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Minimal radiative neutrino mass

-A systematic study-

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Abstract In this thesis, we systematically study neutrino mass models that only generate the dimension-5 Weinberg operator at loop-level. After an introduction to representation theory, fermion masses and see-saw mechanisms, we search for radiative neutrino mass models with a minimal number of new multiplets which do not require a new symmetry beyond the Standard Model gauge group. New coloured fields are also allowed. It can be shown that there is no radiative neutrino mass model with only two new fermionic multiplets. In addition, there exists a unique model with three new fermionic multiplets. A subsequent discussion of potential dark matter candidates present in some of the models reveals that all of them decay too quickly to account for dark matter. In the three-fermion model, they even decay too quickly to lead to displaced vertex signatures at the Large Hadron Collider (LHC). In addition, a connection between coloured neutrino mass models and B-physics anomalies is discussed. Furthermore, we consider radiative neutrino mass models from a $SU(5)$ grand unified theory perspective and study their compatibility with unification and proton decay bounds. We were able to find out that for a group of models with new vector-like fermion unification in agreement with proton decay bounds is possible.

Zusammenfassung In dieser Arbeit werden systematisch Neutrinomassen-Modelle untersucht, die den 5-dimensionalen Weinberg-Operator in erster Ordnung der Störungstheorie erzeugen. Die Basis bildet eine Einführung in Darstellungstheorie, Fermionmassen sowie in die See-Saw-Mechanismen. Gesucht werden radiative Neutrinomassen-Modelle mit einer minimalen Zahl neuer Multiplets, ohne dabei neue Symmetrien zusätzlich zur Eichsymmetrie des Standardmodells zu fordern. Dabei sind auch neue farbige Felder zugelassen. Wir zeigen mit unserer Untersuchung, dass es kein radiatives Neutrinomassen-Modell mit nur zwei neuen fermionischen Multipletts gibt. Zudem weisen wir nach, dass genau ein Modell mit drei neuen Fermionen existiert. Bei der sich daran anschlieSSenden Diskussion zu möglichen Kandidaten für dunkle Materie in einigen Modellen wird deutlich, dass diese Kandidaten zu schnell zerfallen, um dunkle Materie zu sein. Der Kandidat im drei-Fermionen Modell zerfällt sogar zu schnell, um am Large Hadron Collider versetzte Vertex-Signaturen zu erzeugen. Zusätzlich werden in dieser Arbeit Verbindungen zwischen farbigen Modellen für Neutrinomassen und Anomalien in der B-Physik analysiert. Außerdem betrachten wir radiative Neutrinomassen-Modelle aus der Perspektive einer $SU(5)$ vereinheitlichten Theorie. Bezogen hierauf untersuchen wir die Vereinbarkeit der Neutrinomassen-Modelle mit einer Vereinheitlichung der Kopplungen und des Protonzerfall-Limits. Dabei können wir feststellen, dass die Vereinheitlichung unter der Einhaltung des Protonzerfall-Limits in einer Gruppe von Modellen mit neuen, vektorartigen Fermionen möglich ist.

Contents

1 Introduction

For many years, a collection of effective quantum field theories based on the gauge group

$$G_{\text{SM}} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \ , \tag{1.0.1}$$

called the Standard Model of particle physics, or Standard Model, has been incredibly successful in describing a large variety of particle physics phenomena. Despite of its huge success, hints have been accumulating that the Standard Model is insufficient to describe all of particle physics up to arbitrarily high energy scales.

Today, there is a number of issues which range from cosmological implications of particle physics, such as the nature of dark energy [1, 2] and the inflation field [3, ch. 22], over flavour anomalies measured in accelerator experiments [4–6], to questions regarding the nature of its high-energy completion. Here, we will highlight three shortcomings of the Standard Model, which are of of interest to the following work:

The first one is called the baryon asymmetry of the Universe. The baryon density in our observable Universe is measured to be $\Omega_b h^2 = 0.02233 \pm 0.00015$, where h is the reduced Hubble constant [7]. In addition, we know that all baryonic matter we observe is in fact matter and not antimatter. If there were regions filled with antimatter, one could observe the gamma rays from the matter-antimatter annihilation [8, 9]. The fact that there is an excess of matter over antimatter in the Universe is named the baryon asymmetry of the Universe. The reason why this is puzzling is that due to CPT-invariance, the best-working model for the evolution of the Universe contains the production of equal amounts of matter and antimatter after a period called inflation. To create an asymmetry between matter and antimatter from this initial equilibrium, the Sakharov conditions [10] must be fulfilled. These conditions are violation of baryon number B, charge conjugation C, and the particle-antiparticle symmetry CP, as well as deviation from thermal equilibrium. The Standard Model fulfils the first two criteria via non-perturbative sphalerons and complex phases in the quark Yukawa matrices. However, the third one is not fulfilled, and the CP violation in the Standard Model is too small. A different explanation is needed for this asymmetry [11].

The second issue is called dark matter. As early as 1933, F. Zwicky observed that the galaxies in the Coma cluster rotated much faster than he expected [12]. A number of different measurements were executed to determine the matter density and find the source of the discrepancy [13]. Among others, one important observation was that of the bullet-cluster 1E0657-558 [14]. It was detected that in the collision of the two clusters, the gravitational potential, measured by gravitational lensing maps, followed the distribution of galaxies rather than that of the plasma, even though the plasma represents the bulk of the luminous matter. This leads to the conclusion that, even though some effects can very well be explained by changes in the laws of gravity [15], there should probably be additional matter which is not observed electromagnetically. In order to clarify the nature of this matter, astronomers have searched for planet-like objects, neutron stars or black holes as potential dark matter candidates. They found that there are too few of these objects to explain the complete amount of dark matter [16]. Thus there is still a large amount of matter in the Universe whose nature we do not know yet. One solution is a new, still unobserved electrically neutral particle. This makes dark matter a problem that concerns the Standard Model.

The strongest evidence for beyond the Standard Model physics is probably given by the third problem, neutrino oscillations. They have not only been observed [17], but the majority of the parameters describing them has been determined at the ten percent level [18]. Neutrino oscillations cannot be explained by the Standard Model, since they require the neutrinos to be massive, albeit they are massless in the Standard Model. Since neutrino oscillations are only sensitive to mass differences, at least two of the three neutrino species should be massive, while one neutrino may remain massless. As a consequence, some new physics has to be added to the Standard Model to generate a mass for at least two of the neutrinos. Studying the various possibilities to give mass to neutrinos might be an important step in finding a more complete theory of particle physics.

One surprising detail is that neutrino masses are so much smaller than all the other particle masses we know. The strongest bounds on the neutrino mass[1] tell us that it is at least five orders of magnitude smaller than the mass of the electron [7]. As a result, the new physics should not simply introduce neutrino masses, but also contain a mechanism explaining why they are so tiny compared to other masses.

So far, we do not know what this new physics looks like. One possibility is to simply introduce right-handed neutrinos [21–24]. Another very simple possibility adds a certain scalar to the Standard Model to produce neutrino mass [25–28]. In these cases, the smallness of neutrino masses is explained by the so-called see-saw mechanism, which will be described in the next section. However, the standard version of this mechanism requires the new particles to be very heavy. Their large mass makes it very difficult to experimentally observe these particles. Moreover, considering other problems of the Standard Model, such as dark matter or the baryon asymmetry of the Universe, additional new physics might be needed. New problems, for example the so-called neutrino hierarchy problem, which will be discussed in section (3.1), may arise. Considering this, it is not obvious whether one of these simple models based on a see-saw is really Nature's choice. Thus, systematic studies of alternative models are an important tool for theoretical and experimental physicists trying to discover the right mechanism of neutrino mass generation.

In this thesis, we aim to systematically study next-to-minimal neutrino mass models, generating neutrino masses at loop-level or via operators of dimension larger than 5. This type of models will be called radiative neutrino mass models. The additional suppression factor from the loop or the higher dimension in these models helps to lower the mass scale

[1] In fact, different experiments measure different effective neutrino mass parameters. For example, cosmology constrains the sum of neutrino masses [7], while beta-decay experiments such as KATRIN [19], and $0\nu\beta\beta$ experiments such as GERDA [20], measure or constrain $m_{ee}^2 = \Sigma_{i=1}^3 \|U_{ei}\|^2 m_i^2$.

of the new fields, making them more accessible to experiments.

There have been a number of works that study radiative neutrino mass models in a systematic manner. In doing so, they use mainly two criteria to find and organise different models. The first one is a list of higher-dimensional operators which can generate neutrino mass [29, 30], while the second one is the topology of the neutrino mass diagram, or the topology of the higher-dimensional operator including the new fields [31–35].

This work approaches the topic from a different angle, thereby complementing earlier works on this topic. Assuming that there are no new symmetries beyond the ones of the Standard Model, we use the combinatorics of the G_{SM} representations and try to construct lepton number violating interaction terms. Since the Majorana-like neutrino mass term breaks lepton number by two units, we find all models producing a Majorana neutrino mass term. We limit ourselves to two new G_{SM}-multiplets to keep the results readable. The consideration of lepton number violation also enables us to show in a very simple way that one cannot produce explicit lepton number violation with only two new fermionic representations, apart from the ones used in the see-saw models. This scan and its result have also been published in ref. [36].

The thesis is structured as follows: In chapter 2 we give a basic introduction to representations of $SU(N)$, with a focus on $SU(2)$ and $SU(3)$. Moreover, we state some basic properties of neutrino masses and describe the see-saw mechanisms. We will also emphasise different types of fermions arising in our considerations. Chapter 3 contains a detailed explanation of our scan, including its results. In chapter 4, we give an outlook to which other problems neutrino mass could be linked. This includes dark matter, collider phenomenology, and B-anomalies. In chapter 5, we change perspective and discuss the embedding of radiative neutrino mass into grand unified theories based on $SU(5)$. We also test for unification of couplings for a class of models with vector-like fermions. We conclude this work in chapter 6.

Unless stated otherwise, we will work in Natural Units

$$\hbar = c = 1\,. \tag{1.0.2}$$

2 Theory

In this chapter, we describe the theoretical framework for this thesis, including an introduction to representation theory as it is used in particle physics, different kinds of fermions and their relation, and the see-saw mechanism.

2.1 Introduction to representation theory

2.1.1 Symmetries, groups, and algebras

Symmetry principles have been and may still be one of the most important principles in modern physics. They do not only help us to identify conserved currents and charges via Noether's theorem [37]. The development of quantum field theories based on local symmetries, also called gauge symmetries, lead to the construction of very successful theories of nature, such as quantum electrodynamics (QED) [38, 39], quantum chromodynamics (QCD) (see [40] and references therein), and finally the Standard Model of particle physics (see, e.g. [41, 42]).

In field theory at the classical level, symmetries are transformations of the fields in the Lagrangian, under which the action does not change. As an example, the Lagrangian

$$\mathcal{L} = \phi^\dagger(\partial^2 - m^2)\phi \tag{2.1.1}$$

of a complex scalar field ϕ is invariant under global phase rotations $\phi \to \mathrm{e}^{i\alpha}\phi$, where α is any real number.

How is this connected to groups? Groups consist of a set $\mathcal{G}$ and an operation $*$: $\mathcal{G}\times\mathcal{G} \to \mathcal{G}$, such that [43, 44]

1. $\forall\, g, h \in \mathcal{G}$, $g * h \in \mathcal{G}$.
2. $\forall\, g, h, k \in \mathcal{G}$, $(g * h) * k = g * (h * k)$.
3. $\forall\, g \in \mathcal{G}$, $\exists\, e \in \mathcal{G}$, such that $g * e = e * g = g$. e is the unit element, or identity.
4. $\forall\, g \in \mathcal{G}$, $\exists\, g^{-1} \in \mathcal{G}$, such that $g * g^{-1} = g^{-1} * g = e$. g^{-1} is the inverse of g.

The symmetry operations can then be identified as group elements. To be more precise, the object that is to be transformed should be regarded as a vector in a n-dimensional vector space V. To act on it with an element g of the group $\mathcal{G}$, we need a group homomorphism[1] $\mathcal{R}$: $\mathcal{G} \to GL(V)$, $g \to R(g)$, that connects every element g of the group

[1] A function $f : (\mathcal{G}, *) \to (\mathcal{H}, \cdot)$ between two groups $(\mathcal{G}, *)$ and $(\mathcal{H}, \cdot)$ is a group homomorphism if $\forall$ $g_1, g_2 \in \mathcal{G}$: $f(g_1 * g_2) = f(g_1) \cdot f(g_2)$ [45].

to a general linear operator $R(g)$ on the vector space V. The action of an element g of the group $\mathcal{G}$ on an element v of the vector space V is then given by $v \to R(g)v$. $\mathcal{R}$ is called the representation of the Group $\mathcal{G}$ on the vector space V[2]. Every group has a one-dimensional trivial representation $\mathcal{R}_t$, that maps any $g \in \mathcal{G}$ to 1 [43]. A representation is called *faithful*, if the homomorphism $\mathcal{R}$ is bijective. A representation is called *reducible*, if there exists a basis of the vector space in which for any $g \in \mathcal{G}$, $R(g)$ can be written in the form

$$\begin{pmatrix} R_1(g) & A \\ 0 & R_2(g) \end{pmatrix} . \tag{2.1.2}$$

Here, $R_1(g)$ and $R_2(g)$ are m- and n-dimensional representations of g. The zero is to be understood as a zero matrix of dimension $n \times m$, and A is a matrix of dimension $m \times n$ respectively. If $A_{ij} = 0 \; \forall \; i, j$, then the representation is called *fully reducible*, and can be written as the sum of the two representations $\mathcal{R}_1$ and $\mathcal{R}_2$. A representation that is not reducible is called *irreducible* [44].

The word representation is sometimes also used to denote the vector space that the group acts on, as well as the image of the group under the representation homomorphism.

In the example with the simple Lagrangian given in eq. (2.1.1), the symmetry group would be $U(1)$. The scalar field transforms under $U(1)$ non-trivially, while the complete Lagrangian transforms trivially under $U(1)$.

All groups we will consider in this thesis are Lie groups. In general, a Lie group is a set which has a not only a group structure, but is also an analytic manifold, and should satisfy the condition that for any $g, h \in \mathcal{G}$, the mappings $\phi(g, h) = g \cdot h$ and $\varphi(g) = g^{-1}$ are analytic functions [45].

However, the Lie groups we are interested in are all linear Lie groups. They are all subgroups of $GL(\mathbb{C}^n)$, and can be defined in a simpler way [43, 45]:

1. The group $\mathcal{G}$ possesses at least one faithful representation D of finite dimension m. Then $D(\mathcal{G})$ is a metric space with the distance measure between group elements given by
$$d(g, h) = [\sum_{i,j=1}^{m} |D(g)_{ij} - D(h)_{ij}|^2]^{\frac{1}{2}} . \tag{2.1.3}$$

2. $\exists \; \delta > 0$, such that every $g \in V_\delta \equiv \{g \in \mathcal{G} : d(g, e) < \delta\}$, can be parametrized by one and only one point $x = (x_1, \dots, x_n) \in \mathbb{R}^n$, such that e corresponds to $x_1 = x_2 = \dots = x_n = 0$. n is the dimension of the Lie group.

3. $\exists \; \epsilon > 0$, such that every $x \in \mathbb{R}^n$ satisfying
$$\sum_{a=1}^{n} x_a^2 < \epsilon^2 \tag{2.1.4}$$

[2]Note that by this definition we restrict ourselves to linear representations. More generally, representations are defined as group homomorphisms between the group $\mathcal{G}$ and the group of operators on the vector space V [44].

corresponds to some $g \in V_\delta$, making the mapping one-to-one.

4. The functions $D_{ij}(x_1, \ldots, x_n) \equiv [D(g(x_1, \ldots, x_n))]_{ij}$ are analytic functions $\forall\, x \in \mathbb{R}^n$ obeying eq. (2.1.4).

For these groups, one can write the matrices $D(g)$ from a small neighbourhood of the unit matrix as[3]

$$D(g) = \mathrm{e}^{ix_a T^a} \ . \tag{2.1.5}$$

Here, x_a is a set of real parameters in a neighbourhood of zero, and

$$(T^a)_{ij} \equiv i^{-1} \frac{\partial D_{ij}(x_1, \ldots, x_n)}{\partial x_a}|_{x_1 = x_2 = \cdots = x_n = 0} \tag{2.1.6}$$

are the so called generators of the Lie group [46]. The generators T^a form a basis for the real Lie algebra $\mathfrak{g}$ associated to the (neighbourhood of the e in the) Lie group $\mathcal{G}$. A real Lie algebra is a vector space with an additional bilinear, associative inner product $[\cdot, \cdot]$: $\mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$, the Lie bracket. The Lie bracket is anti-commuting ($[x, y] = -[y, x]\ \forall\ x$, $y \in \mathfrak{g}$) and satisfies the Jacobi identity ($[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0\ \forall\ x$, y, $z \in \mathfrak{g}$) [45].

The Lie bracket of two of the generators is given by

$$[T^a, T^b] = i f_c^{ab} T^c \ , \tag{2.1.7}$$

where f_c^{ab} are the structure constants of the algebra [46].

Since we are often interested in infinitesimal elements of the group, it will be convenient to study the Lie algebra and its representations instead of the group.

Let us make some comments on Lie algebras, before proceeding to the special case of $SU(N)$ [47]:

- The *dimension* of an algebra is the number of linearly independent generators.

- The *rank* of an algebra is the number of simultaneously diagonalisable generators. That is also the dimension of the *Cartan subalgebra*, the maximal abelian subalgebra.

- If we want to embed an algebra or the corresponding group into a larger group, the rank of the larger algebra must be at least as large as the rank of the algebra we want to embed.

This will become important later on, when we will consider embeddings of the Standard Model in grand unified theories.

[3] The factor of i is a convention often utilized in physics, since it makes the generators of unitary representations hermitian.

2.1.2 SU(N) representations and Young tableaux

In particle physics, the most common Lie groups we deal with are the abelian group $U(1)$, $SU(2)$ and $SU(3)$. When discussing unified theories, $SU(5)$ and even larger groups such as $SO(10)$ and E_6 become relevant as well. In this section, we will concentrate on $SU(N)$, since these groups will be most relevant to this work.

When we consider a group as a symmetry of a field theory, any term in the Lagrangian describing this theory should be an invariant, i.e. transform as the trivial representation of the group. However, there will be at least some fields that do transform under a faithful representation, since otherwise the symmetry could not be identified. So one important task in understanding the interactions in a theory with some kind of symmetry is finding out how to construct invariant products out of non-trivially transforming fields. For $SU(N)$, Young tableaux can be used to simplify this task. After listing some properties of the representations of $SU(N)$, we will explain how Young tableaux can be used to decompose tensor product representations into a sum of irreducible representations.

$SU(N) \subset GL(\mathbb{C}^N)$ is the group of special unitary transformations. It can be represented by $N \times N$ matrices U with unit determinant, satisfying $U^\dagger U = UU^\dagger = e$ [4]. This representation, acting on an N-dimensional complex vector space, is called the fundamental representation [41]. It will be denoted as N. The generators T_F^a for this representation are $N \times N$ traceless, hermitian matrices. There are $N^2 - 1$ generators, so the dimension of the corresponding algebra $\mathfrak{su}(N)$ is $N^2 - 1$. In general, the generators of the fundamental representation are complex, and the complex conjugate matrices $(T_R^a)^*$ generate a distinct representation of the same dimension, called the anti-fundamental representation $\bar{N}$.

Additionally, the structure constants form a set of generators $(T_{adj}^c)_{ab} = -if_{ab}^c$ for a representation of dimension $N^2 - 1$, since the Lie bracket satisfies the Jacobi identity. This representation is called the adjoint representation. The adjoint representation is always real, so T_{adj}^c and $(T_{adj}^c)^*$ generate the same representation [45].

Surprisingly, the trivial, (anti-)fundamental, and adjoint representation are the most common representations, and the only ones that are present in the Standard Model of particle physics.

After this wrap-up of the most important representations of $SU(N)$, we introduce the formalism of Young tableaux which allows us to study the tensor product of irreducible representations.

A Young tableau is a pictorial tool that can be used to represent the different irreducible $SU(N)$ representations[5]. It consists of a number of boxes ordered into rows and columns, such that in each row (going from top to bottom) and in each column (going from left to

[4] The group of unitary transformations in N dimensions, $U(N)$, is given by $U(N) = \{g \in GL(\mathbb{C}^N) : gg^\dagger = g^\dagger g = e\} \subset GL(\mathbb{C}^N)$. $SU(N)$ can then also be defined as $SU(N) = \{g \in U(N) : \det(g) = 1\} \subset U(N) \subset GL(\mathbb{C}^N)$. [45]

[5] Young tableaux are actually more general than this. They originated from descriptions of the group S_n of symmetric permutations of n objects [44, ch. 4]. Besides the application shown here, they can for example be used to decompose representations of $SU(M+N)$ into representations of $SU(M) \times SU(N)$ [46, ch. 13].

right), there are not more boxes than in the previous row or column:

$$\begin{array}{cccc} \square & \square & \cdots & \square \\ \square & \square & \cdots & \\ \vdots & \vdots & & \\ \square & & & \end{array} \tag{2.1.8}$$

A column can at most contain N boxes. Except for the case where no boxes would be left, any column with N boxes can be removed. A column with N boxes corresponds to the trivial representation, called 1.

$$N \left\{ \begin{array}{ccc} \square & \square & \square \\ \square & \square & \\ \vdots & & \\ \square & & \end{array} \right. \equiv \begin{array}{cc} \square & \square \\ \square & \end{array} \, , \qquad N \left\{ \begin{array}{c} \square \\ \square \\ \vdots \\ \square \end{array} \right. \equiv 1 \tag{2.1.9}$$

The fundamental representation is depicted by a single box, $\square$.

For any representation, the complex conjugate representation can be constructed by changing the number of boxes in each column from x to $N-x$. Thus, the anti-fundamental representation is depicted as a column of $N-1$ boxes.

$$\begin{array}{cc} \square & \square \\ \square & \end{array} \xrightarrow{\text{compl. conj.}} N-1 \left\{ \begin{array}{cc} \square & \square \\ \vdots & \vdots \\ \square & \square \\ \square & \end{array} \right. \, , \qquad N-1 \left\{ \begin{array}{c} \square \\ \square \\ \vdots \\ \square \end{array} \right. \equiv \bar{N} \tag{2.1.10}$$

The dimension of the representation can be found as the following quotient. The numerator is constructed as follows [48, app. A]: label the box in the upper left corner of the tableau by N. The other boxes are labelled by increasing the number by one if you go to the right or upwards, and decreasing it by one if you go to the left or downwards. The numerator num is the product of the labels. For example:

$$\begin{array}{|c|c|c|} \hline N & N{+}1 & N{+}2 \\ \hline N{-}1 & N & \\ \cline{1-2} N{-}2 & & \\ \cline{1-1} \end{array} \rightarrow num = N \cdot (N+1) \cdot (N+2) \cdot (N-1) \cdot N \cdot (N-2)\ . \tag{2.1.11}$$

For the denominator, label every box in the diagram by the number of boxes below it, plus the number of boxes to its right, plus one. The denominator den is the product of these labels. For the diagram above,

$$\begin{array}{|c|c|c|} \hline 5 & 3 & 1 \\ \hline 3 & 1 & \multicolumn{1}{c}{} \\ \cline{1-2} 1 & \multicolumn{2}{c}{} \\ \cline{1-1} \end{array} \rightarrow den = 5 \cdot 3 \cdot 1 \cdot 3 \cdot 1 \cdot 1 \ , \tag{2.1.12}$$

so the dimension of this representation is

$$R = \frac{num}{den} = \frac{N \cdot (N+1) \cdot (N+2) \cdot (N-1) \cdot N \cdot (N-2)}{5 \cdot 3 \cdot 1 \cdot 3 \cdot 1 \cdot 1} \ . \tag{2.1.13}$$

If the fundamental representation acts on vectors v_i, $i = 1, \ldots, N$, in an N-dimensional vector space, the higher-dimensional representations can be understood as acting on tensors $T_{i_1,\ldots,i_k}$, $i_x = 1, \ldots N$ of higher order with certain symmetry properties. The symmetry properties can also be read from the Young tableau of a representation. Each box represents one index. The tensor belonging to that representation is symmetric in any indices in the same row, and totally antisymmetric with respect to exchange of the indices in the same column. That also explains the maximal length of the column: since the indices run from 1 to N, there can be at most N indices which are totally antisymmetric [45, 46, 48].

The tensor product representation of two representations can be split up as follows [46]:

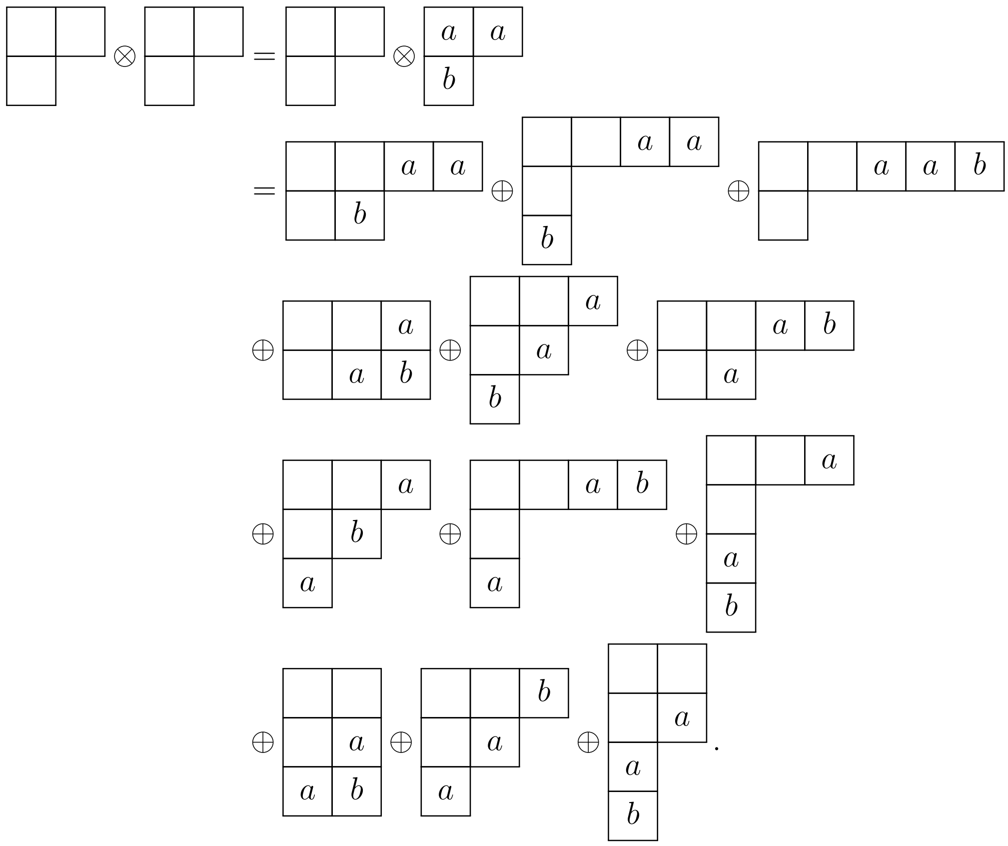
- Label the boxes of the second representation by a in the first row, b in the second row, and so on.

- Add the boxes from the second representation to the Young tableau of the first representation one at a time. Start with the ones from the first row, then continue with the second row and so on. The result should still be a valid Young tableau, and there should not be more than one of each index a, b, $\ldots$ in any column.

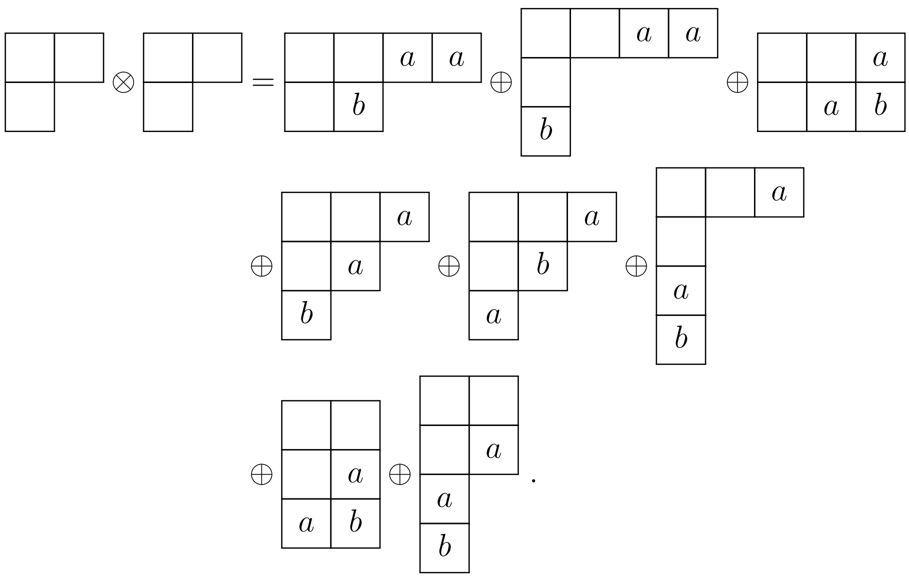
- Repeat the step above for any possible way of attaching the boxes. Discard equivalent results only if they are also equivalent in the distribution of the labels a, b, $\ldots$.

- Look at the new Young tableau. Starting from top to bottom and from *right* to *left*, count the boxes with label a, b, $\ldots$. At any step, there should be less boxes with label b than with label a, or with label c than with label b and so on. Discard any tableau where this is not the case.

Here is an example application. Assume we are in $SU(N)$ with $N \geq 3$:



Note that in the case $N = 3$, the third diagram in rows three and four are absent, since the first column is longer than three boxes. From these diagrams, we have to discard the third one in the first and second row, as well as the second diagram in the third and fourth row according to the index counting rule given in the last step of the recipe. So our result is



In $SU(3)$ this would correspond to

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27 \ . \tag{2.1.14}$$

With this recipe, we can now decompose the tensor product of any two faithful irreducible representations of $SU(N)$ into irreducible representations. In particular, we are able to find out whether the tensor product of two representations contains an invariant, i.e. the fundamental representation 1.

2.1.3 Combinatorics of Standard Model representations

The Standard Model is a chiral field theory, which can be written in terms of left- and right-handed fermions, scalars, and vector-like gauge bosons, as given in tab. (2.1). We

Name	Label	Representation
Left-handed lepton doublet	ℓ_L	(1, 2, -1/2)
Right-handed charged lepton	e_R	(1, 1, -1)
Left-handed quark doublet	Q_L	(3, 2, 1/6)
Right-handed up-quark	u_R	(3, 1 ,2/3)
Right-handed down-quark	d_R	(3, 1, -1/3)
Higgs boson	H	(1, 2, 1/2)
$SU(3)_C$ gauge boson	G	(8, 1, 0)
$SU(2)_L$ gauge boson	W	(1, 3, 0)
$U(1)_Y$ gauge boson	B	(1, 1, 1)

Table 2.1: Field content of the Standard Model and its gauge group representation. The representations are denoted as $(a,\, b,\, c)$, with a the $SU(3)_C$, b the $SU(2)_L$ and c the $U(1)_Y$ representation.

denote the representations as $(a,\, b,\, c)$, with a the $SU(3)_C$, b the $SU(2)_L$ and c the $U(1)_Y$ representation. The representations are always labelled by their dimension, as introduced above. The normalisation of the hypercharge Y is chosen such that the electric charge Q is given as $Q = I_3 + Y$, where I_3 is the third component of the weak isospin.

Any term appearing in the Standard Model Lagrangian or any of its extensions must be invariant under the Standard Model symmetry group. Hence, one can construct the renormalisable Lagrangian by finding all combinations of fields up to mass dimension four which transform trivially under the gauge group. To do so, we can use the results of the previous subsection, and the fact that scalar and vector fields have mass dimension one, while fermions have mass dimension 3/2.

Since the Standard Model gauge group is a tensor product of different groups itself, we can look at every part of the product group separately. The following results have all been acquired with the help of the Young tableau formalism established in the previous section.

