Observations on edge GAM-turbulence interactions in ASDEX Upgrade

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1. Introduction

The interaction of Geodesic Acoustic Modes (GAMs), Zonal Flows and small-scale turbulence is an important topic in magnetic confinement studies. In recent years much progress has been made in the area of measurement, interpretation and numerical simulation. Previously on ASDEX Upgrade tokamak (AUG) microwave Doppler reflectometry (DR) has been used to investigate the temporal and spatial behaviour of coherent edge GAMs [1]. Here, new results are presented on the non-linear interaction of GAMs with both the background incoherent flow $u_\perp$ fluctuations and the ambient density $n_e$ turbulence.

GAMs (few kHz, $m = n = 0$, coherent $E \times B$ flow oscillations) generally exist in the tokamak edge between the separatrix and the $n_e$ pedestal. For weak pedestals they may extend significantly further in, depending on Landau damping. Fig. 1(a) shows a perpendicular velocity $u_\perp = 2\pi f_D/k_\perp$ profile, plus DR shift $f_D$ and amp. $A_D \propto \tilde{n}_e$ spectra for a low density, NBI heated, lower-single-null (LSN) L-mode. The GAM p.t.p. can reach $>30\%$ of $u_\perp$ at the tokamak outer mid-plane [1a].

2. Zonal structure

The GAM radial structure is zonal with either a continuum frequency ($f_{GAM} \propto c_s$) or one or more eigenmodes ($f_{GAM} =$ constant), as in fig. 1 (shaded region), or even multi-mode, as in fig. 5 for a circular, limiter L-mode. What determines the zonal structure is a topic of current investigation. The GAM amplitude $A_{GAM}$ profile is asymmetric, generally peaking towards the outer zonal boundary. In fig. 1(c) the maxima is in the $n_e$ gradient region where the turbulence drive, i.e. pressure gradient $\nabla_r P$, is large but the flow shear $\nabla_r u_\perp$ is weak. The zonal width is usually a few cm, consistent with meso-scale wavelengths $\lambda_r \sim \sqrt{a \rho_i} \sim 3 - 4$ cm, where $a$ and $\rho_i$ are plasma minor radius and ion Larmor radius.

![Fig. 1: (a) $u_\perp$, (b) $f_{GAM}$, (c) $A_{GAM}$ and flow $\sigma_{f_D}$ profiles for LSN L-mode #34930.](image-url)
3. Non-linear drive

Bispectral & biphase analysis has been used widely to study the non-linear drive of the GAM via 3-wave coupling, cf. [2-5]. The squared bicoherence is given by:

\[ b^2(f_1, f_2) = |B(f_1, f_2)|^2 / \langle |X(f_1)X(f_2)|^2 \rangle \langle |X(f_3)|^2 \rangle, \]

with \( B(f_1, f_2) = \langle X(f_1)X(f_2)X^*(f_3) \rangle \) the bispectrum, \( f_3 = f_1 \pm f_2 \), and Fourier mode \( X(f_n) \).

Fig. 2 shows the flow auto-bicoherence at the GAM radial peak in #34930. The coupling of the broad-band fluctuations to the GAM appears as ridges at \( f_{GAM} \) in \( f_1, f_1 \pm f_2, f_3 \), cf. \( b^2(f_3) \) spectra. The total bicoherence \( \overline{b^2} = \sum_{f_1} \sum_{f_2} b^2(f_1, f_2) / N \), i.e. \( b^2 \) integrated over \( f_1 \) and \( f_2 \), is an indicator of the total non-linear activity. In fig. 3(a) the radial profiles of \( \overline{b^2} \) and \( b^2(f_3) = \sum_{f_3=f_1+f_2} b^2(f_1, f_2) \) at \( f_3 = f_{GAM} \) (GAM coupling) show maxima aligning with the GAM intensity \( A_{GAM} \) peak, fig. 1(c). This suggests the GAM is driven at the location of maximum turbulence and non-linear coupling, and then spreads radially inward and outward towards the zonal boundaries, cf. the geometrical optics description [6].

Towards the core, as the GAM weakens, the \( f_D \) flow spectra flattens and the flow fluctuation RMS level \( \sigma_{f_D} \) rises, fig. 1(c), coinciding with a significant increase in the total (full-band) \( \overline{b^2} \) coupling - open triangles in fig. 3(a) - although the \( n_e \) turbulence falls.

