Waves from the Centre: Probing PBH and other Macroscopic Dark Matter with LISA

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A significant fraction of cosmological dark matter can be formed by very dense macroscopic objects, for example primordial black holes. Gravitational waves offer a promising way to probe of these kinds of dark matter candidates, in a parameter space region that is relatively untested by electromagnetic observations. In this work we consider an ensemble of macroscopic dark matter with masses in the range 10^{-13} – $10^{3}\,M_{\odot}$ orbiting a super-massive black hole. While the strain produced by an individual dark matter particle will be very small, gravitational waves emitted by a large number of such objects will add incoherently and produce a stochastic gravitational-wave background. We show that LISA can be a formidable machine for detecting the stochastic background of such objects orbiting the black hole in the center of the Milky Way, Sgr A*, if a dark matter spike of the type originally predicted by Gondolo and Silk forms near the central black hole.

According to the current standard cosmological model, approximately 25 % of the energy density of the Universe is in the form of so-called cold dark matter — non-relativistic objects which collectively act as a perfect fluid of negligible pressure. The leading candidates have long been axions [1–3] as well as weakly-interactive massive particles (WIMPs), heavy ($\gtrsim 1\,\mathrm{GeV}$) particles outside the Standard Model of particle physics, possessing very small scattering cross-sections on each other and on Standard-Model particles.

With the continued non-detection of WIMP dark matter, and the failure of long-predicted Beyond the Standard-Model physics to materialize at the Large Hadron Collider, the case for alternative, and especially Standard Model, candidates has grown stronger, and attracted increasing attention. It has also long-been recognized that there are viable dark-matter candidates of much greater mass, notably primordial black holes (PBH) [4, 5] (see also Refs. [6–27]) and objects of nuclear density (e.g. Ref. [28–32]), either of which could potentially be the result of Standard-Model physics in the early Universe. For the purposes of this paper, we will refer to all such macroscopic dark-matter candidates, including PBHs, generically as *macros*.

There are well-known limits on macros from microlensing of Milky Way and Magellanic Cloud stars [33–36] limiting the abundance of macros above approximately $4\times10^{24}\,\mathrm{g}$. The failure to observe femtolensing of gamma-ray bursts [37] means that dark matter cannot be composed entirely of macros between approximately $2\times10^{17}\,\mathrm{g}$ and $2\times10^{20}\,\mathrm{g}$. However, between about $2\times10^{20}\,\mathrm{g}$ and $4\times10^{24}\,\mathrm{g}$ there is an unconstrained window for anything of approximately ordinary matter density or greater [38]. Candidates of approximately nuclear or greater density are also unconstrained from $55\,\mathrm{g}$ to $2\times10^{17}\,\mathrm{g}$ [38], although if, as expected, primordial black holes emit Hawking radiation, then they would have evaporated before now if their masses were below approximately $10^{15}\,\mathrm{g}$.

With the dawn of gravitational-wave astronomy, it is interesting to use gravitational-wave observations to probe the nature of dark matter. The Advanced Laser Interferometer Gravitational-wave Observatory (LIGO) [39] and Advanced Virgo [40] detectors, operating in approximately the 10 – $10^{3}\,\mathrm{Hz}$ band, is now regularly detecting the merger of compact binaries whose total masses are of order a few to tens of solar masses [41–47]. There has been significant attention to the possibility that multi-solar-mass black holes, such as those detected by Advanced LIGO, could be the dark matter [17, 48–51]. The Laser Interferometer Space Antenna (LISA) [52], is a space-based gravitational-wave detector which will operate in the 10^{-4} – $10^{-1}\,\mathrm{Hz}$ band. Clesse *et al.* [53] have suggested that if the dark matter is composed of multi-solar-mass black holes that LISA could detect their merger.

In this work, we explore whether LISA can also explore lighter macros, in the mass range 10^{-13} – $10^{3}\,M_{\odot}$ ($2\times10^{20}\,\mathrm{g}$ – $2\times10^{36}\,\mathrm{g}$). Because of its relative proximity, a promising target is Sagittarius A* (Sgr A*), a super-massive black hole (SMBH) with a mass of $4\times10^{6}\,M_{\odot}$ at the center of our Galaxy [54]. Even though the strain associated with the gravitational waves from an individual macro orbiting Sgr A* is below the sensitivity threshold of any near-term gravitational-wave detectors such as LISA, the *collective* signal from a large number of such macros might be detectable as a stochastic gravitational-wave background.