For $SU(3)_C$, we need only a small number of different tensor product representations. The most important ones are

$$\begin{aligned} 3 \otimes 3 &= \bar{3} \oplus \bar{6} & \qquad 3 \otimes \bar{3} &= 1 \oplus 8 \\ \bar{6} \otimes 3 &= 8 \oplus 10 & \qquad 3 \otimes 8 &= 3 \oplus 6 \oplus 15 \ . \end{aligned} \tag{2.1.15}$$

Moreover, for any representation R the combination $\bar{R} \otimes R$ contains the trivial representation. Note that the representation 8, the adjoint representation, is real, so the conjugate representation is equivalent to the representation itself. Successive application of the Young tableaux formalism gives the decomposition of the tensor product of more that two representations, such as $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$. Note that the singlet in this product is a completely antisymmetric combination of the three fields, as can be seen from the Young tableau.

For $SU(2)_L$, we need a larger number of representations, but the structure is very simple. $SU(2)$ has only real irreducible representations, so that we do not need to worry about conjugation. Using the formalism from the previous section, one finds that for any pair of irreducible representations of dimensions A and B, their tensor product can be split up as

$$A \otimes B = \bigoplus_{N=\frac{1}{2}|A-B+1|}^{\frac{1}{2}|A+B-1|} 2N \ . \tag{2.1.16}$$

For $SU(2)$, it is often very convenient to form the tensor product of different representations by using Clebsch-Gordan coefficients. The Clebsch-Gordan coefficients are a very efficient way to deconstruct those tensor-representations component-wise. For example, considering the corresponding Clebsch-Gordan coefficients it is apparent that the singlet $1 \subset E \otimes E$, where E are even-dimensional representations, is a completely antisymmetric combination of the two fields (see e.g. [3, ch. 44]).

Finally, we calculate tensor products of different $U(1)$ representations by simply summing up the charges. A tensor product transforms trivially if the sum of charges is zero. Note that complex conjugation changes the sign of the charge.

We are now equipped with all the tools we need to study the possible interactions of a theory given the transformation properties of the different fields under its symmetries.

2.2 Introduction to neutrino masses

2.2.1 Weyl, Majorana, and Dirac fermions

Before we introduce the three see-saw models, we remember some important facts about the different fermions and their mass terms. This consideration closely follows [49] and [50].

The dynamics of a fermion are described by the Dirac equation,

$$(i\partial_\mu \gamma^\mu - m)\Psi = 0 \ . \tag{2.2.1}$$

Here, m is the mass of the corresponding particle, and γ^μ are 4×4 matrices, namely a Dirac representation of the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \tag{2.2.2}$$

where $\{A, B\} = AB + BA$ is the anti-commutator and $\eta^{\mu\nu}$ are the components of the Minkowski metric in four dimensions [51]. We use the convention (+,-,-,-). In addition, for the corresponding Hamiltonian to be hermitian, the Dirac matrices should satisfy (no sum over μ in the last equation)

$$\gamma^0\gamma^\mu\gamma^0 = \gamma^{\mu\dagger}, \qquad \text{or} \qquad \gamma^{\mu\dagger} = g^{\mu\mu}\gamma^\mu\,. \tag{2.2.3}$$

The Dirac equation can be interpreted as the Euler-Lagrange equation belonging to the Lagrange density

$$\mathcal{L} = \overline{\Psi}(i\partial_\mu\gamma^\mu - m)\Psi \tag{2.2.4}$$

with the notation $\overline{\Psi} = \Psi^\dagger\gamma^0$.

The general solution to the Dirac equation is a complex 4-vector transforming under the spin representation of the Lorentz group. However, this solution is not irreducible.

Let us introduce the chirality projection operators

$$P_{L/R} = \frac{1}{2}(\mathbf{1} \mp \gamma_5)\ , \tag{2.2.5}$$

where the matrix γ_5 is defined via

$$\{\gamma_5, \gamma^\mu\} = 0\ , \tag{2.2.6}$$

and $\mathbf{1}$ is the 4×4 unit matrix. γ_5 can be written as $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and satisfies

$$\gamma_5^\dagger = \gamma_5 \qquad \text{and} \qquad (\gamma_5)^2 = \mathbf{1}\,. \tag{2.2.7}$$

These projection operators can be used to split every field satisfying the Dirac equation into two parts, $\Psi = \Psi_L + \Psi_R$, called the chiral components. Writing out the Dirac Lagrangian in chiral components, we find

$$\mathcal{L} = i\overline{\Psi_L}\partial_\mu\gamma^\mu\Psi_L + i\overline{\Psi_R}\partial_\mu\gamma^\mu\Psi_R + \overline{\Psi_L}m\Psi_R + \overline{\Psi_R}m\Psi_L\ . \tag{2.2.8}$$

Hence, in the case where the mass term is absent, we find that the Dirac equation splits up into two separate equations for the single chiral components. The solutions to these equations are called Weyl fermions. In contrast to the general solution, they have only two degrees of freedom and can be used as building blocks for the general (massive) fermion solution.

Next, we introduce the particle-antiparticle conjugation operator

$$\hat{C} : \Psi \to \Psi^c = C\overline{\Psi}^T\ , \tag{2.2.9}$$

with matrix C having the properties

$$C^\dagger = C^T = C^{-1} = -C, \qquad\qquad C\gamma^\mu C^{-1} = -(\gamma^\mu)^T. \tag{2.2.10}$$

Note that the action of $\hat{C}$ on a Weyl fermion flips its chirality: $(\Psi_L)^c = (\Psi^c)_R$, and that the properties of C imply $(\Psi^c)^c = \Psi$. We can now use this operator to define a Majorana fermion as a massive fermion satisfying the additional Majorana condition

$$\Psi^c = \eta\Psi \ , \tag{2.2.11}$$

where η is a phase factor[6]. Since the action of $\hat{C}$ inverses all charge-like quantum numbers, Majorana fermions must carry zero charge. Let us now consider the sum of a Weyl fermion Ψ_L and its conjugate $(\Psi_L)^c = (\Psi^c)_R$. Then the conjugate of that sum is given by

$$(\Psi_L + (\Psi^c)_R)^c = (\Psi^c)_R + (\Psi^{c^c})_L = \Psi_L + (\Psi^c)_R \ . \tag{2.2.12}$$

Hence, a Majorana fermion can be written as the sum of a Weyl fermion and its complex conjugate. In contrast to that, a general Dirac fermion is the sum of two independent Weyl fermions of opposite chirality.

If we now reconsider the mass terms of the Dirac Lagrangian, we find that the Majorana mass term takes the form

$$\begin{aligned} \overline{\Psi_L} m \Psi_R + \overline{\Psi_R} m \Psi_L &= \overline{\Psi_L} m (\Psi_L)^c + \overline{(\Psi_L)^c} m \Psi_L \\ &= \Psi_L^\dagger \gamma^0 m C (\gamma^0)^T \Psi_L^* + \Psi_L^T (\gamma^0)^* C^\dagger \gamma^0 m \Psi_L \\ &= \Psi_L^\dagger (-C) m \Psi_L^* + \Psi_L^T C m \Psi_L \\ &= \Psi_L^T C m \Psi_L + h.c. \ . \end{aligned} \tag{2.2.13}$$

In the last step, we assume that m is real and symmetric.

By similar manipulations, the Dirac mass terms can be rewritten as

$$\overline{\Psi_R} m \Psi_L + \overline{\Psi_L} m \Psi_R = \frac{1}{2}\left[(\Psi^c)_L^T C m \Psi_L + \Psi_L^T C m^T (\Psi^c)_L\right] + h.c. \ . \tag{2.2.14}$$

As this relation indicates, a Dirac fermion consists of two Majorana fermions with same mass and opposite CP parity, which are maximally mixed.

The mass term of a Majorana fermion can also be utilized to reason why a Majorana fermion cannot be charged: assume the Majorana fermion Ψ has the charge a under some $U(1)$ symmetry, i.e. it transforms as $\Psi \to (1 + i\theta a)\Psi$. Here we use the infinitesimal transformation. Considering the Majorana mass term $\mathcal{L}_M$ in eq. (2.2.13), it transforms under this $U(1)$ as

$$\mathcal{L}_M \to \mathcal{L}_M^{'} = \mathcal{L}_M + 2i\theta a \mathcal{L}_M \ . \tag{2.2.15}$$

[6]Neglecting the phase factor, another point of view on the Majorana condition is that it corresponds to a reality condition in the Majorana basis, where the Dirac matrices are purely imaginary and the Dirac equation is a real equation.

This is a non-trivial transformation, and hence the corresponding $U(1)$ is not a symmetry of the Lagrangian. If the rest of the Lagrangian transforms trivially under this $U(1)$, i.e. the $U(1)$ is a symmetry in the absence of the Majorana mass term, we say that the Majorana mass term breaks the symmetry. We further call it a breaking by $2a$ units, since the charge of the Majorana mass term is $2a$. If one turns this argument around, it also implies that it is always possible to include a Majorana mass term for a fermion without charge into the Lagrangian, unless this term is forbidden by some other symmetry.

After this excursus we are now ready to consider neutrino masses.

2.2.2 The see-saw mechanisms

Now, we can turn to the see-saw mechanism. Regarding the Standard Model as an effective field theory, we should include higher-dimensional, non-renormalisable operators. In an effective field theory, these operators encode the effect of the heavy degrees of freedom which have been integrated out. The lowest-dimensional, non-renormalisable operator build out of standard model fields, which is invariant under G_{SM}, is the so-called dimension-5 Weinberg operator

$$\frac{c_W \ell_L H \ell_L H}{\Lambda} \ . \tag{2.2.16}$$

Here, c_W is a dimensionless coupling constant, and Λ is the scale of the new physics. Writing the Weinberg operator in this manner, we have suppressed the contraction of Lorentz indices as well as $SU(2)$ indices. As pointed out by Ma [52], there are exactly three possibilities to contract the $SU(2)$ indices, all leading to the same operator. However, they indicate the three possibilities to generate this operator at tree-level. We will now discuss these possibilities.

The first possibility to contract the $SU(2)$ indices in the Weinberg operator is

$$\ell_{L,i} H_j \epsilon_{ij} \ell_{L,k} H_m \epsilon_{km} \ , \tag{2.2.17}$$

where ϵ_{ij} is the completely antisymmetric tensor. This version of the operator can come from a theory, in which a fermionic, electrically neutral $SU(2)_L$ and $SU(3)_C$ singlet, ν_R, is added to the Standard Model. The new particle is then considered as the right-handed component of the neutrino. In addition to its couplings to the Standard Model via Yukawa couplings,

$$\mathcal{L} \supset y_\ell \bar{\ell}_L \tilde{H} \nu_R \tag{2.2.18}$$

the new particle can also have a Majorana mass term

$$\mathcal{L} \supset M_{\nu_R} \overline{\nu_R^c} \nu_R \,, \tag{2.2.19}$$

since it is not charged under any $U(1)$ symmetry. Due to this mass term, the Weinberg operator will be realised via the diagram[7] fig. (2.1).

[7]Note that these diagrams with explicitly drawn Higgs vacuum expectation values are not actual Feynman diagrams, but rather a very useful tool for the visualisation of neutrino mass mechanisms.

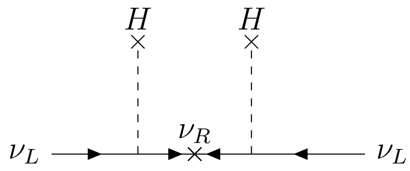


Figure 2.1: Tree-level generation of the Weinberg operator in see-saw type I.

Let us consider the mass matrix of one neutrino generation for simplicity. With the help of eq. (2.2.13) and eq. (2.2.14), we can rewrite eq. (2.2.19) and eq. (2.2.18) as [50]

$$\mathcal{L}_m = -\frac{1}{2}\left(\nu_L^T,\ [(\nu_R)^c]^T\right) C \begin{pmatrix} 0 & m_D \\ m_D^T & M_{\nu_R}^* \end{pmatrix} \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} , \tag{2.2.20}$$

where m_D is the usual Dirac mass, the product of the neutrino Yukawa coupling and the Higgs vacuum expectation value. The mass M_{ν_R} is usually expected to be large, since it is not protected by any symmetry[8]. When diagonalising this matrix approximately in the limit $M_{\nu_R} \gg m_D$, one finds that the eigenvalue of the state made predominantly of ν_L gets the mass

$$m_L \simeq \frac{m_D^2}{M_{\nu_R}} , \tag{2.2.21}$$

while the other eigenstate, which predominantly consists of ν_R, has the mass eigenvalue

$$M_R \simeq M_{\nu_R} . \tag{2.2.22}$$

In this sense, the mass of the active or left-handed neutrinos is suppressed by the heavy mass of the right-handed neutrinos. This is called type I see-saw [21–24].

The second possible contraction of the $SU(2)$ indices in the Weinberg operator is

$$\ell_{L,i}\sigma^a_{ij}H_j\ell_{L,k}\sigma^a_{km}H_m \ , \tag{2.2.23}$$

where σ^a are the Pauli matrices. This case is very similar to the previous one. This operator can be generated by a $SU(2)$ triplet fermion Σ with hypercharge zero [53]. The corresponding diagram is the same as fig. (2.1), with ν_R replaced by Σ. The electrically neutral component of Σ can be considered as the right-handed neutrino. It can have a Majorana, and, together with the left-handed neutrino, a Dirac mass term. Apart from the different contraction of the $SU(2)$ indices, this case works completely analogous to type I see-saw. It is called type III see-saw.

The third possibility to contract the $SU(2)$ indices is

$$\ell_{L,i}\sigma^a_{ij}\ell_{L,j}H_k\sigma^a_{km}H_m \ . \tag{2.2.24}$$

This contraction of indices in the Weinberg operator can be realised by adding a scalar

[8] For a discussion see e.g. [42, ch. 22.6.3]

$SU(2)$ triplet Δ with hypercharge 1 to the Standard Model [25–28]. In this case, there is no Dirac mass term for the neutrinos, but the Weinberg operator is generated at tree-level, as shown in fig. (2.2). This extension of the Standard Model is called see-saw type II. The

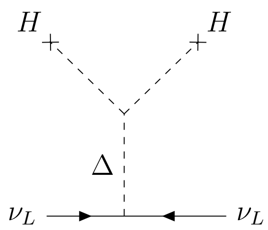


Figure 2.2: Tree-level generation of the Weinberg operator in see-saw type II

mass term for the neutrinos in this case reads

$$\mathcal{L} \supset -y_\nu (\ell_L)_i^T C \epsilon_{ij} \sigma^a_{jk} \Delta_a (\ell_L)_k \,. \tag{2.2.25}$$

The scalar potential contains the terms

$$V \supset M_\Delta^2 \Delta_a^\dagger \Delta^a + \mu H_i^T \epsilon_{ij} \sigma^a_{jk} \Delta^*_a H_k + h.c. \,. \tag{2.2.26}$$

After electroweak symmetry breaking, these terms will lead to a vacuum expectation value for the electrically neutral component of Δ, which is approximately given by [54]

$$v_\Delta \simeq \frac{\mu v^2}{\sqrt{2} M_\Delta^2} \,. \tag{2.2.27}$$

The resulting neutrino mass is then given by $y_\nu v_\Delta$. This mass is suppressed not only by the Yukawa coupling y_ν but also by the large mass of the scalar Δ and the coupling constant μ of the ΔH^2 term.

These three extensions of the Standard Model are the most simple ones that can explain small neutrino masses. They all have been studied extensively. There are a lot of variants and interesting points in their parameter space that exhibit special features, such as pseudo-Dirac neutrinos.

Note that the Weinberg operator is not the only higher-dimensional operator which can lead to Majorana-type neutrino mass, but the only one of mass dimension 5. Every odd mass dimension larger than five has a number of such operators. They split into two categories: One contains Weinberg-like operators,

$$\frac{\ell_L H \ell_L H (H^\dagger H)^n}{\Lambda^{(2n+1)}} \,, \tag{2.2.28}$$

and one contains other operators, which produce neutrino mass by connecting some of the external lines to loops. An example of this second operator type is the dimension-7 operator

$$\frac{\ell_L \ell_L \overline{e_R} \ell_L H}{\Lambda^3} \ . \tag{2.2.29}$$

These two categories have two things in common: one, all operators generate the Weinberg operator radiatively, and two, they break lepton number by two units. The first point is fairly easy to see for the Weinberg-like operators: you simply contract the additional $H^\dagger H$ pairs leading to a scalar loop. In the other operators, such as in our example, additional fermion pairs may appear besides the Higgs pairs. These can be connected into a loop by an insertion of the fermion mass.

In our search for neutrino mass models, we will make use of the second property, the breaking of lepton number by two units. The details will be explained in the next chapter.

3 Scan for minimal radiative neutrino mass models

As explained earlier, the aim of this thesis is to study radiative models of neutrino mass generation in a systematic way. That means we try to find an exhaustive list of possible models within the limitations of our scan. Such surveys have been performed earlier [29–35, 52, 55–57]. They mostly organise neutrino mass models, and limit the considered theory space, by utilizing either the effective operator by which the neutrino mass is generated, the loop order at which the dimension-5 Weinberg operator appears or the topology of the resulting neutrino mass diagram. In this work, we make an alternative approach following the idea of ref. [58]. Instead of one of the mentioned features, we search for minimal radiative neutrino mass models, not making any assumptions on the loop-level or the effective operator. Our minimality conditions are somewhat relaxed compared to ref. [58]. We will discuss them in the next section, while the results are given in sections (3.2) and (3.3). The fndings have also been published in ref. [36].

3.1 Scan conditions

For the scan, we employ the following set of assumptions:

- There is only a minimal number of new G_{SM} multiplets involved in the mass generation of a single neutrino family.
- The Weinberg operator, eq. (2.2.16), appears only at loop-level.
- There are no new symmetries beyond the Standard Model symmetry group G_{SM}.

Condition one states that we consider minimal models in the sense that they contain only a minimal number of new G_{SM} multiplets. To clarify this notion of minimality, we need to define how new multiplets are counted. In particular, we have to define how we handle new fermions in comparison to new scalars.

First of all, the Standard Model, or more precisely its gauge group, should stay anomaly-free. To achieve that, we can either introduce only Majorana fermions with hypercharge zero or vector-like Dirac fermions[1]. A vector-like fermion is made up of a pair of Weyl fermions with the same G_{SM} representation but opposite chirality. As explained in section (2.2.1), new Dirac fermions have twice as many degrees of freedom as new Majorana

[1]There are also other combinations of fermions that leave the Standard Model anomaly free, such as a fourth family of Standard Model fermions. Anyhow, when adding only one fermionic representation to the Standard Model, these are the only possibilities, see e.g. [42, ch. 30.4].

fermions. Nonetheless, we will count both, a new Majorana fermion and a new vector-like Dirac fermion, as one new fermionic multiplet. It will be stated explicitly which combination of Weyl fermions is introduced.

Secondly, there exist three families of fermions in the Standard Model, three copies of each particle that only differ by their mass. Neutrino oscillations allow us to measure the differences of the three neutrino masses, telling us that the masses of all three neutrinos must differ from each other. So in principle, the neutrino mass mechanism should not generate a single mass, but rather at least two distinct non-zero neutrino masses and the corresponding mixing between the flavour- and the mass eigenstates. To do so, multiple copies of the new fields are usually required. Yet, the number of copies can vary depending on the kind of field. For example, in the see-saw types I and III at least two copies of the new fermions are required to produce the correct mixing, while in see-saw type II only one new scalar suffices. To simplify the counting, we only consider the mass generation for one neutrino when counting the multiplets, and comment on the number of copies afterwards. This also includes additional copies of the Standard Model Higgs boson, which we do not count as new multiplets.

After clarifying how we count multiplets, we can now find out the minimal number of new multiplets for neutrino mass generation.

The neutrino mass models with the fewest new representations are clearly the three see-saw mechanisms presented earlier. But there is another model that only introduces one new G_{SM} multiplet and a second Higgs doublet. This is the well-studied Zee model [59]. Besides the second Higgs doublet, it adds a charged scalar singlet $\phi_{(1,1)} \sim (1, 1, 1)$ to the Standard Model. This is the singlet version of the new scalar in type II see-saw. The second Higgs is necessary since the coupling

$$\phi_{(1,1)} H^\dagger \tilde{H} = \phi_{(1,1)} H_i^* \epsilon_{ij} H_j^* \tag{3.1.1}$$

vanishes otherwise. Following our method of counting, this makes the minimal number of new multiplets one, leaving us with one model only. In order to find a larger number of models, we will also consider models which generate neutrino masses with two new multiplets, even though they incorporate, very strictly speaking, a next-to-minimal number of new multiplets.

The second condition excludes the see-saw mechanisms from our consideration. The motivation to do so is twofold. Firstly, it is connected to our notion of minimality. Beyond a minimal number of new multiplets, we also require that all new representations in these models are necessary for the generation of neutrino mass. This ensures minimality in the sense of irreducibility. Since the new fields in the see-saw mechanism can produce neutrino mass by themselves, any model containing them and a second new field can be reduced to one of the see-saw mechanisms. Excluding the see-saw fields from our scan ensures that the models with two new fields genuinely need both of them for the generation of neutrino mass.

Secondly, we explicitly aim for radiative neutrino mass models. By radiative we mean that the Weinberg operator is only generated at loop-level, or the effective operator generated at tree-level is a higher-dimensional Weinberg-type operator.

The advantage of this type of mass model is that, compared to the see-saw mechanism, we get additional suppression from loop factors or the higher dimension of the operator. This additional suppression will in general allow the energy scale of the new physics to be lower than in the see-saw mechanism, making such theories more accessible to experiments. Testing these Standard Model extensions will give an important contribution to narrowing down the theory space for beyond the Standard Model physics.

In addition, the right-handed neutrinos from the see-saw mechanism can also be used to explain the baryon asymmetry of the Universe. The standard version of this solution, often called vanilla leptogenesis [60], requires the right-handed neutrinos to be very heavy. If their mass is large, the right-handed neutrinos will induce radiative corrections to the Higgs mass which are of the order of the Higgs mass itself. This tension between the Higgs mass and leptogenesis is called the neutrino hierarchy problem. For recent discussions see [61–72]. Since the mass scale of the new physics is in general lower in radiative neutrino mass models, they might help to relax the neutrino hierarchy problem. Still, the potential realisation of leptogenesis in radiative neutrino mass models has to be re-examined carefully.

Moreover, a lower mass range of new physics in the radiative models could allow the new fields in radiative neutrino mass models to be candidates for dark matter, the third large puzzle in particle physics. While the right-handed, sterile, neutrinos appearing in the usual see-saw type I can also be considered as dark matter, only a part of the parameter space allows for a sterile neutrino that is light ($\sim$ keV) and long-lived (small mixing). The parameter space is further narrowed by strong restrictions from γ-ray observation, and structure formation [73]. Radiative neutrino masses might therefore lead to valuable alternative approaches to the connection between neutrino masses and dark matter.

The last condition of our scan, allowing no new symmetries, is very restrictive. It makes sure that any renormalisable interaction allowed by the Standard Model gauge symmetry can in principle appear. For example, without this condition, we could always include right-handed Standard Model singlets ν_R and give them a non-trivial representation under a new Z_2 or $U(1)$ symmetry to avoid the Weinberg operator. So without the third condition, the second one would not be equivalent to excluding the three see-saw fields ν_R, Σ and Δ from the scan. Furthermore, excluding new symmetries restricts the type of new fields to scalars and fermions, because new vector fields are usually associated with new gauge symmetries. In addition, the exclusion of new symmetries restricts our consideration to Majorana neutrino masses. This aspect will be discussed in the next section.

3.1.1 Lepton number violation as a guiding principle

Our third scan condition states that we do not enforce any new symmetries. Let us discuss what exactly this means with an example that will be important later.

In the Standard Model, one can assign a lepton number of one to ℓ_L and e_R, and zero to all other fields. It turns out that the Standard Model Lagrangian is invariant under a global $U(1)$ rotation according to these charges. Thus, lepton number is, at least at the classical level, a conserved charge belonging to a symmetry of the Standard Model. Taking into account quantum effects, one finds that lepton number is not only anomalous, but also broken by non-perturbative effects called electroweak sphalerons [74, 75]. The symmetry is called accidental, since it was not imposed in constructing the Standard Model, but is rather a result of gauge invariance. The situation is similar for baryon number B. Here, the charges are assigned according to $B(Q_L) = B(d_R) = B(u_R) = 1/3$ and $B = 0$ for all other fields.

Interestingly, $(B - L)$, the difference of baryon- and lepton number, is not only conserved classically, but is also respected by the electroweak sphalerons [74, 75]. In addition, it can be made anomaly-free by adding three right-handed (G_{SM} singlet) neutrinos ν_R with lepton number one to the Standard Model. This makes $(B - L)$ a popular candidate for a new $U(1)$ gauge symmetry, which can for example be employed to generate Dirac neutrino masses [76–79]. Nonetheless, at the level of the Standard Model it is only an accidental symmetry.

Some would argue that making such an accidental symmetry a guiding principle is just a better motivated version of introducing a new symmetry. Others would say that accidental or not, observed symmetries at low energies should be respected in model-building. Here, we will adopt the first point of view. In order not to impose new symmetries, we will not enforce accidental symmetries, in the sense of using them to forbid otherwise allowed terms in the Lagrangian.

What does that mean for the nature of the neutrinos? Let us assume that the right-handed neutrino is identified with the Standard Model singlet ν_R or the neutral component of the $SU(2)_L$ triplet of zero hypercharge Σ. In this case, it is obvious that the neutrinos will be Majorana particles in the absence of new symmetries, since these particles are not charged. In the absence of additional symmetries, they will generically have a Majorana mass, and hence generate the Weinberg operator at tree-level. This argument is valid for any fermion with electric-, and hypercharge equal to zero, and transforming as a real representation of $SU(3)_C$.

However, there is one loophole: the right-handed neutrino is required to have vanishing electric and colour charge, but the hypercharge is only fixed if we demand in addition that the left- and right-handed neutrinos should be coupled via the Higgs doublet. If we allow for a second scalar multiplet in a different representation, the right-handed neutrino might well be the electrically neutral part of a multiplet with non-zero hypercharge[2]. This would exclude an explicit Majorana mass term for the new fermion ψ of the form $M\psi^T C\psi$. Staying within the limitations of our scan, only one new fermionic representation is allowed if we introduce a new scalar into the model. In order to avoid gauge anomalies, the new right-handed neutrino must be contained in either a fermion multiplet with zero hypercharge or a vector-like pair of Weyl fermions. Since we excluded the first possibility, the right-handed neutrino must be contained in a vector-like representation, allowing for a vector mass of the new fermion multiplet. We can now use eq. (2.2.14) to write the mass term of the electrically neutral components ψ of the new fermion and the

[2]Non-zero hypercharge but zero charge implies a non-trivial transformation under $SU(2)_L$.

neutrino as

$$\mathcal{L}_M \supset \frac{1}{2}(\psi^T\ ,\ \psi^{c^T}\ ,\ \nu^T)_L \begin{pmatrix} 0 & CM & 0 \\ CM^T & 0 & Cm \\ 0 & Cm^T & 0 \end{pmatrix} \begin{pmatrix} \psi \\ \psi^c \\ \nu \end{pmatrix}_L + h.c.\,, \tag{3.1.2}$$

where M is the vector-like mass of the new fermion and m the Dirac mass of the neutrino. The determinant of the mass matrix is zero, meaning that there is one massless eigenstate. Assuming that the vector-like mass is much larger than the neutrino mass, this massless eigenstate mainly consists of the Standard Model neutrino. If this is not the case and the massless component is mainly composed of the new multiplet, we would get at least one additional massless fermion taking part in weak interactions. The presence of such a fermion is for example excluded by the Z-boson decay width [3, ch.10]. So either the neutrino stays massless or the phenomenology of the model contradicts observations.

This indicates that in the absence of any new symmetries and within the limitation of our scan, neutrinos are Majorana fermions. Hence, we explicitly search for Majorana neutrino mass terms. These violate lepton number and $(B - L)$ conservation by two units. Any extension of the Standard Model that introduces Majorana neutrino masses must therefore also contain lepton number violation by two units. For lepton number violation to appear, at least one of the new fields has to couple to the Standard Model leptons. Otherwise, one can just set the lepton number of all new fields to zero, and lepton number is conserved. This restriction allows us to use lepton number as a guiding principle in our search for radiative neutrino masses.

Finding potential Standard Model extensions which lead to Majorana neutrino masses is now mainly combinatorics. We start by listing the fermionic bilinears that have a non-vanishing lepton number L. Their combined representations are possible candidates for new scalars. The results from section (2.1.3) are used to decompose the tensor product of representations. We distinguish between the purely leptonic bilinears

$$\overline{\ell_L^c}\ell_L \sim (1,1,-1) \oplus (1,3,-1) \qquad\qquad \overline{e_R^c}e_R \sim (1,1,-2) \tag{3.1.3}$$

and bilinears containing one lepton and one quark

$$\begin{aligned} \overline{\ell}_L u_R &\sim (3,2,7/6) & \qquad \overline{(e_R)^c}u_R &\sim (3,1,-1/3) \\ \overline{\ell}_L d_R &\sim (3,2,1/6) & \qquad \overline{(e_R)^c}d_R &\sim (3,1,-4/3) \\ \overline{e}_R Q_L &\sim (3,2,7/6) & \qquad \overline{(\ell_L)^c}Q_L &\sim (3,1,-1/3) \oplus (3,3,-1/3)\ . \end{aligned} \tag{3.1.4}$$

For fermionic candidates, we consider all representations that result from coupling the Standard Model leptons to the Higgs bosons. The result is

$$\begin{aligned} \ell_L H &\sim (1,1,0) \oplus (1,3,0) & \qquad e_R^c H &\sim (1,2,3/2) \\ \ell_L \tilde{H} &\sim (1,1,-1) \oplus (1,3,-1) & \qquad e_R^c \tilde{H} &\sim (1,2,1/2)\,. \end{aligned} \tag{3.1.5}$$

This is the set of candidates for the first new multiplet: a new scalar in one of the representations given in eq. (3.1.3) or eq. (3.1.4), or a fermion in one of the representations given in eq. (3.1.5).

After finding the first candidate, we scan its couplings to the Standard Model fields. A possible candidate for the second new multiplet must fulfil the following set of conditions:

- The second candidate must either couple to the first one or to the Standard Model.
- Including the interactions of the second candidate, there must be at least three interaction terms involving non-trivial lepton numbers, i.e. fields with lepton number $L \neq 0$.
- There must be at least one Majorana fermion or one vertex which inverts the fermion number flow.

All pairs of fields satisfying these conditions are examined. If a unique and consistent choice for the lepton numbers of the new fields is not possible, the combination is a potential neutrino mass model. The lists with the potential representations of the second candidate are given in appendix A.1.

Note that we do not cover all possible models in this way. In particular, we miss one class of models where only the combination of the two new particles can couple to the Standard Model leptons. Note also that some models, even though they violate lepton number, do not induce neutrino masses. We will comment on the exceptional model class and $(\Delta L = 1)$-models later on.

3.2 Scan results

To make the results of our scan as clear as possible, we divide the scan into two parts. The first one will cover all possible models that only contain new colour singlet fields, while the second part covers all models in which at least one of the new multiplets transforms non-trivially under $SU(3)_C$.

To allow for a quick identification of the models and the corresponding fields, we will employ the following terminology:

- We differentiate three types of models. Type A contains models with two new scalars. Type B models include one new scalar and one new vector-like Dirac fermion. Models with one new scalar and one new Majorana fermion are called type C. If at least one of the new fields carries colour, a small c is added in front of the type, giving type cA, cB and cC.
- Within any type, the models are ordered by the highest-dimensional representations of the G_{SM} subgroups. $SU(3)_C$ is considered first, $SU(2)_L$ second, and $U(1)_Y$ last. If the multiplet of highest representation coincide for two models, the representations of the second new field are considered in the same order.

- New scalars are denoted as $\phi_{(a,b,c)}$ where a and b denote the $SU(3)_C$ and $SU(2)_L$ representations, and c the hypercharge. In the first part of the scan, all new multiplets are $SU(3)_C$ singlets, wherefore the index a will be omitted and the colourless scalars are denoted $\phi_{(b,c)}$.

- For clarity, new fermions will always be introduced as the sum of their chiral components. The chiral components are written as $\psi_X^{(a,b,c)}$. Here, a, b, and c denote the $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ representations respectively. X can be either L or R, denoting the chirality. In the first part, the $SU(3)_C$ representation will be omitted, yielding the notation $\psi_X^{(b,c)}$.

- Yukawa coupling constants are named y. The trilinear and quartic scalar couplings are labelled μ and λ. The couplings μ have mass dimension one. Vector-like masses for fermions are called m, while Majorana masses are named M.

