Maximumprinzip, Hopf-Bifurkation und Ergodentheorie: Warum sind nur manche seiner Ergebnisse nach Eberhard Hopf benannt?*

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Abstract

First the naming habits of scientific results in mathematics, the sciences and technology are discussed in general, and then specifically the namings of Hopf's results. This requires some knowledge about Hopf's curriculum vitae.

Zunächst werden ganz allgemein die Benennungen von wissenschaftlichen Ergebnissen in Mathematik, Naturwissenschaften und Technik diskutiert, und dann speziell der Umgang mit den Hopf'schen Ergebnissen. Dafür ist es wichtig, einiges über den Lebensweg von Hopf zu wissen.

In dem Artikel *Hidden Authors - Verborgene Autoren*[1] werden publizierte Texte diskutiert, in denen die aufgeführten Autoren nicht die tatsächlichen oder nicht die einzigen sind. Der Bogen spannt sich von simplem Abschreiben bis zu der stolzen Behauptung: 'Gute Künstler kopieren, große Künstler stehlen. - Ich habe gestohlen!'. Interessanter sind aber die weniger bekannten, subtileren Facetten dieses Phänomens wie: Ausnützung hierarchischer Strukturen, Einfluß von Gruppendynamik, Benutzung von Tarnkappen zur Ausübung oder zum Unterlaufen von Diskriminierung, Verschweigen oder Erfinden von (Ko-)Autoren, realen und virtuellen. Unter anderem wurden auch Beispiele für die Benennung mathematischer Objekte untersucht. In diesem Artikel hier steht die Benennung/nicht-Benennung nach *Eberhard Hopf (1902-1983)* im Zentrum. Doch werden zunächst kurz allgemeiner Benennungen nach Personen betrachtet.

In der Entwicklung von Technik und Ökonomie reicht eine großartige Idee zu einer neuen Erfindung einschließlich der Herstellung eines Prototyps im allgemeinen nicht aus. Es muß sich ein Innovationsprozess anschließen, d.h. zusätzliche Forschung und Entwicklung, die zu einem robusten, preiswerten, vermarktungsfähigen Produkt führen.

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[1] Meyer-Spasche 2017 [9]

Dieser Innovationsprozess erfordert im allgemeinen andere Fähigkeiten als die Erfindung selbst und wird deshalb häufig von anderen Personen durchgeführt. In Wirtschaftstheorie und Technikgeschichte [2] sind erfolgreiche Innovationen typisch Kombinationen von mehreren, überwiegend bekannten Technologien und allgemein genug, um vielen potentiellen Benutzern große Vorteile zu bieten. Kreative Nachahmer sind häufig im Markt erfolgreicher als die technischen Pioniere.

Dies widerspricht der Sicht der Öffentlichkeit auf Wissenschaft und Technik: das romantische Ideal des einsamen Künstlers ist für die Öffentlichkeit attraktiver als ein verwickelter Prozess mit vielen verschiedenen Akteuren. So bevorzugt die Öffentlichkeit auch einzelne heroische Erfinder wie James Watt (Dampfmaschine), Carl Benz (Automobil) usw.. Außerdem ist der öffentliche Blick auf Erfindungen häufig national voreingenommen (Wengenroth: "er schmeichelt Sprachgruppen"): der Erfinder des Telefons war Phillip Reis (D), Innocenzo Manzetti (I) oder Alexander Bell (USA), je nachdem, wer sich äußert.[3] Auch in der Wissenschaftsgeschichte gibt es viele Beispiele für *The winner takes it all.* So ist z.B. *Mark Kac (1914-1984)* 1980 nach Polen gereist, um eine Gedenkrede für den wenig bekannten Physiker *Marian Smoluchowski (1872-1917)* zu halten. Dabei hat er betont, daß die Arbeiten Smoluchowskis zur Brownschen Bewegung wegen des Matthäus-Effektes nicht die ihnen gebührende Aufmerksamkeit gefunden haben: er stand im Schatten von *Albert Einstein (1879-1955)*, der auch dazu veröffentlicht hat. Der *Mathew-Effect* wurde 1968 von dem US-Wissenschaftshistoriker *Robert K Merton (1910-2003)* eingeführt, in Anlehnung an die Worte

> Denn wer da hat, dem wird gegeben werden, daß er die Fülle habe; wer aber nicht hat, dem wird auch das genommen, was er hat.

aus dem Matthäus-Evangelium. *Margaret Rossiter (*1944)* hat dann den *Matilda-Effect* eingeführt: in vielen Fällen stehen/standen Wissenschaftlerinnen im Schatten ihrer männlichen Kollegen.[4]

Auch in der Mathematik gibt es viele Beispiele für erste Versionen einer Idee oder eines Resultats (eine Vermutung, ein Satz, ein Algorithmus oder ...) und spätere Verschärfungen, Verallgemeinerungen oder andere Verbesserungen. Wer bekommt die Ehre? Wessen Name wird damit in Verbindung gebracht? Der erste? Derjenige, der den wichtigsten Beitrag geleistet hat? Derjenige, der schon wegen anderer Ergebnisse der Berühmteste ist?