In this work, we aim for a conservative estimate on the gravitational-wave detection prospects of a possible macro dark-matter distribution in our Galaxy. We assume that we can treat the objects non-relativistically, so we use the Peters-Mathews formula [55] for the time-averaged power emitted by one macro of mass μ orbiting Sgr A* (with mass M) at a distance a

$$P = \frac{32}{5}\,\frac{G^4}{c^5}\,\frac{M^2\mu^2\,(M+\mu)}{a^5} \approx \frac{32}{5}\,\frac{q^2\,G^4 M^5}{a^5 c^5}\,, \qquad (1)$$

where $q \equiv \mu/M$ is the mass ratio. For simplicity, we assume that all macros have the same ellipticity; specifically, we assume circular orbits. We have checked that up to eccentricities of 0.99, the signal to noise ratio of elliptical orbits is larger than for circular orbits, and therefore assuming circular orbits gives a conservative estimate for the detectability.

At sufficiently large macro mass, macros near the black hole will lose energy due to gravitational wave emission and plunge into the central black hole. To account for this, we exclude orbits where the timescale for the orbital frequency to change, $\tau = \nu/\dot{\nu}$, is larger than a cutoff $\tau_{\rm min}$ (note that to within a factor of $\mathcal{O}(1)$, the time change of orbital frequency is equal to the time to coalescence.). Here and throughout the text, ν refers to the emitted gravitational-wave frequency. We estimate the inspiral time using [56]

$$\tau \equiv \frac{\nu}{\dot{\nu}} = \frac{M}{\mu}\,\frac{5}{96\,\pi^{8/3}}\,\frac{c^5}{(GM)^{5/3}}\,\nu^{-8/3}\,. \qquad (2)$$

Imposing the condition $\tau > \tau_{\rm min}$ leads to a μ-dependent maximum frequency at which gravitational-waves are emitted. We consider two cases for $\tau_{\rm min}$. First, we consider a highly conservative option $\tau_{\rm min} = H_0^{-1}$, which guarantees that the dark matter profile is stable and the macros do not fall into the black hole due to gravitational wave emission within the age of the universe. Second, we optimistically assume that the dark matter in orbits close to the black hole is refilled efficiently, and take $\tau_{\rm min}$ equal to the observational time of the LISA mission, which we take to be 5 years. In either case, the macros we consider follow very stable orbits and each macro emits gravitational waves at approximately a single frequency.

The stochastic gravitational-wave signal seen by LISA is dominated by orbits which pass very close to the central black hole. We consider several scenarios for the dark distribution. First, we consider a Navarro-Frenk-White (NFW) profile [57], which is given by

$$\rho_{\rm NFW}(r) = \rho_0\,\frac{4r_0}{r\left(1+\frac{r}{r_0}\right)^2}\,, \qquad (3)$$

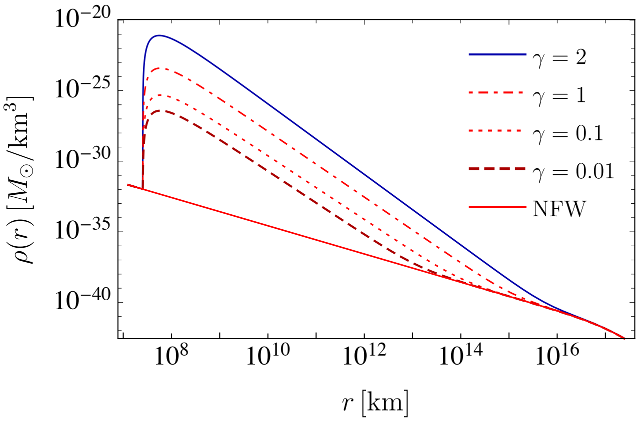

FIG. 1: Dark-matter density profile as a function of radius. Depicted are a pure NFW profile as well as the onset of the spike density profiles Eq. (4) with $\gamma = 0.01$, 0.1, 1, 2 (bottom to top) for $M = M_{\rm Sgr\,A^*}$.

