Supplemental Material

Universality in the Spatial Evolution of Self-Aggregation of Tropical Convection

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Supplementary Material

1. Details on theoretical models and parameter estimation

a. Details on Model introduced by Bretherton et al. (2005)

To obtain a closed equation for the time evolution of a humidity perturbation from the model presented in Bretherton et al. (2005), their Eq. 9

$$\partial_t r = [c_s + c_r - \alpha_h (r - r_h)][P(r) - P_{RCE}]/W_* \tag{1}$$

is combined with the dependence of the precipitation rate on humidity (their Eq. 2)

$$P(r) = P_{RCE} \mathrm{e}^{a_m (r - r_{RCE})}, \tag{2}$$

where r is the column relative humidity, c_s, c_r, α_h, r_h and a_m are fitting parameters for the forcing terms, W_* is the saturation water-vapor-path, P_{RCE} is the horizontal mean radiative-convective equilibrium rain rate and r_{RCE} the corresponding column relative humidity[1]. All parameters were estimated by Bretherton et al. (2005) using their RCE simulation and are summarized in table 1.

Combining Eq. 1 with Eq. 2 and replacing $r - r_{RCE}$ by r' yields the following source term:

$$R_B(r') = [c_s + c_r - \alpha_h (r' - r_h + r_{RCE})][P_{RCE}(\mathrm{e}^{a_m r'} - 1)]/W_* \tag{3}$$

Eq. 3 can be rewritten in the form shown in the main manuscript by combining the different parameters:

$$k_1^b = a_m \qquad = 16.6 \tag{4}$$

$$k_2^b = \frac{P_{RCE}}{W_*}[c_s + c_r - \alpha_h (r_{RCE} - r_h)] \qquad = 7.8 \times 10^{-8}\ \mathrm{s}^{-1} \tag{5}$$

$$k_3^b = \frac{P_{RCE}}{W_*}\alpha_h \qquad = 1.3 \times 10^{-6}\ \mathrm{s}^{-1} \tag{6}$$

and by referring to the column relative humidity r as q.

b. Details on model introduced by Craig and Mack (2013)

The local feedback on the humidity content introduced by Craig and Mack (2013) is given by the first two terms on the right hand side of their Eq. 14:

$$R_C(I_v/I_v^*) = -\alpha I_v/I_v^* + \frac{a(t)}{I_v^*}\left(\frac{I_v^*}{\beta I_v} - 1\right)(e^{bI_v/I_v^*} - 1). \tag{7}$$

where I_v is the vertically integrated free tropospheric moisture content and I_v^* is the corresponding saturation value. Note that we have divided both sides of the equation by I_v^* to obtain a measure of relative humidity, ranging from zero to one. The first term on the right hand side of Eq. 7, represents the *subsidence drying term* which accounts for the loss of humidity due to subsidence, with subsidence rate α.

The second term is the *convective moistening term* and represents the increase in humidity due to convection. Note that Craig and Mack (2013) introduced a time dependent parameter $a(t)$, which ensures that the total amount of precipitation averaged over the area is constant. As noted in the paper, this global constraint is dropped in the present model and $a(t)$ is replaced by a constant parameter a. Based on the dependence of the precipitation rate on humidity found by Bretherton et al. (2005), see Eq. 2, the term $a(\exp(bI_v/I_v^*) - 1)$ is introduced to represent the amount of precipitation for a given value of (I_v/I_v^*), and $(1/(\beta I_v/I_v^*) - 1)$ is the complement of the precipitation efficiency and gives the amount of humidity that does not drop out as precipitation. The parameters a, b and β are fit parameters determined in the following.

The parameters are estimated from the subsidence drying and the convective moistening rates determined by Kempf (2014) from a RCE simulation, see Fig. 1[2]. Assuming that subsidence drying and convective moistening result from the vertical transport of moisture, both were estimated from the net vertical moisture flux into the free troposphere. Dividing the domain into regions with and without convection, convective moistening and subsidence drying as a function of humidity content were approximated as the mean change in humidity at a given humidity value.

Fitting the subsidence drying and the convective moistening term from Eq. 7 to the numerically determined rates allows us to estimate the relevant parameters. Note that we use the results from Bretherton et al. (2005) as first guess values for fitting the convective moistening part in Fig. 1. In addition to the fit, which gives only a value for the fraction of a/I_v^*, we extract the mean saturation water vapor content $I_v^* = 29.6$ kg m^{-2} from the simulation, which allows us to estimate a separately. Finally, we estimate the initial humidity content I_v^{RCE}. As the initially uniform humidity distribution in RCE simulations is unstable to perturbations, we estimate its value using $R_C(I_v^{RCE}/I_v^*) = 0$ and $\partial_{I_v} R_C(I_v/I_v^*)|_{I_v^{RCE}} > 0$ which yields $I_v^{RCE} = 12.6$ kg m^{-2}. In particular, we consider I_v^{RCE} as the homogeneous humidity content found in the absence of self-aggregation, usually determined by a corresponding small domain RCE simulation. The parameter values are summarized in table 2.

