

# The Cognitive Architecture of Recursion: Behavioral and fMRI Evidence from the Visual, Musical and Motor Domains

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## Abstract

In this manuscript, we summarize the results of our research program aiming at describing the cognitive architecture underlying the representation of recursive hierarchical embedding. After conducting a series of behavioral and fMRI experiments in the visual, musical and motor domains, we found that, behaviorally, the acquisition of recursive rules seems supported by cognitive resources that are general across domains. However, when we test well-trained participants in the fMRI, their representation of recursion seems supported by activating schemas stored in (visual, musical and motor) domain-specific repositories. This suggests that the resources necessary to acquire recursive rules are different from those necessary to utilize these rules after extensive training.

**Keywords:** recursion; hierarchy; embedding; visual; motor; music

Recursion is a fascinating concept that has inspired researchers from many disciplines because of its associations with language, music and mathematics, which are uniquely available to the cognitive repertoire of humans (Hauser, Chomsky, & Fitch, 2002).

The definition of recursion is much discussed, and the term is currently used with many different possible meanings (Fitch, 2010). In the original mathematical terminology, recursive functions are those that take their own output as input for the next iteration, such as the function generating the natural numbers:

$$N_0 = 1$$

$$N_i = N_{i-1} + 1, \text{ for } i > 0$$

One of the properties of these functions is the capacity to generate an infinite set of outputs.

Empirically, this property of infinity is impossible to verify (Lobina, 2011). However, there are other properties of recursion that make it interesting for empirical cognitive sciences. For instance, when we combine recursion with hierarchical embedding, we can generate complex hierarchies using simple rules (Martins, 2012). In fact, recursion has been discussed as a necessary condition to generate hierarchical structures of unbounded depth, and the most efficient procedure to generate multiple hierarchical levels (Berwick & Chomsky, 2016). For instance, in language, even though the use of recursive computations cannot be directly verified, it is inferred from the ability to generate hierarchical structures, as it is thought to be the only plausible mechanism to generate sets of sets (Berwick & Chomsky, 2016), without which there

can be no multiple levels of embedding. The same standard has been proposed for the visual-spatial domain (Martins, 2012).

Complex hierarchical structures occur in language, music and action planning (Fitch & Martins, 2014). In these domains, it is difficult to establish the empirical boundaries of the generative capacity. This is especially true when external memory and recording devices are available, as for example, in written language or in large scale engineering projects, such as those involved in building a particle accelerator. Independently of how complex a base structure is, it is always possible to embed it within a higher-order hierarchy.

The investigation of these properties of human cognition pose several challenges, which we tried to address in a systematic 6-year long research program. The first challenge was the definition of a clear theoretical framework to make recursion empirically tractable and consistent with a number of different domains (Martins, 2012; Martins & Fitch, 2014). Crucially, the availability of recursion must be tested experimentally and neither simply assumed nor deduced from pure analytical methods (Martins & Fitch, 2015). The crucial behavioral signature of a computational capacity of recursion is the ability to generate multiple new hierarchical levels (Martins, 2014; Berwick & Chomsky, 2016). This relation is independent of particular algorithmic and biological implementations (Berwick & Chomsky, 2016).

Thus, the second challenge was the development of experimental techniques that could be used to test the ability to represent recursive hierarchical embedding in different domains. Once these two challenges were met, we could start answering two central questions: (1) in which domains of cognition recursion is available, and (2) if recursion were available in more than one domain, would it be instantiated by a domain-general capacity, or by multiple domain-specific abilities?

## Recursion in the visuo-spatial domain

In our previous work, we were able to establish that in addition to language (Roeper, 2011), the ability to represent recursive hierarchical embedding is available in the visual domain (Martins, Martins, & Fitch, 2015) (Figure 1). We have shown that both human adults and children (Martins, Martins, & Fitch, 2015; Martins, Laaha, Freiberger, Choi, & Fitch, 2014) are able spontaneously acquire a hierarchical self-similarity rule of the kind  $A \rightarrow A [A]$  and to use it to make judgments about well-formed visual fractals and violations

(Figure 2A). Crucially, this ability differed cognitively from a control, iteration task (Martins, Martins, & Fitch, 2015), in which participants made judgments about similar fractals which were not generated via a hierarchical embedding rule (Figure 2B). In particular, while visual iteration correlated strongly with visuo-spatial working memory and non-verbal intelligence, recursion correlated weakly with these measures, correlating instead with performance in the Tower of Hanoi, a recursive planning task (Martins, Martins, & Fitch, 2015).

