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Softly broken conformal symmetry with quantum gravitational corrections

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We show that a previously proposed new mechanism to eliminate quadratic divergences for scalar masses is self-consistently compatible with corrections induced by perturbative quantum gravity, provided the theory embeds consistently into a UV completion at the Planck scale.

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Introduction. Over the years various schemes have been devised to solve the electroweak hierarchy problem and thereby to explain the stability of the electroweak scale with regard to the Planck scale. Among these, the most prominent proposal is based on low energy ($N = 1$) supersymmetry. However, in the light of recent LHC results showing no evidence whatsoever of low energy supersymmetry after a decade of taking data, there is clearly a need for alternative ideas. Among these, ansätze relying on (some variant of) conformal symmetry have recently received a lot of attention, see [1–10] and references therein. In one way or another, all these proposals aim for a 'minimalistic' solution of the problem, taking seriously the possibility that the Standard Model (SM) may well survive modulo some minor modifications all the way to the Planck scale, for which there is now accumulating evidence.

Here we follow up on a recent proposal [11] invoking *softly broken conformal symmetry* (SBCS) which is equivalent to demanding the cancellation of quadratic divergences in terms of bare parameters at a distinguished and very large scale Λ. This scale serves as an effective cutoff: it is a basic assumption that at this scale, a proper theory of quantum gravity 'takes over' so that the cutoff Λ gets identified with a physical scale and is never taken to infinity. For this reason we usually assume that $\Lambda \sim M_{\rm PL}$. With the assumption that the pure matter theory is renormalizable, the bare couplings are identified with the running couplings evaluated at this distinguished scale:

$$\lambda_i \equiv \lambda_i^{\rm bare} = \lambda_i(\mu)\Big|_{\mu=\Lambda} \tag{1}$$

The key requirement then reads

$$f_i^{\rm quad}(\{\lambda_j\}) = 0 \tag{2}$$

where $f_i^{\rm quad}$ are the coefficient functions of the quadratic divergences accompanying the mass renormalizations of the scalar fields, with one such function for each independent physical scalar field,

$$\delta m_i^2 \, = \, \left(\frac{\Lambda^2}{16\pi^2}\right) f_i^{\rm quad}(\{\lambda_j\}) \, + \, \mathcal{O}\big(\log(\Lambda)\big) \tag{3}$$

and where $\{\lambda_i\}$ is the collection of all couplings of the model under consideration. We note that for the unmodified SM the possible vanishing of the quadratic mass divergence as a function of the running scalar self-coupling was already investigated in [12], with the result that $\Lambda \sim 10^{24}$ GeV, several orders of magnitude above the Planck scale. More recently, the above criterion was applied in [11, 13] to a slightly extended version of the SM with right-chiral neutrinos and one extra complex scalar field, requiring $\Lambda \sim M_{\rm PL}$, and in such a way that neither Landau poles nor instabilities of the effective potential appear up to that scale, thus ensuring the survival of the SM essentially *as is* up to that scale. The term 'SBCS' derives its justification from the fact that, as a consequence of requiring the absence of quadratically divergent contributions in (3) the physical masses can be kept consistently small in a perturbative treatment as their quantum corrections depend at worst logarithmically on the cutoff Λ (we will comment below on the reformulation of this statement in terms of running masses). Thus, when viewed from the cutoff scale Λ, (the matter part of) the theory looks effectively conformally invariant because $m_i^2 \lll \Lambda^2$. Indeed, our criterion is somewhat similar to softly broken supersymmetry, where one also allows for explicit symmetry breaking terms, on condition that these terms do not spoil the cancellation of quadratic divergences. Our procedure also shows how matter coupled Einstein gravity can give rise to a conformally invariant low energy flat space limit (with the gravitational coupling $\kappa \to 0$) even though Einstein gravity itself is not conformally invariant (see also [14]).

In this note we wish to show that the above criterion can be maintained in a self-consistent manner also if perturbative quantum gravitational corrections are taken into account, provided we assume that at the Planck scale the SM (or rather, some mildly amended version thereof) merges into a UV complete extension (see [15–19] and references therein for previous work concerning perturbative quantum gravity corrections to SM processes). The main point is that with the assumption of hierarchically small masses the gravitational corrections to the coefficient functions $f_i^{\rm quad}$ are of order $(\kappa m)^2$, that is

$$f_i^{\rm quad}(\{\lambda_j\},\kappa) = \, f_i^{\rm quad}(\{\lambda_j\}) \, + \, \mathcal{O}\big((\kappa m)^2\big) \; , \tag{4}$$

hence hierarchically smaller than the contribution of the matter couplings (here assumed to be $< \mathcal{O}(1)$, as is the case for all extended SM couplings up to $\mathcal{O}(M_{\rm PL})$) over the whole range of energies up to the Planck scale, and thus completely negligible. The assumed existence of a UV completion is necessary, because otherwise the theory will be overwhelmed by power law divergences involving arbitrarily high powers of $\kappa\Lambda$ which, if present, would effectively render moot the whole issue of quadratic divergences, as one would expect to be the case for a non-renormalizable theory. We refer readers to [20] for a detailed discussion of the adverse effects of such divergences.

