Modelling of neoclassical toroidal viscosity from internal MHD modes in ASDEX-Upgrade

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Corrugations of tokamak magnetic flux surfaces in the location of a kink mode and/or around the islands created by a tearing mode introduce in combination with toroidal magnetic field inhomogeneity 3D modulations of magnetic field strength within the perturbed flux surfaces and, respectively, give rise to non-ambipolar neoclassical transport [1]. This transport polarizes the plasma changing its rotation, i.e. effectively exerting the neoclassical toroidal viscous (NTV) torque onto it. Generally, NTV torque is a strong function of toroidal plasma rotation frequency with respect to the reference frame where perturbations are (quasi) static. Due to the summary toroidal momentum conservation of plasma and electromagnetic field, any torque onto the plasma from internal modes, which do not interact with plasma exterior, results from electromagnetic momentum change of the modes. Since this momentum is very small, the condition of zero torque determines mode eigenfrequency.

In this work, the typical situation of coupled (3,2) tearing and (2,2) kink modes in ASDEX Upgrade (AUG) is modelled with help of the code NEO-2 [2, 3] for realistic equilibrium field and plasma parameters with perturbation field described in terms of radial flux surface displacement fitted to experimental observations. Namely, two time slices, $t = 2.68$ s and $t = 3.52$ s of AUG shot 35568 (H-mode, $I_p = 1$ MA, $B_t = 2.5$ T, $\kappa = 1.861$, $\delta_o = 0.086$, $\delta_u = 0.475$) with plasma parameters shown in Fig. 1 were studied where a quasi-stable coupled mode has been observed with frequencies 24 kHz and 14 kHz, respectively. The perturbed magnetic field B and cylindrical coordinates (R,Z) expressed as functions of Boozer coordinates $(\rho_{\mathrm{tor}}, \vartheta, \varphi)$ where $\rho_{\mathrm{tor}} = (\psi_{\mathrm{tor}}/\psi^a_{\mathrm{tor}})^{1/2}$ is the normalized toroidal radius were modelled from unperturbed, axisymmetric ones, B_0 and (R_0, Z_0) as follows, $B(\rho_{\mathrm{tor}}, \vartheta, \varphi) = B_0(\rho_{\mathrm{tor}} + \Delta\rho, \vartheta)$, etc. Here, $\Delta\rho = \Delta\rho(\rho_{\mathrm{tor}}, \vartheta, \varphi)$ is radial displacement which for both modes has the form

$$\Delta\rho = \Delta\rho_A(\rho_{\mathrm{tor}}) \cos(m\vartheta - n\varphi + \phi_0) \tag{1}$$

with $(m,n) = (2,2)$, $\phi_0 = 0$ and $(m,n) = (3,2)$, $\phi_0 = \pi$ for kink and tearing mode, respectively. Typical displacement amplitudes together with corrugated flux surfaces in presence of both modes (displacements are added) are shown in Fig. 2. Obviously, the above model assumes that non-axisymmetric modulation of the magnetic field strength B within perturbed flux surfaces comes purely from surface meandering which is a dominant effect in case of mainly transversal