3.2.1 Minimal radiative neutrino mass models without colour

In the first part of our scan, we consider only models without new coloured fields. The complete list of models can be found in table (3.1) for models of type A, and in table (3.2) for models of type B and C. The column "Loops" denotes the loop-level at which the dimension-5 Weinberg operator can be generated and the column "Eff.dim." gives the dimension of the Weinberg-like operator which is generated at the lowest loop-level.

Model	New fields	Loops	Eff. dim.	Relevant interactions	New?	Comments
A0	$\phi_{(1,1)}$	1	5	$y_1\phi_{(1,1)}\overline{\ell_L^c}\ell_L$ $+y_2\overline{\ell_L}H_1e_R$ $+\mu H_2^\dagger\tilde{H}_1\phi_{(1,1)}$	no	Zee model [59]; requires two Higgs doublets
A1	$\phi_{(1,1)}$, $\phi_{(1,2)}$	2	5	$y_1\overline{\ell_L^c}\phi_{(1,1)}\ell_L$ $+y_2\overline{e_R^c}\phi_{(1,2)}e_R$ $+\mu\phi_{(1,2)}^*\phi_{(1,1)}\phi_{(1,1)}$	no	Zee-Babu model [80, 81]
A2	$\phi_{(1,1)}$, $\phi_{(2,3/2)}$	2	5	$y\overline{\ell_L^c}\phi_{(1,1)}\ell_L$ $+\mu\phi_{(1,1)}^*H^\dagger\phi_{(2,3/2)}$ $+\lambda\tilde{H}^\dagger\phi_{(2,3/2)}(\phi_{(1,1)}^*)^2$ $+\frac{1}{2}\lVert D_\mu\phi_{(2,3/2)}\rVert^2$	no	discussed in [58]; no proper mixing; ruled out
A3	$\phi_{(1,2)}$, $\phi_{(2,3/2)}$	2	5	$y\overline{e_R^c}\phi_{(1,2)}e_R$ $+\mu\phi_{(2,3/2)}^\dagger\tilde{H}\phi_{(1,2)}$ $+\lambda H^\dagger\phi_{(2,3/2)}H^\dagger\tilde{H}$	no	discussed in [58]; requires two Higgs doublets

Model	New fields	Loops	Eff. dim.	Relevant interactions	New?	Comments
A4	$\phi_{(1,1)}$, $\phi_{(3,0)}$	2	7	$y\overline{\ell^c_L}\phi_{(1,1)}\ell_L$ $+\mu(H^\dagger\sigma_a H)\phi_{(3,0)}$ $+\lambda\phi^*_{(1,1)}(\tilde{H}^\dagger\sigma_a H)\phi_{(3,0)}$	no	discussed in [58]; no proper mixing; ruled out
A5	$\phi_{(1,1)}$, $\phi_{(4,1/2)}$	3	9	$y\overline{\ell^c_L}\phi_{(1,1)}\ell_L$ $+\mu\tilde{\phi}^\dagger_{(4,1/2)}\phi_{(4,1/2)}\phi^*_{(1,1)}$ $+\lambda\phi^\dagger_{(4,1/2)}HH^\dagger H$	no	discussed in [58]

Table 3.1: Radiative neutrino mass models with two scalars carrying no colour charge. The second column contains the beyond the Standard Model multiplets. Column three denotes the loop-level at which the dimension-5 Weinberg operator can be generated, and column 4 gives the dimension of the Weinberg-like operator which is generated at the lowest loop-level. The interactions relevant for neutrino mass generation are given in column 5. Column 6 tells whether the model has been discussed previously, and the last column contains some general comment.

The first model in table 3.1 is the Zee model [59] discussed earlier. As explained, two copies of the Standard Model Higgs-field are needed in this model. In addition, both of these Higgs doublets have to develop a vacuum expectation value and have to be allowed to couple to the leptons. Otherwise, the model produces a flavour-space mass matrix which is traceless at leading order. This is ruled out by the experimental data. A schematic diagram of the neutrino mass term in the Zee model is given in fig. (3.1)

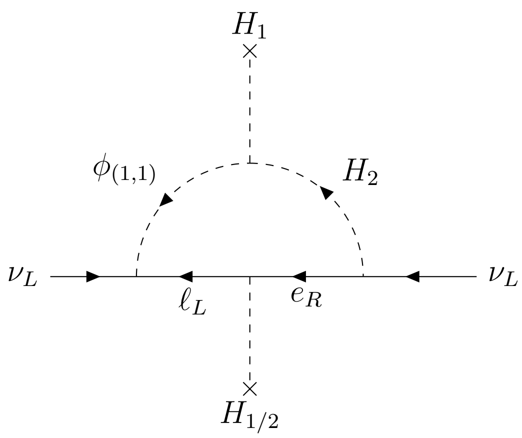


Figure 3.1: Schematic diagram of the neutrino mass term in the Zee model (A0).

Similarly to the Zee model, the models (A2), (A4) and (A5) also rely on the coupling $y\overline{\ell^c_L}\phi_{(1,1)}\ell_L$, but the lepton number violation in the scalar potential is generated in different ways.

The model (A2) contains a scalar $SU(2)$ doublet with hypercharge 3/2. This doublet does not have an electrically neutral component, and hence cannot develop a vacuum expectation value. Moreover, it cannot couple to Standard Model leptons via Yukawa couplings. There are also less possible couplings of the Higgs doublet to this field than there are for two Higgs doublets to each other. As a result, this model has less free parameters than the Zee model (A0). For this reason it was considered as a simpler variant of the Zee model in ref. [58]. However, it was also noted that this model leads to a neutrino mass matrix which is traceless in flavour space. Just as the restricted version of the Zee model, this model is therefore ruled out by experiment.

The model (A4) suffers from the same problem. In this model, a $SU(2)$ triplet scalar with hypercharge zero is introduced. The neutral component of this triplet can develop an effective vacuum expectation value via the trilinear coupling $\mu(H^\dagger\sigma_a H)\phi_{(3,0)}$, and it allows the scalar coupling $\lambda\phi^*_{(1,1)}(\tilde{H}^\dagger\sigma_a H)\phi_{(3,0)}$. Thus, the scalar triplet effectively induces the trilinear scalar term involving one copy of $\phi_{(1,1)}$ and two Higgs fields. The resulting neutrino mass matrix in flavour space is traceless, as in the previous case, and the model is experimentally excluded.

The model (A5) is a variant of the Zee model utilizing a pair of scalar quadruplets to induce the trilinear coupling $\mu\tilde{\phi}^\dagger_{(4,1/2)}\phi_{(4,1/2)}\phi^*_{(1,1)}$. Here, the quadruplets will develop an effective vacuum expectation value and an effective mixing with the Higgs boson via the coupling $\lambda\phi^\dagger_{(4,1/2)}HH^\dagger H$. This model has two competing contributions to the neutrino mass. One is a one-loop realisation of the dimension-9 operator $\ell_L\ell_L HH(H^\dagger H)^2/\Lambda^5$, which is displayed in fig. (3.2). The second is the dimension-5 Weinberg operator, which is generated at the three-loop level. It can be constructed from the diagram in fig. (3.2) by connecting two pairs of external Higgs doublets. The dominating contribution depends on the mass scale of the new particles. A similar remark is also valid for (A4), which generates the dimension-5 Weinberg operator at two-loop level and the dimension-7 operator $\ell_L\ell_L HH(H^\dagger H)/\Lambda^3$ at one-loop level.

Note that a restricted version of model (A5), where only one of the two quadruplets can couple to three Higgs fields, will again result in a traceless neutrino mass matrix [58]. As noted in ref. [58], the tracelessness of the neutrino mass matrix is a general feature of models effectively inducing the coupling $\mu H^\dagger\tilde{H}\phi_{(1,1)}$. We will later find a number of other models doing so via fermion loops. These will be excluded due to the mismatch with the measured structure of the neutrino mass matrix, too.

Model (A1) is the well-known Zee-Babu model [80, 81]. It utilizes both fermionic bilinears from eq. (3.1.3), $\overline{\ell^c_L}\ell_L$ and $\overline{e^c_R}e_R$. To do so, a singly and a doubly charged $SU(2)_L$ singlet scalar are added to the Standard Model. Their trilinear coupling $\mu\phi^*_{(1,2)}\phi_{(1,1)}\phi_{(1,1)}$ then enables a two-loop realisation of the Weinberg operator, as shown in fig. (3.3). We will see in the next section that this mechanism can also be realised with coloured fields.

In the last model, (A3), the doubly charged scalar singlet and the scalar doublet with hypercharge 3/2 are combined. The Weinberg operator is generated at two-loop level. To break lepton number, the coupling $\lambda H^\dagger\phi_{(2,3/2)}H^\dagger\tilde{H}$ is required, which vanishes identically if there is only one Higgs doublet. Hence, two Higgs doublets are needed in this model. The resulting neutrino mass diagram is similar to the one for the Zee-Babu model

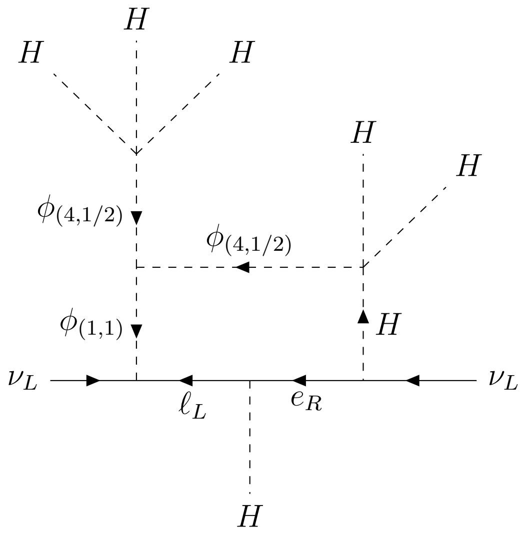


Figure 3.2: Schematic diagram of the one-loop realisation of the dimension-9 operator $\ell_L \ell_L H H (H^\dagger H)^2/\Lambda^5$ in model (A5).

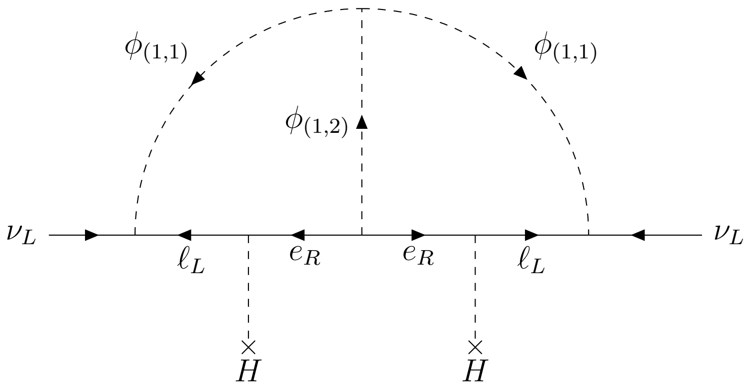


Figure 3.3: Schematic diagram of the neutrino mass term in the Zee-Babu model (A1).

(A1), but with the Higgs vacuum expectation values attached to one common point along one of the outer lines of the scalar loops.

Model	New fields	Loops	Eff. dim.	Relevant interactions	New?	Comments
B1	$\phi_{(1,1)}$, $\psi_L^{(2,-3/2)}$ $+\psi_R^{(2,-3/2)}$	2	5	$y_1 \overline{\ell_L^c} \phi_{(1,1)} \ell_L$ $+y_2 \overline{\psi_R^{(2,-3/2)}} \phi^*_{(1,1)} \ell_L$ $+y_3 \overline{e_R} H \psi_L^{(2,-3/2)}$ $+m \overline{\psi_L^{(2,-3/2)}} \psi_R^{(2,-3/2)}$	no	introduced in [33]; no proper mixing

Model	New fields	Loops	Eff. dim.	Relevant interactions	New?	Comments
B2	$\phi_{(2,3/2)}$, $\psi_L^{(1,-1)}$ $+\psi_R^{(1,-1)}$	1	5	$y_1\overline{\ell^c_L}\phi_{(2,3/2)}\psi_L^{(1,-1)}$ $+y_2\overline{\ell_L}H\psi_R^{(1,-1)}$ $+\lambda H^\dagger\phi_{(2,3/2)}H^\dagger\tilde{H}$ $+m\overline{\psi_L^{(1,-1)}}\psi_R^{(1,-1)}$	yes	requires two Higgs doublets
B3	$\phi_{(2,3/2)}$, $\psi_L^{(2,-1/2)}$ $+\psi_R^{(2,-1/2)}$	2	5	$y_1\overline{e^c_R}\phi_{(2,3/2)}\psi_R^{(2,-1/2)}$ $+y_2\overline{e_R}H^\dagger\psi_L^{(2,-1/2)}$ $+\lambda H^\dagger\phi_{(2,3/2)}H^\dagger\tilde{H}$ $+m\overline{\psi_L^{(2,-1/2)}}\psi_R^{(2,-1/2)}$	yes	requires two Higgs doublets
B4	$\phi_{(2,3/2)}$, $\psi_L^{(3,-1)}$ $+\psi_R^{(3,-1)}$	1	5	$y_1\overline{\ell^c_L}\phi_{(2,3/2)}\psi_L^{(3,-1)}$ $+y_2\overline{\ell_L}H\psi_R^{(3,-1)}$ $+\lambda H^\dagger\phi_{(2,3/2)}H^\dagger\tilde{H}$ $+m\overline{\psi_L^{(3,-1)}}\psi_R^{(3,-1)}$	yes	requires two Higgs doublets
B5	$\phi_{(4,3/2)}$, $\psi_L^{(3,-1)}$ $+\psi_R^{(3,-1)}$	1	7	$y_1\overline{\ell^c_L}\phi_{(4,3/2)}\psi_L^{(3,-1)}$ $+y_2\overline{\ell_L}H\psi_R^{(3,-1)}$ $+\lambda\phi_{(4,3/2)}\tilde{H}H^\dagger\tilde{H}$ $+m\overline{\psi_L^{(3,-1)}}\psi_R^{(3,-1)}$	no	discussed in [82] with two copies of fermions
C1	$\phi_{(4,1/2)}$, $\psi_R^{(5,0)}$	1	7	$y\overline{\ell_L}\phi^\dagger_{(4,1/2)}\psi_R^{(5,0)}$ $+\lambda\phi_{(4,1/2)}\tilde{H}H^\dagger H$ $+M\overline{(\psi_R^{(5,0)})^c}\psi_R^{(5,0)}$	no	discussed by [83–88]
C2	$\phi_{(n,1/2)}$, $\psi_R^{(n\pm1,0)}$ $n>4$, n even	1	5	$y\overline{\ell_L}\tilde{\phi}_{(n,1/2)}\psi_R^{(n\pm1,0)}$ $+\lambda H^\dagger\phi_{(n,1/2)}H^\dagger\phi_{(n,1/2)}$ $+M\overline{(\psi_R^{(n\pm1,0)})^c}\psi_R^{(n\pm1,0)}$	yes	case $\phi_{(6,1/2)}$ $+\psi_R^{(5,0)}$ discussed in [89]; case $\phi_{(3,1/2)}$ $+\psi_R^{(2,0)}$ in [90]

Table 3.2: Colourless radiative neutrino mass models with one scalar and one fermion. The beyond the Standard Model multiplets are listed in column two. Column three contains the loop-level at which the dimension-5 Weinberg operator can be generated, and the dimension of the Weinberg-like operator which is generated at the lowest loop-level is given in column 4. Column 5 contains the interactions relevant for neutrino mass generation. Column 6 indicates whether the model has been discussed previously, and in the last column we give some general comment.

To produce the correct mixing with B- and C-type models, one needs in general two copies of the new scalar or of the new fermion. This is in contrast to A-type models, where one copy of the new scalars can be sufficient to produce two independent neutrino masses. The reason for this is that a non-trivial flavour structure of the neutrino mass matrix requires that the particles running in the loops in the neutrino mass diagrams have such a non-trivial structure themselves. In A-type models, this flavour structure is provided by the Standard Model fermions, and a single copy of the new field is enough. In B-and C-type models, where new fermions and new scalars are typically running in the loop, at least one of them needs to provide a non-trivial flavour structure. This requires at least two copies of that field.

Model (B1) is a variant of the Zee model where the trilinear coupling $\mu H^\dagger \tilde{H} \phi_{(1,1)}$ is generated via a loop including a new fermion. Just like model (A4), this model produces a traceless neutrino mass matrix. Thus the model is excluded by experimental data.

The other B-type models all follow a common mechanism. The models (B2)-(B4) include the scalar doublet $\phi_{(2,3/2)}$ and a vector-like version of a Standard Model lepton (or the triplet version of the right-handed charged lepton in the case of (B4)). These fermions couple to the charge conjugate of a Standard Model fermion via the new scalar. In addition, there are Yukawa couplings of the vector-like fermions with Standard Model leptons including the usual Higgs doublet. Lepton number violation is ensured by the term $\lambda H^\dagger \phi_{(2,3/2)} H^\dagger \tilde{H}$ in the scalar potential. As mentioned earlier, this term requires two Higgs doublets to be non-vanishing. The Weinberg operator is generated at one-loop in (B2) and (B4), and at two-loop in (B3) respectively. The one-loop diagram is shown in fig. (3.4), while the two-loop variant includes an additional W-boson or charged Higgs loop, entangled with the one in fig. (3.4).

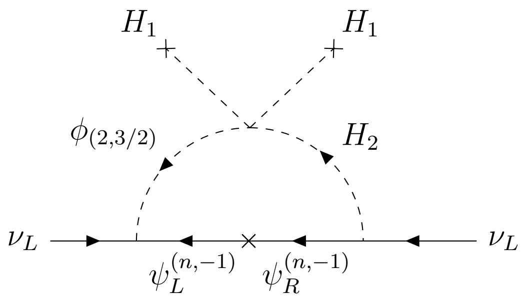


Figure 3.4: Schematic diagram of the neutrino mass term in models (B2) and (B4), including the scalar $\phi_{(2,3/2)}$ and a vector-like fermion $\psi_{(n,-1)} = \psi_L^{(n,-1)} + \psi_R^{(n,-1)}$, $n = 1$ for model (B2) and $n = 3$ for model (B4).

Model (B5) is a variation of (B4) where the scalar doublet is replaced by a scalar quadruplet. The biggest difference to model (B4) is that the scalar quadruplet has an electrically neutral component, and hence can develop a vacuum expectation value via the coupling $\lambda H^\dagger \phi_{(4,3/2)} H^\dagger \tilde{H}$. Therefore, the model realises the dimension-7 operator $\ell_L \ell_L H H (H^\dagger H)/\Lambda^3$ at tree-level. Additionally, no second copy of the Higgs field is required. The phenomenology of this model with two copies of the new fermion has been studied in ref. [82].

The models (C1) and (C2), which is really a class of models, could also be listed together. They both add a new Majorana fermion in an odd representation of $SU(2)_L$ and a matching scalar, which allows a Yukawa coupling between the Majorana fermions and the Standard Model neutrinos. The scalars $\phi_{(n,1/2)}$ have hypercharge 1/2, allowing the term $\lambda H^\dagger \phi_{(n,1/2)} H^\dagger \phi_{(n,1/2)}$ in the scalar potential. This leads to the neutrino mass diagram in fig. (3.5).

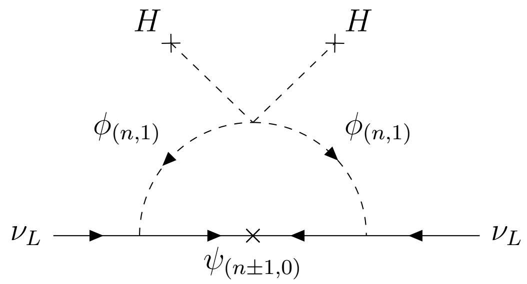


Figure 3.5: Neutrino mass diagram resulting from model (C2).

The case $n = 4$ is special, because the quadruplet scalar is the highest scalar representation that can also couple to three copies of the Higgs field. As a result, besides the one-loop realisation of the dimension-5 Weinberg operator, the dimension-7 operator $\ell_L \ell_L HH(H^\dagger H)/\Lambda^3$ is generated at tree-level. Majorana fermion in smaller odd-dimensional representations are excluded, since they correspond to ν_R and Σ. (C1) has been studied as a starting point for models systematically generating higher-dimensional Weinberg-like operators at tree-level in ref. [83].

A possible question arising on this general class of models is whether it can be extended to include Majorana fermions in even $SU(2)$ representations. Though such models can be written down, the Majorana mass term $M\overline{(\psi_R^{(m,0)})^c}\psi_R^{(m,0)}$ must be antisymmetric in flavour space. Together with the total symmetry of the corresponding diagrams, the resulting mass matrix for neutrinos will be antisymmetric. Since only the symmetric part of the mass matrix contributes to the Majorana mass, no neutrino mass is generated.

The models of class (C2) all contain a potential candidate for dark matter – the neutral component of the Majorana fermion. Since the new fermion and the new scalar can only couple to the Standard Model leptons together, the models possess an accidental Z_2 symmetry stabilising the neutral component of the new fermion. The example of $\phi_{(6,1/2)} \sim (1, 6, 1/2)$ and $\psi_R^{(5,0)} \sim (1, 5, 0)$ has been studied in this regard in ref. [89]. A more detailed account is given in the next chapter.

All C-type models, as well as their coloured versions (cC4)-(cC6) discussed in the next section, cannot be found by the formalism described in section (3.1.1). The reason is that the new fields only couple to the Standard model leptons in pairs. Nonetheless, a way to find them is to check all Majorana fermions as a first candidate in addition to the ones listed in (3.1.3), (3.1.4), and (3.1.5). This check will automatically lead to the model class (C2), as well as to (cC4)-(cC6).

3.2.2 Minimal radiative neutrino mass models with colour

In this subsection all models with at least one new coloured field are presented. They are generated by considering the scalar representations from eq. (3.1.4) as the first candidate or by taking into account couplings to quarks when looking for a second candidate. Note that since $SU(3)$ also has complex representations, taking the conjugate of a field will not only change its hypercharge from Y to $-Y$, but also its $SU(3)_C$ representation from C to $\bar{C}$, where $\bar{C}$ is the conjugate representation of C. The models of type cA can be found in table (3.3), the models of type cB in table (3.4), and the models of type cC in table (3.5). The column "B-viol.?" in these tables states whether baryon number is violated in the model. The contraction of the $SU(3)$ indices is suppressed in the given interaction terms.

Model	New fields	Relevant Interactions	B-viol.?	New?	Comment
cA1	$\phi_{(3,1,-1/3)}$, $\phi_{(3,1,2/3)}$	$y_1\overline{\ell^c_L}\phi^\dagger_{(3,1,-1/3)}Q_L$ $+y_2\overline{Q^c_L}\phi_{(3,1,-1/3)}Q_L$ $+y_3\overline{d^c_R}\phi_{(3,1,2/3)}d_R$ $+\mu\phi_{(3,1,2/3)}(\phi_{(3,1,-1/3)})^2$	yes	yes	scalar up- and down-quark; two copies of $\phi_{(3,1,-1/3)}$ required
cA2	$\phi_{(3,1,-1/3)}$, $\phi_{(3,2,1/6)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $y_2\overline{\ell^c_L}\phi^\dagger_{(3,1,-1/3)}Q_L$ $+\mu\phi^\dagger_{(3,2,1/6)}\phi_{(3,1,-1/3)}H$	yes	no	discussed in [29, 91]; embedding in $SU(5)$ and $SO(10)$ [92]
cA3	$\phi_{(3,3,-1/3)}$, $\phi_{(3,1,2/3)}$	$y_1\overline{\ell^c_L}\phi^\dagger_{(3,3,-1/3)}Q_L$ $+y_2\overline{Q^c_L}\phi_{(3,3,-1/3)}Q_L$ $+y_3\overline{d^c_R}\phi_{(3,1,2/3)}d_R$ $+\mu\phi_{(3,1,2/3)}(\phi_{(3,3,-1/3)})^2$	yes	yes	scalar up-quark and triplet version of scalar down-quark; two copies of $\phi_{(3,3,-1/3)}$ required
cA4	$\phi_{(3,3,-1/3)}$, $\phi_{(3,2,1/6)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $y_2\overline{\ell^c_L}\phi^\dagger_{(3,3,-1/3)}Q_L$ $+\mu\phi^\dagger_{(3,2,1/6)}\phi_{(3,3,-1/3)}H$	yes	no	mentioned in [33]; embedding in $SU(5)$ and $SO(10)$ [92]; discussed in the context of the R_K anomaly [93, 94]
cA5	$\phi_{(3,1,-1/3)}$, $\phi_{(6,1,2/3)}$	$y_1\overline{\ell^c_L}\phi^\dagger_{(3,1,-1/3)}Q_L$ $+y_2\overline{Q^c_L}\phi_{(3,1,-1/3)}Q_L$ $+y_3\overline{d^c_R}\phi_{(6,1,2/3)}d_R$ $+\mu\phi_{(6,1,2/3)}(\phi_{(3,1,-1/3)})^2$	yes	no	discussed in [95]

Model	New fields	Relevant Interactions	B-viol.?	New?	Comment
cA6	$\phi_{(6,1,-1/3)}$, $\phi_{(3,2,1/6)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $+y_2\overline{Q^c_L}\phi_{(6,1,-1/3)}Q_L$ $+\mu\phi_{(6,1,-1/3)}(\phi_{(3,2,1/6)})^2$	no	yes	scalar quark doublet and colour sextet; two copies of $\phi_{(3,2,1/6)}$ required
cA7	$\phi_{(3,3,-1/3)}$, $\phi_{(6,1,2/3)}$	$y_1\overline{\ell^c_L}\phi^\dagger_{(3,3,-1/3)}Q_L$ $+y_2\overline{Q^c_L}\phi_{(3,3,-1/3)}Q_L$ $+y_3\overline{d^c_R}\phi_{(6,1,2/3)}d_R$ $+\mu\phi_{(6,1,2/3)}(\phi_{(3,3,-1/3)})^2$	yes	yes	variation of cA1
cA8	$\phi_{(6,3,-1/3)}$, $\phi_{(3,2,1/6)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $+y_2\overline{Q^c_L}\phi_{(6,3,-1/3)}Q_L$ $+\mu\phi_{(6,3,-1/3)}(\phi_{(3,2,1/6)})^2$	no	yes	scalar quark doublet and exotic colour sextet with non-trivial $SU(2)_L$ charge

Table 3.3: List of neutrino mass models with two new scalars including non-trivial $SU(3)_C$ representations. The first four columns contain the name of the model in our notation, the new beyond the Standard Model fields, and the interactions relevant for neutrino mass generation. Column 5 and 6 indicate whether baryon number is violated in the model, and whether it has been discussed before. A general comment is given in the last column.

The models of type cA can be divided into two categories. The first one is model (cA1) with the new scalars $\phi_{(3,1,-1/3)}$ and $\phi_{(3,1,2/3)}$, and its variants with higher-dimensional representations, (cA3), (cA5), and (cA7). They are all coloured versions of the Zee-Babu model discussed earlier. The corresponding mass diagram will look just like the one in fig. (3.3), but the fermions running in the two loops are now quarks instead of leptons. Of this first group, only model (cA5) has been discussed previously in ref. [95] in connection to lepton flavour violation.

The second category is model (cA2) with the new scalars $\phi_{(3,1,-1/3)}$ and $\phi_{(3,2,1/6)}$, and the versions with higher $SU(3)_C$ and $SU(2)_L$ representations, (cA4), (cA6), and (cA8). All models in this category also have a Zee-Babu-like contribution to the neutrino mass, similar to the first class. But in addition, the models (cA2) and (cA4) also have a one-loop contribution to neutrino mass, which is based on the interaction terms

$$\mathcal{L} \supset y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R + y_2\overline{\ell^c_L}\phi^\dagger_{(3,n,-1/3)}Q_L + \mu\phi^\dagger_{(3,2,1/6)}\phi_{(3,n,-1/3)}H\,, \quad n = 1,3\,. \tag{3.2.1}$$

The corresponding neutrino mass diagram looks like fig. (3.1), where the leptons in the loops are replaced by down quarks, and the scalars running in the loop are the two new leptoquarks. The embedding of these two models in $SU(5)$ and $SO(10)$ unified theories has been discussed in [92]. A variation of (cA2) with only soft L breaking has been studied in ref. [91].

Let us denote the scalar of larger hypercharge by ϕ_a, and the scalar of lower hypercharge by ϕ_b for any of these models. For the scalar trilinear coupling $\mu\phi_a\phi_b^2$ to be non-vanishing, the fully-contracted interaction term should be symmetric under the exchange of the two copies of ϕ_b. Studying the symmetry properties of the group index contractions, we find that the models (cA1), (cA3), and (cA6) need two copies of ϕ_b.

Some of the scalar leptoquarks appearing in these mass models have also been considered in connection to B-anomalies [4–6]. Some of them are also connected to proton decay. The connection of neutrino mass models to other signs of new physics will be covered in the next chapter.