Let us also emphasize that, in contrast to Veltman's original proposal [21], our condition of canceling the quadratic divergences with fixed and finite Λ is an *RG invariant statement* in the effective field theory; consequently, in terms of running couplings, the condition (2) itself becomes scale dependent (and can be easily obtained by expressing the bare couplings in terms of running couplings [11]). The whole scheme becomes well defined by the assumed finiteness of the Planck scale theory. As far as sub-Planckian physics is concerned, our scheme also bears some similarity with ideas proposed in the framework of the asymptotic safety program (see *e.g.* [4, 5, 22–24] and references therein) where the UV completeness would follow from the assumed existence of a non-trivial UV fixed point.

The model: gravity coupled to a set of scalar fields. We consider the Lagrangian

$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{-g}R \, + \, \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu S^A \partial_\nu S^A - V(S)\right) \tag{5}$$

where S^A are real scalar fields ($A, B, ... = 1, ..., n$) with potential $V(S)$. We can ignore fermions at this stage, because at the one-loop order considered here their contribution to $f_i^{\rm quad}$, opposite in sign to the contribution of the bosonic matter fields, remains the same as in the absence of gravitational couplings. By contrast, for scalar fields there appear terms mixing them with h, and these are ones we have to worry about, see below. For the perturbative expansion we split the fields into their background values and fluctuations

$$\begin{aligned} g_{\mu\nu}(x) \; &= \; \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) \;\; , \quad h_{\mu\nu} = H_{\mu\nu} + \frac{1}{4}\eta_{\mu\nu} h \quad (\eta^{\mu\nu}H_{\mu\nu} = 0 \, , \, h \equiv h^\mu{}_\mu) \\ S^A(x) \; &= \; \varphi_{cl}^A(x) + s^A(x) \end{aligned} \tag{6}$$

with the Minkowski metric $\eta_{\mu\nu}$ and background scalar fields $\varphi_{cl}^A(x)$; $\kappa = M_{\rm PL}^{-1}$ is the gravitational coupling. For establishing the mass renormalization it is in fact sufficient to take a static $\varphi_{cl}(x)$. The full Lagrangian also includes a gauge fixing term

$$\mathcal{L}_{gf} = \frac{1}{\xi}\left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h\right)^2 \tag{7}$$

We will set $\xi = 1$ for simplicity (but note that the coefficient functions will depend on the gauge choice). We neglect gravitational ghost fields as they do not couple to scalar fields at the one-loop order, and thus make no contribution to the coefficient of quadratic divergences. The full action is thus

$$S = \int d^4x (\mathcal{L} + \mathcal{L}_{gf}) \tag{8}$$

Our aim then is to perform a perturbative quantization in order to extract the coefficient of quadratic divergences from both matter and gravitational perturbations. The effective action is given by

$$\exp\big(i\Gamma(\varphi_{cl})\big) = \int dh ds \exp\left(iS(\varphi_{cl} + s, \eta + h) \, - \, i\frac{\partial\Gamma(g,\varphi_{cl})}{\partial{\varphi_{cl}}^A}\bigg|_{g=\eta} s^A \, - \, i\frac{\partial\Gamma(g,\varphi_{cl})}{\partial h_{\mu\nu}}\bigg|_{g=\eta} h_{\mu\nu} \right) \tag{9}$$

The subtraction removing the linear terms in the fluctuation fields effectively eliminates tadpoles (see e.g. [25]). Furthermore, at the relevant order we can exploit the well known fact that $\Gamma(\varphi_{cl}) = S(\varphi_{cl}) + \mathcal{O}(\hbar)$ to replace the derivatives of Γ by derivatives of S. Note that it is not necessary to impose the equations of motion on the background fields φ_{cl}.