[1]Note that P_{RCE} and r_{RCE} are determined by fitting an exponential dependence to the scatterplot of precipitation and column-relative humidity, averaged over (72km)^2. Though the fitted relation depends on the stage of self-aggregation, Bretherton et al. (2005) note that Eq. 2 adequately fits the relation throughout the simulation.

[2]As Kempf (2013) and Kempf (2014) are Master theses and not publicly available we shortly summarize the simulation setup and the key results in the next section.

TABLE 1. Parameters for positive feedback loop based on Bretherton et al. (2005)

	c_s	c_r	α_h	r_h	P_{RCE}	W_*	r_{RCE}	a_m
Value	0.12	0.17	1.8	0.62	3.5	57	0.72	16.6
Unit	1	1	1	1	mm day^{-1}	mm	1	1

TABLE 2. Parameters for positive feedback loop based on Craig and Mack (2013)

	α	β	a	b	I_v^*	I_v^{RCE}
Value	$2.0 \cdot 10^{-6}$	1.1	$1.7 \cdot 10^{-7}$	11.4	29.6	12.6
Unit	s^{-1}	1	kg m^{-2} s^{-1}	1	kg m^{-2}	kg m^{-2}

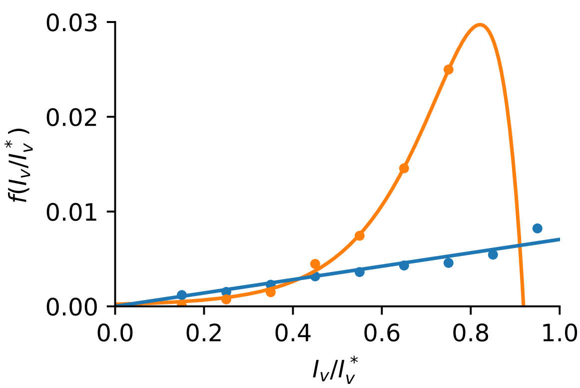

FIG. 1. Absolute values of the subsidence drying (blue) and convective moistening (orange) terms as a function of vertically integrated free tropospheric humidity content determined by Kempf (2014) (markers) with corresponding fit (solid lines).

Apart from replacing $a(t)$ by a constant value, the only difference between Eq. 7 and the corresponding equation in the main text, is that we have combined and renamed the parameters to unify the notation with the other models:

$$k_1^c = \alpha = 2.0 \cdot 10^{-6}\ \mathrm{s}^{-1} \quad (8)$$

$$k_2^c = \frac{a}{I_v^*} = 5.7 \cdot 10^{-9}\ \mathrm{s}^{-1} \quad (9)$$

$$k_3^c = b = 11.4 \quad (10)$$

$$k_4^c = \beta = 1.1 \quad (11)$$

and refer to the free tropospheric column relative humidity, I_v/I_v^*, as q.

c. Details on model introduced by Emanuel et al. (2014)

The time-evolution equation for humidity perturbations derived by Emanuel et al. (2014) accounts for the effect of small perturbations in convective transport, advection and radiative heating on the humidity. As radiative heating depends not only on the vertically integrated moisture content but also on the moisture profile, Emanuel et al. (2014) introduce a linearized two-layer model for the moisture perturbations:

$$L_v \begin{pmatrix} \partial_t q_1' \\ \partial_t q_2' \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} q_1' \\ q_2' \end{pmatrix}, \quad (12)$$

where L_v is the latent heat of vaporization, q_i' is the deviation from the mean RCE specific humidity in the lower ($i = 1$) and upper layer ($i = 2$) and c_{ij} are the four linearization coefficients, derived from the dependence of radiative heating, convective transport and vertical advection on humidity.

In this study, our focus is to compare different models which lead to self-aggregation. Emanuel et al. (2014) show that their model leads to self-aggregation only in the limit of high sea-surface temperatures. Applying this limit in the equations for c_{ij} given in Emanuel et al. (2014), we find that $c_{11} \to 0$ and $c_{21} \to 0$ and thus the perturbation humidity content of the second layer (q_2') decouples from the first layer. Its time evolution and therefore the local source term R_E for the model introduced by Emanuel et al. (2014) can be expressed as:

$$R_E(q_2'/q_2^\star) = \frac{1}{L_v} c_{22}(q_2'/q_2^\star). \quad (13)$$

Note that we have divided both sides of Eq. 12 by the saturation specific humidity of the upper layer $q_2^\star$ to obtain the evolution equation for the nondimensional quantity: $q_2'/q_2^\star$ to which we refer as q in the main text.