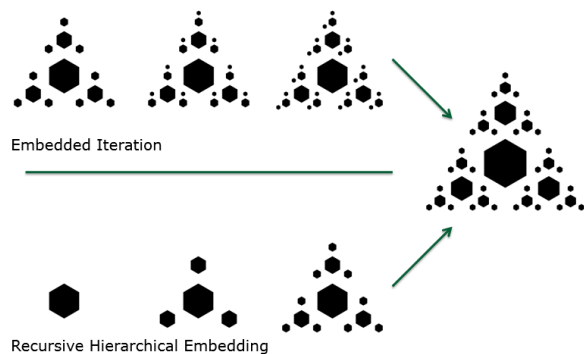


Figure 1: Examples of both ‘recursive’ and ‘iterative’ processes generating a visual fractal. While recursive hierarchical embedding steps generate new hierarchical levels, embedded iteration adds elements to fixed hierarchical levels, without generating new.

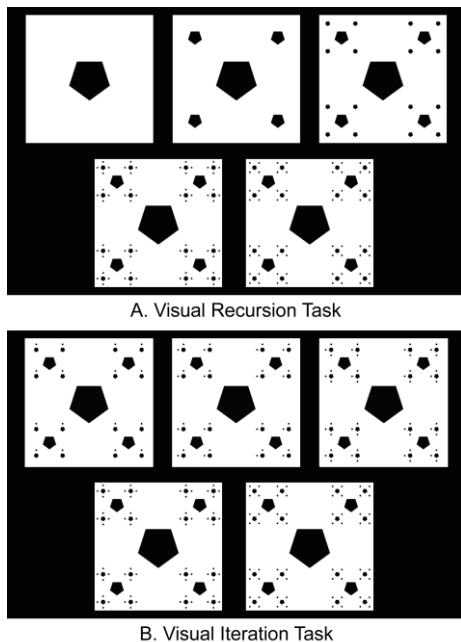


Figure 2: In both tasks participants are exposed to the first three steps generating a fractal (top row) and asked which image from the bottom row is the correct continuation. Despite using the same pairs of test images, the visual recursion and iteration tasks correlated with different abilities (Martins, Martins, & Fitch, 2015).

As in language (Roeper, 2011), the development path towards the acquisition of visual recursion requires the induction of simpler iterative (conjunctive) representations first, before recursive embedding becomes available (Martins, Laaha, Freiberger, Choi, & Fitch, 2014). However, we also found that the representation of visual recursion is independent of verbal resources (Martins, Mursic, Oh, & Fitch, 2015) and that it is not instantiated by the classical brain networks supporting language (Martins, Fischmeister, et al., 2014). Using fMRI, we found that the capacity to represent recursion in the visual domain is supported by the visual ventral stream and by structures (e.g. Medial Temporal Lobe) associated with the episodic memory system (Figure 3). Interestingly, we found that in comparison with non-recursive iterative procedures, recursion hinges mostly on top-down, internal representations (Fischmeister, Martins, Beisteiner, & Fitch, 2016), associated with the Default Mode Network and Semantic Memory. These findings provide some cues concerning the basic mechanisms underlying recursive hierarchical embedding and its usefulness for human cognition: by providing strong top-down priors, recursion can facilitate the processing of complex hierarchical structures.

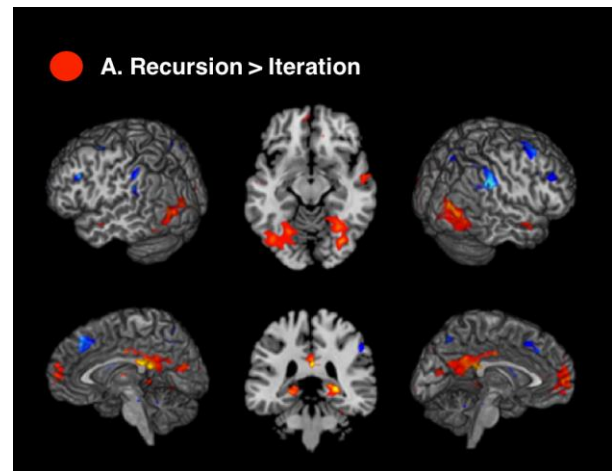


Figure 3: Brain networks supporting the representations of Recursion>Iteration (red) and Iteration>Recursion (blue) in the visuo-spatial domain (thresholded at a voxel-wise  $FDR\text{-adjusted } p < .05$  with a 10-voxel extent threshold).