Einige Namensentwicklungen wurden in großem Detail untersucht, z.B. *Gauß-Elimination* und *Zorn'sches Lemma,* und es gab eine ganze Tagung zu diesem Thema[5]. Das Endergebnis ist manchmal sehr überraschend, aber ein allgemeines Muster oder zusätzliche Effekte sind schwer zu erkennen, außer: Benennungen entstanden häufig in Unkenntnis der Vorgeschichte und in Unkenntnis schon existierender Benennungen an anderen Orten und/oder in anderen Disziplinen, für die die Mathematik eine Hilfswissenschaft ist. Hinzu kommt, daß manche mathematische Objekte schon ziemliche lokale

[2] Schumpeter 1912, [17]; Wengenroth 2015, [19]

[3] Wengenroth 2015 [19]

[4] Rossiter 1993 [15]

[5] Binder 2016 [2], Campbell 1978 [3], Grcar 2011 [4], Meyer-Spasche 2017 [9]

Bekanntheit besitzen, wenn ein neuer Äquivalenzsatz zeigt, daß zwei unterschiedlich benannte Objekte eigentlich dasselbe sind. Campbell kam zu dem Schluß, daß es vorwiegend Unkenntnis war, die zu dem Namen *Zorn'sches Lemma* geführt hat, und daß es letztlich müßig ist, bei Benennungen historische Korrektheit zu erwarten:

> The history of mathematics is rife with a variety of misattributions, whose continued propagation by oral and written tradition is variously due to widespread ignorance of the historical facts, accepted convention, or just plain complication of the situation. All of these play their part in "Zorn's Lemma"; even the term is used by different people to denote different propositions, logically – but not historically – equivalent.
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> This paper does not offer an authoritative version of the history of the interrelations of the tribe of maximal principles that pass for "Zorn's Lemma". The families of principles whose genealogies are discussed here have multiplied further, with contributions from [...] [6]

Diese Unkenntnis wird heute natürlich durch die verbesserte interdisziplinäre, weltweite Kommunikation stark reduziert. Es sollte aber auch nicht der vereinheitlichende, prägende Einfluß von weit verbreiteten, nicht immer historisch korrekten mathematischen Lehrbüchern unterschätzt werden.

Es gibt aber auch einen Algorithmus, der zunächst von Ingenieuren zur Eigenwertberechnung bei 2-Punkt-Randwert-Problemen formuliert wurde, dann von Mathematikern zur Eigenwertberechnung bei Matrizen, dann von Physikern zur Eigenwertberechnung bei Dirichletproblemen und dann ganz allgemein von Mathematikern funktionalanalytisch formuliert wurde: dabei erhielt er jedesmal einen neuen Namen, obwohl die vorherige Quelle jedesmal bekannt war und benannt wurde.[7]

Doch wenden wir uns jetzt Eberhard Hopf zu. Der folgende Text ist aus dem Artikel *Hidden Authors* übernommen.[8] Eine ausführlichere Darstellung mancher Details findet sich in Meyer-Spasche 2018 [8].

Before we consider Hopf bifurcation in some detail, we will look at Eberhard Hopf's vita. Also, we will discuss cases in which the mathematical subjects were *not* named after Hopf, for two completely different reasons.

In 2002, Morawetz, Serrin and Sinai edited *Selected works of Eberhard Hopf with Commentaries.* In the Foreword they wrote:

> Hopf (1902-1983) was a founding father of ergodic theory and produced many beautiful and now classical results in integral equations and partial differential equations. In fact so basic, for example, is his maximum principle that it is often used without reference to its author. Born in Austria, trained in Germany, Hopf spent several years at the Harvard Observatory and at M.I.T., returned for a permanent professorship in Leipzig in 1936 (to

[6] Campbell 1978, p.85, [3]

[7] Meyer-Spasche 2016 [10]

[8] Mit herzlichem Dank an Gudrun Wolfschmidt, die Herausgeberin des Bandes Wolfschmidt 2017 [21], in dem der Artikel steht.