where $r_0 = 2.5 \times 10^{17}$ km is the distance to the Galactic Center, and $\rho_0 = 2.7 \times 10^{-43}\,M_\odot\,{\rm km}^{-3}$ is the local dark matter density. Second, we consider that a dark matter spike can form by adiabatic collapse, following the model of Gondolo and Silk [58], including relativistic corrections by Sadeghian *et. al.* [59]. In this scenario, the dark matter density near the Galactic Centre is enhanced due to an NFW profile due to adiabatic accretion of dark matter by Sgr A*. Sadeghian *et. al.* suggest the following simple analytic approximation for the dark matter spike density

$$\rho_{\rm sp}(r) \approx (1-\epsilon)\,\rho_R\left(1-\frac{2R_{\rm S}}{r}\right)^3\left(\frac{R_{\rm sp}}{r}\right)^{\gamma_{\rm sp}}\,, \qquad (4)$$

where $\epsilon = 0.15$, and where $2\,R_{\rm S} < r < R_{\rm sp}$. Here, $R_{\rm S} = 2\,G\,M_{\rm Sgr\,A^*}/c^2 \simeq 3\,(M_{\rm Sgr\,A^*}/M_\odot)$ km is the Schwarzschild radius of Sgr A*, and $R_{\rm sp} \equiv \alpha_\gamma\,r_0\,[M_{\rm Sgr\,A^*}/(\rho_0\,r_0^3)]^{1/(3-\gamma)}$, where the normalization α_γ is numerically derived for each power-law index γ. Above, $\rho_R \equiv \rho_0\,(R_{\rm sp}/r_0)^{-\gamma}$, where $\gamma_{\rm sp} \equiv (9-2\gamma)/(4-\gamma)$ (see Refs. [58, 60] for details). Fig. 1 depicts the radial behaviour of the halo profile (4). We note that this formula assumes that Sgr A* has negligible spin. We expect this to be a conservative assumption, as the signal-to-noise ratio is dominated by the smallest orbits, and the innermost stable circular orbit for prograde orbits around a spinning black hole is smaller than for a non-spinning black hole. However, we leave a detailed consideration of the effect of black hole spin for future work.

The stochastic background is characterized in terms of the energy density per unit logarithmic frequency interval,

$$\Omega_{\rm GW}(\nu) = \frac{\nu}{\rho_c}\,\frac{{\rm d}\rho_{\rm GW}}{{\rm d}\nu}\,, \qquad (5)$$

where $\rho_{\rm GW}$ is the energy density in gravitational waves, and $\rho_c = 3\,H_0^2/(8\pi G)$ is the critical energy density needed to have a spatially flat universe. We take the value $H_0 = 67.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ for the present-day value of the Hubble parameter as measured by Planck [61]. We will in turn focus on continuous emission of a number of N macros, all with the same mass μ. The generalisation to an extended mass distribution is straight forward and shall not be given here. For simplicity, since we are only interested in an order of magnitude estimate for the strain power needed for detection, we assume that the macros are isotropically distributed on the sky at the distance of Sgr A*, and an isotropic distribution of orbital planes. For circular orbits, which emit gravitational waves at a single frequency ν, $\Omega_{\rm GW}$ is given by incoherently summing the power emitted by each macro

$$\begin{aligned}\Omega_{\rm GW}(\nu) &= \sum_k \frac{\nu P_k}{4\pi\, c\, \rho_c\, r^2}\,\delta(\nu-\nu_k)\theta(\nu_{\rm max}-\nu)\\ &\approx \frac{512\,\pi}{45\,H_0^2\,c^8}\,\frac{\mu^2}{M^2}\,\frac{(GM)^{9/2}}{r^2\,a^{1/2}\,\mu}\,\nu\,\rho_{\rm macro}(a)\theta(\nu_{\rm max}-\nu)\ ,\end{aligned} \tag{6}$$

where in the first line the index k labels the macros, the power P_k is given by Equation (1), and to obtain the second line we have replaced a sum over individual macros with an integral over the macro density and have done the integral. The frequency cutoff $\nu_{\rm max}$ is defined as $\nu_{\rm max} = \min(\nu_{\rm plunge}, \nu_\rho)$, where $\nu_{\rm plunge}$ is found by inverting Equation 2 with $\tau = \tau_{\rm min}$, and ν_ρ is the gravitational wave frequency emitted by an orbit with radius $2\,R_{\rm S}$. The theta function $\theta(\nu_{\rm max}-\nu)$ thus imposes the constraints that the orbits contributing to $\Omega_{\rm GW}$ should be included in the density distribution Equation (4), and also should not plunge into the black hole due to GW emission on a timescale shorter than $\tau_{\rm min}$. The orbital radius a is related to the emitted gravitational wave frequency ν by $a \equiv (GM)^{1/3}/(\pi\nu)^{2/3}$, and r is the distance from Earth to the galactic center. The quantity $\rho_{\rm macro}(a)$ is the macro mass density, which will be assumed to follow the dark-matter distribution, $\rho_{\rm macro} = f_{\rm macro}\,(\rho_{\rm NFW} + \rho_{\rm sp})$, where $f_{\rm macro}$ is the fraction of dark matter residing in macros.