4. Energy transfer

While the bispectra shows GAM-turb. coupling, it does not give the energy transfer direction of the coupling. One technique, based theory-wise on the high freq. \( n_e \) turbulence being modulated by the GAM, involves high-pass filtering the \( A_D \) (turb.) signal then extracting the envelope using a Hilbert transform \( \text{Env}(A_D) = |A_D + iH(A_D)| \) [2,4]. Fig. 4 shows segments of band-pass filtered \( f_D (f_{GAM} \pm 1 \text{ kHz}) \) flow and \( \text{Env}(A_D [30-300] \text{ kHz}) \) signals which are correlated (cf. dashed lines) confirming the HF \( n_e \) fluctuations are modulated at the GAM frequency.

Fig. 2: (a) Flow \( \tilde{u}_\perp \) auto-bicoherence \( b^2 \), (b) \( b^2(f_3) \) spectrum at GAM maxima, #34930.

Fig. 3: (a) \( \overline{b^2} \) and \( b^2(f_{GAM}) \) and (b) flow-turb. coherence \( \gamma \) & cross-phase \( \Theta \) profiles, #34930.
and thus non-linear coupled. From the relative phase the GAM generally leads the turbulence, but the phase delay shifts and reverses with time and radial position. Fig. 3(b) shows the profile of coherence $\gamma$ and cross-phase $\Theta$ between $f_D$ and $\text{Env}(A_D)$ at $f_{GAM}$. As with $b^2(f_3)$, the $\gamma$ also peaks at the GAM outer edge while $\Theta$ progressively varies (and reverses) across the GAM peak - consistent with radial propagation velocities of a few hundred ms$^{-1}$ [7] - implying the GAM and turbulence decorrelate as the GAM propagates.

5. GAM frequency modulation
At its spatial maxima the GAM is highly coherent with a spectral width $\Delta f_{\text{fwhm}}$ (indicative of the GAM lifetime) of only a few hundred Hz. However, at the zonal boundaries the GAM spectral peak broadens to several kHz as it weakens and decays faster, as shown by the $\Delta f$ profile (down-triangles) in fig. 5(c) for the circular, limiter L-mode #34953.

In this discharge there is a radial region where two continuum GAMs appear to overlap and, as shown by the amplitude profiles, switch dominance. Nevertheless, even for single eigenmode or continuum GAMs, the GAM spectral peak will exhibit a fine frequency splitting - as shown in fig. 6(a) for LSN L-mode #29725. The effect is more clearly seen in the corresponding band-pass filtered $f_D$ auto-correlation of fig. 6(b) by double decay times $\tau_x$, with $\rho \propto \exp(-|\tau/\tau_x|)$. Where a single GAM is dominant, fig. 5(a), the initial short decay $\tau_S^{-1} \sim f_{GAM}$, is set by the GAM frequency, while the later long decay is inverse to the spectral width, $0.5/\tau_L \sim \Delta f_{\text{fwhm}}$. With multiple GAM peaks there is strong inter-modulation and $1/\tau_S \ll f_{GAM}$. This behaviour is indicative of a low frequency modulation (FM) of the GAM.

6. GAM amplitude modulation
The GAM amplitude $A_{GAM}$ also displays a low frequency (typically between 40 – 400 Hz) intermittency or ‘breathing’ with up to 50% modulation, roughly correlated, but phase-shifted to a corresponding modulation of $f_{GAM}$ [1a].
The GAM temporal modulation is more clearly seen in the $f_D$ (flow) spectrogram of fig. 7(a) for #29725. Below, are time traces of integrated power over the GAM peak ($P_{f_D(GAM)} : 12 - 18$ kHz) and the high frequency HF band $P_{f_D(HF)} > 18$ kHz, as well as the corresponding HF $n_e$ turbulence $P_{A_{D(HF)}}$. The adjacent power spectra show the GAM and $n_e$ turb. primarily LF modulated, around 47 Hz with other peaks at 124 Hz and 260 Hz. Generally, $P_{f_D(GAM)}$ and $P_{f_D(HF)}$ are out of phase [1a], i.e. energy moves within the flow spectrum, while $P_{f_D(GAM)}$ and $P_{A_{D(HF)}}$ are in-phase at 47 Hz, but otherwise phase-shifted.

### 7. Conclusions

Bispectral analysis shows the non-linear coupling of the broad-band flow $u_\perp$ and $n_e$ turbulence to the GAM at a narrow radial location, corresponding to a strong $\nabla P$ and weak $\nabla u_\perp$ region. Towards the zonal edges the GAM decays faster - suggesting the energy spreads radially away from the GAM maxima as the GAM propagates. While the GAM tends to lead the $n_e$ turbulence (i.e. GAM modifies turb.) their relative phase varies and reverses in time. Taken with the spectral behaviour this suggests the energy moves back and forth between the GAM and the turbulence both temporally and spectrally. The low frequency modulation of the GAM and the HF turbulence is suggestive of either a LF zonal flow oscillation, or close frequency beating between two radially overlapping GAMS.

### References