Model	New fields	Relevant Interactions	B-viol.?	New?	Comment
cB1	$\phi_{(1,1,1)}$, $\psi_L^{(3,1,-1/3)}$ $+\psi_R^{(3,1,-1/3)}$	$y_1\overline{\ell^c_L}\phi_{(1,1,1)}\ell_L$ $+y_2\overline{Q_L}H\psi_R^{(3,1,-1/3)}$ $+y_3\overline{u_R}\phi_{(1,1,1)}\psi_L^{(3,1,-1/3)}$ $+m\overline{\psi_L^{(3,1,-1/3)}}\psi_R^{(3,1,-1/3)}$	no	no	mentioned in [33]; singly charged scalar and vector-like down-type quark no proper mixing; ruled out
cB2	$\phi_{(1,1,1)}$, $\psi_L^{(3,1,2/3)}$ $+\psi_R^{(3,1,2/3)}$	$y_1\overline{\ell^c_L}\phi_{(1,1,1)}\ell_L$ $+y_2\overline{Q_L}H^\dagger\psi_R^{(3,1,2/3)}$ $+y_3\overline{d_R}\phi^\dagger_{(1,1,1)}\psi_L^{(3,1,2/3)}$ $+m\overline{\psi_L^{(3,1,2/3)}}\psi_R^{(3,1,2/3)}$	no	no	mentioned in [33]; singly charged scalar and vector-like up-type quark no proper mixing; ruled out
cB3	$\phi_{(1,1,1)}$, $\psi_L^{(3,2,-5/6)}$ $+\psi_R^{(3,2,-5/6)}$	$y_1\overline{\ell^c_L}\phi_{(1,1,1)}\ell_L$ $+y_2\overline{d_R}H\psi_L^{(3,2,-5/6)}$ $+y_3\overline{Q_L}\phi_{(1,1,1)}\psi_R^{(3,2,-5/6)}$ $+m\overline{\psi_L^{(3,2,-5/6)}}\psi_R^{(3,2,-5/6)}$	no	no	mentioned in [33] no proper mixing; ruled out
cB4	$\phi_{(1,1,1)}$, $\psi_L^{(3,2,7/6)}$ $+\psi_R^{(3,2,7/6)}$	$y_1\overline{\ell^c_L}\phi_{(1,1,1)}\ell_L$ $+y_2\overline{u_R}H^\dagger\psi_L^{(3,2,7/6)}$ $+y_3\overline{Q_L}\phi^\dagger_{(1,1,1)}\psi_R^{(3,2,7/6)}$ $+m\overline{\psi_L^{(3,2,7/6)}}\psi_R^{(3,2,7/6)}$	no	no	considered in [33] no proper mixing; ruled out
cB5	$\phi_{(3,2,1/6)}$, $\psi_L^{(1,2,1/2)}$ $+\psi_R^{(1,2,1/2)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $+y_2\overline{u_R}\phi_{(3,2,1/6)}\psi_L^{(1,2,1/2)}$ $+y_3\overline{e^c_R}H\psi_R^{(1,2,1/2)}$ $+m\overline{\psi_L^{(1,2,1/2)}}\psi_R^{(1,2,1/2)}$	yes	no	mentioned in [33]; scalar quark doublet and vector-like lepton doublet; gauge unification at 10^{14} GeV [96]

Model	New fields	Relevant Interactions	B-viol.?	New?	Comment
cB6	$\phi_{(3,2,1/6)}$, $\psi_L^{(3,1,-1/3)}$ $+\psi_R^{(3,1,-1/3)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $+y_2\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}\psi_R^{(3,1,-1/3)}$ $+y_3\overline{Q^c_L}\phi_{(3,2,1/6)}\psi_L^{(3,1,-1/3)}$ $+\lambda H^\dagger\phi^3_{(3,2,1/6)}$ $+m\overline{\psi_L^{(3,1,-1/3)}}\psi_R^{(3,1,-1/3)}$	yes	yes	scalar quark doublet and vector-like down-type quark; two copies of $\phi_{(3,2,1/6)}$ required
cB7	$\phi_{(3,2,1/6)}$, $\psi_L^{(3,1,2/3)}$ $+\psi_R^{(3,1,2/3)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $+y_2\overline{\ell^c_L}\phi^\dagger_{(3,2,1/6)}\psi_L^{(3,1,2/3)}$ $+y_3\overline{Q_L}H^\dagger\psi_R^{(3,1,2/3)}$ $+m\overline{\psi_L^{(3,1,2/3)}}\psi_R^{(3,1,2/3)}$	yes	no	discussed in [97]; scalar quark doublet and vector-like up-type quark
cB8	$\phi_{(3,1,-1/3)}$, $\psi_L^{(3,2,-5/6)}$ $+\psi_R^{(3,2,-5/6)}$	$y_1\overline{\ell^c_L}\phi^\dagger_{(3,1,-1/3)}Q_L$ $+y_2\overline{\ell_L}\phi^\dagger_{(3,1,-1/3)}\psi_R^{(3,2,-5/6)}$ $+y_3\overline{d_R}H\psi_L^{(3,2,-5/6)}$ $+m\overline{\psi_L^{(3,2,-5/6)}}\psi_R^{(3,2,-5/6)}$	yes	no	discussed in [33, 98]
cB9	$\phi_{(3,2,1/6)}$, $\psi_L^{(3,2,1/6)}$ $+\psi_R^{(3,2,1/6)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $+y_2\overline{d^c_R}\phi_{(3,2,1/6)}\psi_R^{(3,2,1/6)}$ $+y_3\overline{d_R}H^\dagger\psi_L^{(3,2,1/6)}$ $+\lambda H^\dagger\phi^3_{(3,2,1/6)}$ $+m\overline{\psi_L^{(3,2,1/6)}}\psi_R^{(3,2,1/6)}$	yes	yes	scalar quark doublet and vector-like quark doublet; two copies of $\phi_{(3,2,1/6)}$ required
cB10	$\phi_{(3,2,1/6)}$, $\psi_L^{(3,2,7/6)}$ $+\psi_R^{(3,2,7/6)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $+y_2\overline{e^c_R}\phi^\dagger_{(3,2,1/6)}\psi_R^{(3,2,7/6)}$ $+y_3\overline{u_R}H^\dagger\psi_L^{(3,2,7/6)}$ $+m\overline{\psi_L^{(3,2,7/6)}}\psi_R^{(3,2,7/6)}$	yes	no	mentioned in [33]
cB11	$\phi_{(3,2,1/6)}$, $\psi_L^{(3,3,-1/3)}$ $+\psi_R^{(3,3,-1/3)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $+y_2\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}\psi_R^{(3,3,-1/3)}$ $+y_3\overline{Q^c_L}\phi_{(3,2,1/6)}\psi_L^{(3,3,-1/3)}$ $+\lambda H^\dagger\phi^3_{(3,2,1/6)}$ $+m\overline{\psi_L^{(3,3,-1/3)}}\psi_R^{(3,3,-1/3)}$	yes	yes	scalar quark doublet and triplet version of down-quark; two copies of $\phi_{(3,2,1/6)}$ required
cB12	$\phi_{(3,2,1/6)}$, $\psi_L^{(3,3,2/3)}$ $+\psi_R^{(3,3,2/3)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $+y_2\overline{\ell^c_L}\phi^\dagger_{(3,2,1/6)}\psi_L^{(3,3,2/3)}$ $+y_3\overline{Q_L}H^\dagger\psi_R^{(3,3,2/3)}$ $+m\overline{\psi_L^{(3,3,2/3)}}\psi_R^{(3,3,2/3)}$	yes	no	mentioned in [33]

Model	New fields	Relevant Interactions	B-viol.?	New?	Comment
cB13	$\phi_{(3,3,-1/3)}$, $\psi_L^{(3,2,-5/6)}$ $+\psi_R^{(3,2,-5/6)}$	$y_1\overline{\ell^c_L}\phi^\dagger_{(3,3,-1/3)}Q_L$ $+y_2\overline{\ell_L}\phi^\dagger_{(3,3,-1/3)}\psi_R^{(3,2,-5/6)}$ $+y_3\overline{d_R}H\psi_L^{(3,2,-5/6)}$ $+m\overline{\psi_L^{(3,2,-5/6)}}\psi_R^{(3,2,-5/6)}$	yes	no	listed in [33]
cB14	$\phi_{(3,2,1/6)}$, $\psi_L^{(8,2,1/2)}$ $+\psi_R^{(8,2,1/2)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $+y_2\overline{u_R}\phi_{(3,2,1/6)}\psi_L^{(8,2,1/2)}$ $+y_3\overline{d^c_R}\phi^\dagger_{(3,2,1/6)}\psi_R^{(8,2,1/2)}$ $+m\overline{\psi_L^{(8,2,1/2)}}\psi_R^{(8,2,1/2)}$	yes	yes	colour octett version of cB5

Table 3.4: Neutrino mass models with one new scalar and one new vector-like Dirac fermion with at least one coloured field. The table contains the new beyond the Standard model fields, as well as the interaction terms necessary for neutrino mass generation. Some comment on baryon number violation and previous discussions in the literature are given. Finally, the last column contains some general remarks.

The type cB contains the largest number of models. However, most of them can be assigned to one of four classes of models.

The first class of models are variations of the Zee model, (cB1)-(cB4). Like the Zee model, they all contain a charged scalar singlet $\phi_{(1,1,1)}$. The trilinear scalar coupling $\mu H^\dagger \tilde{H}\phi_{(1,1,1)}$ is generated by a fermion loop containing quarks and a new, coloured, vector-like Dirac fermion. As discussed earlier, this type of model leads to a traceless neutrino mass matrix in flavour space. Hence these models are excluded by experimental data. The models (cB1)-(cB4) are the only ones of type cB that do not break baryon number B in addition to lepton number. The models (cB1)-(cB4) and some aspects of their collider phenomenology have been discussed in [33].

The second class of models is (cB8), and its triplet version (cB13). Apart from the Zee-like models (cB1)-(cB4), these are the only two models of type cB which do not include the scalar leptoquark $\phi_{(3,2,1/6)}$. They are also the only two models in type cB which generate the dimension-5 Weinberg operator at one-loop. The corresponding diagram is shown in fig. (3.6). While (cB13) has to our knowledge not been discussed yet, (cB8) was considered as a connection of B-anomalies, neutrino masses, and unification in ref. [98].

The third group of models contains (cB6), (cB9), and (cB11). To our best knowledge, this group of models has not been discussed before. They have in common that due to the couplings present, the vector mass term of the new fermion breaks lepton number by one instead of two units. To get $\Delta L = 2$, an additional lepton number breaking is needed. The quartic scalar coupling $\lambda H^\dagger \phi^3_{(3,2,1/6)}$ can provide the necessary lepton number violation. However, this term vanishes identically if there is only one copy of $\phi_{(3,2,1/6)}$. Therefore, two copies of the scalar leptoquarks are needed in these models. Fig. (3.7) shows the corresponding neutrino mass diagram of (cB6) as an example. For

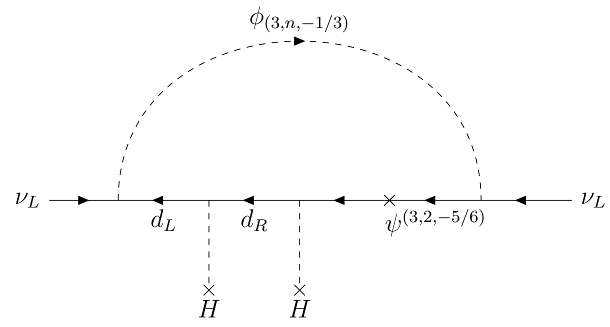


Figure 3.6: Schematic diagram of the neutrino mass term in the model (cB8) and (cB13), including the new scalar $\phi_{(3,n,-1/3)}$, where $n = 1, 3$, and the new fermion $\psi_{(3,2,-5/6)}$.

(cB9), the loop structure stays the same, but the mass insertions in the fermion line are placed differently.

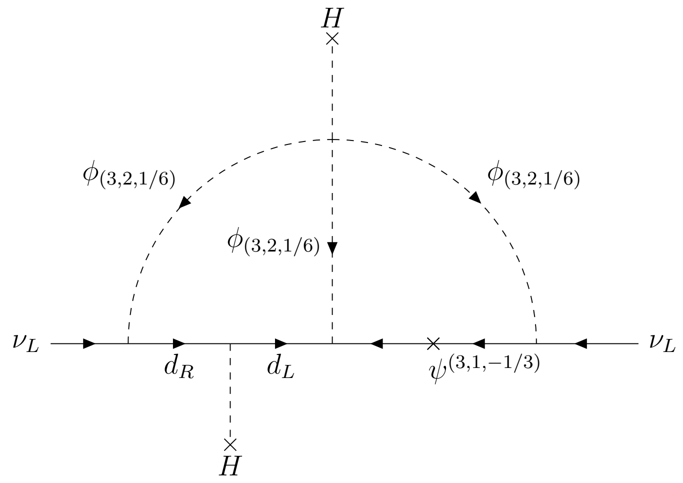


Figure 3.7: Schematic diagram of the neutrino mass term in model (cB6), containing $\phi_{(3,2,1/6)}$ and $\psi_{(3,1,-1/3)}$ as new fields. For (cB9), the loop structure stays the same, while the mass insertions in the fermion line are placed differently. For (cB11) the diagram is identical.

The last group of models contains (cB5), (cB7), (cB10), (cB12), and (cB14). These models also contain $\phi_{(3,2,1/6)}$ as a new scalar, but the vector mass of the new fermion breaks lepton number by two units. Hence the quartic scalar coupling $\lambda H^\dagger \phi^3_{(3,2,1/3)}$ is not necessary for neutrino mass generation, and one copy of the new leptoquark is sufficient. The models generate the dimension-5 Weinberg operator via a chain-like two-loop diagram, except for the model (cB14). While (cB14) is the octet version of (cB5), due to the $SU(3)_C$ representation of the new fermion, the two-loop contribution vanishes and the dimension-5 Weinberg operator is only generated at the three-loop level. One possible mass diagram of the model (cB5) is shown in fig. (3.8) as an example, the other diagrams have a similar loop structure, while the placement of mass insertions along the fermion line can vary. (cB14) is the only model in this class which has not been discussed previously. (cB5) was considered in the context of unification in [96]. It was shown that this model admits successful unification in $SU(5)$. (cB7) was discussed in detail in ref. [97]. The other models appear in the scan in ref. [33].

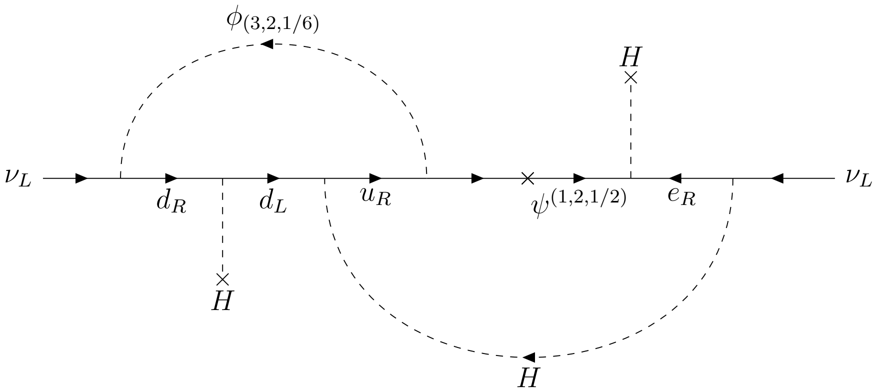


Figure 3.8: One possible neutrino mass diagram in the model (cB5), with the new multiplets $\phi_{(3,2,1/6)}$ and $\psi^{(1,2,1/2)}$.

Model	New fields	Relevant Interactions	B-viol.?	New?	Comment
cC1	$\phi_{(3,1,-1/3)}$, $\psi_L^{(8,1,0)}$	$y_1\overline{\ell^c_L}\phi^\dagger_{(3,1,-1/3)}Q_L$ $+y_2\overline{d_R}\phi_{(3,1,-1/3)}\psi_L^{(8,1,0)}$ $+M\overline{(\psi_L^{(8,1,0)})^c}\psi_L^{(8,1,0)}$	yes	no	discussed in [99, 100]; gluino and down-type squark
cC2	$\phi_{(3,2,1/6)}$, $\psi_R^{(8,1,0)}$	$y_1\overline{\ell_L}\phi^\dagger_{(3,2,1/6)}d_R$ $+y_2\overline{Q_L}\phi_{(3,2,1/6)}\psi_R^{(8,1,0)}$ $+M\overline{(\psi_R^{(8,1,0)})^c}\psi_R^{(8,1,0)}$	no	yes	squark doublet and gluino
cC3	$\phi_{(3,3,-1/3)}$, $\psi_L^{(8,3,0)}$	$y_1\overline{\ell^c_L}\phi^\dagger_{(3,3,-1/3)}Q_L$ $+y_2\overline{d_R}\phi_{(3,3,-1/3)}\psi_L^{(8,3,0)}$ $+M\overline{(\psi_L^{(8,3,0)})^c}\psi_L^{(8,3,0)}$	yes	yes	triplet version of cC1
cC4	$\phi_{(\mathcal{C},2,1/2)}$, $\psi_R^{(\mathcal{C},1,0)}$, $\mathcal{C}$ real $SU(3)$ rep.	$y\overline{\ell_L}\phi^\dagger_{(\mathcal{C},2,1/2)}\psi_R^{(\mathcal{C},1,0)}$ $+\lambda H^\dagger\phi_{(\mathcal{C},2,1/2)}H^\dagger\phi_{(\mathcal{C},2,1/2)}$ $+M\overline{(\psi_R^{(\mathcal{C},1,0)})^c}\psi_R^{(\mathcal{C},1,0)}$	no	no	discussed in [90]

Model	New fields	Relevant Interactions	B-viol.?	New?	Comment
cC5	$\phi_{(\mathcal{C},2,1/2)}$, $\psi_R^{(\mathcal{C},3,0)}$, $\mathcal{C}$ real $SU(3)$ rep.	$y\overline{\ell_L}\phi^\dagger_{(\mathcal{C},2,1/2)}\psi_R^{(\mathcal{C},3,0)}$ $+\lambda H^\dagger\phi_{(\mathcal{C},2,1/2)}H^\dagger\phi_{(\mathcal{C},2,1/2)}$ $+M\overline{(\psi_R^{(\mathcal{C},3,0)})^c}\psi_R^{(\mathcal{C},3,0)}$	no	no	discussed in [90]
cC6	$\phi_{(\mathcal{C},n,1/2)}$, $\psi_R^{(\mathcal{C},n\pm1,0)}$, $\mathcal{C}$ real $SU(3)$ rep., $n \geq 4$, n even	$y\overline{\ell_L}\phi^\dagger_{(\mathcal{C},n,1/2)}\psi_R^{(\mathcal{C},n\pm1,0)}$ $+\lambda H^\dagger\phi_{(\mathcal{C},n,1/2)}H^\dagger\phi_{(\mathcal{C},n,1/2)}$ $+M\overline{(\psi_R^{(\mathcal{C},n\pm1,0)})^c}\psi_R^{(\mathcal{C},n\pm1,0)}$	no	yes	model C2 with coloured fields;

Table 3.5: Neutrino mass models with one new scalar and one new Majorana fermion with at least one new coloured field. Besides the name of the model in our notation, the table contains the new fields as well as the interaction terms relevant for neutrino mass generation. We also make a comment on baryon number violation and previous appearances in the literature. In addition, some general remarks are given.

The models of type cC split up into two groups. The first one is (cC1)-(cC3). The mechanism of neutrino mass generation in these models is similar to the last group of models of type cB. The mass term of the new fermion breaks lepton number by two units, and the dimension-5 Weinberg operator is generated by a chain-like two-loop diagram. As an example, one possible neutrino mass diagram for the model (cC1) is shown in fig. (3.9). The model (cC1) has been studied in connection with lepton flavour violation [99] and B-anomalies [100]. In a context of supersymmetry, the new fields in the models (cC1) and (cC2) could be identified as squarks and gluinos. Using this identification, (cC1) introduces a down-type squark and a gluino, while (cC2) introduces a squark doublet and a gluino. Even though this is an interesting point of view, a complete study of how these models could be embedded into a supersymmetrical framework is beyond the scope of this work.

The models (cC4) and (cC5) are coloured versions of the type I and III see-saw mechanisms. The new fields are a coloured version of the Higgs doublet, $\phi_{(\mathcal{C},2,1/2)}$, and a coloured version of the right-handed fermion singlet or triplet, $\psi_R^{(\mathcal{C},1,0)}$ or $\psi_R^{(\mathcal{C},3,0)}$. Here, $\mathcal{C}$ must be a real representation of $SU(3)$ for the Majorana mass term of the new fermion to be gauge invariant. The non-trivial colour representation of the new fields forbids the tree-level generation of the dimension-5 Weinberg operator. It is instead generated at one-loop level. The neutrino mass diagram has the same form as the one for model class (C2), fig. (3.5), with the new fields replaced accordingly. These models have been considered in ref. [90].

The models (cC4) and (cC5) are also the lowest-dimensional versions of the model class (cC6). (cC6) is the extension of (C2) to coloured fields. As explained above, the

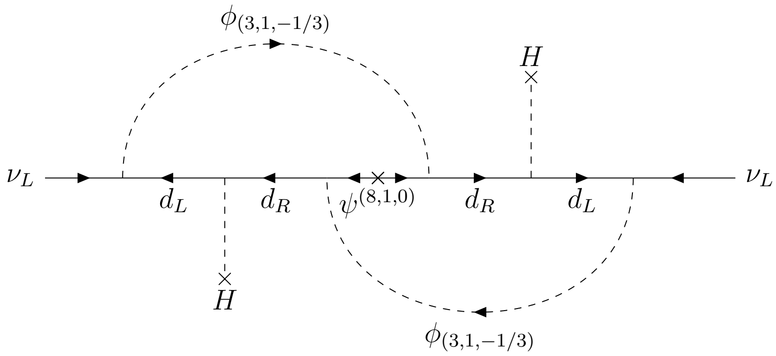


Figure 3.9: One possible realisation of the neutrino mass diagram in the model (cC1). The model includes the new scalar $\phi_{(3,1,-1/3)}$, and the new Majorana fermion $\psi^{(8,1,0)}$. The possible diagrams for the models (cC2) and (cC3) look similar.

$SU(3)$ representation of the new fields must be real. Since the new fields are coloured, the models do not contain fundamental dark matter candidates.

3.2.3 Unsuccessful models

When scanning for the list of models presented in the previous sections, we used a system based on lepton number violation. Some models leading to lepton number violation do not lead to a realisation of the Weinberg operator, though. In this subsection we want to comment on these models, and try to explain why they did not lead to neutrino mass.

The unsuccessful models all appear when new coloured representations are involved. Let us consider two examples, one with two new scalars and one with a new scalar and a new Dirac fermion. They will illustrate the mechanism stopping these models from inducing neutrino mass.

The first example is a model containing the two new scalars $\phi_{(1,1,1)}$ and $\phi_{(3,1,-1/3)}$. Including these two scalars, we can write down the following terms leading to lepton number violation:

$$\mathcal{L} \supset y_1 \overline{\ell^c_L} \phi^\dagger_{(3,1,-1/3)} Q_L + y_2 \overline{Q^c_L} \phi_{(3,1,-1/3)} + y_3 \overline{\ell^c_L} \phi_{(1,1,1)} \ell_L + \lambda \phi^3_{(3,1,-1/3)} \phi_{(1,1,1)} \ . \quad (3.2.2)$$

Taking into account that the lepton number of ℓ_L is one and the baryon number of Q_L is 1/3, we find that by assigning the baryon- and lepton number to the two new scalars as

$$\begin{aligned} L(\phi_{(1,1,1)}) &= -2 & B(\phi_{(1,1,1)}) &= 0 \\ L(\phi_{(3,1,-1/3)}) &= 1 & B(\phi_{(3,1,-1/3)}) &= 1/3 \ , \end{aligned} \quad (3.2.3)$$

the second and fourth interaction in eq. (3.2.2) break lepton and baryon number by one unit each. As a result, the difference, $(B-L)$, is conserved. Since the Weinberg operator does not only break L, but also $(B-L)$, it is not generated in this case.

A similar observation can be made in the second example. Here, we add the scalar

$\phi_{(3,1,-8/6)}$ and the fermion $\psi_L^{(3,2,7/6)} + \psi_R^{(3,2,7/6)}$ to the Standard Model. In this case, the interactions relevant for lepton number violation are

$$\begin{aligned}\mathcal{L} \supset &\overline{e^c_R}\phi^\dagger_{(3,1,-8/6)}d_R + \overline{u^c_R}\phi_{(3,1,-8/6)}u_R \\ &+ \overline{u_R}H^\dagger\psi_L^{(3,2,7/6)} + \overline{Q^c_L}\phi_{(3,1,-8/6)}\psi_L^{(3,2,7/6)} \ .\end{aligned} \tag{3.2.4}$$

Choosing the lepton and baryon numbers as

$$\begin{aligned} L(\phi_{(3,1,-8/6)}) &= 0 \qquad & B(\phi_{(1,1,1)}) &= -2/3 \\ L(\psi_L^{(3,2,7/6)}) &= 0 \qquad & B(\psi_L^{(3,2,7/6)}) &= 1/3 \ , \end{aligned} \tag{3.2.5}$$

the only lepton number violating term is the first one in eq. (3.2.4). It violates lepton number by one unit. Similarly, this term is also the only one violating Baryon number, and it violates baryon number by one. Again, $(B-L)$ is conserved, and there is no neutrino mass.

The two examples clearly show the difficulty with coloured models. If the new fields carry colour, quarks are involved in their interactions. When quarks are involved, not only lepton number, but also baryon number can be broken. In this case one has to make sure that there is no configuration of the baryon- and lepton-numbers of the new fields that conserves $(B-L)$. The reason is that the Majorana neutrino mass breaks L as well as $(B-L)$. So any model leading to Majorana neutrino masses must break both of them.

3.3 Minimal radiative neutrino mass with new fermions only

The minimal models for neutrino masses which only include new fermionic representations are the see-saw models of type I and type III. In the see-saw mechanisms the dimension-5 Weinberg operator is generated at tree-level. This was excluded in our scan for what we call radiative models. In the scan, we found that, apart from the Zee model, the minimal number of new multiplets needed for radiative neutrino masses is two. However, all models we found contained at least one new scalar multiplet. One might ask the question whether there exists a minimal radiative neutrino mass model with two new fermionic representations.

In this section, we will take a closer look at that question. Since we are not referring to the chirality of the new fermions in our considerations in the next section, we will omit the chirality index for brevity.

3.3.1 Radiative neutrino mass with two new fermions

In the following, we study radiative neutrino mass models with new fermions only. Considering the conditions for minimality and our definition of a radiative model excludes the right-handed neutrino ν_R and the triplet Σ from any such model. It turns out that if we conduct our model scan as described in sections (3.1), we do not find any model with just two new fermionic representations inducing the dimension-5 Weinberg operator only at the loop-level. One can even argue that such a model cannot exist.

The proof will use lepton number violation as a guiding principle. Let us first make some general observations before applying them to potential extensions of the Standard Model.

Assume that we have some fermion ψ_a fulfilling the condition

$$\|Y(\psi_a)\| \geq \|Y(H)\| \tag{3.3.1}$$

on its hypercharge. Then we can infer from hypercharge conservation that for any pair of fermions complying with eq. (3.3.1), any Yukawa coupling or mass term takes the form

$$\begin{aligned} \overline{\psi_a}\Phi\psi_b: &\quad \mathrm{sign}\left(Y(\psi_x)\right) = \mathrm{sign}\left(Y(\psi_y)\right) \,, \\ \overline{\psi_a^c}\Phi\psi_b: &\quad \mathrm{sign}\left(Y(\psi_a)\right) \neq \mathrm{sign}\left(Y(\psi_b)\right) \,, \end{aligned} \tag{3.3.2}$$

where Φ stands for H, $\tilde{H}$, or m. Assigning a lepton number L_a to each fermion ψ_a , we can rewrite eq. (3.3.2) as a condition for lepton number conservation,

$$\mathrm{sign}\left(Y(\psi_a)\right) L_a - \mathrm{sign}\left(Y(\psi_b)\right) L_b = 0\,. \tag{3.3.3}$$

Considering the leptons in the Standard Model, they all fulfil condition (3.3.1). Furthermore, they are of negative hypercharge and have a lepton number of one. Putting this into (3.3.3), one finds that for any new fermion ψ_a coupling to Standard Model leptons and matching condition (3.3.1), the lepton number fixed by lepton number conservations is

$$L_a = -\mathrm{sign}\left(Y(\psi_a)\right) \,. \tag{3.3.4}$$

Finally, we note that if a field only couples to fields with vanishing lepton number, its lepton number can be set to zero and lepton number is conserved.

In the next step, we turn to models with two new fermions generating the dimension-5 Weinberg operator only at loop-level. Let us assume we have a pair ψ_a, ψ_b of fermions that lead to such a model. More specifically, adding them to the Standard Model breaks lepton number. We now try to find conditions for lepton number conservation which cannot be fulfilled.

First, observe that the representation of a new fermion coupling to a Standard Model lepton must be one of those listed in eq. (3.1.5). It should be mentioned that fields of different $SU(3)_C$ representations cannot be coupled to each other in this way, since there is no coloured scalar in the Standard Model. As a consequence, we can drop the $SU(3)$ representation in the index of the new fields as in section (3.2.1). Note also that the representations $(1,0)$ and $(3,0)$ are excluded, since they lead to see-saw type I and III. The rest of the representations are potential candidates. Moreover, the list of all fermions coupling to one of the candidates via Yukawa coupling is

$$
\begin{aligned}
\psi^{(1,-1)}H &\sim (2,-1/2)\,, & \psi^{(1,-1)}\tilde{H} &\sim (2,-3/2)\,, \\
\psi^{(2,-1/2}H &\sim (1,0)\oplus(3,0)\,, & \psi^{(2,-1/2)}\tilde{H} &\sim (1,-1)\oplus(3,-1)\,, \\
\psi^{(2,-3/2)}H &\sim (1,-1)\oplus(3,-1)\,, & \psi^{(2,-3/2)}\tilde{H} &\sim (1,-2)\oplus(3,-2)\,, \\
\psi^{(3,-1)}H &\sim (2,-1/2)\oplus(4,-1/2)\,, & \psi^{(3,-1)}\tilde{H} &\sim (2,-3/2)\oplus(4,-3/2)\,.
\end{aligned}
\tag{3.3.5}
$$

$(1,0)$ and $(3,0)$ are again excluded. If we neglect representations overlapping with (3.1.5), there are four representations left:

$$
\psi^{(1,2)} \sim (1,2)\,, \quad \psi^{(3,2)} \sim (3,2)\,, \quad \psi^{(4,1/2)} \sim (4,1/2)\,, \quad \psi^{(4,3/2)} \sim (4,3/2)\,. \tag{3.3.6}
$$

We observe that all allowed representations in (3.1.5) and (3.3.6) obey eq. (3.3.1). So if there are only two new fermionic representations, we can fix their lepton numbers as follows:

1. If the representation of ψ_a appears in (3.1.5), the lepton number is fixed to $L_a = -\mathrm{sign}\,(Y(\psi_a))$ according to eq. (3.3.4).
2. If the representation of ψ_a appears in (3.3.6), test to which representations form eq. (3.1.5) it can couple. Have a look whether a field with this representation is present in your model. If true, the lepton number is fixed to $L_a = -\mathrm{sign}\,(Y(\psi_a))$ according to eq. (3.3.4). Otherwise, the lepton number is set to $L_a = 0$.
3. If the representation of ψ_a does not appear in (3.1.5) or (3.3.6), the lepton number is set to $L_a = 0$.

Using this scheme lepton number is always conserved. Hence there is no model with only two fermionic representations that can generate the dimension-5 Weinberg operator only at loop-level.

3.3.2 Radiative neutrino mass with three new fermions

Going to three new fermionic representations, we can show that there is exactly one unique model inducing the dimension-5 Weinberg operator only at the loop-level, up to a change of sign for the hypercharges. Some aspects of this model have been discussed in ref. [57]. In order to show this, we use the relations (3.3.1)-(3.3.4) established in the previous section. In addition, we need a third set of representations, namely the ones for potential new fermions coupling to the ones in eq. (3.3.6). The complete list is

$$
\begin{aligned}
\psi^{(1,-2)}H &\sim (2,-3/2)\,, & \psi^{(1,-2)}\tilde{H} &\sim (2,-5/2)\,, \\
\psi^{(3,-2)}H &\sim (2,-3/2)\oplus(4,-3/2)\,, & \psi^{(3,-2)}\tilde{H} &\sim (2,-5/2)\oplus(4,-5/2)\,, \\
\psi^{(4,-1/2)}H &\sim (3,0)\oplus(5,0)\,, & \psi^{(4,-1/2)}\tilde{H} &\sim (3,-1)\oplus(5,-1)\,, \\
\psi^{(4,-3/2)}H &\sim (3,-1)\oplus(5,-1)\,, & \psi^{(4,-3/2)}\tilde{H} &\sim (3,-2)\oplus(5,-2)\,.
\end{aligned}
\tag{3.3.7}
$$

Observe that apart from $(3, 0)$, which we exclude, there is exactly one representation not obeying eq. (3.3.1), the representation $(5, 0)$. Now we can study models with three new fermionic representations, which generate the dimension-5 Weinberg operator only at loop-level.

Let us first assume that none of the new fermions is in the representation $(5, 0)$. Then each fermion in one of the allowed representations from eq. (3.1.5), eq. (3.3.6), or eq. (3.3.7) satisfies eq. (3.3.1). As a result, we can use an extended version of the description above to fix all lepton numbers:

1. If the representation of ψ_a appears in eq. (3.1.5), the lepton number is fixed to $L_a = -\mathrm{sign}\,(Y(\psi_a))$ according to eq. (3.3.4).

2. If the representation of ψ_a appears in eq. (3.3.6), check whether a fermion ψ_b in a representation from eq. (3.1.5) which ψ_a couples to is also present in your model. If true, the lepton number of ψ_a is fixed to $L_a = -\mathrm{sign}\,(Y(\psi_a))$ according to eq. (3.3.4). Otherwise, the lepton number is set to $L_a = 0$.

3. If the representation of ψ_a appears in eq. (3.3.7), check whether a field ψ_b in the representation from eq. (3.3.6) which ψ_a couples to is also present in your model. If it is and has non-zero lepton number (see previous step), the lepton number is fixed to $L_a = -\mathrm{sign}\,(Y(\psi_a))$ according to eq. (3.3.4). Otherwise, the lepton number is set to $L_a = 0$.

4. If the representation of ψ_a does not appear in eq. (3.1.5), eq. (3.3.6), or eq. (3.3.7), the lepton number is set to $L_a = 0$.

This description will always ensure lepton number conservation, so in these cases no Majorana neutrino mass is generated.

Let us now assume that one of the new fermions is $\psi^{(5,0)}$. Then we can write down a Majorana mass term

$$\mathcal{L} \supset M\overline{(\psi^{(5,0)})^c}\psi^{(5,0)} \ . \tag{3.3.8}$$

In order for lepton number to be conserved, we have $L(\psi^{(5,0)}) = 0$. The rest of the lepton numbers can again be assigned according to the scheme above. Since this will lead to a consistent lepton number for the other two fields, the inconsistency must arise for $\psi^{(5,0)}$. If $\psi^{(5,0)}$ does only couple to fermions of hypercharge zero, no inconsistency can arise from $L(\psi^{(5,0)}) = 0$. Hence, it must couple to some fermion that, at least potentially, has a non-zero lepton number, or, in other words, to one of the representations in eq. (3.1.5), eq. (3.3.6), or eq. (3.3.7). The only one it can couple to is $(4, 1/2)$. So the second fermion of the model must be $\psi^{(4,-1/2)}$. For this field, the above description tells us that its lepton number is only non-vanishing if it can couple to a fermion in one of the representations in eq. (3.1.5). Comparing to eq. (3.3.5), the only matching representation is $(3, 1)$. This fixes the third new fermion to $\psi^{(3,-1)}$.