Bayesian data analysis methods have been developed to study the stochastic background of LISA [62], including subtraction of the white-dwarf foreground [63]. In the context of constraining phase-transition models of the early Universe, Ref. [64] estimated that an SNR of 10 corresponded to a detection assuming a detector with six one-directional laser links between the spacecraft. We use this same detection threshold here. The signal-to-noise ratio (SNR) of the traditional stochastic (cross-correlation) search can be written as [65]

$$\mathrm{SNR} = \sqrt{T\int_0^{\nu_{\rm max}} \mathrm{d}\nu\,\frac{\mathcal{R}^2(\nu)\,S_{\rm h}^2(\nu)}{P^2(\nu)}}\ , \tag{7}$$

where $\mathcal{R}(\nu)$ is the overlap reduction function for LISA (see Ref. [62]), $S_{\rm h}(\nu)$ is the strain power spectrum of the signal, which is related to $\Omega_{\rm GW}(\nu)$ by (see Ref. [65])

$$S_{\rm h}(\nu) = \frac{3\,H_0^2}{2\pi^2}\,\frac{\Omega_{\rm GW}(\nu)}{\nu^3}\ , \tag{8}$$

and $P(\nu)$ is the detector-power spectral density (PSD). We use the parameterization given in [52]

$$P(\nu) = \frac{1}{L^2}\left[S_{\rm x}(\nu) + \frac{1}{4\pi^2\nu^2}\,S_{\rm a}(\nu)\right]\ , \tag{9}$$

where the arm length is $L = 2.5\times10^6\,\mathrm{km}$, and where the acceleration noise $S_{\rm a}$ and position noise $S_{\rm x}$ spectra are given by

$$\begin{aligned}S_{\rm a}(\nu)^{1/2} &= 3\times10^{-15}\,\frac{\mathrm{m}}{\mathrm{s}^2\sqrt{\mathrm{Hz}}}\sqrt{1+\left(\frac{0.4\,\mathrm{mHz}}{\nu}\right)^2}\\ &\quad\times\sqrt{1+\left(\frac{\nu}{8\,\mathrm{mHz}}\right)^4}\\ S_{\rm x}(\nu)^{1/2} &= 10^{-11}\,\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}\sqrt{1+\left(\frac{2\,\mathrm{mHz}}{\nu}\right)^4}\ .\end{aligned} \tag{10}$$

The collective signal-to-noise integral, Eq. (7), can be computed numerically. As above, we will use $M = M_{\rm Sgr\,A^*} \approx 4\times10^6\,M_\odot$, $\mu/M \approx 2.5\times10^{-17}$, $\sigma \approx 2.3\times10^{31}\,\mathrm{km}^2$, and the LISA PSD above. We will use $\nu_{\rm min} = 10^{-6}\,\mathrm{Hz}$ as the lower cutoff of the LISA band, although lowering this by an order of magnitude does not affect the results. These are depicted in Fig. 2. One interesting feature is that the signal is dominated by orbits close to the central black hole. This opens the possibility of directly probing the dark matter distribution very close to the galactic center. Finally, in Fig. 3 we show the minimum value of $f_{\rm macro}$ that LISA can detect as a function of macro mass.

We therefore conclude that LISA has the potential to be a robust detector of primordial-black hole dark-matter candidates. The same holds true for other macroscopic dark-matter candidates of approximately nuclear or higher density. The precise utility depends sensitively on the rate at which the orbits nearest to the black hole are replenished, and on the details of the dark-matter spike (if any).

In this work we have aimed at a conservative estimate of the gravitational-wave emission. There are several approximations we have made which can be improved in future work. First, due to the shape of the LISA power spectrum, the SNR is dominated by frequencies near 1 mHz. In practice, this means that the signal is dominated by the orbits very close to the central black hole, which are relativistic ($v/c > 0.1$). In future work it will be interesting to explore the effect of relativistic corrections. We have also ignored effects of inclination relative to the observer, which we expect could introduce an $\mathcal{O}(1)$ factor into the final result. Ref. [66] has placed