The relevant linearization coefficient c_{22} is given in Emanuel et al. (2014) as

$$c_{22} = \frac{\partial \varepsilon_2}{\partial q_2} \sigma T_2^4 \frac{1}{H} \left(\frac{\varepsilon_p}{\rho_1} \frac{S_2}{S_1} + \frac{1}{\rho_2} \left(\left(\frac{T_1}{T_2} \right)^4 - 2 \right) (1 - \varepsilon_p) \right), \quad (14)$$

with the Stefan-Boltzmann constant $\sigma = 5.7 \cdot 10^{-8}$ W m^{-2} K^{-4}, H the height of one layer, ρ_i, T_i

and S_i the density, temperature and dry static stability of layer i. As $\partial\varepsilon_2/\partial q_2$ denotes the dependence of the emissivity of the second layer ε_2 on humidity of the second layer q_2, the term outside the brackets describes the change in radiative cooling of the upper troposphere due to a change in humidity. We do not go into detail on the meaning of the remaining terms, but note that ε_p is the ratio of the updraft mass-flux to the total mass-flux, i.e. the mass-flux also including convective downdrafts, introduced in the context of determining the amount of subsidence necessary to compensate the updraft convective mass-flux. The corresponding parameters values, summarized in table 3, are estimated in the following.

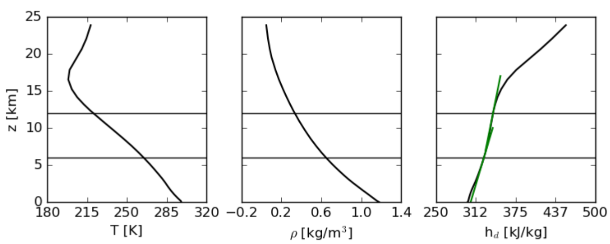

FIG. 2. Mean annual West Indies sounding of temperature (left), density (middle) and dry static energy (right) determined from Jordan (1958).

To estimate the parameters, we start with the mean annual West Indies sounding data for isobaric surfaces obtained by Jordan (1958). Choosing the heights of the two layers as $H_1 = 6$ km and $H_2 = 12$ km, the first layer is located within the lower troposphere, while the second layer is within the upper troposphere, as sketched in Fig. (1) in Emanuel et al. (2014). Evaluating the temperature and density soundings, shown in Fig. 2, at $z = 6$ km and $z = 12$ km, gives the values $T_1 = 264.7$ K, $T_2 = 220.4$ K, $\rho_1 = 0.647\,\mathrm{kg\,m^{-3}}$ and $\rho_2 = 0.333\,\mathrm{kg\,m^{-3}}$.

To estimate the dry static stability we first need to calculate the dry static energy as a function of height z. The dry static energy is given by

$$h_d(z) = c_p T(z) + gz \tag{15}$$

where c_p is the specific heat capacity of air at constant pressure and g is the gravitational acceleration. Using the temperature profile shown above, the calculated profile of $h_d(z)$ is also shown in Fig. 2. Determining the gradient at the two height levels, gives the following estimates for the dry static stability: $S_1 = 3.5\,\mathrm{J\,K^{-1}\,m^{-1}}$ and $S_2 = 2.3\,\mathrm{J\,K^{-1}\,m^{-1}}$.

In addition to the temperature, density and static stability values, we need to estimate how the emissivity in the upper model layer depends on the humidity within this layer ($\partial_{q_2}\varepsilon_2$). To this end we use Eq. 28 from Emanuel et al. (2014):

$$\frac{\partial \dot{Q}_2}{\partial q_2} = \frac{\bar{\dot{Q}}_2}{\varepsilon_2}\frac{\partial \varepsilon_2}{\partial q_2}, \tag{16}$$

solve it for $\partial_{q_2}\varepsilon_2$ and approximate each term separately:

$$\frac{\partial \varepsilon_2}{\partial q_2} = \frac{\partial \dot{Q}_2}{\partial q_2}\frac{\varepsilon_2}{\bar{\dot{Q}}_2} \tag{17}$$

$$\approx \left(\frac{\Delta \dot{Q}_2}{\dot{Q}_2}\right)\left(\frac{q_2}{\Delta q_2}\right)\frac{\varepsilon_2}{q_2} \tag{18}$$

