Quantitative study of kinetic ballooning mode theory in magnetically confined toroidal plasmas

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The basic theory of kinetic ballooning modes was developed in Ref. [1]. Here, the authors solve the gyrokinetic equation by expanding in $\varepsilon = v_{thi}^2/\omega^2 l_c^2 \ll 1$, where $v_{thi} = \sqrt{2T_i/m_i}$ is the ion thermal speed, l_c the connection length and $\omega \ll v_{the}/l_c$ is the mode frequency, with v_{the} the electron thermal speed. The general KBM equation below retains magnetic drift resonances, gyro-averaging and magnetic compressibility effects, but trapped particles are neglected. The result is a second order differential equation for the electrostatic potential ϕ that reads:

$$\frac{1}{\beta_i B}\frac{v_{ths}^2/l_c^2}{\omega^2}\frac{\partial}{\partial z}bB\frac{\partial \phi}{\partial z} = K\phi, \tag{1}$$

with

$$K = \left\{\left[Q - \left(1 - \frac{\omega_{*i}}{\omega}\right)\right]\left[\alpha_{0,e}\left(1 + \frac{\beta_i}{2}R\right) - \alpha_{1,e}\tau Q'\frac{\beta_i}{2}\right]\right.$$
$$\left. - \frac{\beta_i}{2}\left(Q' + \alpha_{1,e}\right)\left[\alpha_{0,e}Q' + \alpha_{1,e}\left(1 + \tau - \tau Q\right)\right]\right\} \times \left\{\left(1 + \tau - \tau Q\right)\left(1 + \frac{\beta_i}{2}R\right) + \tau Q'^2\frac{\beta_i}{2}\right\}^{-1}$$
$$- \alpha_{1,e}\frac{\omega_\kappa + \omega_B}{\omega}, \tag{2}$$

where we defined the connection length l_c and the coordinate z along the field is defined so that $l_c\nabla_\parallel = \partial_z$, $\beta_i = 8\pi p_i/B_a^2$, $b = k_\perp^2 v_{thi}^2/2\Omega_i^2 B$, $\mathbf{k}_\perp$ is the wavevector across the equilibrium magnetic field, $\Omega_i(B) = m_i c/(eB)$ is the ion cyclotron frequency, $\alpha_{n,j} = 1 - (\omega_{*i}/\omega)\left(1 + n\eta_j\right)$, $\tau = T_e/T_i$, $\eta_i = L_{n_i}/L_{T_i}$, $\omega_{*i,e} = \frac{1}{2}k_y\rho_{i,e}v_{th}/L_n$, $\omega_B = (\mathbf{k}_\perp\rho_s/2)\cdot v_{ths}\hat{\mathbf{b}}\times\nabla B/B$, $\rho_s = v_{ths}/\Omega_s(B_a)$, where B_a is a reference constant magnetic field, $\omega_\kappa = (\mathbf{k}_\perp\rho_s/2)\cdot v_{ths}\hat{\mathbf{b}}\times(\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}})$, $L_{Ti,e}$ and L_n are the characteristic gradient lengths for temperature and density and Q, Q', R are velocity-space integrals (defined in [1]).

In the recently published work [2] the problem of kinetic ballooning mode instability was revisited in simple tokamak geometry. The authors derived an appropriate β-ordering, which allows Eq.(1) to be simplified greatly. The results of the kinetic instability analysis and its comparison with numerics lead to a natural distinction between "high-temperature-gradient" and "moderate-temperature-gradient" KBM regimes.