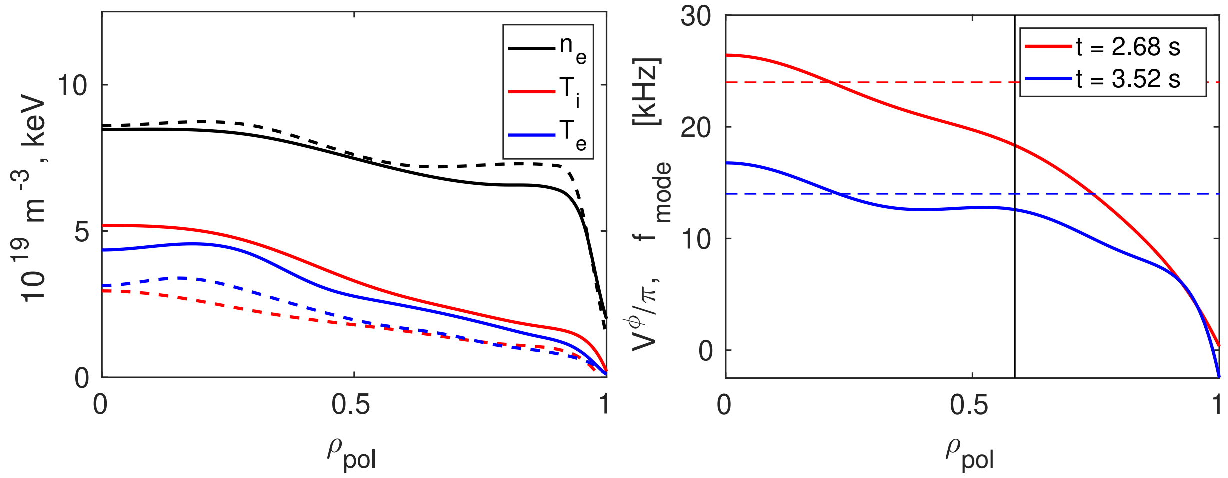

Figure 1: Left - density, ion and electron temperature for $t = 2.68$ s (solid) and $t = 3.52$ s (dashed) as functions of normalized poloidal radius $\rho_{\text{pol}} = \left(\psi_{\text{pol}}/\psi^a_{\text{pol}}\right)^{1/2}$. Right - toroidal rotation velocity V^φ in terms of mode frequency, $nV^\varphi/(2\pi)$. Black line indicates position of $q = 3/2$ surface, dashed lines - observed mode frequencies.

field perturbations. NTV torque density T^{NA}_φ has been computed via a flux force relation in

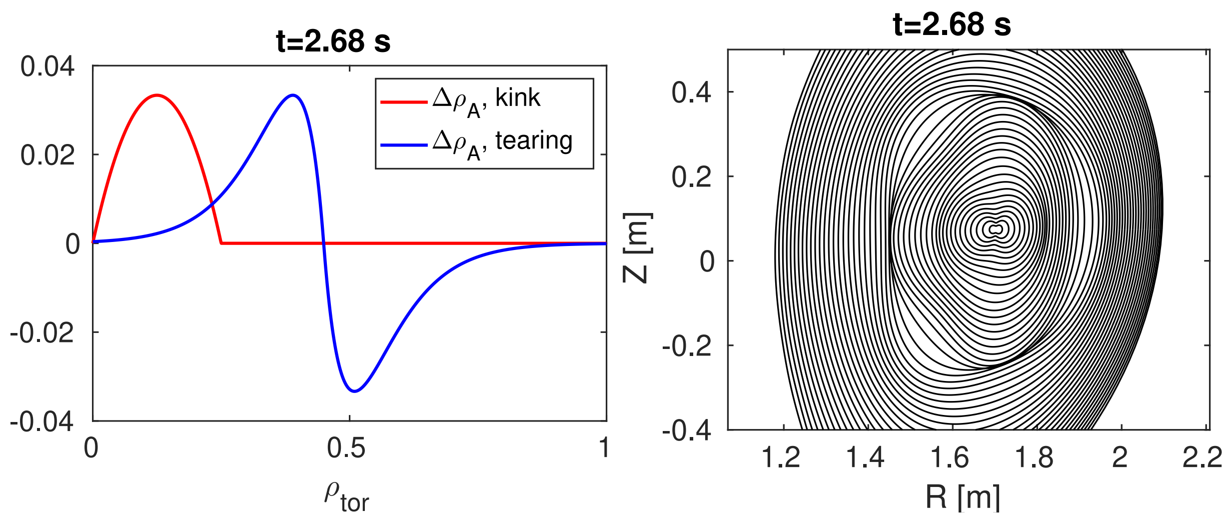

Figure 2: Amplitudes of radial displacements vs toroidal radius (left) and corrugated flux surfaces in the cross-section $\varphi = 0$ (right).

the reference frame rotating toroidally together with the perturbation (with angular frequency $\Omega = 2\pi f/n$ where f is the ordinary $n = 2$ perturbation frequency) as follows,

$$T^{NA}_\varphi = -\frac{1}{c}\frac{\mathrm{d}\psi_{\text{pol}}}{\mathrm{d}r}\sum_\alpha e_\alpha \Gamma_\alpha, \qquad \Gamma_\alpha = -n_\alpha \sum_{\alpha'}\sum_{k=1}^{3} D^{\alpha\alpha'}_{1k} A^{\alpha'}_k, \tag{2}$$

where $\mathrm{d}\psi_{\text{pol}}/\mathrm{d}r$ is the derivative of poloidal flux over effective radius r, $\alpha, \alpha' = e, i$ denote particle species, Γ^{NA}_α is α-species flux density averaged over the magnetic surface, $D^{\alpha\alpha'}_{jk}$ are neoclassical transport coefficients computed by NEO-2, and A^α_k are thermodynamic forces related to gradients of plasma parameters as follows,

$$A^\alpha_1 = \frac{1}{n_\alpha}\frac{\partial n_\alpha}{\partial r} - \frac{e_\alpha E_r}{T_\alpha} - \frac{3}{2T_\alpha}\frac{\partial T_\alpha}{\partial r}, \qquad A^\alpha_2 = \frac{1}{T_\alpha}\frac{\partial T_\alpha}{\partial r}, \qquad A^\alpha_3 = \frac{e_\alpha}{T_\alpha}\frac{\langle E_\parallel B\rangle}{\langle B^2\rangle}. \tag{3}$$