Hence, up to inversion of the hypercharges, the model is uniquely fixed to two vector-like Dirac fermions [3]

$$\psi_L^{(3,-1)} + \psi_R^{(3,-1)} \sim (1,3,-1) \quad \text{and} \quad \psi_L^{(4,-1/2)} + \psi_R^{(4,-1/2)} \sim (1,4,-1/2)\,, \tag{3.3.9}$$

and a single Majorana fermion

$$(\psi_R^{(5,0)})^c + \psi_R^{(5,0)} \sim (1,5,0)\,. \tag{3.3.10}$$

The relevant interactions in this model are given by

$$\mathcal{L} \supset M\overline{(\psi_R^{(5,0)})^c}\psi_R^{(5,0)} + y_1\overline{\psi^{(3,-1)}}_R\tilde{H}\ell_L + y_2\overline{\psi^{(4,-1/2)}}_L H\psi_R^{(3,-1)} + y_3\overline{\psi^{(5,0)}}_R H\psi_L^{(4,-1/2)}\,. \tag{3.3.11}$$

The Majorana mass term of $\psi_R^{(5,0)}$ breaks lepton number by two units. The dimension-5 Weinberg operator is induced at two-loop, but there is also a one-loop realisation of the dimension-7 operator $\ell_L\ell_L HH(H^\dagger H)/\Lambda^3$, and a tree-level realisation of the dimension-9 operator $\ell_L\ell_L HH(H^\dagger H)^2/\Lambda^5$. One diagram contributing to the dimension-5 Weinberg operator is the chain-like diagram which also appears in the model (cB5). It is shown in fig. (3.10).

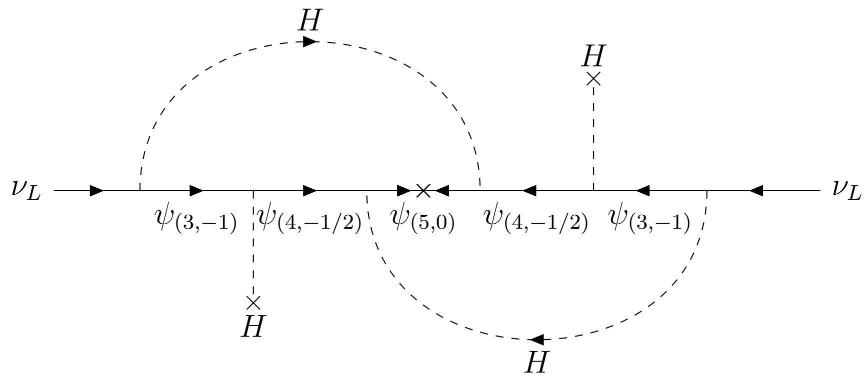


Figure 3.10: One possible realisation of the Weinberg operator in the unique model with only three new fermionic representations generating neutrino mass at loop-level.

As the proof might have revealed, this model can also be viewed as a prototype for a more general mechanism of neutrino mass suppression via fermionic representation chains. The starting point is a Majorana fermion in a $SU(2)_L \otimes U(1)_Y$ representation of the form $(N,0)$ with N odd. This Majorana fermion is then connected to the Standard Model lepton-doublet by a chain of fermions, of which only neighbouring ones can couple to each other via Yukawa couplings. Note that an increasing number of intermediate

[3]Though this model contains several fermionic representations, their contribution to the gauge anomalies of the Standard Model do not cancel, even if different numbers of copies of the two fields are added. Hence, they must be added as vector-like Dirac fermions.

fermions is needed to complete the chain. This leads to an increasing number of loops l in the realisation of the dimension-5 Weinberg operator,

$$l = N - 3\,, \quad N \geq 3\,. \tag{3.3.12}$$

In addition, the dimension d of the higher operator generated at tree-level increases as

$$d = 2N - 1\,, \quad N \geq 3\,. \tag{3.3.13}$$

This chain of fermions which can only interact with their next neighbours resembles the Froggatt-Nielsen mechanism [101] and the Clockwork mechanism [102, 103]. Here, the neutrino mass is suppressed with respect to the Majorana mass of the heavy fermion not only by a Yukawa coupling for each element of the chain, but also by either a loop order or a factor of $\frac{v^2}{\Lambda^2}$.

One illustrating example is to start with a septet Majorana fermion $\Xi \sim (1, 7, 0)$. This can be connected to the Standard Model in two different ways. Using the variant with lower overall hypercharge, we find the intermediate fermions $\chi_1 \sim (1, 3, -1)$, $\chi_2 \sim (1, 4, -1/2)$, $\chi_3 \sim (1, 5, -1)$, and $\chi_4 \sim (1, 6, -1/2)$. One typical contribution to the dimension-5 Weinberg operator is shown in fig. (3.11).

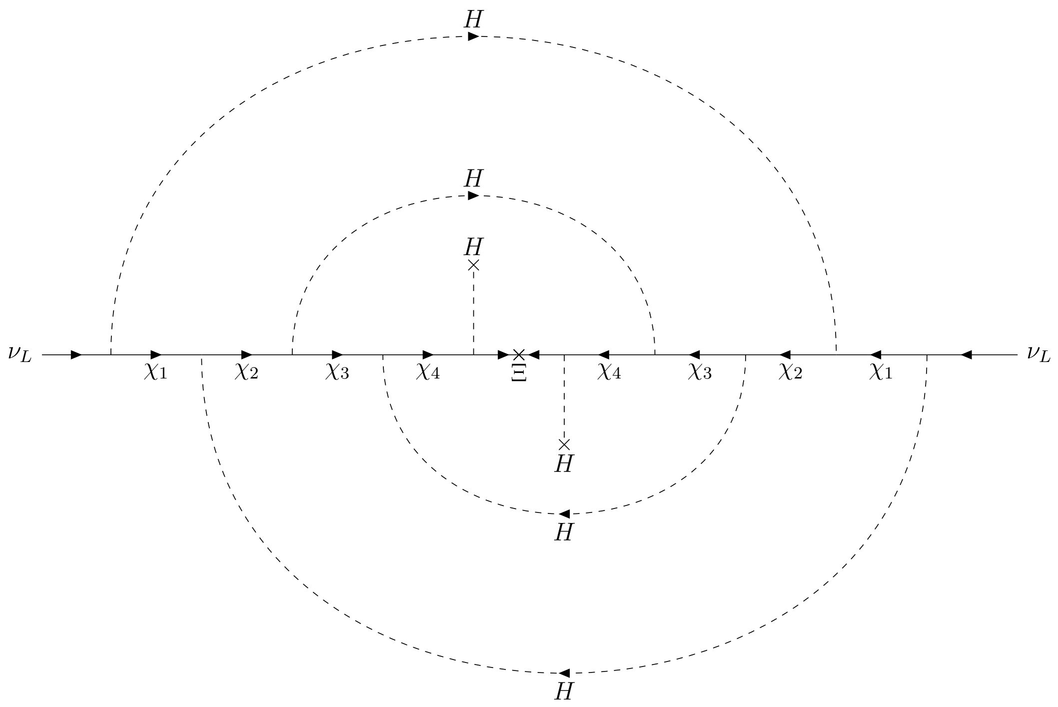


Figure 3.11: "Chain" diagram for neutrino mass based on a model with the new fermions $\Xi \sim (1, 7, 0)$, $\chi_1 \sim (1, 3, -1)$, $\chi_2 \sim (1, 4, -1/2)$, $\chi_3 \sim (1, 5, -1)$, and $\chi_4 \sim (1, 6, -1/2)$. This is an example for a mechanism of systematic loop suppression of the neutrino mass.

4 Phenomenological aspects

In this chapter, we elaborate on some phenomenological aspects of the neutrino mass models found in our classification. In particular, we comment on the connection between neutrino physics and other areas of beyond the Standard Model physics, including dark matter, the search for long-lived particles at the Large Hadron Collider (LHC), anomalies in B-physics, and proton decay.

4.1 Neutrino mass and dark matter

Apart from the neutrino mass, dark matter is one of the biggest shortcomings of the Standard Model. So far, we do not know what the majority of the dark matter is made of. The literature contains a long list of proposals (see [104–106] for reviews), of which the idea of a weakly interacting massive particle, called WIMP, is considered very promising. This is connected to the simple production mechanism for such a particle. Using a rough estimate on the cross section, one finds an abundance for the WIMP which is in the right ballpark to account for the dark matter by simple thermal production and freeze-out (see, e.g. [107]).

WIMPs can be motivated in many theoretical contexts, and they can be realised as different representations of the underlying symmetry group. One example of a dark matter candidate that does not need any new symmetries besides the Standard Model gauge group is the so-called minimal dark matter [108–110]. The idea is to add only a single new field in a large G_{SM}- representation to the Standard Model. If the representation is large enough, and the lightest component of the multiplet is electrically neutral, this neutral component is approximately stabel and could be the bulk of the dark matter[1]. The most promising multiplet is a fermionic $SU(2)_L$ quintuplet with zero hypercharge and without colour. This allows us again to drop the $SU(3)$ representation index in the notation of our fields in this and the following section. Looking through the list of models from the previous section, we find that the models (C1), (C2), as well as the three-fermion-model, contain the fermionic quintuplet.

For the model (C1), the scalar quadruplet develops an effective vacuum expectation value, allowing the fermionic quintuplet to decay quickly. As a result, in this model the neutral component of the quintuplet cannot be dark matter [112].

Next, consider model (C2), or more specific the version with $\phi_{(6,1/2)}$ and $\psi_R^{(5,0)}$. This model was considered as a minimal combination of radiative neutrino mass and dark matter in [89]. In this case, the quintuplet is stabilised at tree-level by an accidental Z_2

[1] If the representation is too large, the theory will become non-perturbative at a low energy scale. This disfavours any $SU(2)_L$ representations larger than 5 for fermions [111].

symmetry. However, as was first pointed out in [113], the scalar potential contains a term of the form $\phi^3_{(6,1/2)}H$. This term breaks the accidental Z_2 symmetry and leads to a one-loop decay of the fermionic quintuplet. As a result, the neutral component of the fermion multiplet is too short lived to be the bulk of dark matter. This result was also verified in [112]. In fact, in [112] the study of possible decays of the fermionic quintuplet was generalised to generic one-loop neutrino mass models. The authors of [112] find that all one-loop neutrino mass models including $\psi_R^{(5,0)}$ include some decay mechanism which is too fast for $\psi_R^{(5,0)}$ to be the bulk of dark matter.

Since our unique model with three new fermionic representations generates the dimension-5 Weinberg operator only at two loop, one could hope that this does not apply to the three-fermion model. In addition, in the limit where the Yukawa coupling y_1 between the Standard Model lepton and the new triplet is zero, the model has an accidental Z_2 symmetry

$$\begin{aligned} \psi^{(3,-1)} &\to -\psi^{(3,-1)} \quad \text{and} \quad \overline{\psi^{(3,-1)}} \to -\overline{\psi^{(3,-1)}}\,, \\ \psi^{(4,-1/2)} &\to -\psi^{(4,-1/2)} \quad \text{and} \quad \overline{\psi^{(4,-1/2)}} \to -\overline{\psi^{(4,-1/2)}}\,, \\ \psi^{(5,0)} &\to -\psi^{(5,0)}\,. \end{aligned} \tag{4.1.1}$$

Thus, the Yukawa coupling y_1 can be naturally small, stabilising the lightest of the neutral components of the new multiplets. To check whether this is sufficient to lead to a viable dark matter candidate, we roughly estimate the lifetime for the fermionic quintuplet in this model. To do so, we make the assumption that the dominant decay channel of the neutral component of the quintuplet is $\psi^{(5,0)} \to H\psi^{(4,-1/2)} \to HH\psi^{(3,-1)} \to HHH^\dagger \ell_L$, where the intermediate states are no resonances, but far off shell. The diagram for this decay is given in fig. (4.1). We will also assume that the vector masses m_4 and m_3 of

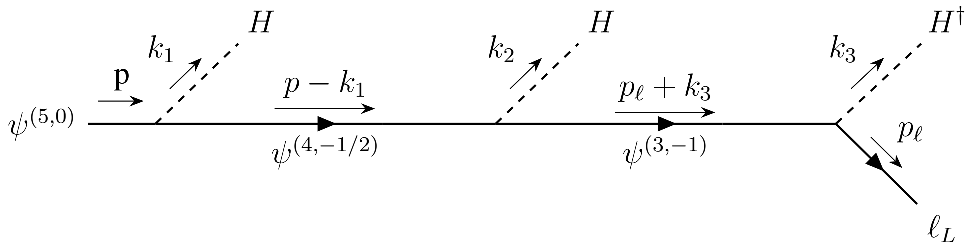


Figure 4.1: Tree-level four-body decay of the fermion quintuplet $\psi_R^{(5,0)}$ in the three-fermion model.

the two intermediate fermions $\psi^{(4,-1/2)}$ and $\psi^{(3,-1)}$ are much larger than the mass of the scalar quintuplet and that the Higgs bosons and the final state lepton can be considered as massless.

The matrix element for this diagram is given by

$$i\mathcal{M} = i\bar{u}(p_\ell)y_1 \frac{\not{p}_\ell + \not{k}_3 + m_3}{(p_\ell + k_3)^2 - m_3^2} y_2 \frac{\not{p} - \not{k}_1 + m_4}{(p - k_1)^2 - m_4^2} y_3 u(p)\ . \tag{4.1.2}$$

When taking the absolute value squared, we will average over initial spins and sum over final spins. We find as a final result

$$|\overline{\mathcal{M}}|^2 = \frac{|y_1|^2|y_2|^2|y_3|^2}{4[(p_\ell+k_3)^2-m_3^2]^2[(p-k_1)^2-m_4^2]^2}\times \\ \times \not{p}_\ell(\not{p}_\ell+\not{k}_3+m_3)(\not{p}-\not{k}_1+m_4)(\not{p}+M)(\not{p}-\not{k}_1+m_4)(\not{p}_\ell+\not{k}_3+m_3)\ , \tag{4.1.3}$$

where M is the mass of the quintuplet[2]. Taking into account that the masses of the intermediate fermions are assumed to be large compared to the mass of the quintuplet, we approximate the internal propagators by $1/m_3$ and $1/m_4$ respectively. Thus, we keep only the leading order terms in m_3 and m_4. After computing the trace over the remaining Dirac matrices with package X for Mathematica [114, 115], we find the result

$$|\overline{\mathcal{M}}|^2 = \frac{p\cdot p_\ell}{m_3^2 m_4^2}\ . \tag{4.1.4}$$

The partial decay width is then given by

$$\Gamma = \int \frac{|\overline{\mathcal{M}}|^2}{2M}\mathrm{d}\Phi_4\ , \tag{4.1.5}$$

where $\int \mathrm{d}\Phi_4$ is the integral over the 4-particle phase space. To write out the phase space, we make the following definitions

$$m_{23\ell}^2 = (p-k_1)^2 \qquad\qquad m_{3\ell}^2 = (p_\ell+k_3)^2 = (p-k_1-k_2)^2\ , \tag{4.1.6}$$

and use the Källén function

$$\lambda(a^2,b^2,c^2) = (a^2-b^2-c^2)-4b^2c^2\ . \tag{4.1.7}$$

Applying a sequential decay parametrisation, the differential phase space volume can be written as

$$\mathrm{d}\Phi_4 = \frac{1}{(16\pi^2)^3}\frac{\sqrt{\lambda(M^2,0,m_{23\ell}^2)}}{M^2}\frac{\sqrt{\lambda(m_{23\ell}^2,0,m_{3\ell}^2)}}{m_{23\ell}^2}\frac{\sqrt{\lambda(m_{3\ell}^2,0,0)}}{Mm_{3\ell}^2}\times \\ \times\, m_{23\ell}\mathrm{d}m_{23\ell}\mathrm{d}m_{3\ell}\mathrm{d}\Omega_{1,23\ell}\mathrm{d}\Omega_{2,3\ell}\mathrm{d}\Omega_{3,\ell}\ , \tag{4.1.8}$$

where $\mathrm{d}\Omega_{a,b}$ denotes the differential unit sphere element in the $a+b$ rest frame. Our result

[2] To be precise, the mass eigenstate made mainly of the quintuplet is the one lying in this mass range. The quintuplet does not have a definite mass. Since we consider the mixing to be small, we stick to the slightly incorrect, but shorter notation.

for Γ is thus

$$\begin{aligned}\Gamma =& \frac{1}{2(16\pi^2 M)^3} \int \frac{p\cdot p_\ell(M^2 - m_{23\ell}^2)(m_{23\ell}^2 - m_{3\ell}^2)m_{3\ell}}{m_3^2 m_4^2 m_{23\ell}} \times \\ & \times \mathrm{d}m_{23\ell}\mathrm{d}m_{3\ell}\mathrm{d}\Omega 1, 23\ell \mathrm{d}\Omega 2, 3\ell \mathrm{d}\Omega_{3,\ell} \ .\end{aligned} \tag{4.1.9}$$

Before computing the integral, we need to express the product $p \cdot p_\ell$ in terms of the variables used in the integration. Using momentum conservation, we find that in the quintuplet rest frame ($p = (M, 0, 0, 0)^T$)

$$(p - k_1) = \begin{pmatrix} \sqrt{[(M^2 - m_{23\ell}^2)/4M^2] + m_{23\ell}^2} \\ [(M^2 - m_{23\ell}^2)/2M]\sin\theta_1\cos\theta_1 \\ [(M^2 - m_{23\ell}^2)/2M]\sin\theta_1\sin\theta_1 \\ [(M^2 - m_{23\ell}^2)/2M]\cdot\cos\theta_1 \end{pmatrix} . \tag{4.1.10}$$

Since we averaged over initial spins, there is no preferred direction, and we can rotate our coordinate system such that $\theta_1 = 0$. Doing the same for the subsequent results, we find

$$(p_\ell + k_3) = \begin{pmatrix} \sqrt{[(m_{23\ell}^2 - m_{3\ell}^2)/4m_{23\ell}^2] + m_{3\ell}^2} \\ [(m_{23\ell}^2 - m_{3\ell}^2)/2m_{23\ell}]\sin\theta_2\cos\theta_2 \\ [(M^2 - m_{23\ell}^2)/2M]\sin\theta_2\sin\theta_2 \\ [(M^2 - m_{23\ell}^2)/2M]\cos\theta_2 \end{pmatrix} \tag{4.1.11}$$

in the rest frame of the intermediate $\psi^{(4,-1/2)}$ ($p - k_1 = (m_{23\ell}, 0, 0, 0)^T$), and

$$p_\ell = \frac{1}{2}m_{3\ell}\begin{pmatrix} 1 \\ \sin\theta_3\cos\theta_3 \\ \sin\theta_3\sin\theta_3 \\ \cos\theta_3 \end{pmatrix} \tag{4.1.12}$$

in the rest frame of the intermediate $\psi^{(3,-1)}$ ($p - k_1 - k_2 = (m_{3\ell}, 0, 0, 0)^T$).

We can now boost the result for p_ℓ into the rest frame of the quintuplet. After multiplying the result to p we can carry out the integral. We integrate over $m_{23\ell}$ from 0 to M, and $m_{3\ell}$ from 0 to $m_{23\ell}$. The result for this integral from Mathematica [114] is

$$\Gamma \approx 3.2 \times 10^{-6}|y_1|^2|y_2|^2|y_3|^2\frac{M^5}{m_3^2 m_4^2} \ . \tag{4.1.13}$$

This can be converted into an approximation of the lifetime τ by utilizing [3, ch. 1]

$$\hbar = 6.58 \times 10^{-25}\,\mathrm{GeV\,s} \tag{4.1.14}$$

and

$$\tau = \frac{\hbar}{\Gamma} \ . \tag{4.1.15}$$

The result is

$$\tau \lesssim 2.0 \times 10^{-19}\,\mathrm{s}\frac{1}{|y_1|^2}\frac{1}{|y_2|^2}\frac{1}{|y_3|^2}\frac{m_3^2 m_4^2}{M^4}\frac{\mathrm{GeV}}{M}\ . \tag{4.1.16}$$

In the low mass regime of the new fermions, a naive estimate of the neutrino mass in the three-fermion model can be made from the tree-level contribution, yielding [57]

$$m_\nu \simeq y_1^2 y_2^2 y_3^2 \frac{v^6}{M m_3^2 m_4^2}\ . \tag{4.1.17}$$

If we use this relation to replace the Yukawa couplings in the quintuplet lifetime (4.1.16), we find

$$\tau \lesssim \frac{2.0 \times 10^{-10}\,\mathrm{s}}{m_\nu(\mathrm{eV})}\left(\frac{v}{M}\right)^6\ . \tag{4.1.18}$$

We can estimate the neutrino mass by its lower limit from the atmospheric mass splitting [18],

$$m_\nu \gtrsim 0.05\,\mathrm{eV}\ , \tag{4.1.19}$$

leaving us with a prefactor of 4×10^{-7}. In order to have a large lifetime, the mass of the quintuplet would have to be far below the mass of Higgs vacuum expectation value. In fact, since the lifetime of the universe is approximately 10^{26} s, the mass of the quintuplet should be six orders of magnitude smaller than v for the lifetime to be at most of the order of the age of the universe. Our approximations, such as setting the Higgs mass to zero, are no longer valid in this regime, and other decay channels become dominant. Besides, the fermionic quintuplet does also contain singly and doubly charged components. If the quintuplet mass was six orders of magnitudes smaller than v, around $100\,\mathrm{keV}$, the charged components should have been observed by now. So the lifetime is much too small for the quintuplet to be dark matter, even without taking into account other potential decay mechanisms.

4.2 The three-fermion model at the LHC

In the previous section, we found that the new fermions in the three-fermion model are too short-lived to account for the bulk of the dark matter. But they could still be long-lived enough to lead to displaced vertex signatures at the Large Hadron Collier (LHC). This could facilitate the direct search for these particles.

If we want to observe the new fermions at the LHC via displaced vertex signatures, at least one of them should be in a mass range between $300\,\mathrm{GeV}$ and $3\,\mathrm{TeV}$, as it will be explained in the following. We assume that the quintuplet is the particle lying in this mass range, while the quadruplet and the triplet are heavier. For the neutral and singly charged component of the quintuplet, we can estimate the mass limit from the ATLAS search for chargino and neutralino pair production [116]. The lightest super partner in the final state of the chargino/neutralino search has the same collider phenomenology as a neutrino if its mass is close to zero. However, the branching ratio will be smaller than 100% as assumed in ref. [116]. In our case, the charged component of the quintuplet can also decay into a Higgs or Z-boson and a charged lepton, and the neutral component of the 5-plet can decay into a W-boson and a charged lepton, or a Z-boson and a neutrino. Consequently, we expect that in the limit where the mass of the quintuplet is much larger than the Higgs mass, the branching ratio will approach 1/6. Nonetheless, observation of other decay channels might partially compensate the small branching ratio, leading to a rough estimate for the mass limit on the neutral and singly charged components of the 5-plet of a few hundred GeV. In the following, we assume it to be $300\,\mathrm{GeV}$, which is a bit smaller than the limit found in ref. [116] for a vanishing mass of the lightest super partner. Hence, if the mass of the quintuplet was lower, it should have been observed already. If the quintuplet was much than roughly $3\,\mathrm{TeV}$, the production cross section would become too small.

Since the result in eq. (4.1.18) is close to what can be observed as a displaced vertex at the LHC [117], we also take into account other contributions, such as the two-body decay $\psi^{(5,0)} \to H\ell_L$. As for the neutrino mass, we will use the tree-level approximation of this process, which is described by the diagram (4.1), but with all but one external Higgs replaced by the vacuum expectation value.

Having only two external particles hugely simplifies the phase space calculation. As a rough approximation, we consider only one of the three possibilities to attach the physical Higgs boson, and we again use the relation (4.1.17). The matrix element is then similar to the one for the four-body decay, but with $k_1 = k_2 = 0$. In addition, a factor v is inserted at the first and second vertex, replacing the external Higgs boson. The phase space is reduced to the usual two-body phase space

$$\mathrm{d}\Phi_2 = \frac{1}{16\pi^2}\frac{\sqrt{\lambda(M^2, m_H^2, 0)}}{2M^2}\mathrm{d}\Omega\,, \tag{4.2.1}$$

where m_H is the mass of the Higgs boson. We still assume the final state lepton to be massless. The result for the decay width is

$$\Gamma = \frac{|y_1|^2|y_2|^2|y_3|^2}{32\pi}\frac{v^4(M^2 - m_H^2)^2}{M^3(M - m_3)^2(M - m_4)^2}\,, \tag{4.2.2}$$

leading to

$$c\tau = \frac{2.0 \times 10^{-3}\,\mathrm{cm}}{m_\nu(\mathrm{eV})}\frac{v^2M^2(M - m_3)^2(M - m_4)^2}{(M^2 - m_H^2)^2m_3^2m_4^2}\,, \tag{4.2.3}$$

by applying again eq. (4.1.17), and including the speed of light [3, ch. 1]

$$c = 299\,792\,458\,\mathrm{ms}^{-1}\,. \tag{4.2.4}$$

Estimating the neutrino mass again from its lower bound eq. (4.1.19), and applying the approximations $(M^2 - m_3^2) \approx -m_3^2$ and $(M^2 - m_4^2) \approx -m_4^2$, we can evaluate the lifetime of the quintuplet in the considered mass range. The largest lifetime that can be reached within this mass range is $c\tau \approx 0.04\,\mathrm{cm}$ for a mass of $M = 300\,\mathrm{GeV}$ of the quintuplet.

Indeed, the lifetime increases monotonously towards the lower bound of M. The result indicates that the lifetime is too short for it to be "long-lived" at the LHC, since the minimal proper decay length for displaced vertices is of the order of $0.1\,\mathrm{cm}$ [117].

So far, we have only considered the electrically neutral or singly charged component of the quintuplet. A slightly different picture could emerge for the doubly charged component because it cannot mix with the Standard Model lepton doublet. Neglecting the charged Higgs, which is absent in unitary gauge, a W boson could be emitted in the decay, converting the doubly charged into a singly charged component. The singly charged component is then mixed with the Standard Model lepton.

In the tree-level approximation, there are three different ways the W boson can be emitted:

$$\begin{aligned}
&\psi_{(5,0)} \to \psi_{(5,0)}W \to \psi_{(4,-1/2)}W \to \psi_{(3,-1)}W \to \ell_L W \\
&\psi_{(5,0)} \to \psi_{(4,-1/2)} \to \psi_{(4,-1/2)}W \to \psi_{(3,-1)}W \to \ell_L W \\
&\psi_{(5,0)} \to \psi_{(4,-1/2)} \to \psi_{(3,-1)} \to \psi_{(3,-1)}W \to \ell_L W
\end{aligned} \tag{4.2.5}$$

Here, we use the same approach to describe the mass mixing as in the two-body decay. Similar to the previous case, we deal with a two particle phase space, with one massless lepton and a massive vector boson W of mass m_W. The mixing of the multiplets via Yukawa interactions is described by explicit insertions of the Higgs vacuum expectation value and the Yukawa coupling, while the momentum remains unchanged. In contrast to the previous case we take into account all three possibilities to emit the W boson. The relative strength of the couplings in the different diagrams due to the different contraction of $SU(2)$ indices is accounted for using Clebsch-Gordan coefficients for the contraction of the $SU(2)$ indices. The absolute square of the matrix element then contains also the interference terms of the three diagrams, and the final result for the decay width is

$$\begin{aligned}
\Gamma_{5_{--}} = &\frac{y_1^2 y_2^2 y_3^2 g_W^2 v^6 (M^2 - m_W^2)^2 (M^2 + 2m_W^2)}{1920\pi m_W^2 M^5 m_3^2 m_4^2 (M - m_3)^2 (M - m_4)^2} \times \\
&\times \left[c_1 M^4 + c_2 m_3^2 m_4^2 - 2M m_3 m_4 (c_3 m_3 - c_4 m_4) - 2M^3 (c_5 m_3 - c_6 m_4) \right. \\
&\left. + M^2 (c_7 m_3^2 + c_8 m_3 m_4 + c_9 m_4^2)\right],
\end{aligned} \tag{4.2.6}$$

where the coefficients are given by

$$\begin{aligned}
&c_1 = 13 + 4\sqrt{10} \quad && c_2 = 5 \quad && c_3 = 5 + 2\sqrt{10} \\
&c_4 = 5\sqrt{3} - 5 \quad && c_5 = 13 + 4\sqrt{10} \quad && c_6 = (5 + 2\sqrt{10})(\sqrt{3} - 1) \\
&c_7 = 13 + 4\sqrt{10} \quad && c_8 = (4 - 2\sqrt{3})(5 + 2\sqrt{10}) \quad && c_9 = 20 - 10\sqrt{3}\,.
\end{aligned} \tag{4.2.7}$$

These factors arise from the Clebsch-Gordan coefficients in the coupling of the $SU(2)$ multiplets.

In the limit where the masses m_3 and m_4 of the triplet and quadruplet are much larger than the mass M of the quintuplet, we can approximate this by its leading term in $\frac{M^2}{m_{3/4}^2}$ as

$$\Gamma_{5_{--}} \approx \frac{y_1^2 y_2^2 y_3^2 g_W^2 v^6 (M^2 - m_W^2)^2 (M^2 + 2m_W^2)}{384\pi m_W^2 M^5 m_3^2 m_4^2} \,. \tag{4.2.8}$$

To estimate the resulting lifetime, we utilize again eq. (4.1.17) to replace the Yukawa couplings. The weak coupling constant squared can be expressed as

$$g_W^2 = \frac{4\pi\alpha}{\sin^2\theta_W} \approx 0.4\,, \tag{4.2.9}$$

where [3, ch. 10]

$$\alpha = \frac{1}{127.955 \pm 0.01} \tag{4.2.10}$$

is the finestructure constant, and [3, ch. 1]

$$\sin^2\theta_W = 0.2312 \pm 0.0004 \tag{4.2.11}$$

is the sine squared of the Weinberg angle connecting the electric and weak couplings. The mass of the W boson is [3, ch. 1]

$$m_W = (80.38 \pm 0.01)\,\mathrm{GeV}\,. \tag{4.2.12}$$

Assuming the intermediate triplet and quadruplet to be slightly heavier than the quintuplet, we find the results shown in fig. (4.2). We clearly see that the lifetime of the quintuplet decreases as its mass increases. This decrease of the lifetime is faster if the triplet and quadruplet are lighter. Even for a mass of the quintuplet as low as $300\,\mathrm{GeV}$, the lifetime does not exceed $1\,\mathrm{mm}$.

Thus, the additional suppression factor for the doubly charged component is not large enough to make it long-lived on LHC scales, unless it is very light. In this case, assuming similar exclusion limits as for the other components of the quintuplet, it should be detectable without the specific search for displaced vertices.

4.3 Neutrino mass and B anomalies

In our scan, we have found a number of models containing so-called scalar leptoquarks. These scalar fields can couple leptons to quarks via Yukawa interactions. In particular, the scalar $\phi_{(3,2,1/6)}$ appears in the models (cA2), (cA4), (cA6), (cA8), (cB5)-(cB7), (cB9)-(cB12), (cB14) and (cC2). Moreover, the scalar leptoquark $\phi_{(3,1,-1/3)}$ is contained in the models (cA1), (cA2), (cA5), (cB8) and (cC1), while its triplet version $\phi_{(3,3,-1/3)}$ is featured in (cA3), (cA4), (cA7), (cB13) and (cC3). Besides their application in neutrino physics, these scalar leptoquarks have all been discussed as a possible solution to the anomalies appearing in B-physics, leading to an interplay of these two areas of particle physics. This has also been discussed in ref. [93, 94].

