 5}\\ 0 & e_5{}^{\dot 5}=\exp -P\end{pmatrix}, \tag{16}$$

where the scalar field has been written as $\exp P$ for later convenience.

After the following redefinitions and Weyl rescalings [7] for the bosonic fields:

$$\begin{aligned}
&e_{4\mu}{}^{\alpha}=\exp(\tfrac12 P)\,e_\mu{}^\alpha\,,\quad e_{4\alpha}{}^{\mu}=\exp(-\tfrac12 P)\,e_\alpha{}^\mu\,,\\
&G_{4\alpha\beta}{}^5=2\exp(P)\,e_{4[\alpha}{}^{\mu}e_{4\beta]}{}^{\nu}\partial_\mu[\exp(-P)\,B_\nu{}^5]\,,\\
&A'_\mu=A_\mu-B_\mu{}^5A_5\,,\quad A_5=e_5{}^{\dot 5}A_{\dot 5}\,,\\
&F'_{\mu\nu}=2\partial_{[\mu}A'_{\nu]}\,,\quad F_{4\mu\nu}=F'_{\mu\nu}+G_{\mu\nu}{}^5A_5\,,\quad F_{\alpha\beta}=\exp(P)\,F_{4\alpha\beta}\,,
\end{aligned} \tag{17}$$

we obtain the reduced lagrangian

$$\begin{aligned}
\mathcal{L}_{\rm bosonic}=&-\tfrac14 e_4R_4(\omega_0)+\tfrac38 e_4(\partial_{4\alpha}P)^2+\tfrac1{16}e_4(G_{4\alpha\beta}{}^5)^2\\
&-\tfrac14 e_4[\exp(P)\,F_{4\mu\nu}F_{4\rho\sigma}g_4{}^{\mu\rho}g_4{}^{\nu\sigma}+2g_4{}^{\dot5\dot5}\,\partial_\mu A_{\dot5}\,\partial_\nu A_{\dot5}\,g_4{}^{\mu\nu}]\\
&-(2\sqrt3)^{-1}\exp(P)\,\epsilon^{\mu\nu\rho\sigma}(F_{4\mu\nu}F_{4\rho\sigma}+F_{4\mu\nu}G_{\rho\sigma}{}^5A_5-\tfrac13 G_{\mu\nu}{}^5G_{\rho\sigma}{}^5A_5^2)\,A_5\,.
\end{aligned} \tag{18}$$

For the fermionic sector we need the following redefinitions:

$$\chi_4=\psi_{\dot5}e_5{}^{\dot5}\exp(\tfrac14 P)\,,\quad \chi_{4\alpha}=\psi_Me_\alpha{}^M\exp(-\tfrac14 P)\,,\quad \psi_{4\mu}=e_{4\mu}{}^{\alpha}(\chi_{4\alpha}-\tfrac12\Gamma^5\gamma_\alpha\chi_4)\,. \tag{19}$$

For instance, the last redefinition diagonalizes the kinetic term for the fermions which now takes the form

$$-\tfrac12 ie_4[\bar\psi_{4\mu}\gamma^{\mu\nu\rho}\mathrm{D}_{4\nu}(\omega_0)\,\psi_{4\rho}+\bar\psi_{4\rho}\overleftarrow{\mathrm{D}}_{4\nu}(\omega^0)\,\gamma^{\mu\nu\rho}\psi_{4\mu}+\tfrac32\bar\chi_4\gamma^\mu\mathrm{D}_{4\mu}(\omega_0)\chi_4+\tfrac32\bar\chi_4\overleftarrow{\mathrm{D}}_{4\mu}\gamma^\mu\chi_4]\,. \tag{20}$$