Here, e_α, n_α and T_α are α-species charge density and temperature, respectively, E_r is radial electric field in the rotating frame, $E_\parallel$ is the inductive electric field, and $\langle \ldots \rangle$ is magnetic surface average. NTV torque density (2) can also be conveniently cast to the generic form

$$T_\varphi^{NA} = -\nu_s m_i n_i \left\langle R^2 \left(V^\varphi - V_{\rm in}^\varphi - \Omega \right) \right\rangle \tag{4}$$

where m_i is ion mass, V^φ is toroidal angular plasma rotation frequency and ν_s and $V_{\rm in}^\varphi$ are slowing down rate and NTV intrinsic rotation velocity ("NTV off-set"), see, e.g., Ref. [2]. Integrating

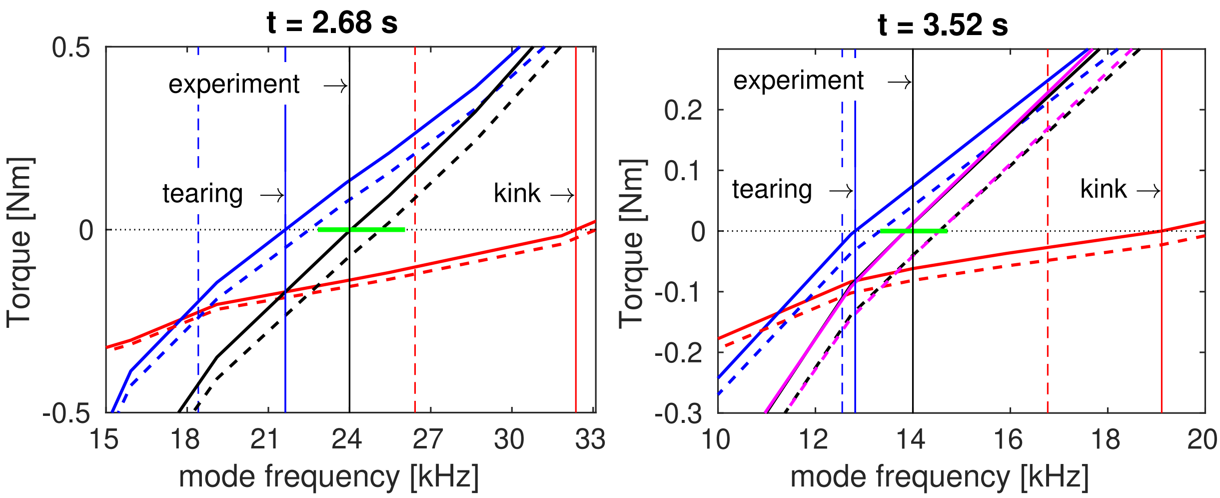

Figure 3: Integral torque at plasma boundary, $T_\varphi^{\rm int}(1)$, as function of mode frequency for kink mode alone (red) tearing mode alone (blue) and their sum (black) together with the torque with superimposed modes (magenta). Summary torque (ion+electron) and ion contribution alone are shown by thick solid and dashed lines, respectively. Solid vertical lines indicate eigenfrequencies for kink (red) and tearing (blue) modes alone and the measurement (black). Dashed vertical lines show rotation frequencies at magnetic axis (red) and at $q = 3/2$ surface (blue). Green segment shows the range of eigenfrequencies if relative mode amplitudes are varied between 2/3 and 3/2.