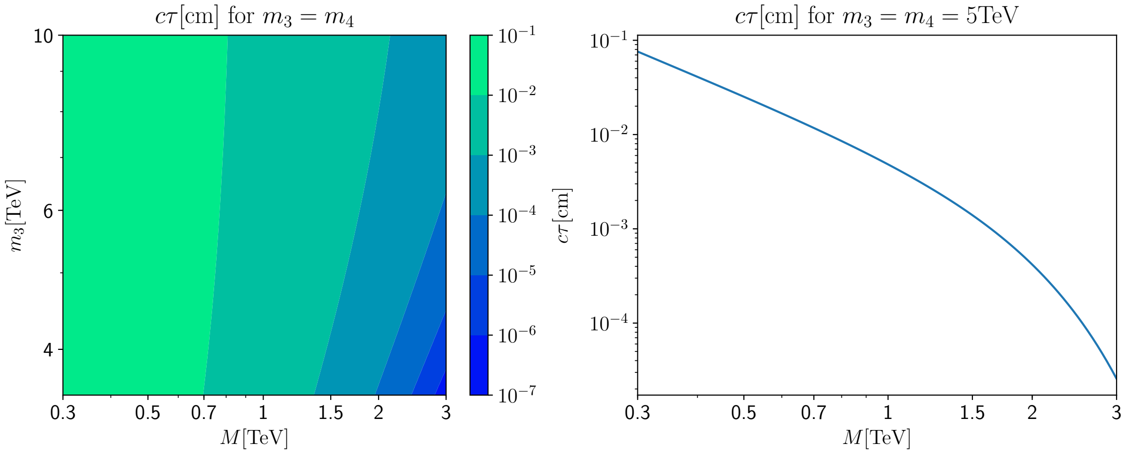


Figure 4.2: Lifetime τ of the doubly charged component of $\psi_{(5,0)}$ times c in cm as a function of the mass of the quintuplet in GeV in the three-fermion model. The masses m_3 and m_4 of the triplet and quadruplet have been chosen equal. In the left panel, m_3 is allowed to vary between $3.5\,\mathrm{TeV}$ and $10\,\mathrm{TeV}$. In the right panel, it is fixed to $5\,\mathrm{TeV}$.

In B-physics, among other quantities, the branching ratios of the decays of the B-meson $B^+ = u\bar{b}$ are studied. Semi-leptonic decays are of special interest, since they proceed via flavour changing currents. This makes them especially sensitive to new physics. In fact, deviations from the Standard Model prediction were found in two of the measured parameters. The first one is

$$R_K = \frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B \to K^+e^+e^-)} \ , \tag{4.3.1}$$

the ratio of the branching fractions of the B meson decay into a Kaon $K = u\bar{s}$ and a muon-antimuon-pair, or electron-positron pair. In the Standard Model, lepton universality of the weak interactions predicts a value of $R_K = 1$. The measurement of R_K at the LHCb experiment gave the value $R_K = 0.745^{+0.090}_{-0.074}$ (stat.) $\pm\, 0.036$ (syst.) [4].

One solution for this deviation is adding the scalar leptoquark $\phi_{(3,2,1/6)}$ to the Standard Model [118–120]. Allowing the new particle to couple differently to each of the three generations of charged leptons, the effective operators generated by integrating out the new scalar can push the Standard Model value for R_K towards the measured value. But this scalar can not only improve the value of this one ratio. In ref. [98], and in ref. [121] with the aid of a right handed neutrino, $\phi_{(3,2,1/6)}$ is used to explain the anomaly in the parameter

$$R_{D^{(*)}} = \frac{\mathcal{B}(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell)}\Big|_{\ell=\mu,e} \ . \tag{4.3.2}$$

$R_{D^{(*)}}$ has also been found to deviate from its Standard Model value [5, 6, 122–124]. For example, the Belle collaboration reports $R_D = 0.375 \pm 0.064$ (stat.) ± 0.026 (syst.) and $R_{D^*} = 0.293 \pm 0.038$ (stat.) ± 0.015 (syst.) [6], which is slightly lower than the results obtained by the BaBar collaboration [5], and deviates from the Standard model values $R_D^{SM} = 0.297 \pm 0.0017$ and $R_{D^*}^{SM} = 0.252 \pm 0.003$ [5] by 1.4σ and 1.8σ.

Another potential solution to the anomalies is the scalar leptoquark $\phi_{(3,1,-1/3)}$. This leptoquark has also been considered as an explanation for the R_K anomaly [118]. In addition, there was a discussion whether this leptoquark can also ease the tension in $R_{D^{(*)}}$, without getting into conflict with other observables [120, 125]. It seems that $\phi_{(3,1,-1/3)}$ can at least reduce the tension with the Standard Model prediction for both, $R_{D^{(*)}}$ and R_K, to 2σ [100].

Combined with the new fermion $\psi^{(3,2,-5/6)}$ as in model (cB8), the scalar leptoquark $\phi_{(3,1,-1/3)}$ might even improve gauge unification [98].

In ref. [118], the triplet leptoquark $\phi_{(3,3,-1/3)}$ is also considered as a solution to the R_K-anomaly.

Note that the leptoquark $\phi_{(3,1,-1/3)}$ allows the Yukawa couplings

$$\begin{aligned}\mathcal{L}_Y \supset & y_1 \overline{\ell_L^c} \phi^*_{(3,1,-1/3)} Q_L + y_2 \overline{Q_L^c} \phi_{(3,1,-1/3)} Q_L \\ & + y_3 \overline{e_R^c} \phi^*_{(3,1,-1/3)} u_R + y_4 \overline{u_R^c} \phi_{(3,1,-1/3)} d_R \ .\end{aligned} \tag{4.3.3}$$

This combination of Yukawa couplings implies not only Lepton-, but also Baryon-number violation and leads to proton decay via the four-point interactions $uu \to \overline{d}e^+$, and $ud \to \overline{u}e^+$. The resulting antiquark and the remaining spectator quark from the proton then form a neutral pion, resulting in the decay $p \to \pi^0 e^+$. Since there are strong experimental bounds on proton decay, the leptoquark must be quite heavy, about 10^{11} GeV [126]. This lower mass bound is dependent on the exact couplings present in the model and can even become as large as 10^{15} GeV for couplings of order one. A similar bound should apply to the triplet leptoquark $\phi_{(3,3,-1/3)}$, since it leads to similar interactions.

This discussion about leptoquarks and B-anomalies is still ongoing, even after latest results from experiments. Some tensions with the Standard Model results are shrinking. In particular, the values of R_D and R_{D^*} are getting closer to their Standard Model values [127]. Nonetheless, other observables, especially R_K and

$$R_{K^*} = \frac{\mathcal{B}(B^0 \to K^{*0} \mu^+ \mu^-)}{\mathcal{B}(B^0 \to K^{*0} e^+ e^-)} \tag{4.3.4}$$

still seem to call for an explanation [128]. Moreover, their connection to neutrino masses is still investigated [129]. So B-anomalies remain an important door to beyond the Standard Model physics.

5 Neutrino mass in the context of unification

In our scan, we have studied radiative neutrino mass models from a low-energy perspective. In our definition of minimality, we have restricted the number of new multiplets of the Standard Model symmetry group. The whole approach has been bottom-up: We have started from the low-energy theory, the Standard Model, and have studied where we could go from there, without specific expectations on the theory at high scales. It is interesting to investigate how the picture changes in a top-down perspective. In such an approach, some hypothesis on the UV completion of the Standard Model will be made. Then the neutrino mass production will be studied in the context of this high-energy theory.

One potential class of high-energy theories are grand unified theories (GUTs). In these models, the Standard Model gauge group is embedded into a larger group with equal or larger rank (see section 2.1.1). The Standard Model gauge group has rank 4 (two from $SU(3)$, one from $SU(2)$, and one from $U(1)$), so the smallest group (which is not itself a tensor product of different groups) which allows for an embedding is $SU(5)$. The first $SU(5)$ GUT model was proposed by Georgi and Glashow [130]. While their minimal $SU(5)$ model has been ruled out [131], certain models with a larger field content ([132–136], for example) as well as GUTs based on larger groups such as SO(10), often including supersymmetry, [137] are still viable.

Since the minimal SO(10) models automatically include right-handed neutrinos [48], we focus on $SU(5)$. Our goal in this section is to change the perspective on our search for neutrino mass mechanisms. Having explored which combinations of representations of the Standard Model gauge group allow for neutrino masses, we use this list of models to find the corresponding list of $SU(5)$ models which predict non-zero neutrino masses. In order to do so, we first introduce the embedding of the Standard Model field content into $SU(5)$. We find that embedding the Standard Model gauge group into $SU(5)$ changes our counting of multiplets and results in a different list of radiative neutrino mass models. A certain class of models will be examined in more detail. In particular, we will check whether we find successful unification in agreement with neutrino- and charged fermion masses.

5.1 The Standard Model in SU(5)

Before we discuss beyond the Standard Model fields, let us have a look how the Standard Model fits into representations of $SU(5)$ in the minimal model of Georgi and Glashow. It can be demonstrated [46, 48], that the fundamental representation 5 of $SU(5)$ can be decomposed into $SU(3) \otimes SU(2) \otimes U(1)$ as

$$5 = (3, 1, -2) \oplus (1, 2, 3) \ . \tag{5.1.1}$$

Rescaling the $U(1)$ charges accordingly, one can fit the Standard Model fermions

$$\Psi_{\overline{5}} = \begin{pmatrix} d_R^c \\ \ell_L \end{pmatrix} \tag{5.1.2}$$

into a left-handed Weyl fermion transforming as the $\overline{5}$ of $SU(5)$. From here, one can use either tensor products [48], or Young tableaux [46] to find the decomposition of larger $SU(5)$ multiplets into those of the Standard Model. We find that the 10 of $SU(5)$ can be decomposed as

$$10 = (3, 2, 1) \oplus (\overline{3}, 1, -4) \oplus (1, 1, 6) \ . \tag{5.1.3}$$

Thus, a left-handed Weyl fermion transforming as a 10 under $SU(5)$ can be identified with Standard model fields via

$$\Psi_{10} = \begin{pmatrix} u_R^c & -Q_L^T \\ Q_L & e_R^c \end{pmatrix} \ , \tag{5.1.4}$$

where we denote the representation 10 as an antisymmetric tensor. Thus, the complete fermion content of the Standard Model is contained in three copies of left-handed, fermionic $\overline{5}$'s and 10's.

The gauge bosons of the Standard Model can be embedded into the adjoint representation of $SU(5)$. Indeed, decomposing the adjoint representation 24, one finds

$$24 = (1, 1, 0) \oplus (1, 3, 0) \oplus (8, 1, 0) \oplus (3, 2, -5) \oplus (\overline{3}, 2, 5) \ . \tag{5.1.5}$$

In addition to the Standard Model gauge bosons, there are two new fields. These new gauge bosons must be very heavy since they mediate proton decay [126].

In order to give a large mass to the new gauge bosons, while leaving those of the Standard model massless, we need to break $SU(5)$ down to the Standard Model group spontaneously. One scalar field representation enabling this is the 24 [48]. If this scalar develops a vacuum expectation value proportional to its Standard Model singlet component, $SU(5)$ is spontaneously broken, while the group of the Standard Model remains unbroken. The breaking of $SU(5)$ leads to a mass for the new gauge bosons of the order of the unification scale. In addition to the $SU(5)$-breaking scalar, we need a scalar multiplet containing the Standard Model Higgs boson. The smallest representation containing a Higgs doublet is the 5.

All together, the content of the minimal Georgi-Glashow model is given by [130]:

- three copies of a left-handed Weyl fermion transforming as a $\overline{5}$, and three copies of a left-handed Weyl fermion transforming as a 10,

- a gauge boson transforming as the adjoint representation, 24,

- a scalar field transforming as a 24, and a second one transforming as a 5.

Note that this minimal model leads to wrong mass relations between the charged fermions. In addition, unification cannot be achieved with the minimal model, since the gauge couplings of the three Standard Model forces do not run into one common point. Moreover, just as the Standard Model, the minimal Georgi-Glashow model contains massless neutrinos. These three points are considered as the biggest flaws of the minimal model [131, 135, 136, 138].

In order to cure the charged fermion mass relations, one can either include higher-dimensional operators [139], or extend the scalar content of the theory by another scalar multiplet transforming as the 45 of $SU(5)$ [140]. This leads to a two-Higgs-Doublet model in the low energy theory. Possible solutions to the other shortcomings of the minimal model will be discussed in the next two sections.

5.2 Neutrino mass models in SU(5)

Next, we will try to give an overview of neutrino masses in $SU(5)$ GUT. In order to do so, we first look for the $SU(5)$ representations containing the new fields for our radiative neutrino mass models of type A and cA from tables (3.1) and (3.3). We restrict ourselves to representations up to the representation 75 of $SU(5)$. A list of decompositions for all $SU(5)$ representations under consideration can be found in appendix A.2. Then we check whether all necessary interactions for neutrino masses are allowed. The list of viable models is given in table (5.1). Similarly to our scan result, they are ordered by size and number of representations appearing.

Note, however, that we first consider all models without the scalar 45-plet, and then the ones including it, because for a viable renormalisable $SU(5)$ model, the 45-plet is part of the minimal scalar content. Note also that some of the models displayed are reducible in the sense that neutrino mass could already be included by a smaller number of scalar fields. We decided to include these models here in order to show the embedding of all models of type A and cA. Compared to the list of models we have found from the low-energy perspective, the list of $SU(5)$ models is rather short. In total, we find three different classes of models with similar mechanisms for neutrino mass generation.

The first class of models contains (S1) and (S3)-(S6). The model (S1) is the minimal model that allows for a radiative generation of neutrino mass. The model is contained in (S3)-(S5), and in (S6). In that sense, the models (S3)-(S6) are all variants of this model with different additions.

The models (S3)-(S5) are extensions of (S1) containing additional radiative neutrino mass models at low scales. The additional low-energy model in (S3) compared to (S1) is (A2), while the additional model in (S4) is the Zee-Babu model. (S5) contains (A5) as an additional contribution.

As mentioned earlier, the models (S1)-(S5) all still contain the wrong mass relations for the charged fermions at the renormalisable level. They need higher-dimensional operators to correct these relations, which would also include the Weinberg-operator. There is only

Model	Scalar content	Neutrino mass model
S1	ϕ_5, ϕ_{24}, and ϕ_{10}	cA2,(cA1), (A4)
S2	ϕ_5, ϕ_{24}, and ϕ_{15}	See-saw type II, (cA5)
S3	ϕ_5, ϕ_{24}, ϕ_{10}, and ϕ_{40}	cA2,(cA1), (A4),(A2)
S4	ϕ_5, ϕ_{24}, ϕ_{10}, and ϕ_{50}	cA2, (A1),(cA1), (A4)
S5	ϕ_5, ϕ_{24}, ϕ_{10}, ϕ^1_{70}, and ϕ^2_{70}	cA2,(A5),(cA1), (A4)
S6	ϕ_5, ϕ_{24}, ϕ_{45}, and ϕ_{10}	A0, cA2, cA4, (A4), (cA1), (cA3), (cA6)
S7	ϕ_5, ϕ_{24}, ϕ_{45}, and ϕ_{15}	See-saw type II, (cA5), (cA7)
S8	ϕ_5, ϕ_{24}, ϕ_{45}, ϕ_{40}, and ϕ_{50}	A3

Table 5.1: Neutrino mass in $SU(5)$ GUT, extending the scalar sector only. Mass models in brackets are either ruled out already (e.g. (A2)), or have a higher loop order, or effective operator dimension (e.g. (cA1)).

one model in which this is not obvious: (S5). Since it contains two copies of scalar 70-plets, it also contains two additional copies of the Higgs doublet. But these two Higgs doublets reside in a 70-plet of $SU(5)$, while the scalar multiplets present in the charged fermion mass term must transform as $\overline{5} \otimes 10 = 5 \oplus 45$ or $10 \otimes 10 = 5 + 45 + 50$ [47]. Thus, the Higgs doublet from a scalar 70-plet cannot solve the charged fermion mass ratio problem. Hence, higher-dimensional operators are also needed for (S5).

The model (S6) extends the model (S1) by a scalar 45-plet. This new field does not only add a realisation of the Zee-model (A0) at low energies, but also a potential solution to the charged fermion mass problem without higher-dimensional operators. It was discussed in ref. [132], focussing on neutrino mass generation via the Zee-model (A0). The realisation of the models (cA2) and (cA4) in this theory was examined in ref. [92]. This model is the minimal model that addresses the charged fermion masses, as well as the neutrino masses. There are further higher order contributions to the neutrino masses coming from different two-loop mechanisms of which (cA3) and (cA6) are not present in the minimal model (S1).

The second group of models contains (S2) and (S7). Model (S2) has been discussed in ref. [141] as a minimal realistic $SU(5)$ model including higher-dimensional operators. The dimension-5 Weinberg operator can be generated at tree-level by the scalar triplet $\Delta \sim (1, 3, 1)$ in the scalar 15-plet. This enables a realisation of type-II see-saw. The radiative neutrino mass mechanisms, which are also realised, are most probably only subdominant in this case. Thus this model, as well as its variant (S7), is not a radiative neutrino mass model.

The model (S7) is the extension of (S2) by the scalar 45-plet. The main difference between the models is that in model (S2), higher-dimensional operators are needed to get the correct relation of the charged fermion masses, while in model (S7) this can be achieved by the extra scalar 45-plet. Model (S7) also has an additional 2-loop contribution compared to model (S2).

Model (S8) has been discussed recently in ref. [135]. It is the only model of which there is no version without the scalar 45-plet. It is also the only model which realises neutrino masses only at 2-loop level, via the mechanism of model (A3). Since the presence of the scalar 45-plet is a necessary ingredient for neutrino masses in this case, the charged fermion and neutrino masses are connected in this model.

When adding new fermions, the number of possible realisations of neutrino mass in $SU(5)$ becomes larger. However, all models considered here also require additions to the minimal scalar content. In some cases, this alternation of the scalar content by itself already leads to neutrino masses. The resulting neutrino mass models including also the new fermions are then no longer minimal. These cases will be omitted in the following list. The different realisations of neutrino mass in $SU(5)$ including new fermionic multiplets are given in table (5.2). They are again ordered by the number of representations, counting first the scalar and then the fermionic content, and by the dimension of the representation. Again, we first list models without the scalar 45-plet. Note that all fermionic multiplets are left-handed Weyl fermions. Contributions of higher loop order or operator dimension are again written in brackets.

It should be mentioned that as in the purely scalar cases, we take into account only representations of $SU(5)$ smaller than the 75. In contrast to the scalar models, this excludes some of the models containing fermions. Consider in particular the Majorana fermion $\psi_R^{(1,5,0)}$ which is present in model (C1), one case of (C2), and the three-fermion model. Its first appearance in $SU(5)$ in the standard embedding is within a 200-plet. As a result, the embedding of these models will not be included in table (5.2).

Model	Scalar content	Fermionic content	Neutrino mass model
F1	ϕ_5, and ϕ_{24}	$\psi_{\overline{5}}$, ψ_{10}, and ψ_{24}	See-saw type I+III, (cB8), (cC1)
F2	ϕ_5, ϕ_{24}, and ϕ_{35}	$\psi_{\overline{5}}$, ψ_{10}, ψ_{15}, and $\psi_{\overline{15}}$	B5
F3	ϕ_5, ϕ_{24}, and ϕ_{45}	$\psi_{\overline{5}}$, ψ_{10}, and ψ_{24}	See-saw type I+III, (cB8), (cB13), (cC1)
F4	ϕ_5, ϕ_{24}, and ϕ_{45}	$\psi_{\overline{5}}$, ψ_{10}, and ψ_{75}	See-saw type I, (cB8), (cB13), (cC1), (cC3)
F5	ϕ_5, ϕ_{24}, ϕ_{45}, and ϕ_{40}	$\psi_{\overline{5}}$, ψ_{10}, ψ_5, and $\psi_{\overline{5}}$	B3
F6	ϕ_5, ϕ_{24}, ϕ_{45}, and ϕ_{40}	$\psi_{\overline{5}}$, ψ_{10}, ψ_{10}, and $\psi_{\overline{10}}$	B2
F7	ϕ_5, ϕ_{24}, ϕ_{45}, and ϕ_{40}	$\psi_{\overline{5}}$, ψ_{10}, ψ_{15}, and $\psi_{\overline{15}}$	B4
F8	ϕ_5, ϕ_{24}, ϕ_{45}, and ϕ_{40}	$\psi_{\overline{5}}$, ψ_{10}, ψ_{45}, and $\psi_{\overline{45}}$	B3

Table 5.2: Neutrino masses in $SU(5)$ GUT extending both, the scalar and the fermionic sector. Contributions in brackets are of higher loop order or operator dimension. Models in which a change of the scalar sector only leads to neutrino masses are omitted.

The models with new fermions and scalars can be divided into three groups. The first one includes the models (F1), (F3), and (F4). The models (F1), (F3), and (F4) all add a right-handed neutrino $\nu_R \sim (1, 1, 0)$, allowing for type I see-saw. However, there are some differences between these models: The biggest difference between (F1) and (F3) or (F4) is that (F3) and (F4) also contain the scalar 45-plet, thus solving the problem of the charged fermion mass relations at the renormalisable level. (F4), in contrast to (F1) and (F3), does not include the uncharged fermionic triplet $\Sigma \sim (1, 3, 0)$, which is responsible for type III see-saw. Instead, the model (F4) has more higher order contributions. Nonetheless, all these models work similarly and, more importantly, generate the Weinberg operator at tree-level. Hence they are not radiative neutrino mass models.

The second group of models is made up of a single model, (F2). The model (F2) is, apart from (F1), the only other model not including the scalar 45-plet. In addition, the representation 35 of $SU(5)$, though not very large, is rather exotic considering its decomposition into representations of the Standard Model group: $35 = (1, 4, -3/2) \oplus (\overline{3}, 3, -2/3) \oplus (6, 2, 1/6) \oplus (\overline{10}, 1, 1)$. Thus higher-dimensional operators, as well as fields in very exotic representations, are required to make this model work.

The models (F5)-(F8) form the last group. They are all very similar. The minimal scalar content is enlarged by a scalar 45-plet and a scalar 40-plet. The 45-plet does not only solve the charged fermion mass problem, but the second Higgs doublet present in the 45-plet also enables neutrino mass generation. In addition, a new vector-like fermion is added. These models will be the focus of the next section. We will test whether they can also improve other issues of the minimal $SU(5)$ model besides the absence of neutrino masses.

5.3 Unification and neutrino mass

In the previous section, we have found that many radiative neutrino mass models already include the scalar 45-plet. Hence, they do not only lead to neutrino masses, but also give an answer to the charged fermion mass relation problem at the renormalisable level. In this section, we want to examine whether some of these models can also allow for unification in $SU(5)$.

Indeed, in ref. [132] it is argued that the model (S6) can lead to successful unification of the gauge couplings, while satisfying current bounds from proton decay and collider searches. However, the parameter space for successful unification is very close to current bounds. Ref. [135] states that the model (S8) can also lead to unification, leaving the proton sufficiently stable.

Here, we will focus on the models (F5)-(F8), and test whether they can lead to successful unification. In order to do so, we explain how one can test unification before applying this test to the models (F5)-(F8). A special emphasis is on (F6), for which we estimate the neutrino mass to further restrict the parameter space in our search for unification.

5.3.1 Unification conditions

In quantum field theory, the renormalisation procedure inserts a mass scale dependence into the theory. This mass scale dependence is then absorbed into the coupling constants g, y, λ and μ, and masses m. As a result, the renormalised coupling constants g_R now depend on the energy scale, they are said to run with energy. This running is described by the β-functions [42]

$$\beta(g_R) = \mu \frac{d}{d\mu} g_R \,. \tag{5.3.1}$$

For unification, it is conventional to use the β- functions of $\alpha = \frac{g^2}{4\pi}$:

$$\beta(\alpha_R) = \frac{g_R}{2\pi} \beta(g_R) \,. \tag{5.3.2}$$

To leading order in powers of α_R, the β-function is conventionally written as

$$\beta(\alpha_R) = -\frac{\alpha_R^2}{2\pi} \beta_0 \,, \tag{5.3.3}$$

where β_0 is a constant independent of α_R. This corresponds to the one-loop contribution in a perturbative expansion. In the following, we will always talk about the renormalised coupling, so we will drop the subscript R unless it is needed for clarification. The solution to this equation is

$$\alpha(\mu) = \frac{2\pi}{\beta_0} \frac{1}{\ln(\frac{\mu}{\Lambda})} \,, \tag{5.3.4}$$

where Λ is an integration constant fixed by the boundary conditions. If the coupling is measured at one scale, for example at the Z-boson mass scale M_Z, the coupling at any other scale can be calculated via

$$\alpha^{-1}(\mu) = \alpha^{-1}(M_Z) + \frac{\beta_0}{2\pi} \ln\left(\frac{\mu}{M_Z}\right) \,. \tag{5.3.5}$$

The relevant couplings for $SU(5)$ unification are

$$\alpha_1 = \frac{5\alpha}{3\cos^2\theta_W} \,, \qquad \alpha_2 = \frac{\alpha}{\sin^2\theta_W} \,, \qquad \alpha_3 = \alpha_S \,, \tag{5.3.6}$$

where α_S is the strong coupling. The factor of $5/3$ in α_1 is due to the relative normalisation of the generators [142]. If these three couplings run towards a common value at some large mass scale M_{GUT}, the equations

$$\alpha_{GUT}^{-1}(M_{GUT}) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln\left(\frac{M_{GUT}}{M_Z}\right) \,, \; i = 1,2,3 \tag{5.3.7}$$

must be satisfied. Here, α_{GUT} is the $SU(5)$ coupling, and the effective one-loop running, including new heavy particles, is

$$b_i = -\beta_0(\alpha_i) = b_i^{SM} + \Delta b_{i,k} r_k \ , \tag{5.3.8}$$

$$r_k = \frac{\ln(M_{GUT}/M_k)}{\ln(M_{GUT}/M_Z)} \ . \tag{5.3.9}$$

k runs over all new particles with masses below the unification scale. The factor r_k takes into account that heavy particles influence the running only at energy scales above their mass [143]. We now define the relative runnings

$$B_{ij} = b_i - b_j \ . \tag{5.3.10}$$

By setting eq. (5.3.7) for α_1 equal to eq. (5.3.7) for α_2, one finds

$$\ln\left(\frac{M_{GUT}}{M_Z}\right) = \frac{16\pi}{5\alpha(M_Z)} \frac{(3/8 - \sin^2\theta_W(M_Z))}{B_{12}} \ . \tag{5.3.11}$$

Next we set $\alpha_3(M_{GUT}) = \alpha_2(M_{GUT})$ and use the equation above to replace the logarithm. The result is [143]

$$\frac{B_{23}}{B_{12}} = \frac{5}{8}\left(\frac{\sin^2\theta_W(M_Z) - \alpha(M_Z)\alpha_S^{-1}(M_Z)}{3/8 - \sin^2\theta_W(M_Z)}\right) \ . \tag{5.3.12}$$

Inserting the Standard Model values for the couplings at the Z-boson mass, as given in the 2018 review of the particle data group, [3, ch. 1], we find that the unification condition can be expressed as

$$\frac{B_{23}}{B_{12}} = 0.719 \pm 0.005 \ . \tag{5.3.13}$$

The value derived from the Standard Model β-functions

$$b_1^{SM} = \frac{41}{10}\,, \qquad b_2^{SM} = -\frac{19}{6}\,, \qquad b_3^{SM} = -7\,, \tag{5.3.14}$$

is approximately $B_{23}^{SM}/B_{12}^{SM} \approx 0.53$. This value has to be increased by the new fields in our model to achieve gauge coupling unification.

Still, we have to be careful that the scale of gauge coupling unification does not get too low: The unification scale is connected to the mass of the heavy $SU(5)$ gauge bosons $X \sim (3, 2, -5/6)$ and $Y \sim (\overline{3}, 2, 5/6)$ which mediate proton decay. If M_{GUT} becomes smaller, so does the proton lifetime. As a result, a rough estimate of the proton lifetime [126]

$$\tau_P \approx \frac{M_{GUT}^4}{\alpha_{GUT}^2 m_p^5} \tag{5.3.15}$$

translates the experimental lower bound on the proton lifetime [144]

$$\tau_P \geq 1.6 \times 10^{34}\,\mathrm{y} \tag{5.3.16}$$

into a lower bound of the unification scale

$$M_{GUT} \geq (4-6) \times 10^{15}\,\mathrm{GeV}\,. \tag{5.3.17}$$

Here, we use the proton mass [3, ch. 1]

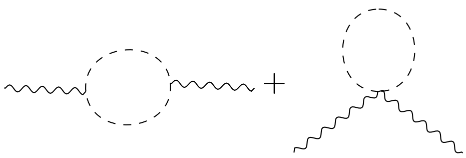
$$m_p = 0.938\,\mathrm{GeV}\,, \tag{5.3.18}$$

and we estimate the coupling $\alpha_{GUT}^{-1} = 20 - 40$. Taking into account eq. (5.3.11), this yields an upper bound for B_{12} [143]:

$$B_{12} \leq 5.8 - 5.9\,. \tag{5.3.19}$$

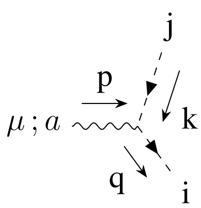
In the following, we will use the more conservative bound of $B_{12} \leq 5.8$. For the Standard Model, we find the value $B_{12} \approx 7.3 > 5.8$. Eq. (5.3.19) and eq. (5.3.13) are the conditions we need to fulfil for a successful unification in agreement with current bounds on the proton lifetime. Comparing to the Standard Model results, we see that we need to decrease B_{12}, while at the same time increasing, or at least less decreasing, B_{23}. In addition, we demand that any new field mediating proton decay should have a mass of the order of M_{GUT}, and that the couplings involved in the decay are not too large, so the gauge boson contributions to proton decay remain the dominant contributions. This way, we should not have to worry about proton decay as long as condition (5.3.19) is satisfied.

The next step is to test whether the new fields in the models (F5)-(F8) can help to satisfy the unification conditions. At one-loop, the new scalars contribute to the running of the couplings only via the vacuum polarisation diagrams

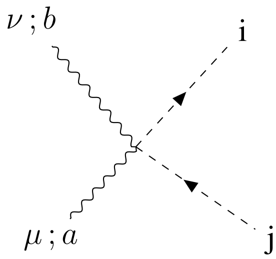


. (5.3.20)

Utilizing the general Feynman rules [42]



$= ig_i(k^\mu + q^\mu)t^a_{ij}$ (5.3.21) and



$$= i g_i^2 t^a_{ik} t^b_{kj} g^{\mu\nu}\,, \tag{5.3.22}$$

where g_i is the coupling constant under consideration, and t^a are the generators of the corresponding gauge group in the representation of the scalar, the results for $\Delta b_{i,k}$ are [133]

$$\Delta b^S_{1,k} = n_C(2j+1)\frac{Y^2}{5} \tag{5.3.23}$$

$$\Delta b^S_{2,k} = n_C s_k \frac{C_{SU(2)}(R_k)}{3} \tag{5.3.24}$$

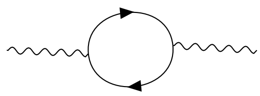
$$\Delta b^S_{3,k} = (2j+1) s_k \frac{C_{SU(3)}(R_k)}{3}\,. \tag{5.3.25}$$

Here, n_C is the number of colours, and j is the weak isospin of the multiplet. The factor s_k is 1 for complex scalars, and $1/2$ for real scalar fields [133]. $C_G(R_k)$ denotes the Dynkin index of the representation R_k of the group G. It is defined as

$$C_G(R)\delta^{ab} = \mathrm{Tr}(T^a_R T^b_R)\,, \tag{5.3.26}$$

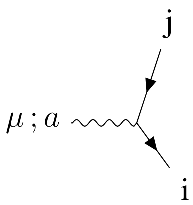
where T^a_R are the generators of the corresponding group in the representation R. For $SU(N)$, the Dynkin index is $1/2$ for the fundamental representation, and N for the adjoint representation. We will also need $C_{SU(3)}(6) = C_{SU(3)}(\overline{6}) = 5/2$ [145, ch. 3.4].

The contributions of the new fermions are very similar. They contribute via the fermion loop



$$, \tag{5.3.27}$$

which can be calculated with the Feynman rule [42]



$$= i g_i \gamma^\mu t^a_{ij}\,. \tag{5.3.28}$$

Taking into account a factor of (-1) for the fermion loop, the results are [133]

$$\Delta b^F_{1,k} = n_C(2j+1)\frac{4Y^2}{5} \tag{5.3.29}$$
$$\Delta b^F_{2,k} = n_C s_k \frac{4C_{SU(2)}(R_k)}{3} \tag{5.3.30}$$
$$\Delta b^F_{3,k} = (2j+1)s_k \frac{4C_{SU(3)}(R_k)}{3}\,. \tag{5.3.31}$$

Here, the factor s_k is $1/2$ for Weyl fermions and 1 for Dirac fermions respectively.