The interaction terms arise from

$$-\tfrac18 ie\bar\psi_M\{\Gamma^{MNP},\Gamma^{AB}\}\,\psi_P\omega^0{}_{NAB}-\tfrac14\sqrt3 ie\bar\psi_MX^{MNPQ}\psi_NF_{PQ}\,, \tag{21}$$

which leads to the reduced vector interaction

$$\begin{aligned}
\mathcal{L}_{\rm V}^{\rm int}=&\,ie_4\exp(\tfrac12 P)(-\tfrac12\sqrt3F_{4\alpha\beta}+\tfrac14\tilde G_{\alpha\beta})\cdot[(\bar\psi_4{}^\alpha\psi_4{}^\beta-\tfrac12\bar\chi_4\gamma^\beta\Gamma^5\psi_4{}^\alpha+\tfrac12\bar\psi_4{}^\alpha\Gamma^5\gamma^\beta\chi_4)\\
&-\tfrac12\epsilon^{\alpha\beta\gamma\delta}(\bar\psi_{4\gamma}\Gamma_5\psi_{4\delta}-\tfrac12\bar\chi_4\gamma_\delta\psi_{4\gamma}+\tfrac12\bar\psi_{4\gamma}\gamma_\delta\chi_4)]\,.
\end{aligned} \tag{22}$$

The reduction of the quartic terms is straightforward but tedious, and we have only checked that the spin 3/2 quartic terms coincide with those of pure SO(2) supergravity.

We are now able to identify the SO(2) submultiplets. A non-trivial mixing, besides the one in (19) between spin 3/2 and spin 1/2, only occurs for the vector fields where the two multiplets "overlap". We find

$$\mathcal{F}_{\mu\nu}(\text{gravity})=-\tfrac12\sqrt3F_{4\mu\nu}+(2/e_4)\,\widetilde{\delta\mathcal{L}_{\rm V}/\delta G_4{}^{\mu\nu}}\,,\quad \mathcal{G}_{\mu\nu}(\text{vector})=\tfrac12F_{4\mu\nu}+(2\sqrt3/e_4)\,\widetilde{\delta\mathcal{L}_{\rm V}/\delta G_4{}^{\mu\nu}}\,, \tag{23}$$

where

$$(2/e_4)\,\widetilde{\delta \mathcal{L}_V/\delta G_4{}^{\alpha\beta}} = \tfrac{1}{4}\tilde{G}_{4\alpha\beta} + \tfrac{1}{2}\mathrm{i}[(\bar{\psi}_{4\alpha}\psi_{4\beta} - \tfrac{1}{2}\bar{\chi}_4\gamma_\beta\Gamma^5\psi_{4\alpha} + \tfrac{1}{2}\bar{\psi}_{4\alpha}\Gamma^5\gamma^\beta\chi_4)$$
$$+ \tfrac{1}{2}\epsilon_{\alpha\beta\gamma\delta}(\bar{\psi}_4{}^\gamma\Gamma_5\psi_4{}^\delta - \tfrac{1}{2}\bar{\chi}_4\gamma^\delta\psi_4{}^\gamma + \tfrac{1}{2}\bar{\psi}_4{}^\gamma\gamma^\delta\chi_4)]\,. \tag{24}$$

A curious, although not unexpected, feature is that the identification (23) is possible *on shell only* because it is only there that $\mathcal{F}_{\mu\nu}$ and $\mathcal{G}_{\mu\nu}$ can be expressed as curls of vector fields. The duality transformation (23) automatically leads to the correct sign for the kinetic terms of the vector fields because

$$-\tfrac{1}{4}(\mathcal{F}^2_{\mu\nu} + \mathcal{G}^2_{\mu\nu}) = -\tfrac{1}{4}F^2_{4\mu\nu} + \tfrac{1}{16}(G_{4\mu\nu}{}^5)^2\,. \tag{25}$$

(This is also true for the $N = 8$ theory [7].)