torque density (2) over the volume $V\left(\rho_{\rm pol}\right)$ limited by the flux surface $S\left(\rho_{\rm pol}\right)$ with given $\rho_{\rm pol}$ one can approximately relate this integral to the electromagnetic momentum flux F_{EM},

$$T_\varphi^{\rm int}\left(\rho_{\rm pol}\right) = \int_{V\left(\rho_{\rm pol}\right)} {\rm d}^3 r\, T_\varphi^{NA} = -F_{EM} = \frac{1}{4\pi} \int_{S\left(\rho_{\rm pol}\right)} {\rm d}\mathbf{S} \cdot \mathbf{B}\,\mathbf{B} \cdot \frac{\partial \mathbf{r}}{\partial \varphi}, \tag{5}$$

where $S\left(\rho_{\rm pol}\right)$ and φ correspond to the unperturbed field, and $\mathbf{B}$ is perturbed. Last equality, which has been obtained ignoring negligible small variation of electromagnetic momentum in time and contribution of electric field to Maxwell stress tensor, is only formal here and indicates that $T_\varphi^{\rm int}$ must be zero at the last closed surface for intrinsic MHD perturbations (eigenmodes). Therefore, condition $T_\varphi^{\rm int}(1) = 0$ on reference frame velocity Ω determines the eigenmode frequency. Fig. 3 shows $T_\varphi^{\rm int}(1)$ as function of mode frequency in some range around observed mode frequencies which includes also eigenfrequencies of kink and tearing modes alone. The best match to the experimental frequency is obtained for combination of kink and tearing mode displacements in Fig. 2 with equal amplitudes which is in agreement with earlier experimental observation that mode amplitudes are of the same order [4]. Note that it is mainly the relation between amplitudes which determines mode frequency but not their absolute value which affects mainly the magnitude of the torque but not its frequency dependence. In case of static mode, $\Omega = 0$, NTV torque for combined modes is -2.2 Nm and -1.2 Nm for 2.68 s and 3.52 s, respectively, which is comparable to the NBI torque in AUG. Torque is predominantly due to

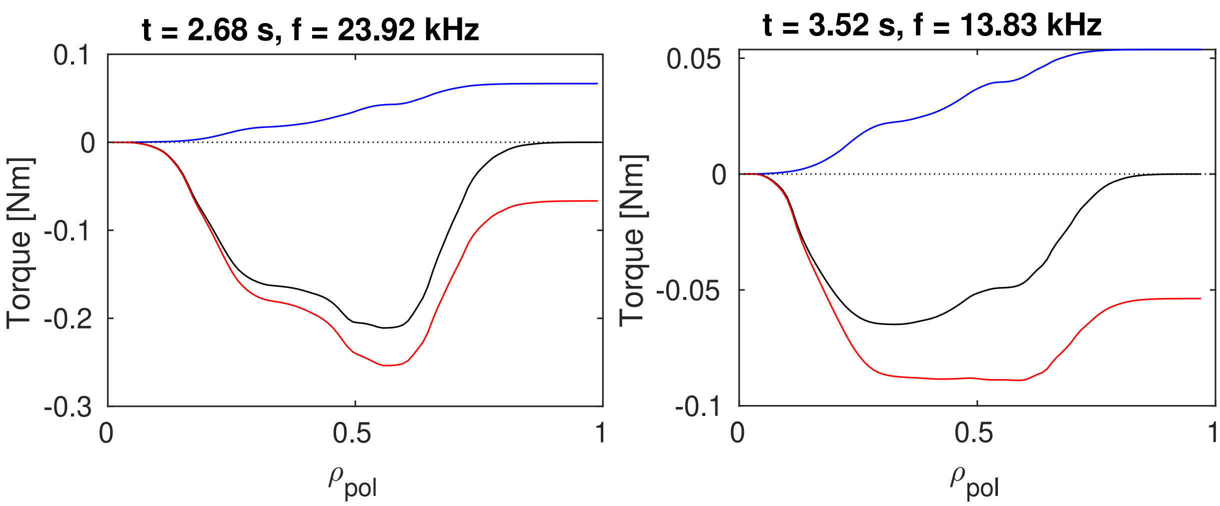

Figure 4: Radial profiles of the integral torque $T_\varphi^{\text{int}}\left(\rho_{\text{pol}}\right)$ at mode eigenfrequency (see the title). Contributions of ions and electrons and the total torque are shown in red, blue and black, respectively.

the non-ambipolar ion transport in the regime of bounce-resonances [3] which leads to mode eigenfrequencies higher than plasma rotation frequency at mode location (negative off-set in (4), $V_{\text{in}}^{\varphi} < 0$). Electrons provide only a small positive contribution to V_{in}^{φ}. Signs of ion and electron contributions are in agreement with experimental observations [5] and NTV theory which predicts outward transport and, respectively, opposite signs of torques due to electrons and ions, see Fig. 4. Since non-ambipolar transport is significant only for low-collisional components it may affect the concentration of high-Z impurities which are almost unaffected by this transport.

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