In the following, we assume that the new vector-like fermions do not influence unification. This is the case if there is no significant mass splitting within the multiplet. The reason is that a complete, mass degenerate $SU(5)$ multiplet changes the running of each coupling by the same amount. Thus, we examine only the effect of the additional scalars, which are all scalars in the 5-, 24-, 45- and 40-plet except for the G_{SM} singlet $\phi_{(1,1,0)} \in 24$ and the Standard Model Higgs $H \in 5$ in all four models (F5)-(F8). The scalar 40-plet contains the following Standard Model multiplets [47]:

$$40 = (1,2,-3/2) \oplus (\overline{3},1-2/3) \oplus (3,2,1/6) \oplus (\overline{3},3,-2/3) \oplus (6,2,1/6) \oplus (8,1,1)\,. \tag{5.3.32}$$

We are now able to compute the $\Delta b_{i,k}$ or $\Delta B_{ij,k} = \Delta b_{i,k} - \Delta b_{j,k}$ for all new scalars k. The complete list is given in table (5.3), and can also be found in ref. [135].

Field	$\Delta b_{1,k}$	$\Delta b_{2,k}$	$\Delta b_{3,k}$	$\Delta B_{12,k}$	$\Delta B_{23,k}$
$\phi_{(1,3,0)} \in 24$	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$
$\phi_{(8,1,0)} \in 24$	0	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
$\phi_{(3,1,-1/3)} \in 5$	$\frac{1}{15}$	0	$\frac{1}{6}$	$\frac{1}{15}$	$-\frac{1}{6}$
$\phi_{(1,2,1/2)} \in 45$	$\frac{1}{10}$	$\frac{1}{6}$	0	$-\frac{1}{15}$	$\frac{1}{6}$
$\phi_{(3,1,-1/3)} \in 45$	$\frac{1}{15}$	0	$\frac{1}{6}$	$\frac{1}{15}$	$-\frac{1}{6}$
$\phi_{(\overline{3},1,4/3)} \in 45$	$\frac{16}{15}$	0	$\frac{1}{6}$	$\frac{16}{15}$	$-\frac{1}{6}$
$\phi_{(\overline{3},2,-7/6)} \in 45$	$\frac{49}{30}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{17}{15}$	$\frac{1}{6}$
$\phi_{(3,3,-1/3)} \in 45$	$\frac{1}{5}$	2	$\frac{1}{2}$	$-\frac{9}{5}$	$-\frac{3}{2}$
$\phi_{(6,1,-1/3)} \in 45$	$\frac{2}{15}$	0	$\frac{5}{6}$	$\frac{2}{15}$	$-\frac{5}{6}$
$\phi_{(8,2,1/2)} \in 45$	$\frac{4}{5}$	$\frac{4}{3}$	2	$-\frac{8}{15}$	$-\frac{2}{3}$
$\phi_{(1,2,-3/2)} \in 40$	$\frac{9}{10}$	$\frac{1}{6}$	0	$\frac{11}{15}$	$\frac{1}{6}$
$\phi_{(\overline{3},1,-2/3)} \in 40$	$\frac{4}{15}$	0	$\frac{1}{6}$	$\frac{4}{15}$	$-\frac{1}{6}$

Field	$\Delta b_{1,k}$	$\Delta b_{2,k}$	$\Delta b_{3,k}$	$\Delta B_{12,k}$	$\Delta B_{23,k}$
$\phi_{(3,2,1/6)} \in 40$	$\frac{1}{30}$	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{7}{15}$	$\frac{1}{6}$
$\phi_{(\overline{3},3,-2/3)} \in 40$	$\frac{4}{5}$	2	$\frac{1}{2}$	$-\frac{6}{5}$	$\frac{3}{2}$
$\phi_{(6,2,1/6)} \in 40$	$\frac{1}{15}$	1	$\frac{5}{3}$	$-\frac{14}{15}$	$-\frac{2}{3}$
$\phi_{(8,1,1)} \in 40$	$\frac{8}{5}$	0	1	$\frac{8}{5}$	-1

Table 5.3: Change of the one-loop running of the gauge couplings $\Delta b_{i,k}$ and their differences $\Delta B_{ij,k} = \Delta b_{i,k} - \Delta b_{j,k}$ for the new scalar fields in the models (F5)-(F8).

In order to improve both unification and proton stability, out of these scalar fields we pick those that give a negative contribution to B_{12} and do not mediate proton decay by themselves. All other fields are assumed to have masses of the order of the unification scale. Hence, they do not contribute to the running of the couplings up to this scale. The fields we choose to be light, and thus contribute to the running of the couplings from the Z-mass to M_{GUT}, are

1. $\phi_{(1,3,0)} \in 24$,
2. $\phi_{(1,2,1/2)} \in 45$,
3. $\phi_{(8,2,1/2)} \in 45$,
4. $\phi_{(3,2,1/6)} \in 40$,
5. $\phi_{(\overline{3},3,-2/3)} \in 40$,
6. $\phi_{(6,2,1/6)} \in 40$.

In addition, we include the scalar $\phi_{(1,2,-3/2)} \in 40$ which takes part in the generation of the neutrino masses as field number 7.

5.3.2 Estimating the neutrino mass

In order to restrain the mass of $\phi_{(1,2,-3/2)}$ in the following search for unification, we take a look at the neutrino mass. In contrast to the previous considerations, this requires to pick one model. We chose to take (F6) in which the neutrino mass is generate at one-loop. The results for (F7) will be similar, while those for the two-loop models (F5) and (F8) are expected to be different.

The part of the Lagrangian of (F6) relevant for the neutrino mass is

$$\begin{aligned}\mathcal{L}_m = \psi_{\bar{5}}^{SM}\psi_{10}^{SM}(y_1^*\phi_5^* - \frac{1}{6}y_2^*\phi_{45}^*) + \psi_{\bar{5}}^{SM}\psi_{10}(y_3^*\phi_5^* - \frac{1}{6}y_4^*\phi_{45}^*) \\ + y_5\psi_{\bar{5}}^{SM}\psi_{\overline{10}}\phi_{40}^* + m_\psi\psi_{\overline{10}}\psi_{10} + \lambda_1\phi_5^2\phi_{45}\phi_{40} + \lambda_2\phi_5\phi_{45}^2\phi_{40}\ .\end{aligned} \tag{5.3.33}$$

Since we are only interested in the magnitude of the neutrino mass, and not in the structure of the mixing matrix, flavour indices are suppressed in the following calculations. When the two Higgs doublets in ϕ_5 and ϕ_{45} develop a vacuum expectation value, their charged components mix according to

$$\begin{pmatrix} H_5^+ \\ H_{45}^+ \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \ . \tag{5.3.34}$$

Here, G^+ is the charged Goldstone boson absorbed by the gauge bosons, and the mixing angle β is given by $\tan\beta = -\frac{v_{45}}{v_5}$, the ratio of the vacuum expectation values of the Higgs doublets embedded in the 5- and 45-plet. The charged Higgs field H^+ further mixes with the singly charged component $\Phi^+ \subset \phi_{(1,2,3/2)} \in 40^*$:

$$\begin{pmatrix} H^+ \\ \Phi^+ \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1^+ \\ h_2^+ \end{pmatrix} \ . \tag{5.3.35}$$

The mixing in this case is given by

$$\begin{aligned} \tan 2\theta &\approx \sin 2\theta \\ &= \frac{2\left(\lambda_1 \cos\beta(\cos^2\beta - 2\sin^2\beta) + \lambda_2 \sin\beta(\sin^2\beta - 2\cos^2\beta)\right) v^2}{m_{h_1^+}^2 - m_{h_2^+}^2} \ . \end{aligned} \tag{5.3.36}$$

Neglecting the mixing of the new vector-like fermions with those from the Standard Model, the terms included in the neutrino mass can be written as

$$\begin{aligned} \mathcal{L}_m \supset & \overline{\psi_R}\nu_L \left\{ \left(y_3^* s_\beta c_\theta (h_1^+)^* + s_\beta s_\theta (h_2^+)^*\right) - \frac{1}{6} y_4^* \left(c_\beta c_\theta (h_1^+)^* + c_\beta s_\theta (h_2^+)^*\right) \right\} \\ & + y_5 \overline{\psi_L^c} \nu_L \left(-s_\theta h_1^+ + c_\theta h_2^+\right) + \left(m_\psi \overline{\psi_L}\psi_R + h.c.\right) \ , \end{aligned} \tag{5.3.37}$$

where we introduced the short notation $s_x = \sin x$ and $c_x = \cos x$ for the sine and cosine, and ψ for the vector-like fermion transforming under G_{SM} as $(1, 1, -1)$. The diagram for neutrino mass generation in (F6) is depicted in fig. (5.1). There are two contributions, one with h_1^+ and one with h_2^+ running in the loop. Their sum can be written as

$$m_\nu = \frac{y_5}{2} \left(y_3^* s_\beta - \frac{1}{6} y_4^* c_\beta \right) s_{2\theta} \left(\mathcal{A}(m_\psi, m_{h_2^+}) - \mathcal{A}(m_\psi, m_{h_1^+}) \right) \ . \tag{5.3.38}$$

Here, $\mathcal{A}(m_1, m_2)$ is the loop amplitude for a loop with a fermion of mass m_1 and a scalar of mass m_2. While the individual loop amplitudes diverge, their sum is finite, and the

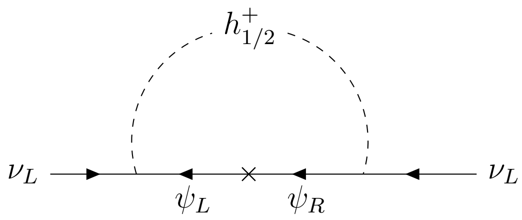


Figure 5.1: Schematic diagram of the neutrino mass term in models (F6) and (F7). While the single diagram leads to a divergent result, the sum of the contributions of h_1^+ and h_2^+ running in the loop leads to a finite result. The fermion ψ is the charged singlet $\sim (1, 1, -1)$ from the vector-like 10-plet in model (F6). A similar diagram is expected in model (F7) where the fermion running in the loop is a component of the $SU(2)_L$ triplet $\sim (1, 3, -1)$ from the vector-like fermionic 15-plet.

neutrino mass is given by

$$\begin{aligned} m_\nu \approx & m_\psi \frac{y_5 y^*_{eff} v^2 \lambda_{eff}}{8\pi (m^2_{h_2^+} - m^2_{h_1^+})(m^2_\psi - m^2_{h_1^+})(m^2_\psi - m^2_{h_2^+})} \times \\ & \times \left[m^2_\psi m^2_{h_2^+} \log\left(\frac{m^2_\psi}{m^2_{h_2^+}}\right) + m^2_\psi m^2_{h_1^+} \log\left(\frac{m^2_{h_1^+}}{m^2_\psi}\right) + m^2_{h_1^+} m^2_{h_2^+} \log\left(\frac{m^2_{h_2^+}}{m^2_{h_1^+}}\right)\right] \\ & + \text{transpose} \, , \end{aligned} \tag{5.3.39}$$

where we have defined the effective couplings

$$y_{eff} = y_3 s_\beta - \frac{1}{6} y_4 c_\beta \tag{5.3.40}$$

$$\lambda_{eff} = \lambda_1 c_\beta (c^2_\beta - 2s^2_\beta) + \lambda_2 s_\beta (s^2_\beta - 2c^2_\beta) \, . \tag{5.3.41}$$

The form of the result is in agreement with the results for a similar one loop model obtained in ref. [146], and, in the appropriate limit, with the results for the Zee-model [59].

Thus, in the limit where $m_{h_2^+}$ is the largest of the masses appearing, the neutrino mass is proportional to

$$m_\nu \propto \frac{m_\psi v^2}{m^2_{h_2^+}} \, . \tag{5.3.42}$$

If the mass m_ψ of the new fermion is the largest one appearing in eq. (5.3.39), the dominant contribution to the neutrino mass is proportional to

$$m_\nu \propto \frac{v^2}{m_\psi} \, . \tag{5.3.43}$$

Hence, depending on the regime, the neutrino mass leads to different bounds on the masses h_2^+ and the new fermions ψ.

5.3.3 Parameter scan for unification

To test whether unification and viable neutrino mass can simultaneously be achieved within model (F6), we now scan over possible r_k-values (see eq. (5.3.9)) and identify all 7-tuples satisfying eq. (5.3.19) and eq. (5.3.13) within 2σ

$$0.709 \leq \frac{B_{23}}{B_{12}} \leq 0.729\,. \tag{5.3.44}$$

Since the mass matrices of the present $SU(5)$ multiplets have enough free parameters to choose the masses of the Standard Model multiplets freely [135], we vary the parameters r_k between 0 and 0.9. This corresponds to a lower mass bound of $\mathcal{O}(1\,\mathrm{TeV})$. This way, we mostly avoid detection bounds on electroweak multiplets. The scan is conducted by generating 7-tuples of r_k values within the given boundaries at random. If the conditions eq. (5.3.19) and eq. (5.3.44) are not satisfied, the 7-tuple is discarded. At the end of the scan a list of successful data points is returned. We conduct several separate scans with different sets of conditions. In the end, we present some promising benchmark points.

In the model (F6), the new fermionic $SU(5)$ multiplets will automatically receive a mass splitting of the order of the electroweak scale due to their Yukawa interactions. To make sure that the fermions do not influence the running of the couplings, and to avoid possible problems with flavour observables, we assume that the fermion mass $m\psi$ is large enough for the splitting to be negligible, $m_\psi \gtrsim 10^5\,\mathrm{GeV}$. Consequently, to get a small enough neutrino mass with $\mathcal{O}(1)$ Yukawa couplings for any fermion mass, the mass of the scalar should be larger than $10^{11}\,\mathrm{GeV}$. Thus we restrict its r-value to $r_7 \leq 0.55$. An upper limit for the scalar mass can be derived from the neutrino mass. In the regime, where the fermion mass m_ψ is larger than $m_{h_2^+}$, the fermion mass is determined by the neutrino mass, and gives an upper bound for the scalar mass for $\mathcal{O}(1)$ Yukawa couplings of $10^{15}\,\mathrm{GeV}$. The same upper limit can also be derived from the regime where $m_{h_2^+}$ is the largest mass scale in the neutrino mass diagrams. In this regime, the factor $\frac{m_\psi}{m_{h_2^+}}$ can at most be one, in which case, for $\mathcal{O}(1)$ Yukawa couplings, the scalar mass is fixed by the neutrino mass to be at most $10^{15}\,\mathrm{GeV}$. This corresponds to a bound on the r-value $0.15 \leq r_7$. As a result, the first condition for the scan is $0.15 \leq r_7 \leq 0.55$, ensuring viable neutrino masses. Note that the exact mass bound does now depend on the resulting unification scale.

The second condition we impose on all scans is that the second Higgs doublet, and hence its charged component, should at least be moderately light, $m_{h_2^+} \lesssim 10\,\mathrm{TeV}$. In our scan, we implement this as the condition $r_2 \geq 0.85$. The lower mass limit for the second Higgs doublet is chosen to be $r_2 \leq 0.95$, or $m_{h_2^+} \gtrsim 500\,\mathrm{GeV}$ [3, p. 33].

We use this set of conditions to produce our first benchmark point. It is a set of parameters leading to a very large unification scale, $M_{GUT} = 7 \times 10^{17}\,\mathrm{GeV}$. Indeed, within our

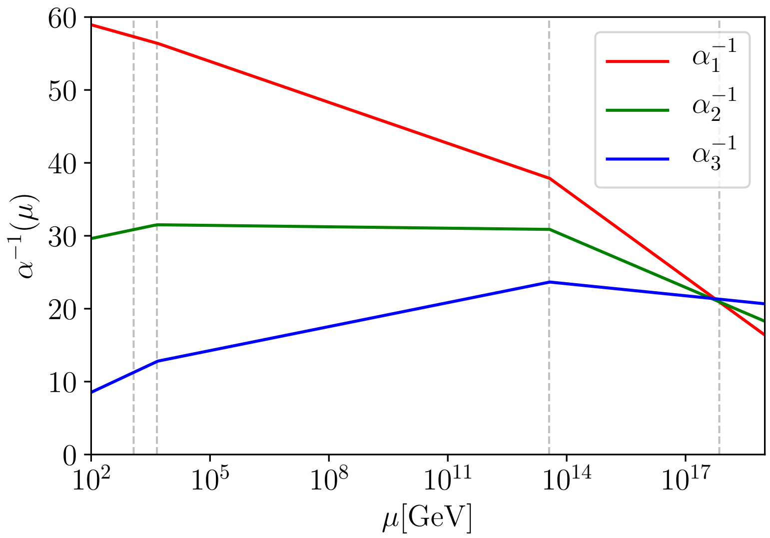


Figure 5.2: Running of the gauge couplings in the model (F6) for the benchmark point $m_{\phi_{(1,2,1/2)}} = 1200\,\text{GeV}$, $m_{\phi_{(1,3,0)}} = m_{\phi_{(8,2,1/2)}} = m_{\phi_{(3,2,1/6)}} = m_{\phi_{(6,2,1/6)}} = 4600\,\text{GeV}$, $m_{\phi_{(\overline{3},3,-2/3)}} = m_{\phi_{(1,2,-3/2)}} = 3.6 \times 10^{13}\,\text{GeV}$, $m_\psi = 3.6 \times 10^{13}\,\text{GeV}$, and $M_{GUT} = 7 \times 10^{17}\,\text{GeV}$. Two generations of the vector-like fermionic 10-plet are included.

scan we find only a small number of models with such large unification scales, and no model with a scale much larger than $10^{18}\,\text{GeV}$.

In order to reach such a high unification scale, almost all particles pushing the scale up need to be very light. At our benchmark point this means $m_{\phi_{(1,2,1/2)}} = 1200\,\text{GeV}$, $m_{\phi_{(1,3,0)}} = m_{\phi_{(8,2,1/2)}} = m_{\phi_{(3,2,1/6)}} = m_{\phi_{(6,2,1/6)}} = 4600\,\text{GeV}$. The two remaining scalars $\phi_{(\overline{3},3,-2/3)}$ and $\phi_{(1,2,-3/2)}$ reside at an intermediate scale $m_{\phi_{(\overline{3},3,-2/3)}} = m_{\phi_{(1,2,-3/2)}} = 3.6 \times 10^{13}\,\text{GeV}$ which is not only close to the upper mass bound for $\phi_{(1,2,-3/2)}$, but additionally coincides with the mass scale of the new fermions $m_\psi = 3.6 \times 10^{13}\,\text{GeV}$. Thus, in the case of a high unification scale, there is a large number of light particles, including a scalar colour octett, $\phi_{(8,2,1/2)}$. The mass of this particle is very close to the current bound from collider search of $m_{\phi_{(8,2,1/2)}} \geq 4.2\,\text{TeV}$ [147], and might be excluded in the near future.

However, we have only a very small number of different scales: a low scale close to the electroweak scale, an intermediate scale connected to neutrino masses, and the unification scale. The running of the couplings for this choice of masses is shown in fig. (5.2). We also include the change of running caused by the fermions to check for unacceptably low Landau poles. We find that two generations of the vector-like fermions do not lead to such a problem.

Even with the constraint from the neutrino masses, we find that there are solutions to the equations (5.3.19) and (5.3.44) for almost any value of r_1. Field 1, $\phi_{(1,3,0)} \in 24$, is the only Standard Model multiplet in the 24-plet which we allowed to be lighter than the unification scale. Thus, it seems reasonable to keep it at the GUT scale and set $r_1 = 0$ as

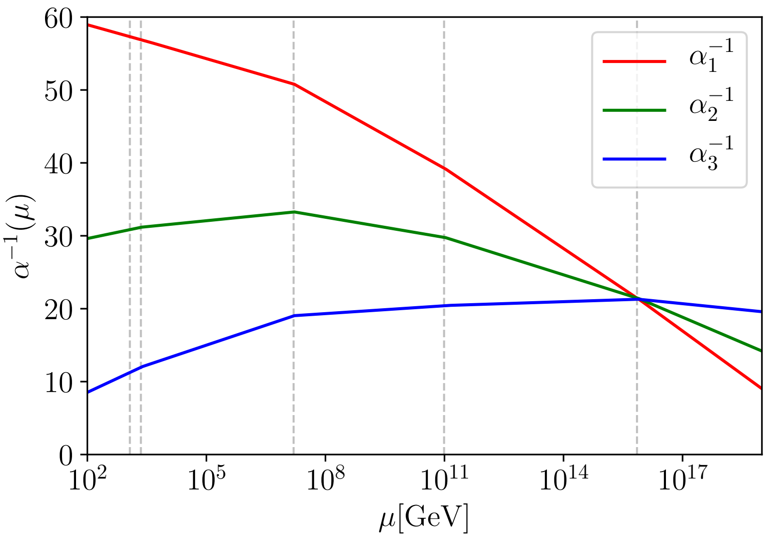


Figure 5.3: Running of the gauge couplings in model (F6) for the benchmark point $m_{\phi_{(1,2,1/2)}} = 1200\,\text{GeV}$, $m_{\phi_{(3,2,1/6)}} = m_{\phi_{(6,2,1/6)}} = 2200\,\text{GeV}$, $m_{\phi_{(\overline{3},3,-2/3)}} = m_{\phi_{(1,2,-3/2)}} = 9.8 \times 10^{10}\,\text{GeV}$, $m_{\phi_{(1,3,0)}} = m_{\phi_{(8,2,1/2)}} = M_{GUT} = 7 \times 10^{15}\,\text{GeV}$, and $m_\psi = 1.5 \times 10^7\,\text{GeV}$, including two copies of the new vector-like, fermionic 10-plet.

an additional scan condition to avoid splitting of the 24-plet.

One benchmark point from this part of the parameter space corresponds to the scalar masses $m_{\phi_{(1,2,1/2)}} = 1200\,\text{GeV}$, $m_{\phi_{(3,2,1/6)}} = m_{\phi_{(6,2,1/6)}} = 2200\,\text{GeV}$, $m_{\phi_{(\overline{3},3,-2/3)}} = m_{\phi_{(1,2,-3/2)}} = 9.8 \times 10^{10}\,\text{GeV}$, and $m_{\phi_{(1,3,0)}} = m_{\phi_{(8,2,1/2)}} = M_{GUT} = 7 \times 10^{15}\,\text{GeV}$, and the mass of the new fermions $m = 1.5 \times 10^7\,\text{GeV}$. Here, r_3 has been set to zero in addition to r_1. As a consequence, the only scalars with masses below the unification scale are the second Higgs doublet and parts of the 40-plet.

The running of the coupling in this case is shown in fig. (5.3). This benchmark point is very close to the parameter point with the highest unification scale, and therefore proton lifetime, with r_1 and r_3 set to zero. Moreover, this choice of parameters corresponds to a rather small number of different scales, even though the intermediate scalar mass scale is not identical to the fermion mass scale as in the previous case. Although the unification scale for this choice of parameters is two orders of magnitude smaller than for the first benchmark point, and the intermediate scales of the scalars and fermions are not identical, this choice of parameters can be of interest, since it reduces the number of light particles.

The results of our scan show that it is even possible to further reduce the number of scalars with a mass below M_{GUT}. A benchmark point with a minimal number of light scalars in the above sense is for example $m_{\phi_{(1,2,1/2)}} = 1200\,\text{GeV}$, $m_{\phi_{(6,2,1/6)}} = 2200\,\text{GeV}$, $m_{\phi_{(\overline{3},3,-2/3)}} = 2.0 \times 10^8\,\text{GeV}$, $m_{\phi_{(1,2,-3/2)}} = 4.7 \times 10^{13}\,\text{GeV}$, $m_{\phi_{(1,3,0)}} = m_{\phi_{(8,2,1/2)}} = m_{\phi_{(3,2,1/6)}} = M_{GUT} = 5.5 \times 10^{15}\,\text{GeV}$, and $m_\psi = 4.6 \times 10^{13}\,\text{GeV}$. In this case, the only scalars with masses below the unification scale are the second Higgs and $\phi_{(1,2,-3/2)}$, which

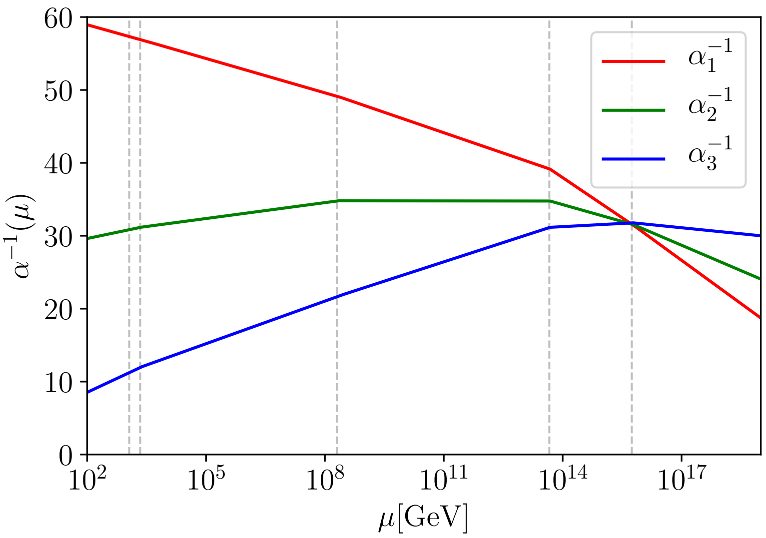


Figure 5.4: Running of the gauge couplings in model (F6) for the benchmark point $m_{\phi_{(1,2,1/2)}} = 1200\,\mathrm{GeV}$, $m_{\phi_{(6,2,1/6)}} = 2200\,\mathrm{GeV}$, $m_{\phi_{(\overline{3},3,-2/3)}} = 2.0 \times 10^8\,\mathrm{GeV}$, $m_{\phi_{(1,2,-3/2)}} = 4.7 \times 10^{13}\,\mathrm{GeV}$, $m_{\phi_{(1,3,0)}} = m_{\phi_{(8,2,1/2)}} = m_{\phi_{(3,2,1/6)}} = M_{GUT} = 5.5 \times 10^{15}\,\mathrm{GeV}$, and $m_\psi = 4.6 \times 10^{13}\,\mathrm{GeV}$, with two copies of the new fermions.

are required to be light by the charged fermion and neutrino masses, and two other fields from the scalar 40-plet. The running in this case is shown in fig. (5.4). Note that it is no longer possible to keep the two scalars $\phi_{(\overline{3},3,-2/3)}$ and $\phi_{(1,2,-3/2)}$ at a common mass scale without violating the lower bound for the unification scale. So a decrease of the number of light fields results in an increase of scales for the scalars. In addition, we find that the more fields are kept at the unification scale, the lower the typical scale of unification becomes. For the benchmark presented here, the estimated proton lifetime $\tau_p \approx 2.7 \times 10^{34}$ y is well within reach for the planned Hyper-Kamiokande experiment [148].

Concluding, one can say that unification is indeed possible in the model (F6) in agreement with current proton decay bounds and neutrino mass measurements. This has been demonstrated by three interesting benchmark parameter points. Notably, these benchmark points can be accessed by different experiments. While one possesses a large number of light particles potentially in reach for the LHC, another one has a low unification scale, placing the proton-lifetime in the reach of Hyper-Kamiokande.

6 Conclusion

In this thesis, we have studied different models for neutrino mass generation, starting with an introduction to different fermion masses and the three see-saw mechanisms. The fields used in the see-saw mechanisms are special in the sense that they are the only three representations that generate the dimension-5 Weinberg operator at tree-level. In the classical see-saw, the mass of the neutrinos is then suppressed by the large mass of these new fields. While minimal extensions of the Standard Model by these fields are well-studied phenomenologically, they are experimentally hard to test. Here, an alternative in the form of models with additional suppression factors for the neutrino mass has been explored. In particular, we have looked for models that only generate the dimension-5 Weinberg operator at loop-level, called radiative neutrino mass models.

We have studied radiative neutrino mass models in a systematic approach that is complementary to the usual operator- or topology-ordering. In our research, minimal models have been considered in the sense that the models only included at most two new G_{SM} multiplets, and did not have new ad-hoc symmetries. The new multiplets were either scalars, Majorana fermions, or vector-like Dirac fermions. In order to find models of this type, the fact that any model generating neutrino masses with a small number of new multiplets and without any new symmetry leads to Majorana neutrino mass and hence lepton number violation has been employed. Searching for models with explicit lepton number violation allowed to construct an exhaustive list of minimal radiative neutrino mass models. The list is given in the tables 3.1, 3.2, 3.3, 3.4, and 3.5.

Among the models found in our scan, there is a large number of models with coloured fields. These models usually involve quarks running in the loop of the neutrino mass diagram, and the new particles lead to baryon number (B) violation in addition to lepton number (L) violation. In such a case, baryon and lepton number must be broken in distinct ways, since otherwise the symmetry $(B - L)$ is conserved and forbids the Majorana neutrino mass term.

Many of the neutrino mass models involving coloured fields contain at least one of the so-called scalar leptoquarks, scalar fields allowing a coupling between leptons and quarks. Leptoquarks have recently drawn attention as potential solutions to the anomalies appearing in B-flavour physics. The ongoing discussion has been briefly summarised.

All the models found in the scan with up to two representations contain at least one new scalar. By considerations involving lepton number violation, we could show that there cannot be a radiative neutrino mass model with only two new fermionic representations. Moreover, it has been shown that there is a unique model with three new fermionic representations, which was first mentioned by ref. [57]. It contains a vector-like $SU(2)_L$ triplet and quadruplet and a Majorana-like $SU(2)_L$ quintuplet. The new fermions are only coupled to the Standard Model via Yukawa couplings. In particular, the Yukawa coupling between the Standard Model leptons and the triplet can be naturally small due to an accidental Z_2 symmetry, under which the new fermions are odd.

A similar accidental Z_2 symmetry has also been observed in the models of type C2, in which a new Majorana fermion in a large $SU(2)$ representation and a matching scalar are added to the Standard Model. The version of this model type with a scalar 6-plet and a fermionic quintuplet have been studied as a potential model unifying neutrino mass and dark matter. But it was shown in ref. [112] that in the presence of the scalar 6-plet the accidental Z_2 symmetry is broken and the quintuplet decays too fast to account for the dark matter.

By estimating the four-body decay rate of the fermionic quintuplet in the three-fermion neutrino mass model, we have discovered that in this model, the quintuplet decays too fast, too. Hence, the model cannot explain dark matter without any further extension of either the particle content or the symmetry. A more detailed analysis including the decay of the doubly charged component of the 5-plet shows that the quintuplet is even too short-lived to produce displaced-vertex signatures at the LHC.

Additionally, the embedding of radiative neutrino mass models into a $SU(5)$ grand unified theory has been examined. $SU(5)$ is chosen since minimal $SO(10)$ models automatically include neutrino masses. This top-down approach complements the bottom-up scan. It turns out that the number of minimal models, which correspond to one of the minimal models from our scan, is much smaller from the $SU(5)$ perspective. For a specific class of models, which included vector-like fermions, we have tested whether they could lead to successful unification of the gauge couplings and viable relations of the charged fermion masses. A parameter scan has revealed that this is indeed possible, and some benchmark points and their features have been presented.

To put it briefly, this work shows that even after years of study, the question of the neutrino mass mechanism is far from being resolved. This is why it is important to organise the possibilities in a systematic way, and study also marginally viable models. The more models we are able to exclude, the smaller the theory space for neutrino mass mechanisms becomes. The bottom-up perspective is a good point to start a systematic study, since it is really based on what we have measured and observed to work. Still, it is also important to consider models in a larger context. One way to do so is to look for implications of the neutrino mass models with regards to other problems of the Standard Model, such as dark matter, the baryon-asymmetry of the Universe, or B-anomalies. Another way is to change perspective and look at the model inside a possible UV-completion of the Standard Model. These considerations have shown that while each of the models can to some extend solve the neutrino mass problem, their connection to other problems of the Standard Model can be vastly different. It will be interesting to observe how next-generation observations or new revelations in theory may change that evaluation, rule out some more of the models presented, and maybe finally come to a conclusion about the nature of the neutrino mass and its generating mechanism.