At quadratic order the fluctuations are

$$\begin{aligned}\mathcal{L}'' \;=\; & -\frac{1}{2}h_{\mu\nu}P^{\mu\nu;\rho\sigma}\left(-\Box + \frac{\kappa^2}{2}\left[\frac{1}{2}(\partial_\mu\varphi_{cl})^2 - V_0\right]\right)h_{\rho\sigma} - \\ & -\frac{1}{2}\kappa\, h\,(-\partial_\mu\varphi_{cl}\,\partial^\mu + V_{0,A})\, s^A - \frac{1}{2}s^A\left[\delta_{AB}\Box + V_{0,AB}\right]s^B\end{aligned} \tag{10}$$

where

$$V_0(\varphi_{cl}) \equiv V\Big|_{S=\varphi_{cl}} \quad , \quad V_{0,A}(\varphi_{cl}) \equiv \frac{\partial V}{\partial S^A}\bigg|_{S=\varphi_{cl}} \quad , \quad V_{0,AB}(\varphi_{cl}) \equiv \frac{\partial^2 V}{\partial S^A \partial S^B}\bigg|_{S=\varphi_{cl}} \tag{11}$$

and

$$P^{\mu\nu;\rho\sigma} \equiv \frac{1}{2}\big(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\big)\,. \tag{12}$$

We also note the simplification

$$-\frac{1}{2}h_{\mu\nu}P^{\mu\nu;\rho\sigma}\left(-\Box + \frac{\kappa^2}{2}\mathcal{L}_0\right)h_{\rho\sigma} = -\frac{1}{8}h\left(-\Box + \frac{\kappa^2}{2}\mathcal{L}_0\right)h + \frac{1}{2}H_{\mu\nu}\left(-\Box + \frac{\kappa^2}{2}\mathcal{L}_0\right)H^{\mu\nu} \tag{13}$$

where $\mathcal{L}_0 \equiv \frac{1}{2}(\partial_\mu\varphi_{cl})^2 - V_0$. Path integration in quadratic fluctuations leads to the functional determinant

$$\mathcal{M} = \det\left(-\Box + \frac{\kappa^2}{2}\mathcal{L}_0\right)^{-\frac{9}{2}} \cdot \det\begin{pmatrix} -\Box + \frac{\kappa^2}{2}\mathcal{L}_0 & \frac{\kappa}{2}(-\partial_\mu\varphi^B_{cl}\,\partial^\mu + V_{0,B}) \\ \frac{\kappa}{2}(-\partial_\mu\varphi^A_{cl}\,\partial^\mu + V_{0,A}) & \delta_{AB}\Box + V_{0,AB}\end{pmatrix}^{-\frac{1}{2}} \tag{14}$$

where the components are split into 9 traceless modes $H_{\mu\nu}$ (giving the first factor), the trace h and the n scalar fields s^A (the contribution of ghosts at this stage would only supply trivial extra factors of $\det(-\Box)$). Specializing to static backgrounds $\varphi^A_{cl} = \text{const}$ and Fourier transforming ($\Box = -p^2$) the last determinant is a degree $n+1$ polynomial in p^2

$$(-)^n \det\begin{pmatrix} p^2 - \frac{\kappa^2}{2}V_0 & \frac{\kappa}{2}V_{0,B} \\ \frac{\kappa}{2}V_{0,A} & -\delta_{AB}p^2 + V_{0,AB}\end{pmatrix} = (p^2)^{n+1} - (p^2)^n\left(\frac{\kappa^2}{2}V_0 + \sum_{A=1}^{n} V_{0,AA}\right) + \mathcal{O}((p^2)^{n-1}) =: \prod_{i=1}^{n+1}(p^2 - M_i^2) \tag{15}$$

from which we learn that $\sum_{i=1}^{n+1} M_i^2 = \frac{\kappa^2}{2}V_0 + \sum_{A=1}^{n} V_{0,AA}$. The effective action in cutoff regularization then follows after Wick rotating as

$$i\Gamma(\varphi_{cl}) = \log\mathcal{M} = -\frac{i}{2}\int_0^\Lambda \frac{d^4p_E}{(2\pi)^4}\left(9\log(p_E^2 - \frac{\kappa^2}{2}V_0) + \sum_{i=1}^{n+1}\log(p_E^2 - M_i^2)\right) \tag{16}$$

where the subscript E indicates that the integral is to be performed in Euclidean signature. We first subtract out the zero point energy using

$$\int_0^\Lambda \frac{d^4p_E}{(2\pi)^4}\,\ln p_E^2 = \frac{\Lambda^4}{32\pi^2}\,(\log\Lambda^2 - \frac{1}{2})\,. \tag{17}$$