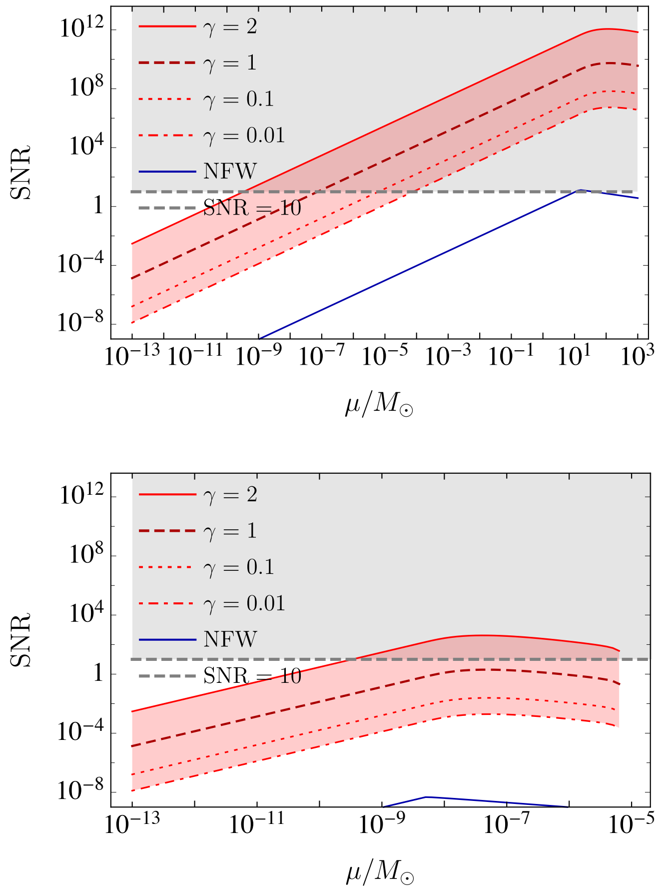

FIG. 2: Signal-to-noise ratio (SNR) for circular orbits macros of mass μ around Sgr A*. The underlying dark-matter distribution is the same as in Fig. 1. The upper panel shows results for $\tau_{\rm min} = 5\,$years, while the lower panel utilises $\tau_{\rm min} = H_0^{-1}$.

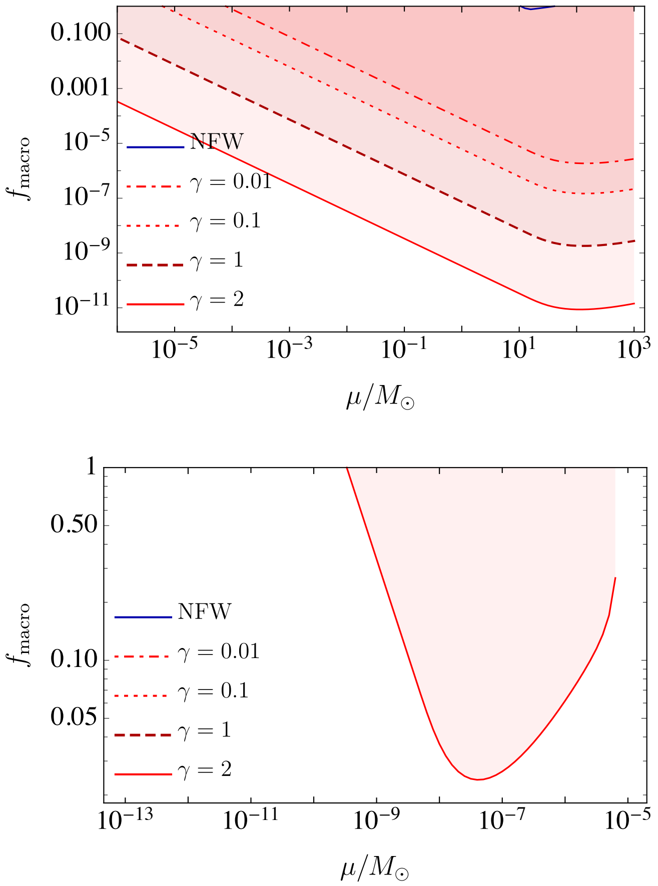

FIG. 3: Minimum value of dark-matter fraction $f_{\rm macro}$ that LISA can detect as a function of macro mass μ, using a signal-to-noise ratio threshold of 10. The underlying dark-matter distribution is the same as in Fig. 1. As in Fig. 2, the upper panel shows results for $\tau_{\rm min} = 5\,$years, while the lower panel utilises $\tau_{\rm min} = H_0^{-1}$.

constraints on the power law index for the dark matter spike using observations of stellar orbits. However, because these observations are sensitive to the behavior of the spike at distances of order ten parsecs or greater from the Galactic Center, these observations do not rule out the possibility of a dark matter spike that falls off faster than a simple power law at these distance scales. Finally we have ignored N-body interactions between the macros themselves, which may cause some orbits to plunge into the black hole.

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