> both dismay and understanding in the mathematical community), moved to Munich in 1944 and was a visiting professor at New York University in 1947. The remainder of his professional life he was a professor at Indiana University.
>
> Hopf was not a prolific writer but a very large fraction of his work remains at the core of the fields he worked in and he wrote with such elegance and clarity that they are of great use today. [. . .] One notes for example that the paper on Burger's equation has been cited 539 times according to the "Web of Science", but there are hardly any references to his very well-known and useful bifurcation results. However, the phrase "Wiener-Hopf" appears in 645 titles in the A.M.S. Math Reviews.[9]

The maximum principle for harmonic functions, the Laplace equation and some other equations has been known for a long time, starting with Riemann's dissertation in 1851 (advisor Gauss), or even earlier. It was then reformulated and generalized by many authors. Hopf proved strong maximum principles for general second-order elliptic operators in 1927. This opened the way to many additional important applications.[10]

There is no need, however, to identify a maximum principle by an additional name: which version is needed depends on the equations treated and on the intended results.[11] For bifurcations this is very different: the same parameter-dependent system may have many different types of bifurcations, depending on the parameter domain considered: pitchfork bifurcation, Hopf bifurcation, period-doubling bifurcation, symmetry-breaking bifurcation, etc.[12] The work of Hopf with respect to bifurcation is brought to mind by the name 'Hopf bifurcation'. That Hopf's paper does not have a record in the "Web of Science" [22] is due to the fact that it appeared in the proceedings of the Saxon academy of sciences. This journal is not in the core collection of WoS and not easily accessed. Also, non-German-speaking people often have problems with a correct citation of this journal.[13]

In the preface to their book *The Hopf Bifurcation and Its Applications* Marsden and McCracken wrote:

> Historically, the subject had its origins in the works of Poincaré [. . .] around 1892 and was extensively discussed by Andronov and Witt [. . .] and their co-workers starting around 1930. Hopf's basic paper [. . .] appeared in 1942. Although the term "Poincaré-Andronov-Hopf bifurcation" is more accurate (sometimes Friedrichs is also included), the name "Hopf Bifurcation" seems more common, so we have used it. Hopf's crucial contribution was the extension from two dimensions to higher dimensions.[14]

[9] Morawetz, Serrin, Sinai 2002, Foreword, [12]
[10] Protter, Weinberger 1984, p. vi; Bibliographical Notes, pp. 156-158, [14]
[11] Lortz, Meyer-Spasche 1982, [6]
[12] Meyer-Spasche 1999, [11]
[13] Hopf 1942, [5]
[14] Marsden, McCracken 1976 pp.vii-viii, [7]

Thus Marsden, McCracken reinforced the use of the concise name "Hopf bifurcation" because it already was more common than the very lengthy name "Poincaré-Andronov-Hopf(-Friedrichs) bifurcation". This longer name also would not be perfectly accurate: there are further contributions to the subject by other authors – also contributions in the book of Marsden, McCracken: it has 25 sections; 15 were written by Marsden and McCracken and 10 were written by 15 other authors, see the list of additional authors on the inner front page and in the table of contents.

Hopf himself was very aware of the work of Poincaré, but maybe not of the publications of Andronov et al before 1942 (The list of references in Marsden, McCracken contains two short papers (1930, 3 pages; 1937, 5 pages) and a book on oscillations in Russian (1937)). In his paper of 1942, Hopf discussed the connections of his results with the work of other authors, especially Poincaré:

> In the literature, I have not come across the bifurcation problem considered on the basis of the hypothesis (1.2) [$\alpha(0) = -\bar{\alpha}(0) \neq 0, \quad \mathrm{Re}(\alpha'(0)) \neq 0$]. However, I scarcely think that there is anything essentially new in the above theorem. The methods have been developed by Poincaré perhaps 50 years ago[15] and belong today to the classical conceptual structure of the theory of periodic solutions in the small. Since, however, the theorem is of interest in non-conservative mechanics it seems to me that a thorough presentation is not without value. In order to facilitate the extension to systems with infinitely many degrees of freedom, for example the fundamental equations of motion of a viscous fluid, I have given preference to the more general methods of linear algebra rather than special techniques (e.g. choice of a special coordinate system).[16]

That some other results of Hopf do not carry his name is related to his return to Germany in 1936:

> Hopf was never forgiven by many people for his moving to Germany in 1936, where the Nazi party was already in power. As a result most of his work on ergodic theory and topology was neglected or even attributed to others in the years following the end of World War II. An example of this was the dropping of Hopf's name from the discrete version of the so-called Wiener-Hopf equations which are currently referred to as "Wiener filter".[17]

Thus the name 'Hopf bifurcation' is a bit surprizing.