In the general case this quartic divergence is multiplied by $(n_B - n_F)$, the difference in the number of bosonic and fermionic degrees of freedom. As we assume that the UV completion will involve supersymmetry in one way or another, this divergence can be ignored. For the determination of quadratic divergences the central integral reads

$$\int_0^\Lambda \frac{d^4p_E}{(2\pi)^4}\log\left(1 - \frac{m^2}{p_E^2}\right) = -\frac{1}{16\pi^2}\Lambda^2 m^2 - \frac{m^4}{32\pi^2}\left[\log\left(-\frac{\Lambda^2}{m^2}\right) - \frac{1}{2}\right] + \mathcal{O}(\Lambda^{-2})\,. \tag{18}$$

Hence the quadratically divergent contributions to the effective action emerging from $\log\mathcal{M}$ may be extracted to be [26]

$$\Gamma^{\rm div}(\varphi_{cl}) = \frac{\Lambda^2}{32\pi^2}\left(\frac{9}{2}\kappa^2 V_0 + \sum_{i=1}^{n+1} M_i^2\right) + \mathcal{O}(\log\Lambda) = \frac{\Lambda^2}{32\pi^2}\left(5\kappa^2 V_0 + \sum_{A=1}^{n} V_{0,AA}\right) + \mathcal{O}(\log\Lambda)\,. \tag{19}$$

Here we are only interested in the mass renormalization (3), so we need only keep the terms quadratic in the background fields φ_{cl}. We also note that the correction term in the second factor of (19) gives only a logarithmically divergent contribution, and can therefore be neglected for our purposes of establishing the quadratically divergent contributions.

Concretely for a general potential

$$V(S) = \frac{1}{2}m_{AB}^2 S^A S^B + \frac{1}{4!}\lambda_{ABCD}S^A S^B S^C S^D + \mathcal{O}(\kappa^2) \tag{20}$$

the relevant divergent contributions to the mass matrix read

$$\delta m_{AB}^2 = -\left(\frac{\Lambda^2}{16\pi^2}\right)\left(\frac{5}{2}\kappa^2 m_{AB}^2 + \frac{1}{2}\sum_{C=1}^{n}\lambda_{CCAB}\right) + \mathcal{O}(\log\Lambda) \tag{21}$$

From this matrix we can extract the coefficient functions $f_i^{\rm quad}$ simply by diagonalization. The main point now is that, after the inclusion of fermionic contributions and with the assumed absence of quadratic divergences, the initial condition $m_{AB}^2 \lll \Lambda^2$ implies $\kappa m \lll \mathcal{O}(1)$, whence the initial condition remains consistent with the quantum gravitational corrections. We thus conclude that the conditions on $f_i^{\rm quad}$ are effectively the same as in the flat space theory, as the gravitational contribution is hierarchically suppressed.

There appears to be no consensus in the literature whether these statements can or cannot be consistently rephrased in terms of running masses. Let us recall that running coupling parameters are merely an auxiliary, though very convenient, device to parametrize the scaling behavior of n-point correlation functions in renormalizable quantum field theory, but it is arguable whether the very notion of a running coupling continues to make sense in the context of non-renormalizable theories [17]. Indeed, while there appears to be no unambiguous way to obtain for the running masses $m^2(\mu)$ a quadratic dependence on the scale parameter μ in the context of renormalized perturbation theory (because the subtraction of a quadratic divergence leaves ambiguous a finite contribution), such a dependence can arise in a Wilsonian treatment, where one integrates out modes with momenta $\mu^2 < p^2 < \Lambda^2$ (this is the point of view adopted in asymptotic safety scenarios, see [22–24] and references therein). In the latter view our condition (2) would eliminate the quadratic dependence of the running masses on μ, and thus ensure a logarithmic running of $m^2(\mu)$, keeping the so defined running masses consistently small over the whole range of energies $\mu \le \Lambda$, in accord with the SBCS hypothesis.

Let us also point out that the existence of quadratic (and higher power) divergences in other parts of the effective action follows directly from the above formulas. For instance, to pick out the quadratic divergences in the wave function renormalization one only needs to expand the above effective action up to quadratic order in the derivatives $\partial_\mu\varphi_{cl}$. Secondly, possible non-renormalizable interactions (with or without derivatives) not written explicitly in the formulas above, i.e. the $\mathcal{O}(\kappa^2)$ terms in (20), will likewise pick up power law divergences. However, these will not modify our condition (2), but instead affect higher order operators, as already pointed out in [17–19].

Conclusions. We have shown that the novel mechanism proposed in [11] to avoid the hierarchy problem of the effective quantum field theory below the Planck scale can be maintained self-consistently in the presence of perturbative quantum gravitational corrections. The main advantage of this proposal is that, unlike low energy supersymmetry, it can make do without the extra baggage of numerous new, and so far unseen, degrees of freedom and the concomitant plethora of new couplings (not to mention the fact that $N = 1$ matter coupled supergravities are just as non-renormalizable as pure gravity, and therefore eventually will also run into the problem of power law divergences). Finally, as already pointed out in [11], and assuming a minimal extension of the SM along the lines proposed here can be validated and the corresponding couplings are known, the condition (2) can be subjected to experimental tests.

