Pellet cloud expansion in hot plasmas

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It has been demonstrated that the injection of cryogenic pellets into a magnetically confined plasma is accompanied by a considerable transfer of thermal energy from the electrons of the background plasma to the ions [1]. This is the result of the ambipolar expansion along the magnetic field line of the cold and dense plasma cloud left behind by the ablated pellet. Although relatively dense, this cloud is essentially transparent for the ambient hot electrons that fly through. Therefore, during this expansion, the cloud is constantly heated by the hot background plasma.

In a simplified model, where the cloud ions are assumed to be cold, such one dimensional expansion is governed by the following system of hydrodynamic equations

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nu)=0, \tag{1}$$

$$m_i\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)=-e\frac{\partial \phi}{\partial x}=-T(t)\frac{\partial \ln n}{\partial x}, \tag{2}$$

$$\frac{d}{dt}\int_{-\infty}^{\infty}\left(\frac{3nT}{2}+\frac{m_i nu^2}{2}\right)dx=\int_{-\infty}^{\infty}Q(t)ndx, \tag{3}$$

where n is the plasma cloud density (assumed to be much greater than the background plasma density), u denotes the ion velocity, ϕ the potential, m_i the mass of the ions, T the cloud electron temperature (assumed uniform) and Q the heating power per particle. Ref. [1] suggested a self-similar solution for the system of hydrodynamic equations (1)-(3):

$$\begin{aligned} n(x,t) &= n_0\sqrt{\frac{3m_i}{8\pi\tau t^3}}\ \exp\left(-\frac{3m_i x^2}{8\tau t^3}\right),\\ u(x,t) &= \frac{3x}{2t},\\ T(t) &= t\tau, \end{aligned} \tag{4}$$

where $\tau=\frac{1}{3n_0}\int_{-\infty}^{\infty}nQ\,dx$ is the heating power (assumed to be constant in time). A notable feature of this solution is that the cloud electron temperature is half of what it would have been if the pellet cloud were stationary. Therefore, half the heating power goes into the ion kinetic energy associated with the expansion of the cloud, significantly affecting the energy balance of the pellet-fuelled plasma.

Expansion with a non constant heating $Q(t)$.

The self-similar solution (4) is only applicable when Q is constant in time, which is not the case in the late stages of pellet cloud evolution, since the collisional energy exchange between the ambient plasma (with temperature T_{am}) and the cloud is proportional to $(T_{am} - T)$, namely

$$nQ(T) \approx \frac{3n(T_{am} - T)}{\tau_{hh}(1 + T/T_{am})^{3/2}}, \tag{5}$$

where τ_{hh} is the electron collisional time in the ambient plasma (Eq. (2.5e) in [2]).

Equations (1) - (3) admit solutions with Gaussian density and linear velocity profiles:

$$n(x,t) = n_0 \sqrt{\frac{a(t)}{\pi}} \exp\left(-a(t)x^2\right), \tag{6}$$

$$u(x,t) = b(t)x. \tag{7}$$

This Ansatz reduces equations (1) - (3) to a set of three ordinary differential equations:

$$\begin{aligned} &\dot{a} + 2ab = 0, \\ &\dot{b} + b^2 = 2aT/m_i, \\ &\frac{3}{2}\dot{T} + bT = Q(t). \end{aligned} \tag{8}$$

Note that the cloud density scale n_0 does not appear in these equations when the heating function $Q(t)$ is defined by the collisional energy exchange expression (5). Therefore, the solution of the system (8) describes the expansion of a cloud of any initial density. Figure 1 shows the numerical solution of the system (8) with (5) for $T_{am} = 1$ keV and the ambient plasma density of $2 \cdot 10^{19} m^{-3}$. Unlike the case of constant heating, the ion energy now exceeds the electron