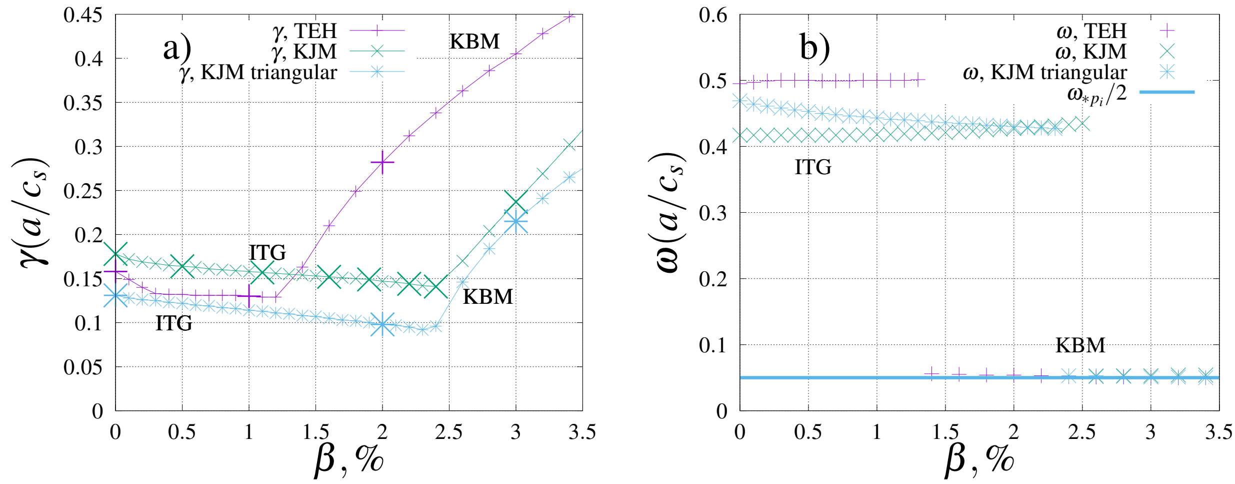

Figure 1: Fastest growing mode dependence on β for the with $a/L_{Ti,e} = 2$ and $k_y\rho_s$ in the range from 0.05 to 0.8, in two different W7-X configurations (KJM and TEH) and two simulation tubes (centred around the outboard midplane of the "bean" and "triangular" plane). (a): growth rate of the instabilities. (b): real frequency of the corresponding modes. Large dots specify where the magnetic equilibrium was varied.

In the first case, instability occurs only for mode frequencies such that $\omega_r = \omega_{pi}/2$, where $\omega_{pi} = \omega_{*i}(1+\eta_i)$. The maximum growth rate is located at very long wavelength. If magnetic drifts are kept consistent with the equilibrium pressure gradient, the gyrokinetic codes GS2 [3, 4] and GENE [5, 6] show excellent quantitative agreement with the familiar ideal-MHD equation with a diamagnetic correction [7]:

$$\frac{1}{\beta_i}\frac{v_{thi}^2}{\omega^2 l_c^2}\frac{\partial}{\partial z}bB\frac{\partial\phi}{\partial z} = -\frac{2\omega_\kappa\omega_p}{\omega^2}\phi - b\left[1-\frac{\omega_{*i}}{\omega}(1+\eta_i)\right]\phi, \tag{3}$$

where $\omega_p = \omega_{*i}(1+\eta_i) - \omega_{*e}(1+\eta_e) \equiv \omega_{pi} + \omega_{pe}$ and $\omega_\kappa \neq \omega_B$ since β is finite. Keeping $\omega_\kappa = \omega_B$ does not give a satisfactory agreement between analytics and numerics.

In more complicated geometries, Eq.(1) is still valid. General KBM equation for stellarator geometry:

$$\frac{1}{\beta_i\sqrt{g_B}}\frac{v_{thi}^2}{\omega^2 l_c^2}\frac{\partial}{\partial\theta}\frac{b}{\sqrt{g_B}}\frac{\partial\phi}{\partial\theta} = K\phi. \tag{4}$$

Here $\sqrt{g_B}$ is the determinant of the Jacobian matrix, K is described in Eq. (2). Therefore, the magnetic geometry enters the KBM equation for stellarators through the magnetic field B, the Jacobian and through the $\mathbf{k}_\perp$ terms. In the framework of a flux tube approach, all of these quantities are functions of the field-line following coordinate. In the present work, linear electromagnetic gyrokinetic numerical simulations of microinstabilities have been performed with the