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[1] W.A. Bardeen, *On Naturalness in the Standard Model*, preprint FERMILAB-CONF-95-391-T; *Mechanisms of Electroweak Symmetry Breaking and the Role of a Heavy Top Quark*, preprint FERMILAB-CONF-95-377-T.

[2] K.A. Meissner and H. Nicolai, Phys. Lett. B **648** (2007) 312; Phys. Lett. B **660** (2008) 260.
[3] M. Holthausen, M. Lindner and M.A. Schmidt, Phys. Rev. **D82** (2010) 055002;
[4] M. Shaposhnikov and C. Wetterich, Phys. Lett. **B683** (2010) 196.
[5] O. Antipin, M. Mojaza and F. Sannino, `arXiv:1310.0957 [hep-ph]`
[6] M. Lindner, S. Schmidt, J. Smirnov, JHEP **1410** (2014) 177.
[7] M. Heikinheimo, A. Racioppi, M. Raidal, C. Spethmann and K. Tuominen, Mod. Phys. Lett. A **29** (2014) 1450077.
[8] S. Oda, N. Okada and D. Takahashi, Phys. Rev. **D 92** (2015) 015026;
A. Das, S. Oda, N. Okada and D. Takahashi, Phys.Rev. **D 93** (2016) 115038
[9] C. Wetterich and M. Yamada, Phys.Lett. **B770** (2017) 268
[10] A. J. Helmboldt, P. Humbert, M. Lindner, and J. Smirnov, JHEP **07** (2017) 113;
V. Brdar, Y. Emonds, A.J. Helmboldt, M. Lindner, `arXiv:1807.11490 [hep-ph]`
[11] P. H. Chankowski, A. Lewandowski, K.A. Meissner and H. Nicolai, Mod.Phys.Lett. **A30** (2015) 1550006
[12] Y. Hamada, H. Kawai and K. Oda, Phys. Rev. **D87** (2013) 053009.
[13] A. Lewandowski, K.A. Meissner and H. Nicolai Phys.Rev. **D97** (2018) 035024
[14] K.A. Meissner and H. Nicolai, Phys.Rev. **D80** (2009) 086005
[15] G. 't Hooft and M. J. G. Veltman, Ann. Inst. H. Poincare Phys. Theor. A **20** (1974) 69;
S. Deser and P. van Nieuwenhuizen, Phys. Rev. D **10** (1974) 401;
J. F. Donoghue, Phys. Rev. D **50** (1994) 3874;
S. Sannan, Phys. Rev. D **34** (1986) 1749.
[16] S. P. Robinson and F. Wilczek, Phys. Rev. Lett. **96** (2006) 231601;
D. Ebert, J. Plefka and A. Rodigast, Phys.Lett. **B660** (2008) 579;
A. Rodigast and T. Schuster, Phys. Rev. Lett. **104** (2010) 081301;
J. E. Daum, U. Harst and M. Reuter, JHEP **1001** (2010) 084;
D. J. Toms, Nature **468** (2010) 56.
[17] M.M. Anber, J.F. Donoghue and M. El-Houssieny, Phys.Rev. **D83** (2011) 124003
[18] M.M. Anber and J.F. Donoghue, Phys.Rev. **D85** (2012) 104016
[19] F. Loebbert and J. Plefka, Mod. Phys. Lett. A **30** (2015) no.34.
[20] A. Smilga, Phys.Part.Nucl.Lett. **14** (2017) 245
[21] M. Veltman, Acta Phys. Pol. **B12** (1981) 437.
[22] A. Eichhorn, `arXiv:1810.07615 [hep-th]`
[23] O. Lauscher and M. Reuter, In Fauser, B. (ed.) et al.: *Quantum gravity* 293-313 `hep-th/0511260`;
N. Christiansen, D. F. Litim, J. M. Pawlowski and M. Reichert, Phys. Rev. D **97** (2018) no.10, 106012.
[24] F. Grabowski, J.H. Kwapisz and K.A. Meissner, `arXiv:1810.08461 [hep-ph]`;
[25] M. D. Schwartz, "Quantum Field Theory and the Standard Model," Cambridge University Press 2014.
[26] We note that in a general gauge (7) on has $5\kappa^2 \to (3+2\xi)\kappa^2$ in the above.