$$\tag{19}$$

We estimate the first two terms on the right from Fig. 5 in Emanuel et al. (2014), where a reduction of the humidity (initially close to saturation) by 20% ($q_2/\Delta q_2 \approx 1.0/0.2$), leads to a radiative cooling perturbation ($\Delta\dot{Q}_2$) of approximately $0.1\,\mathrm{K\,d^{-1}}$. Assuming standard values for the mean radiative cooling over tropical oceans of $\dot{Q}_2 \approx 2\,\mathrm{K\,d^{-1}}$ (e.g. Tompkins and Craig 1998), the mean emissivity of the upper troposphere of $\varepsilon_2 \approx 0.3$ (p. 394 Pierrehumbert 2010) and the specific humidity of approximately $q_2 \approx 1\times10^{-4}\,\mathrm{kg\,kg^{-1}}$ (Jordan 1958), the resulting estimate for $\partial_{q_2}\varepsilon_2$ is 750.

Finally we estimate ε_p which, as noted by Emanuel et al. (2014), is related to a bulk precipitation efficiency. Assuming $\varepsilon_p = 1.1\cdot q_{RCE}$, the relationship between precipitation efficiency and relative humidity proposed by Craig and Mack (2013), and setting $q_{RCE} = 0.72$ (Bretherton et al. 2005) yields $\varepsilon_p \approx 0.8$.

Having estimated all necessary parameter values (see table 3) we can now calculate the linear coefficient, relevant for $R_E(q)$ and given in the main text, from Eq. 14:

$$k^e = \frac{1}{L_v}c_{22} \qquad = 5.8\cdot10^{-6}\ \mathrm{s^{-1}} \tag{20}$$

where we have used $L_v = 2.5\cdot10^6\ \mathrm{J\ kg^{-1}}$.

2. Simulations performed by Kempf (2013) and Kempf (2014)

Kempf (2013) and Kempf (2014) performed RCE simulations using EULAG (Smolarkiewicz and Margolin 1997). A number of simulations were performed, each using a domain with a horizontal extent of 510 km × 510 km, a horizontal resolution of 2km and constant SST. One key goal of Kempf (2013) was to investigate the existence of a critical sea-surface temperature ($\mathrm{SST_c}$) for self-aggregation. Testing three different values, Kempf (2013) found that self-aggregation occured for 300 K and 306 K but not for 294 K, indicating a $\mathrm{SST_c}$ below 300 K and above 294 K. Kempf (2014) investigated the $\mathrm{SST_c}$ in more detail and determined its dependence on varying initial conditions. In particular, they limited the value of the critical sea-surface temperature to a smaller range (between 292.5K and 294.0K) and showed that, if the initial moisture field contains a dry spot, the critical sea-surface temperature decreases to a smaller value. Reducing the humidity content of the initial dry spot further led to an even smaller value for the critical SST.

TABLE 3. Parameters for positive feedback loop based on Emanuel et al. (2014)

	H	T_1/T_2	ρ_1/ρ_2	S_1/S_2	$\partial_{q_2}\varepsilon_2$	ε_p
Value	6000	264.7 / 220.4	0.647 / 0.333	3.5 / 2.3	750	0.8
Unit	m	K	kg m^{-3}	J K^{-1}m^{-1}	1	1

3. Autocorrelation Length

The autocorrelation length l_{cor} is defined as the length-scale at which the radially averaged autocorrelation function has dropped to e^{-1}. The first step in calculating the radially averaged autocorrelation function of the two dimensional binary field $b(t;\vec{x})$ (dry perturbations: 0, moist perturbations: 1) at a given time t is to calculate the two dimensional spatial autocorrelation function $A(\vec{x})$, which is defined as:

$$A(\vec{x}) = \int \mathrm{d}\vec{x}' \, \frac{(b(t,\vec{x}+\vec{x}') - \langle b(t)\rangle)(b(t,\vec{x}') - \langle b(t)\rangle)}{\sigma(b(t))^2}. \tag{21}$$

The $\langle\cdot\rangle$ denotes the mean and $\sigma(\cdot)$ the standard deviation. Note that the autocorrelation function can be efficiently calculated in Fourier-Space using the convolution theorem, e.g. Newman and Barkema (1999). From the 2D autocorrelation function, the radially averaged autocorrelation function is then calculated as:

$$A(r) = \frac{\int_\Gamma A\mathrm{d}s}{2\pi r} \tag{22}$$

where Γ corresponds to a circular path with radius r and center at the origin and ds is the corresponding line element. From $A(r)$, the correlation length l_{cor} is determined by $A(r = l_{cor}) = e^{-1}$.

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