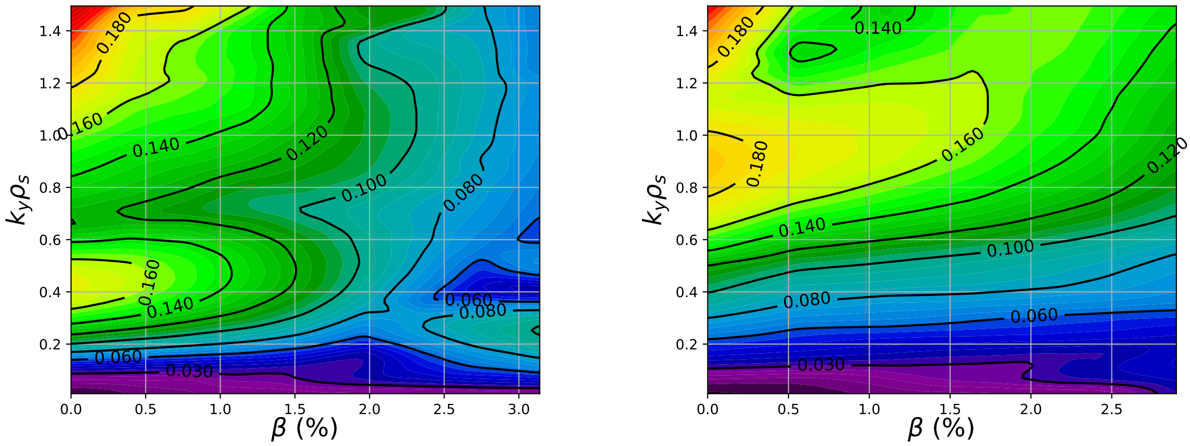

Figure 2: Instabilities growth rates as a function of β and $k_y\rho_s$ (colorcoded), when pressure gradients are consistent with the plasma equilibrium. Left: simulations for standard tokamak case. Right: W7-X standard configuration (KJM, "bean" tube).

GENE code in finite-β plasmas for different geometries in the stellarator device Wendelstein 7-X as well as a generic tokamak model. The results exhibit good agreement of the real frequencies of KBMs with the diamagnetic modification of ideal MHD limit in the large-gradient-regime for W7-X geometry. This finding agrees with a recent KBM study in simple tokamak geometry [2].

Thresholds for KBM mode destabilization in W7-X configurations with different ideal MHD stability properties have been compared and found to be correlated (see Fig. 1). Simulations of the KBM instability in the standard (MHD-optimized) W7-X geometry with $\beta = 3\%$, flat density profile and temperature gradient $a/L_{Ti,e} = 2$ show that the KBM stability threshold ($k_y\rho_s = 0.05$) is about 2.2% (simulation tube centred around the outboard midplane of the "triangular" plane) and 1.9% ("bean") as compared with 0.65% in the non-optimized configuration, which has a much lower MHD stability threshold. KBM critical β values correspond to the point of marginal KBM stability were compared with the reference MHD estimates obtained by ideal ballooning code and were found to be lower in KJM case. Thus we conclude that we may expect a rise of KBM instability before corresponding iMHD threshold.

We further proceed with a comparison of the W7-X results with a generic tokamak case, emphasising the consistency of the local pressure gradient and the plasma equilibrium. In Fig. 2 we show the growth rates obtained from the gyrokinetic calculations performed by GENE code as functions of β and k_y. The comparison of W7-X and a generic tokamak shows that one of the significant features of the KBM in W7-X geometry is that the most unstable mode has $k_y\rho_s = 0$