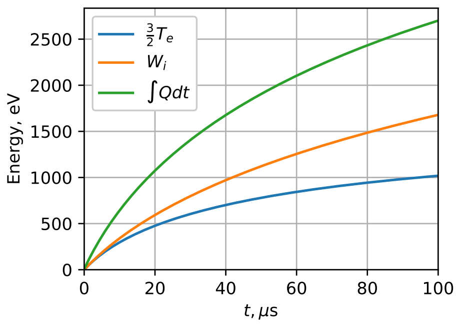

Figure 1. Evolution of the cloud electron thermal energy and ion kinetic energy $W_i = \int_{-\infty}^{\infty} \frac{m_i n u^2}{2} dx = \frac{m_i b^2}{4a}$, derived from the solution of equations (8) with (5).

energy throughout the expansion and more than half the energy transferred to the pellet cloud ends up in the ions.

Comparison with a more complete system.

The two models described above assume zero temperature of the cloud ions, uniform electron temperature, and negligible viscosity. A novel Lagrangian code has been developed to

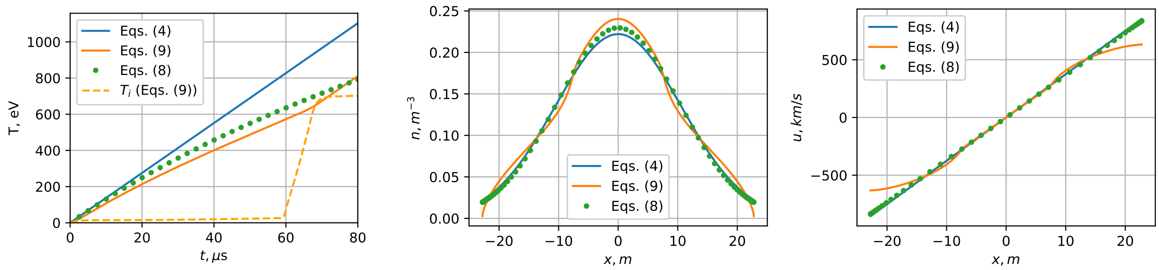

Figure 2. Evolution of the cloud electron (and ion) temperature, and snapshots of the cloud density (n) and the ion velocity (u) from the self-similar solution (4), the solution of the ODEs (8), and the full system (9) at $t = 40$ μs. The ambient plasma temperature is $T_{am} = 4$ keV and the corresponding density is $2 \cdot 10^{19} m^{-3}$.

solve a more complete system of Braginskii equations,

$$\begin{aligned}
&\frac{\partial n}{\partial t} + \frac{\partial nu}{\partial x} = S_n \\
&\frac{\partial m_i nu}{\partial t} + \frac{\partial}{\partial x}\left(m_i nu^2 + n(T_e + T_i) - \frac{4}{3}\eta_i \frac{\partial u}{\partial x}\right) = S_u \\
&\frac{\partial}{\partial t}\left(\frac{3}{2}nT_i\right) + \frac{\partial}{\partial x}\left(\frac{5}{2}nuT_i - \kappa_i \frac{\partial T_i}{\partial x}\right) = S_{Ti} + u\frac{\partial nT_i}{\partial x} + Q_{ei} + \frac{4}{3}\eta_i\left(\frac{\partial u}{\partial x}\right)^2 + \frac{m_i nu^2}{2}S_n - uS_u \\
&\frac{\partial}{\partial t}\left(\frac{3}{2}nT_e\right) + \frac{\partial}{\partial x}\left(\frac{5}{2}nuT_e - \kappa_e \frac{\partial T_e}{\partial x}\right) = S_{Te} + u\frac{\partial nT_e}{\partial x} + Q_{ie},
\end{aligned} \tag{9}$$

where η and κ denote the viscosity and thermal conductivity, respectively, and Q_{ab} is the collisional energy exchange between the species. The sources S are due to collisions with the ambient plasma and are calculated from integrals of the BGK kinetic equation, which is solved self-consiestently with equations (9). Self-collisions within the ambient plasma are ignored when solving this kinetic equation.