The return of Hopf, *Walter Tollmien (1900-1968)* and others to Germany after May 1933 was discussed by Siegmund-Schultze in detail.[18] When so many scientists lost

[15] Les méthodes nouvelles de la mécanique céleste. The above periodic solutions represent the simplest limiting case of Poincaré's periodic solutions of the second type [...]. Poincaré, having applications to celestial mechanics in mind, has only thoroughly investigated these solutions [...] in the case of canonical systems of differential equations, where the situation is more difficult than above. [...] which thereby becomes simpler. [...]

[16] Hopf 1942, [5], English version by Howard, Kopell in: Marsden, McCracken 1976, p.167f, [7]

[17] O'Connor, Robertson et al in: MacTutor History of Mathematics archive, Hopf Eberhard biography, [13], last visit in Jan 2016

[18] Siegmund-Schultze 2009, chapter 7, especially sections 7.S.2 and 7.S.3, [18]

their jobs because of the Nazis, many positions became available in Germany, but even excellent scientists had problems to find adequate positions abroad.

> It should not be very difficult in the near future for a German with flawless [einwandfrei] grandparents to find an adequate position in Germany. Perhaps one should make Germans abroad aware of this fact – something that would in turn open up chances for us abroad.[19]

There were several attempts to offer chairs to Eberhard Hopf: one in Bonn, one in Göttingen, and the one of v.Mises in Berlin. They failed because of the resistance of the 'Dozentenschaft' (the Nazi faculty organization). 'There were accusations that he was friends with an alleged communist during his university studies.' If he could be convinced to move to Bonn 'he could probably be more courageous than others because he would always have the chance to go back to America.'[20]

When Hopf got the offer of the chair in Leipzig, he had a permanent position at MIT, not very well payed. He did not accept the offer for Leipzig spontaneously, but quite fast, too fast for his enemies among the Nazis. He consulted with a number of German refugees and he tried unsuccessfully to improve his position at MIT. Then he accepted the offer, delighted by the prestigious position and the very good payment.[21]

Schlote investigated the interactions of mathematics and physics at Leipzig university during the years 1905-1945. This includes the years when Hopf was there.[22] On August 4, 1936 it was decided in Berlin to send an offer to Hopf. Two detailed letters were sent to him, one by the dean *Paul Koebe (1882-1945)* and one by the ministry. After a short exchange about financial matters, Hopf cabled his agreement, only three weeks after the first letter was sent to him. A few days later, Hopf was denunciated as being friendly with Jews - everything became uncertain again. Since the 'Stellvertreter des Führers' (representative of Hitler) had agreed to the offer, it could not be withdrawn, they had to find another way to resolve the problem. From october 1936 until summer 1937, Hopf was only the administrator of the chair. Positive reviews were written about him regarding his political, professional and personal qualities, and then he became full professor. He stayed at Leipzig university until march 1942, when he was given leave of absence and was sent by the ministry to an aerodynamic research institution in Ainring (Bavaria), too far away from Leipzig for commuting. His work (important for the war) in Ainring was probably the motif for his move from Leipzig university to Munich university in 1944.

Was Hopf a Nazi? *Norbert Wiener (1894-1964)* remembered in 1956:

> Originally he was hostile to Hitler, or at least sympathetic to those on whom Hitler had wreaked his ill will. However, there were strong family influences pulling him to the Nazi side.[23]

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[19] letter in May 1935 from *Kurt Hohenemser (1906-2001)* to von Kármán in: Siegmund-Schultze 2009, section 7.S.2, [18]

[20] letter from Alfred Brauer (Berlin) to Otto Toeplitz (Bonn) in May 1935, in: Siegmund-Schultze 2009, section 7.S.3, [18]

[21] Wiener 1956, [20], Siegmund-Schultze 2009, sec. 7.S.3, [18]

[22] Schlote 2008, pp. 250ff, [16]

[23] Wiener 1956, [20]; Siegmund-Schultze 2009, sec. 7.S.3, [18]

And in Leipzig? Nothing is known from his time in Leipzig (1936-1942) which would prove that he was an active Nazi - on the contrary: Hopf was among those who tried to help *Ernst Hölder (1901-1990)* when Hölder was denied an adequate position because his two sisters were married to Jewish mathematicians. Also, Hopf and *B.L. van der Waerden (1903-1996)* had many disagreements with Koebe who was much closer to the regime and who tried to impose the 'Führerprinzip' - leadership by one person (Koebe) instead of the more democratic joint administration by all three full professors.[24] Hopf's reference to Poincaré's work in his bifurcation paper shows very clearly that he ignored the silly theory of Bieberbach et al about 'German Mathematics' and 'French Mathematics'.

Mein Dank geht an alle, die diesen Artikel durch Kritik, Fragen oder Bemerkungen verbessert haben, und an Gudrun Wolfschmidt, die den Nachdruck des Textes aus *Hidden Authors* erlaubt hat.

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