The supersymmetry transformation rules become, after the reduction (dropping indices 4, so, e.g., $e_4 \equiv e$, etc.)

$$\delta e_\mu{}^\alpha = -\mathrm{i}\bar{\epsilon}\gamma^\alpha\psi_\mu + \mathrm{i}\bar{\psi}_\mu\gamma^\alpha\epsilon + \Omega^\alpha{}_\beta e_\mu{}^\beta\,,$$
$$\Omega^\alpha{}_\beta = \text{compensating gauge rotation}\,,\quad \delta A_5 = -\tfrac{1}{2}\mathrm{i}\sqrt{3}\,(\bar{\epsilon}\chi - \bar{\chi}\epsilon)\exp(-P)\,,\quad \delta P = -\mathrm{i}(\bar{\epsilon}\Gamma^5\chi - \bar{\chi}\Gamma^5\epsilon)\,,$$
$$\delta\chi = \tfrac{1}{2}\gamma^\mu\Gamma_5\epsilon\cdot\hat{\mathrm{D}}_\mu P - (\sqrt{3})^{-1}\exp(-P)\,\gamma^\nu\epsilon\cdot\hat{\mathrm{D}}_\nu A_5 + (2\sqrt{3})^{-1}\exp(\tfrac{1}{2}P)\,\Gamma_5\gamma^{\alpha\beta}\epsilon\cdot\mathcal{G}_{\alpha\beta}\,,$$
$$\delta\psi_\mu = \mathrm{D}_\mu\epsilon - \tfrac{1}{4}\gamma^{\rho\sigma}\gamma_\mu\epsilon\cdot\mathcal{F}_{\rho\sigma} + \ldots\,, \tag{26}$$

where

$$\hat{\mathrm{D}}_\mu P = \partial_\mu P - \tfrac{1}{2}\mathrm{i}(\bar{\psi}_\mu\Gamma_5\chi - \bar{\chi}\Gamma_5\psi_\mu)\,,\quad \hat{\mathrm{D}}_\mu A_5 = \partial_\mu A_5 - \tfrac{1}{2}\mathrm{i}\sqrt{3}\,(\bar{\psi}_\mu\chi - \bar{\chi}\psi_\mu)\,. \tag{27}$$

To get the right parity assignments we perform a further (chiral) redefinition:

$$\chi \to \Gamma^5\chi\,. \tag{28}$$

Moreover, all factors of $\sqrt{3}$ disappear if one rescales

$$\chi \to (\sqrt{3})^{-1}\chi\,,\quad P \to (2/\sqrt{3})\,P\,, \tag{29}$$

so as to obtain the canonical normalization of the corresponding kinetic terms. Splitting χ into $2^{-1/2}(\chi_1 + \mathrm{i}\chi_2)$ where χ_1, χ_2 are Majorana spinors, we get, for instance, using (3)

$$\delta\chi^i = \tfrac{1}{2}\gamma^{\mu\nu}\epsilon^i\mathcal{G}_{\mu\nu} - \mathrm{i}\epsilon^{ij}(\gamma^\mu\epsilon_j\partial_\mu P + \mathrm{i}\gamma^\mu\gamma^5\epsilon_j\partial_\mu A_5) + \ldots\,, \tag{30}$$

which is just the expected transformation law [2,3]. It is not difficult to check that the truncations to the respective SO(2) submultiplets are consistent. The above results are easily seen to agree with ref. [3].

A physically somewhat more interesting situation is obtained in a generalized dimensional reduction [8] if one retains a dependence on the fifth coordinate in the form $\exp(imx_5)$. The supersymmetry is then spontaneously broken because $\partial_5 \neq 0$ and the central charge [9] which is gauged by the A_M field no longer vanishes but is rather proportional to the mass. This may in turn lead to antigravity [10] and we are presently investigating whether this interesting phenomenon indeed occurs for the model of this paper.

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Note added. B. de Wit informed us that the most general couplings of $N = 2$ supergravity have already been constructed, see e.g. M. de Roo, J.W. van Holten, B. de Wit and A. van Proeyen, Nucl. Phys. B173 (1980) 175. After our paper had been accepted, we also learnt of a paper by E. Cremmer, Supergravity in 5 dimensions, LPTENS 80/17 (August 1980), where all five-dimensional supergravities have been constructed and classified.

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