A Appendix

A.1 List of representations for second candidates

The method described in section (3.1.1) leads to the following lists of possible representations for the second candidate:

As second candidates in models of type A we have to take into account candidates appearing in eq. (3.1.3), which are not the first candidate, as well as all representations appearing in the following list [1]: Independently of the first candidate, the representations appearing in scalar potential interactions are

$$\begin{aligned} H^2 &\sim (1,1,0) \oplus (1,1,1) \oplus (1,3,0) \oplus (1,3,1) \\ H^3 &\sim (1,2,1/2) \oplus (1,2,3/2) \oplus (1,4,1/2) \oplus (1,4,3/2) \ . \end{aligned} \tag{A.1.1}$$

Coupling to the Standard Model can also include

$$\overline{e_R}\ell_L \sim \overline{Q_L}u_R \sim \overline{d_R}Q_L \sim (1,2,1/2) \ . \tag{A.1.2}$$

The rest of the interactions involves the first candidate. The terms

$$\phi^2_{(1,1,1)} \sim (1,1,0) \oplus (1,1,2) \qquad H\phi_{(1,1,1)} \sim (1,2,1/2) \oplus (1,2,3/2) \tag{A.1.3}$$

$$\phi^2_{(1,1,2)} \sim (1,1,0) \oplus (1,1,4) \qquad H\phi_{(1,1,2)} \sim (1,2,3/2) \oplus (2,5/2) \tag{A.1.4}$$

result in cubic scalar terms when one more scalar is added and

$$\begin{aligned} H\phi^2_{(1,1,1)} &\sim (1,2,1/2) \oplus (1,2,3/2) \oplus (1,2,5/2) \\ H^2\phi_{(1,1,1)} &\sim (1,1,0) \oplus (1,1,1) \oplus (1,1,2) \oplus (1,3,0) \oplus (1,3,1) \oplus (1,3,2) \\ \phi^3_{(1,1,1)} &\sim (1,1,1) \oplus (1,1,3) \end{aligned} \tag{A.1.5}$$

and

$$\begin{aligned} H\phi^2_{(1,1,2)} &\sim (1,2,1/2) \oplus (1,2,7/2) \oplus (1,2,9/2) \\ H^2\phi_{(1,1,2)} &\sim (1,1,1) \oplus (1,1,2) \oplus (1,1,3) \oplus (1,3,1) \oplus (1,3,2) \oplus (1,3,3) \\ \phi^3_{(1,1,2)} &\sim (1,1,2) \oplus (1,1,6) \end{aligned} \tag{A.1.6}$$

result in quartic scalar terms for $\phi_{(1,1,1)}$ or $\phi_{(1,1,2)}$ as the first candidate respectively. According to ref. [58], a representation should in general appear twice in these lists, choosing

[1] This list includes antisymmetric contractions, such as $(1,1,1) \subset H^2$ or $(1,2,3/2) \subset H^3$. They vanish identically if there is only one Higgs doublet.

either the terms with $\phi_{(1,1,1)}$, or $\phi_{(1,1,2)}$, in order to generate lepton number violation. This is not necessary if the second candidate already appears in eq. (3.1.3).

For models of type cA, the second candidate can either be from eq. (3.1.4), the terms above involving the Higgs doublet only, or appear in the following set of interactions: The terms

$$\begin{aligned} \overline{Q_L^c}Q_L &\sim ([\overline{3}\oplus\overline{6}],[1\oplus 3],1/3) & \overline{u_R^c}u_R &\sim ([\overline{3}\oplus\overline{6}],1,4/3) \\ \overline{u_R^c}d_R &\sim ([\overline{3}\oplus\overline{6}],1,1/3) & \overline{d_R^c}d_R &\sim ([\overline{3}\oplus\overline{6}],1,-2/3) \end{aligned} \tag{A.1.7}$$

generate additional interactions with the Standard Model fermions, independently of the first candidate. Additional cubic or quartic terms in the scalar potential, depending on the first candidate, can be generated from

$$\begin{aligned} H\phi_{(3,1,-1/3)} &\sim (3,2,1/6)\oplus(3,2,-5/6) \\ \phi^2_{(3,1,-1/3)} &\sim (1,1,0)\oplus([\overline{3}\oplus\overline{6}],1,-2/3) \\ H^2\phi_{(3,1,-1/3)} &\sim (3,[1\oplus 3],-1/3)\oplus(3,[1\oplus 3],2/3)\oplus(3,[1\oplus 3],-4/3) \\ H\phi^2_{(3,1,-1/3)} &\sim (1,2,1/2)\oplus([\overline{3}\oplus\overline{6}],2,-1/6)\oplus([\overline{3}\oplus\overline{6}],2,-7/6) \\ \phi^3_{(3,1,-1/3)} &\sim ([3\oplus 6\oplus 15],1,-1/3)\oplus([1\oplus 8\oplus 10],1,-1) \end{aligned} \tag{A.1.8}$$

$$\begin{aligned} H\phi_{(3,1,-4/3)} &\sim (3,2,-5/6)\oplus(3,2,-11/6) \\ \phi^2_{(3,1,-4/3)} &\sim (1,1,0)\oplus([\overline{3}\oplus\overline{6}],1,-8/3) \\ H^2\phi_{(3,1,-4/3)} &\sim (3,[1\oplus 3],-1/3)\oplus(3,[1\oplus 3],-4/3)\oplus(3,[1\oplus 3],-7/3) \\ H\phi^2_{(3,1,-4/3)} &\sim (1,2,1/2)\oplus([\overline{3}\oplus\overline{6}],2,-13/6)\oplus([\overline{3}\oplus\overline{6}],2,-19/6) \\ \phi^3_{(3,1,-4/3)} &\sim ([3\oplus 6\oplus 15],1,-4/3)\oplus([1\oplus 8\oplus 10],1,-4) \end{aligned} \tag{A.1.9}$$

$$\begin{aligned} H\phi_{(3,2,1/6)} &\sim (3,[1\oplus 3],-1/3)\oplus(3,[1\oplus 3],2/3) \\ \phi^2_{(3,2,1/6)} &\sim (1,[1\oplus 3],0)\oplus([\overline{3}\oplus\overline{6}],[1\oplus 3],1/3) \\ H^2\phi_{(3,2,1/6)} &\sim (3,[2\oplus 4],1/6)\oplus(3,[2\oplus 4],-5/6)\oplus(3,[2\oplus 4],7/6) \\ H\phi^2_{(3,2,1/6)} &\sim (1,[2\oplus 4],1/2)\oplus([\overline{3}\oplus\overline{6}],[2\oplus 4],-1/6)\oplus([\overline{3}\oplus\overline{6}],[2\oplus 4],5/6) \\ \phi^3_{(3,2,1/6)} &\sim ([3\oplus 6\oplus 15],[2\oplus 4],1/6)\oplus([1\oplus 8\oplus 10],[2\oplus 4],1/2) \end{aligned} \tag{A.1.10}$$

$$\begin{aligned} H\phi_{(3,2,7/6)} &\sim (3,[1\oplus 3],2/3)\oplus(3,[1\oplus 3],5/3) \\ \phi^2_{(3,2,7/6)} &\sim (1,[1\oplus 3],0)\oplus([\overline{3}\oplus\overline{6}],[1\oplus 3],7/3) \\ H^2\phi_{(3,2,7/6)} &\sim (3,[2\oplus 4],1/6)\oplus(3,[2\oplus 4],7/6)\oplus(3,[2\oplus 4],13/6) \\ H\phi^2_{(3,2,7/6)} &\sim (1,[2\oplus 4],1/2)\oplus([\overline{3}\oplus\overline{6}],[2\oplus 4],11/6)\oplus([\overline{3}\oplus\overline{6}],[2\oplus 4],17/6) \\ \phi^3_{(3,2,7/6)} &\sim ([3\oplus 6\oplus 15],[2\oplus 4],7/6)\oplus([1\oplus 8\oplus 10],[2\oplus 4],7/2) \end{aligned} \tag{A.1.11}$$

$$
\begin{aligned}
H\phi_{(3,3,-1/3)} &\sim (3,[2\oplus 4],1/6)\oplus(3,[2\oplus 4],-5/6) \\
\phi^2_{(3,3,-1/3)} &\sim (1,[1\oplus 3\oplus 5],0)\oplus([\overline{3}\oplus\overline{6}],[1\oplus 3\oplus 5],-2/3) \\
H^2\phi_{(3,3,-1/3)} &\sim (3,[1\oplus 3\oplus 5],-1/3)\oplus(3,[1\oplus 3\oplus 5],2/3)\oplus(3,[1\oplus 3\oplus 5],-4/3) \\
H\phi^2_{(3,3,-1/3)} &\sim (1,[2\oplus 4\oplus 6],1/2)\oplus([\overline{3}\oplus\overline{6}],[2\oplus 4\oplus 6],-1/6)\oplus([\overline{3}\oplus\overline{6}],[2\oplus 4\oplus 6],-7/6) \\
\phi^3_{(3,3,-1/3)} &\sim ([3\oplus 6\oplus 15],[1\oplus 3\oplus 5\oplus 7],-1/3)\oplus([1\oplus 8\oplus 10],[1\oplus 3\oplus 5\oplus 7],-1)\ ,
\end{aligned}
\tag{A.1.12}
$$

once the second scalar candidate is added. Here, we use the short notation $([c\oplus d],[a\oplus b],y)=(c,a,y)\oplus(c,b,y)\oplus(d,a,y)\oplus(d,b,y)$. Again, a suitable second candidate for models of type cA should appear twice in the general lists or the ones associated to the corresponding first candidate.

For models of type B, if the first candidate is one of the scalars in eq. (3.1.3), the representation of the fermionic candidate should appear in eq. (3.1.5), or in the corresponding one of the following lists:

$$
\ell_L\phi_{(1,1,1)} \sim (1,2,-1/2)\oplus(1,2,3/2) \quad e_R^c\phi_{(1,1,1)} \sim (1,1,0)\oplus(1,1,2) \tag{A.1.13}
$$

$$
\ell_L\phi_{(1,1,2)} \sim (1,2,3/2)\oplus(1,2,-5/2) \quad e_R^c\phi_{(1,1,2)} \sim (1,1,-1)\oplus(1,1,3)\ . \tag{A.1.14}
$$

Fermionic candidates should ideally appear twice as well, in order to include them into the fermion line or a fermionic loop in a non-trivial way.

If the fermion is the first candidate and comes from the list in eq. (3.1.5), the scalar candidate should couple to one of the H^n-terms in eq. (A.1.1), or, depending on the fermionic candidate, to

$$
\begin{aligned}
\ell_L\psi_L^{(1,1,1)} &\sim (1,2,1/2) & \ell_L\psi_L^{(1,1,-1)} &\sim (1,2,-3/2) \\
e_R^c\psi_L^{(1,1,1)} &\sim (1,1,2) & e_R^c\psi_L^{(1,1,-1)} &\sim (1,1,0)\ ,
\end{aligned}
\tag{A.1.15}
$$

$$
\begin{aligned}
\ell_L\psi_L^{(1,2,1/2)} &\sim (1,[1\oplus 3],0) & \ell_L\psi_L^{(1,2,-1/2)} &\sim (1,[1\oplus 3],-1) \\
e_R^c\psi_L^{(1,2,1/2)} &\sim (1,2,3/2) & e_R^c\psi_L^{(1,2,-1/2)} &\sim (1,2,1/2)\ ,
\end{aligned}
\tag{A.1.16}
$$

$$
\begin{aligned}
\ell_L\psi_L^{(1,2,3/2)} &\sim (1,[1\oplus 3],1) & \ell_L\psi_L^{(1,2,-3/2)} &\sim (1,[1\oplus 3],-2) \\
e_R^c\psi_L^{(1,2,3/2)} &\sim (1,2,5/2) & e_R^c\psi_L^{(1,2,-3/2)} &\sim (1,2,-1/2)\ ,
\end{aligned}
\tag{A.1.17}
$$

or

$$
\begin{aligned}
\ell_L\psi_L^{(1,3,1)} &\sim (1,[2\oplus 4],1/2) & \ell_L\psi_L^{(1,3,-1)} &\sim (1,[2\oplus 4],-3/2) \\
e_R^c\psi_L^{(1,3,1)} &\sim (1,3,2) & e_R^c\psi_L^{(1,3,-1)} &\sim (1,3,0)\ .
\end{aligned}
\tag{A.1.18}
$$

After adding the scalar candidate to the model, the new fermion should have two distinct Yukawa couplings to Standard Model fermions.

The models of type C are of the exceptional type we do not cover with this method.

Hence we jump directly to models of type cB or cC.

If the first candidate for the type cB or type cC models is a scalar from eq. (3.1.3), the fermionic candidate, in order to include colour, should appear in the model-independent list

$$\begin{aligned} Q_L H &\sim (3,[1\oplus 3],2/3)\oplus(3,[1\oplus 3],-1/3) \\ u_R^c H &\sim (\overline{3},2,-1/6)\oplus(\overline{3},2,-7/6) \\ d_R^c H &\sim (\overline{3},2,-1/6)\oplus(\overline{3},2,5/6) \end{aligned} \tag{A.1.19}$$

or in the corresponding one of the lists

$$\begin{aligned} Q_L\phi_{(1,1)} &\sim (3,2,-5/6)\oplus(3,2,7/6) \\ u_R^c\phi_{(1,1)} &\sim (\overline{3},1,1/3)\oplus(\overline{3},1,-5/3) \\ d_R^c\phi_{(1,1)} &\sim (\overline{3},1,-2/3)\oplus(\overline{3},1,4/3) \end{aligned} \tag{A.1.20}$$

$$\begin{aligned} Q_L\phi_{(1,2)} &\sim (3,2,-11/6)\oplus(3,2,13/6) \\ u_R^c\phi_{(1,2)} &\sim (\overline{3},1,4/3)\oplus(\overline{3},1,-8/3) \\ d_R^c\phi_{(1,2)} &\sim (\overline{3},1,-5/3)\oplus(\overline{3},1,7/3)\ . \end{aligned} \tag{A.1.21}$$

If the first candidate for the type cB model is a scalar from eq. (3.1.4), then the representation of the fermionic candidate should be contained in eq. (3.1.5), eq. (A.1.19), or in the list below belonging to the chosen scalar candidate:

$$\begin{aligned} \ell_L\phi_{(3,1,-1/3)} &\sim (\overline{3},2,-1/6)\oplus(3,2,-5/6) \\ e_R^c\phi_{(3,1,-1/3)} &\sim (3,1,2/3)\oplus(\overline{3},1,4/3) \\ Q_L\phi_{(3,1,-1/3)} &\sim ([1\oplus 8],2,1/2)\oplus([\overline{3}\oplus\overline{6}],2,-1/6) \\ u_R^c\phi_{(3,1,-1/3)} &\sim ([1\oplus 8],1,-1)\oplus([3\oplus 6],1,-1/3) \\ d_R^c\phi_{(3,1,-1/3)} &\sim ([1\oplus 8],1,0)\oplus([3\oplus 6],1,2/3)\ , \end{aligned} \tag{A.1.22}$$

$$\begin{aligned} \ell_L\phi_{(3,1,-4/3)} &\sim (\overline{3},2,5/6)\oplus(3,2,-11/6) \\ e_R^c\phi_{(3,1,-4/3)} &\sim (3,1,-1/3)\oplus(\overline{3},1,7/3) \\ Q_L\phi_{(3,1,-4/3)} &\sim ([1\oplus 8],2,3/2)\oplus([\overline{3}\oplus\overline{6}],2,-7/6) \\ u_R^c\phi_{(3,1,-4/3)} &\sim ([1\oplus 8],1,-2)\oplus([3\oplus 6],1,2/3) \\ d_R^c\phi_{(3,1,-4/3)} &\sim ([1\oplus 8],1,-1)\oplus([3\oplus 6],1,5/3)\ , \end{aligned} \tag{A.1.23}$$

$$\begin{aligned}
\ell_L\phi_{(3,2,1/6)} &\sim (3,[1\oplus 3],-1/3)\oplus(\overline{3},[1\oplus 3],-2/3)\\
e^c_R\phi_{(3,2,1/6)} &\sim (\overline{3},2,5/6)\oplus(3,2,7/6)\\
Q_L\phi_{(3,2,1/6)} &\sim ([1\oplus 8],[1\oplus 3],0)\oplus([\overline{3}\oplus\overline{6}],[1\oplus 3],1/3)\\
u^c_R\phi_{(3,2,1/6)} &\sim ([1\oplus 8],2,-1/2)\oplus([3\oplus 6],2,-5/6)\\
d^c_R\phi_{(3,2,1/6)} &\sim ([1\oplus 8],2,1/2)\oplus([3\oplus 6],2,1/6)\ ,
\end{aligned}\tag{A.1.24}$$

$$\begin{aligned}
\ell_L\phi_{(3,2,7/6)} &\sim (3,[1\oplus 3],2/3)\oplus(\overline{3},[1\oplus 3],-5/3)\\
e^c_R\phi_{(3,2,7/6)} &\sim (\overline{3},2,-1/6)\oplus(3,2,13/6)\\
Q_L\phi_{(3,2,7/6)} &\sim ([1\oplus 8],[1\oplus 3],-1)\oplus([\overline{3}\oplus\overline{6}],[1\oplus 3],4/3)\\
u^c_R\phi_{(3,2,7/6)} &\sim ([1\oplus 8],2,1/2)\oplus([3\oplus 6],2,-11/6)\\
d^c_R\phi_{(3,2,7/6)} &\sim ([1\oplus 8],2,3/2)\oplus([3\oplus 6],2,-5/6)\ ,
\end{aligned}\tag{A.1.25}$$

or

$$\begin{aligned}
\ell_L\phi_{(3,3,-1/3)} &\sim (\overline{3},[2\oplus 4],-1/6)\oplus(3,[2\oplus 4],-5/6)\\
e^c_R\phi_{(3,3,-1/3)} &\sim (3,3,2/3)\oplus(\overline{3},3,4/3)\\
Q_L\phi_{(3,3,-1/3)} &\sim ([1\oplus 8],[2\oplus 4],1/2)\oplus([\overline{3}\oplus\overline{6}],[2\oplus 4],-1/6)\\
u^c_R\phi_{(3,3,-1/3)} &\sim ([1\oplus 8],3,-1)\oplus([3\oplus 6],3,-1/3)\\
d^c_R\phi_{(3,3,-1/3)} &\sim ([1\oplus 8],3,0)\oplus([3\oplus 6],3,2/3)\ .
\end{aligned}\tag{A.1.26}$$

Again, a suitable fermionic candidate should have two different couplings to Standard Model fermions, or one coupling and one Majorana mass term.

If the first candidate is a fermion from the list in eq. (3.1.5), then the scalar should appear twice taking into account the H^n-terms, eq. (A.1.7), and

$$\begin{aligned}
Q_L\psi_L^{(1,1,1)} &\sim (3,2,7/6) & Q_L\psi_L^{(1,1,-1)} &\sim (3,2,-5/6)\\
u^c_R\psi_L^{(1,1,1)} &\sim (\overline{3},1,1/3) & u^c_R\psi_L^{(1,1,-1)} &\sim (\overline{3},1,-5/3)\\
d^d_R\psi_L^{(1,1,1)} &\sim (\overline{3},1,4/3) & d^c_R\psi_L^{(1,1,-1)} &\sim (\overline{3},1,-2/3)\ ,
\end{aligned}\tag{A.1.27}$$

$$\begin{aligned}
Q_L\psi_L^{(1,2,1/2)} &\sim (3,[1\oplus 3],2/3) & Q_L\psi_L^{(1,2,-1/2)} &\sim (3,[1\oplus 3],-1/3)\\
u^c_R\psi_L^{(1,2,1/2)} &\sim (\overline{3},2,-1/6) & u^c_R\psi_L^{(1,2,-1/2)} &\sim (\overline{3},2,-7/6)\\
d^d_R\psi_L^{(1,2,1/2)} &\sim (\overline{3},2,5/6) & d^c_R\psi_L^{(1,2,-1/2)} &\sim (\overline{3},2,-1/6)\ ,
\end{aligned}\tag{A.1.28}$$

$$\begin{aligned}
Q_L\psi_L^{(1,2,3/2)} &\sim (3,[1\oplus 3],5/3) & Q_L\psi_L^{(1,2,-3/2)} &\sim (3,[1\oplus 3],-4/3)\\
u^c_R\psi_L^{(1,2,3/2)} &\sim (\overline{3},2,5/6) & u^c_R\psi_L^{(1,2,-3/2)} &\sim (\overline{3},2,-13/6)\\
d^d_R\psi_L^{(1,2,3/2)} &\sim (\overline{3},2,11/6) & d^c_R\psi_L^{(1,2,-3/2)} &\sim (\overline{3},2,-7/6)\ ,
\end{aligned}\tag{A.1.29}$$

or

$$\begin{aligned}
&Q_L\psi_L^{(1,3,1)} \sim (3,[2\oplus 4],7/6) && Q_L\psi_L^{(1,3,-1)} \sim (3,[2\oplus 4],-5/6) \\
&u_R^c\psi_L^{(1,3,1)} \sim (\overline{3},3,1/3) && u_R^c\psi_L^{(1,3,-1)} \sim (\overline{3},3,-5/3) \\
&d_R^d\psi_L^{(1,3,1)} \sim (\overline{3},3,4/3) && d_R^c\psi_L^{(1,3,-1)} \sim (\overline{3},3,-2/3)\ .
\end{aligned} \tag{A.1.30}$$

If the chosen fermion has only one coupling to the Standard Model fermions via the Standard Model Higgs boson, the chosen scalar must appear in the list above to again generate a second interaction.

If the first candidate is a fermion from eq. (A.1.19), the scalar may also be colourless, and its representation can appear in the H^n-terms in eq. (A.1.1), eq. (A.1.7), eq. (3.1.3), eq. (3.1.4), or in

$$\begin{aligned}
&\ell_L\psi_L^{(3,1,-1/3)} \sim (3,2,-5/6) && \ell_L\psi_L^{(\overline{3},1,1/3)} \sim (\overline{3},2,-1/6) \\
&e_R^c\psi_L^{(3,1,-1/3)} \sim (3,1,2/3) && e_R^c\psi_L^{(\overline{3},1,1/3)} \sim (\overline{3},1,4/3)\ ,
\end{aligned} \tag{A.1.31}$$

$$\begin{aligned}
&\ell_L\psi_L^{(3,1,2/3)} \sim (3,2,1/6) && \ell_L\psi_L^{(\overline{3},1,-2/3)} \sim (\overline{3},2,-7/6) \\
&e_R^c\psi_L^{(3,1,2/3)} \sim (3,1,5/3) && e_R^c\psi_L^{(\overline{3},1,-2/3)} \sim (\overline{3},1,1/3)\ ,
\end{aligned} \tag{A.1.32}$$

$$\begin{aligned}
&\ell_L\psi_L^{(3,2,1/6)} \sim (3,[1\oplus 3],-1/3) && \ell_L\psi_L^{(\overline{3},2,-1/6)} \sim (\overline{3},[1\oplus 3],-2/3) \\
&e_R^c\psi_L^{(3,2,1/6)} \sim (3,2,7/6) && e_R^c\psi_L^{(\overline{3},2,-1/6)} \sim (\overline{3},2,5/6)\ ,
\end{aligned} \tag{A.1.33}$$

$$\begin{aligned}
&\ell_L\psi_L^{(3,2,-5/6)} \sim (3,[1\oplus 3],-4/3) && \ell_L\psi_L^{(\overline{3},2,5/6)} \sim (\overline{3},[1\oplus 3],1/3) \\
&e_R^c\psi_L^{(3,2,-5/6)} \sim (3,2,1/6) && e_R^c\psi_L^{(\overline{3},2,5/6)} \sim (\overline{3},2,11/6)\ ,
\end{aligned} \tag{A.1.34}$$

$$\begin{aligned}
&\ell_L\psi_L^{(3,2,7/6)} \sim (3,[1\oplus 3],2/3) && \ell_L\psi_L^{(\overline{3},2,-7/6)} \sim (\overline{3},[1\oplus 3],-5/3) \\
&e_R^c\psi_L^{(3,2,7/6)} \sim (3,2,13/6) && e_R^c\psi_L^{(\overline{3},2,-7/6)} \sim (\overline{3},2,-1/6)\ ,
\end{aligned} \tag{A.1.35}$$

$$\begin{aligned}
&\ell_L\psi_L^{(3,3,-1/3)} \sim (3,[2\oplus 4],-5/6) && \ell_L\psi_L^{(\overline{3},3,1/3)} \sim (\overline{3},[2\oplus 4],-1/6) \\
&e_R^c\psi_L^{(3,3,-1/3)} \sim (3,3,2/3) && e_R^c\psi_L^{(\overline{3},3,1/3)} \sim (\overline{3},3,4/3)\ ,
\end{aligned} \tag{A.1.36}$$

or

$$\begin{aligned}
&\ell_L\psi_L^{(3,3,2/3)} \sim (3,[2\oplus 4],1/6) && \ell_L\psi_L^{(\overline{3},3,-2/3)} \sim (\overline{3},[2\oplus 4],-7/6) \\
&e_R^c\psi_L^{(3,3,2/3)} \sim (3,3,5/3) && e_R^c\psi_L^{(\overline{3},3,-2/3)} \sim (\overline{3},3,1/3)\ ,
\end{aligned} \tag{A.1.37}$$

depending on the fermionic candidate. Again, the scalar should have two couplings, and the fermion should have two different couplings to the Standard Model.

The models (cC4)-(cC6) are again of the type which is not covered by this method, so they are not represented in this section.

Note that while this list appears rather long, there is only a very limited number of representations appearing here. These representations are

1. $([1 \oplus 8 \oplus 10], [1 \oplus 3], 0)$: These are all real representations, of which the fermionic versions of $(1, 1, 0)$ and $(1, 3, 0)$ are excluded.

2. $([1 \oplus 8 \oplus 10], [2 \oplus 4], n/2)$: In these representations, n is always an odd number, the largest one being 9. In combination with the $SU(3)_C$ 8- and 10-plet, only $n = 1$ and $n = 7$ appear, the 8-plet also allows $n = 3$.

3. $([1 \oplus 8 \oplus 10], [1 \oplus 3(\oplus 5 \oplus 7)], n)$: Here, n is any real number, the largest one being $n = 6$. The largest n for coloured fields is $n = 2$. The scalar version of $(1, 3, 1)$ is excluded.

4. $([3 \oplus 6(\oplus 15)], [1 \oplus 3(\oplus 5 \oplus 7)], n - 1/3)$: Here, $n \in [-2, 3]$.

5. $([3 \oplus 6(\oplus 15)], [2 \oplus 4(\oplus 6)], n + 1/6)$: For these combinations $n \in [-3, 3]$.

The representations in brackets only appear rarely, while the other ones are more common and thus promising as representations for our new fields.

A.2 Decomposition of $SU(5)$ representations

In this section we list the decomposition of all $SU(5)$ representations up to 75 into representations of the Standard Model gauge group in the standard embedding. This list can also be found in ref. [47].

$$
\begin{aligned}
5 &= (1,2,1/2) \oplus (3,1,-1/3) \\
10 &= (1,1,1) \oplus (\bar{3},1,-2/3) \oplus (3,2,1/6) \\
15 &= (1,3,1) \oplus (6,1,-2/3) \oplus (3,2,1/6) \\
24 &= (1,1,0) \oplus (1,3,0) \oplus (8,1,0) \oplus (3,2,-5/6) \oplus (\bar{3},2,5/6) \\
35 &= (1,4,-3/2) \oplus (\bar{3},3,-2/3) \oplus (\bar{6},2,1/6) \oplus (\overline{10},1,1) \\
40 &= (1,2,-3/2) \oplus (3,2,1/6) \oplus (\bar{3},1,-2/3) \oplus (3,2,1/6) \\
&\oplus (\bar{3},3,-2/3) \oplus (\bar{6},2,1/6) \oplus (8,1,1) \\
45 &= (1,2,1/2) \oplus (3,1,-1/3) \oplus (\bar{3},1,4/3) \oplus (\bar{3},2,-7/6) \\
&\oplus (3,3,-1/3) \oplus (\bar{6},1,-1/3) \oplus (8,2,1/2) \\
50 &= (1,1,-2) \oplus (3,1,-1/3) \oplus (\bar{3},2,-7/6) \oplus (6,1,4/3) \\
&\oplus (\bar{6},3,-1/3) \oplus (8,2,1/2) \\
70 &= (1,2,1/2) \oplus (1,4,1/2) \oplus (3,1,-1/3) \oplus (3,3,-1/3) \\
&\oplus (\bar{3},3,4/3) \oplus (6,2,-7/6) \oplus (8,2,1/2) \oplus (15,1,-1/3) \\
70' &= (1,5,-2) \oplus (\bar{3},4,-7/6) \oplus (\bar{6},3,-1/3) \\
&\oplus (\overline{10},2,1/2) \oplus (\overline{15'},1,4/3) \\
75 &= (1,1,0) \oplus (8,1,0) \oplus (8,3,0) \\
&\oplus (3,1,5/3) \oplus (\bar{3},1,-5/3) \oplus (3,2,-5/6) \oplus (\bar{3},2,5/6)
\end{aligned}
\tag{A.2.1}
$$

Additionally, we give the decomposition of the most important tensor product representations of $SU(5)$ into irreducible representations of $SU(5)$. We also give examples of important couplings of G_{SM} multiplets included in these products. This may involve Standard Model fields as well as new multiplets. The decompositions can also be found in ref. [47]. Representations larger than the 75 are omitted and only indicated by dots.

Tensor product	Standard Model example
$5 \otimes 5 = 10 \oplus 15$	$\overline{\ell_L}\ell_L^c$
$5 \otimes \bar{5} = 1 \oplus 24$	$\overline{\ell_L}\ell_L$
$10 \otimes \bar{5} = 5 \oplus 45$	$\overline{e_R}\ell_L$
$\overline{10} \otimes \bar{5} = 10 \oplus 40$	$\overline{\ell_L^c}\psi_L^{(1,1,-1)}$
$10 \otimes 10 = \bar{5} \oplus \overline{45} \oplus \overline{50}$	$\overline{Q_L^c}Q_L$
$10 \otimes \overline{10} = 1 \oplus 24 \oplus 75$	$\overline{e_R}e_R$
$\bar{5} \otimes \overline{15} = 35 \oplus 40$	$\overline{\ell_L^c}\psi_L^{1,3,-1}$
$15 \otimes 15 = \overline{50} \oplus \overline{70'} \oplus \ldots$	$\phi^2_{(3,2,1/6)}$

Tensor product	Standard Model example
$5 \otimes 24 = 5 \oplus 45 \oplus 70$	$\overline{\ell_L}\phi_{(1,3,0)}$
$40 \otimes 40 = \overline{45} \oplus \overline{50} \oplus \overline{70} \oplus \ldots$	$\phi^2_{(3,2,1/6)}$
$\overline{40} \otimes 45 = \overline{45} \oplus \overline{50} \oplus \overline{50} \oplus \overline{70} \oplus \ldots$	$\phi_{(3,1,-1/3)}\phi_{(3,1,2/3)}$
$\bar{5} \otimes \overline{45} = 10 \oplus 40 \oplus \ldots$	$H_1^* H_2^*$
$5 \otimes \overline{45} = 24 \oplus 75 \oplus \ldots$	$\overline{\ell_L} H_2^*$
$10 \otimes \overline{45} = 5 \oplus 45 \oplus 50 \oplus 70 \oplus \ldots$	$\phi_{(1,1,1)} H_2^*$
$45 \otimes 45 = 10 \oplus 15 \oplus 35 \oplus 40 \oplus 40 \oplus \ldots$	H_2^2
$\overline{10} \otimes 50 = \overline{45} \oplus \ldots$	$\phi_{(3,1,-1/3)}\phi_{(3,1,2/3)}$
$\overline{15} \otimes 50 = \overline{50} \oplus \ldots$	$\phi_{(3,1,-1/3)}\phi_{(6,1,2/3)}$
$5 \otimes 5 \otimes 45 = \overline{10} \oplus \overline{15} \oplus \overline{40} \oplus \ldots$	$H_1^2 H_2$
$5 \otimes 45 \otimes 45 = \overline{10} \oplus \overline{15} \oplus \overline{40} \oplus \ldots$	$H_1 H_2^2$

Table A.1: Tensor products of $SU(5)$ representations and examples of important interactions of G_{SM} multiplets contained in that product.

B Lists

B.1 List of Figures

B.2 List of Tables

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Erklärung:

Ich versichere, dass ich diese Arbeit selbstständig verfasst habe und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

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