A comparison of the evolution of the cloud electron temperature between the three models is shown in figure 2. For the case of equations (9), only the central ($x = 0$) value is plotted, but the electron temperature profile remains nearly flat during most of the expansion. The dashed curve in figure 2 shows the evolution of the central ion temperature resulting from the full Braginskii modelling. Density and velocity profiles at $t = 40$ μs are also compared in figure 2.

This comparison shows excellent agreement between the full Braginskii modelling and the simplified equations (8) during most of the evolution. Note that the characteristic size of the cloud reaches 30 m at the time $t = 50$ μs when the models diverge. The reason for the disagreement is an abrupt viscous slowing down of the expansion in the full Braginskii modelling, which occurs when the central ion temperature increases as a result of heat conduction and inter-species heat exchange. The parallel viscosity then becomes very large, but at this point the ion mean free path starts exceeding the characteristic length of the cloud and the hydrodynamic approximation becomes invalid.

Anisotropy of the ion distribution function.

The ion kinetic energy will eventually (after a few ion collision times) end up as thermal ion

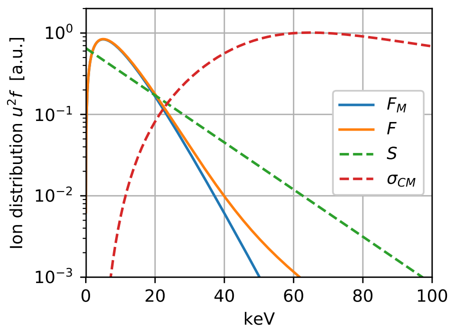

Figure 3. Quasi-steady-state ion distribution function ($F = u^2 f$), Maxwellian distribution ($F_M = u^2 f_M$), the super-thermal ion source ($S = u^2 S(t^*)$) and D–T fusion reactivity (σ_{CM}).

energy, and the electron-ion energy transfer due to ambipolar cloud expansion is a competitive ion heating mechanism. The effect of this mechanism on the global plasma energy balance is, in general, an increase of the ion temperature, as discussed in greater detail in Ref. [1].

Note that the ion distribution function resulting from the expansion is not Maxwellian, but is skewed toward higher energies. Using equations (4), we find the following source term of fast ions;

$$S(t) = 2\frac{n_0}{u^2}\sqrt{\frac{1}{\pi}\frac{m_i}{6\tau t}}\exp\left[-\frac{m_i u^2}{6\tau t}\right]. \tag{10}$$

The presence of such a supra-thermal ion population has a positive impact on fusion reactivity. Although the initial acceleration of the ions is along the field line, we take advantage of the fact that their isotropization is faster than slowing down. The kinetic equation for such super-thermal ions can be written as approximately as

$$\frac{\partial f}{\partial t} = \frac{1}{u^2\tau_s}\frac{\partial}{\partial u}\left[(u_c^3 + u^3)f + \left(u_c^3 + u^3\frac{T_e}{T_i}\right)\frac{T_i}{m_i u}\frac{\partial f}{\partial u}\right] + \nu S(t^*), \tag{11}$$

where ν is the pellet injection frequency, $u_c = (3\sqrt{\pi}m_e/4m_i)^{1/3}u_{T_e}$ and t^* is the cloud homogenization time. A quasi-steady-state ion distribution function is found from the numerical solution of equation (11) for $T_e = T_i = 5$ keV, $t^* = 2\tau_{hh}$ (which is toward the end of the expansion) and a pellet injection frequency ν corresponding to a doubling of the plasma density in one ion slowing-down time $\tau_s = \sqrt{m_i/m_e}(T_e/T_i)^{3/2}\tau_{ii} \approx 1s$. (A uniform sink term keeps the total ion density constant.) Figure 3 shows this quasi-steady-state ion distribution: Its convolution with the D–T fusion cross-section predicts an increase in the fusion rate of up to 30%, falling to below 10% if the plasma is hotter (>10 keV) or the pellet injection rate lower.

References

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