### Recursion in the music and motor domains

Recently, we have extended this research to the musical and motor domains, where the availability of recursion has been previously suggested (Corballis, 2014; Jackendoff & Lerdahl, 2006). We found that both musicians and non-musicians can acquire rules governing recursive embedding in tonal hierarchies (Figure 4A) (Martins, Gingras, Puig-Waldmueller, & Fitch, 2017), and that this capacity shares resources with visual recursion and with recursive action planning (Tower of Hanoi). This suggests some degree of domain-generality. However, when we measured the brain activity underlying this

musical capacity in well-trained participants (using fMRI), we found little overlap between the musical and visual domains.

Here, we exposed 15 non-musicians to the first three steps forming a tonal fractal, either using recursive or iterative rules (Figure 4B and 4C), then gave them 4 seconds to try to imagine how the correct continuation (4<sup>th</sup> step) would sound like. This 4-second period was the ‘generation phase’. After the 4 seconds, they were exposed to a test stimulus (‘test phase’) and asked to judge whether this was a correct continuation or a foil. Overall, participants performed 4 sessions of 18 trials each (6 trials of Fractal, 6 of Iteration and 6 of Repetition)

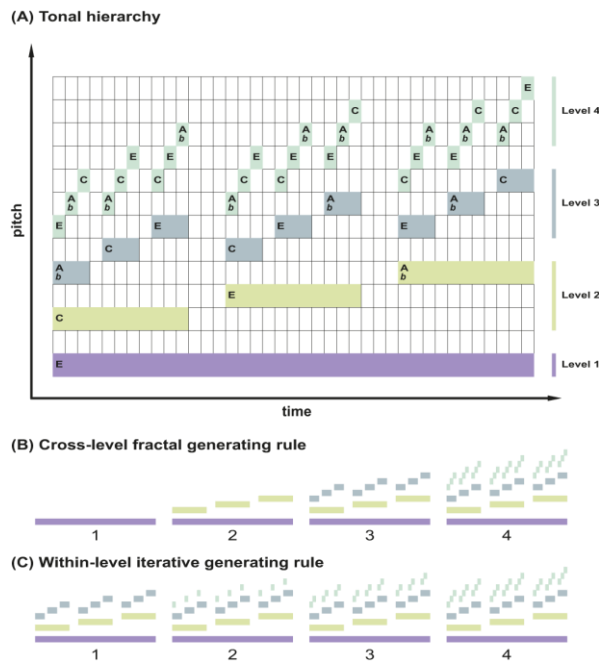


Figure 4: (A) Example of music fractal with tonal relations between levels, which were identical across all levels. (B) Cross-level fractal (recursive) rule, (C) Within-level (iterative) rule.

By contrasting Recursion>Iteration in the “generation phase” (while controlling for the activations in steps I, II and III) we found that recursive hierarchical embedding in music is supported by the primary and secondary Auditory Cortices in the left hemisphere and by the right Superior Temporal Gyrus, an area known to encode complex tonal relations (Figure 5) (Martins, Fischmeister, et al., in prep.). Interestingly, despite differences in the specific pattern of activation, we found again evidence that the representation of recursive embedding is supported by top-down processing: These activations occur in anticipation to a certain tonal sequence before it is played (“generation phase”), but only when the sequence was generated recursively (vs. iteratively). There were no task differences in the “test phase”, only a main effect of correctness (i.e. violation>well-formed structures).

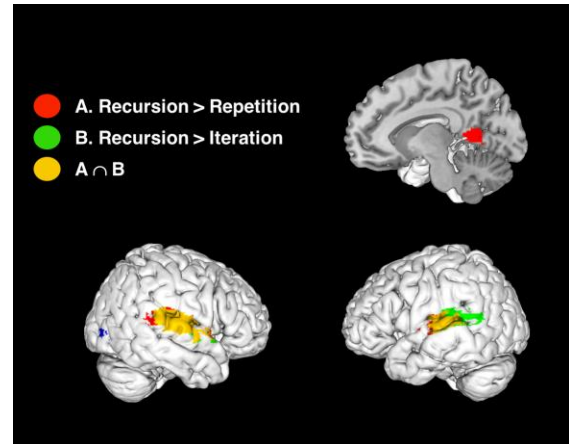


Figure 5: Brain network supporting the representation of recursive hierarchical embedding in tonal sequences during the “generation phase” ( $FWE\text{-adjusted } p_{cluster} < .05, p_{voxel} < .001$ ). The Repetition task was a working memory task, in which participants simply had to buffer step 3 (a complete and well-formed fractal) and then determine whether step 4 was a repetition of step 3. The opposite contrasts (Iteration>Recursion and Repetition>Recursion) yielded no activations.