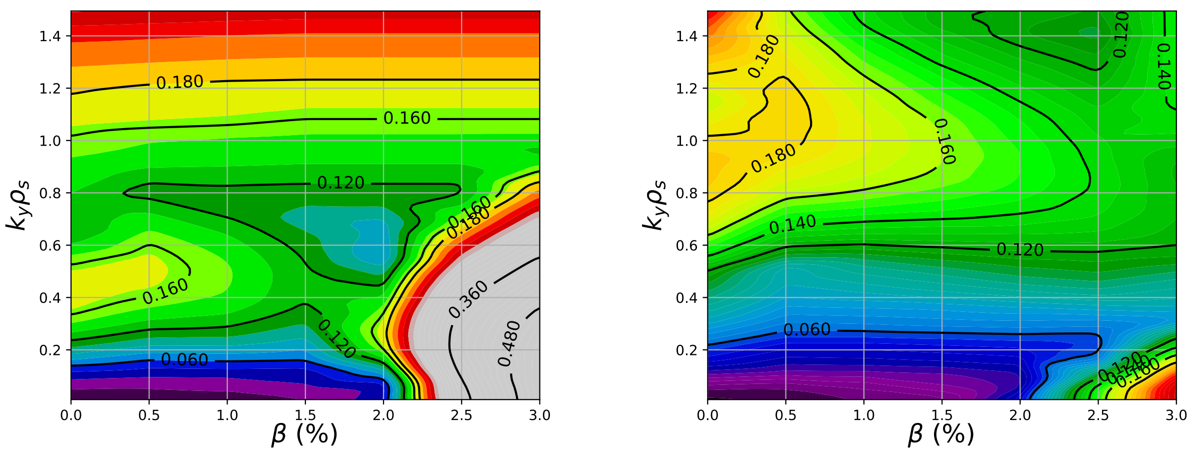

Figure 3: Instabilities growth rates as a function of β and $k_y\rho_s$ (colorcoded), when pressure gradients are not (!) consistent with the plasma equilibrium. Left: simulations for standard tokamak case. Right: W7-X standard configuration (KJM, "bean" tube).

for a wide range of simulation parameters. This is in contrast to the tokamak configuration, where the most unstable mode closer to the marginality has a finite $k_y\rho_s$. In W7-X, the ITG mode is progressively suppressed with increasing β but is not stabilized completely and the critical value for the onset of KBM is about $\beta = 2.5\%$ for $a/L_{Ti,e} = 2$, flat density gradient and and $\hat{s} \approx -0.1$. For the tokamak case, we observe a more significant β stabilization of the ITG mode and more unstable KBMs ($\beta = 2.1\%$). We conclude that this W7-X configuration appears to be more stable than the tokamak case with respect to low k_y modes, including KBMs.

In all these simulations, it is important to keep the magnetic equilibrium geometry consistent with the varying pressure gradient in the gyrokinetic simulations. This is clearly seen when we compare our results with simulations obtained without this sort of concordance (see Fig. 3). It is noted that β stabilization of TEM in the tokamak case appears only if the equilibrium is varied consistently with the pressure gradient, whereas in W7-X calculations, the TEM is also (moderately) stabilized by β even if the equilibrium is kept fixed.

References

[1] W.M. Tang, J.W. Connor, and R. J. Hastie, Nucl. Fusion 20, 1439 (1980)

[2] K. Aleynikova, A. Zocco, Phys. Plasmas 24, 092106 (2017)

[3] M. Kotschenreuther, G. Rewoldt, and W.M. Tang, Comput. Phys. Commun. 88, 128 (1995)

[4] W. Dorland, F. Jenko, M. Kotschenreuther, and B.N. Rogers, Phys. Rev. Lett. 85, 5579 (2000)

[5] F. Jenko, W. Dorland, M. Kotschenreuther, and B.N. Rogers, Phys. Plasmas 7, 1904 (2000)

[6] T. Dannert and F. Jenko, Phys. Plasmas 12, 72309 (2005)

[7] K.V. Roberts and J.B. Taylor, Phys. Rev. Lett. 8, 197 (1962)