Finally, we tested participants in the motor domain (Figure 6). Here we asked 20 (non-musician) participants to execute motor sequences on a 16-keys keyboard as depicted on a computer screen. Similar to the music domain, we exposed participants to the first two steps of a process generating motor fractals, then asked them to plan the next correct step (the “planning phase”) for 6 seconds. After the planning phase they were asked to execute the correct sequence on the keyboard without visual assistance (‘execution phase’). Iteration and Repetition baseline tasks were devised, similar to the music domain. Each participant performed 4 sessions of 20 trials each (8 Fractal, 8 Iteration and 4 Repetition trials).

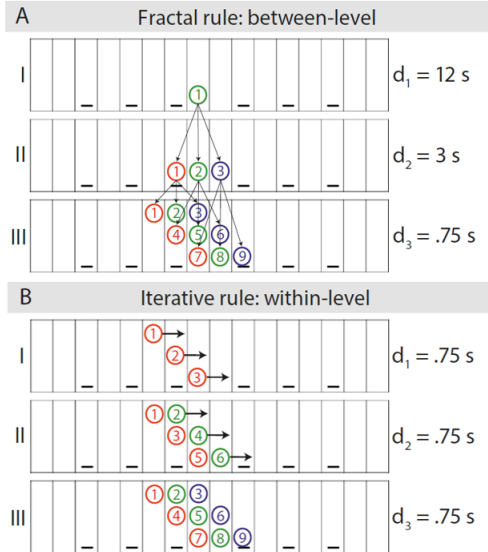


Figure 6: A) Example of a recursive process generating sequences of (silent) finger movements. Red, Green and Blue denote key presses with the thumb, index and middle fingers, respectively. On each step  $N$ , each key press is substituted by a sequence of 3 key presses with less than one third of the duration  $d_n$ . B) Example of an Iterative process generating the same motor fractal.

We found that during the planning of hierarchical motor sequences using recursive rules, participants activated a network known to instantiate motor planning and imagining, comprising the Somato-Motor and Premotor cortices bilaterally, Cerebellum and Basal Ganglia (Figure 7) (Martins, Bianco, Sammler, & Villringer, in prep.). Furthermore, we found that the underlying generating rule (Recursive>Iterative) changed how the execution of identical motor sequences were neurally represented: During the execution of a sequence formed using iterative rules, we found a strong activation in the primary motor hand area ( $x = -52, y = -18, z = 50, Z = 6.06, FWE p_{cluster} < .05, p_{voxel} < .001$ ). In contrast, when the sequence was formed recursively, we did not find a direct activation in the motor cortex, but a modulation of this area from a fronto-striatal cluster (PPI: Recursion>Iteration:  $x = -36, y = -24, z = 48, cluster\ extent = 182\ voxels, Z = 3.96, p_{voxel} = .016$ ).

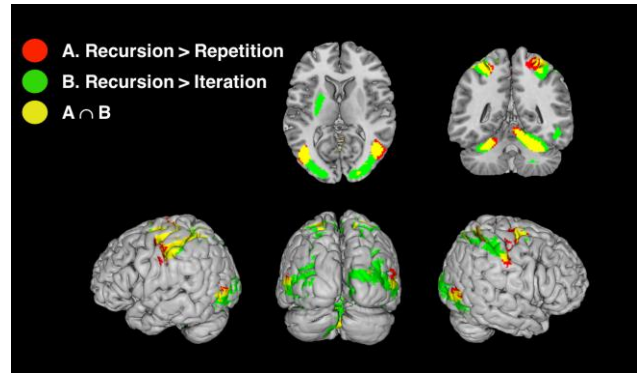


Figure 6: Brain network supporting the representation of recursive hierarchical embedding in the motor domain ( $FWE$ -adjusted  $p_{cluster} < .05, p_{voxel} < .001$ ).

### Discussion

Taken together, these results suggest several things: (1) The acquisition of recursive rules is probably supported by cognitive resources that are general across domains; (2) However, when we test participants that are well-trained and at ceiling performance in the fMRI, their representation of recursion is instantiated by domain-specific neural systems; (3) In contrast with other (iterative) rules applied to hierarchies, recursion seems to allow a controlled top-down processing, in both discrimination (visual and tonal) and production (motor) of well-formed hierarchical structures. This result is consistent across several domains.

The apparent contradiction between points (1) and (2) can be solved if we surmise that the resources necessary to acquire recursive rules are different from those necessary to utilize these rules after extensive training. We hypothesize that acquisition requires domain-general resources, which are perhaps slow and effortful, while expert use is instantiated by activating schemas stored in domain-specific repositories, which are formed after a process of automatization. The answer to this question requires novel research investigating the neural networks supporting the acquisition of recursive rules.

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