Reply to Comment on "Is a Trineutron Resonance Lower in Energy than a Tetraneutron Resonance?" by A. Deltuva and R. Lazauskas

S. Gandolfi,[1, *] H.-W. Hammer,[2, 3, †] P. Klos,[2, 3, ‡] J. E. Lynn,[2, 3, §] and A. Schwenk[2, 3, 4, ¶]
[1] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*
[2] *Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany*
[3] *ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany*
[4] *Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*

In Ref. [1] we presented calculations for three- and four-neutron ($3n$ and $4n$) states in the presence of an external trapping potential. These calculations were extrapolated to the limit of zero trap depth, as in Ref. [2], and showed the remarkable feature that these extrapolations point to a common positive energy scale, independent of the trap geometries considered. Based on calculations for a two-body resonance, where the same extrapolation procedure correctly locates the resonance energy, we suggested that our results support the possible observation of a tetraneutron resonance [3], and provide indications that a $3n$ resonance might also exist at an energy below a possible $4n$ resonance. We did not claim that a $3n$ or $4n$ resonance definitely exists, nor did we quantify their widths.

The question of few-neutron resonances is an interesting and challenging problem with many conflicting theoretical results at present [1, 2, 4–10] including Ref. [11], which we regrettably missed in our Letter and the more recent Ref. [12], which already put forward the arguments raised in the Comment by Deltuva and Lazauskas [13].

The arguments presented in the Comment largely rely on ideas related to the analytic continuation in the coupling constant (ACCC) method, where the Hamiltonian of the system is written as $H(\lambda) = H_0 + \lambda H_{\text{att}}$, with H_{att} the attractive part of the ($\lambda = 1$) original Hamiltonian. However, we point out that ACCC is not the same as applying an external trap, which is the procedure we employ in our Letter. Therefore, while we agree that bound dineutrons emerge early on as the trap depth V_0 is nonzero, we do not agree with the authors' conclusion that "in the presumed $E_{4n} \approx 0$ region ... [t]he tetraneutron states ... are not true bound states." It is not clear what the authors mean by "true bound states." Bound states are states whose wave functions have compact support. This is the case for all of our calculated $3n$ and $4n$ states in the trap. In our Letter, we used the auxiliary field diffusion Monte Carlo method, which converges to the lowest energy eigenstate with the relevant quantum numbers of a given Hamiltonian. There are cases where diffusion Monte Carlo methods have been applied to states that decay. For example, the unbound nucleus ^{8}Be was calculated using the Green's function Monte Carlo method in Ref. [14]. In this case, the states decay asymptotically to two α clusters, and this decay is observed clearly, e.g., in the evolution of the 4^+ energy even after a short imaginary time, as shown in the inset of Fig. 1. As Fig. 1 also shows, for the $4n$ system in the region in question, where $E_{2n} < E_{4n}$ (e.g., Woods-Saxon (WS) well depth $V_0 = -1.25$ MeV and WS radius $R_{\text{WS}} = 6.0$ fm), we observe no such decay in the energy over a very long imaginary time evolution. This suggests that this $4n$ state is more complex than a pair of dineutrons, or a dineutron with a pair of neutrons. Moreover, we have checked that including in the extrapolation only the points where $E_{4n} < E_{2n}$ for $R_{\text{WS}} = 7.5$ fm still identifies the potential $4n$ resonance at approximately 2.5 MeV.

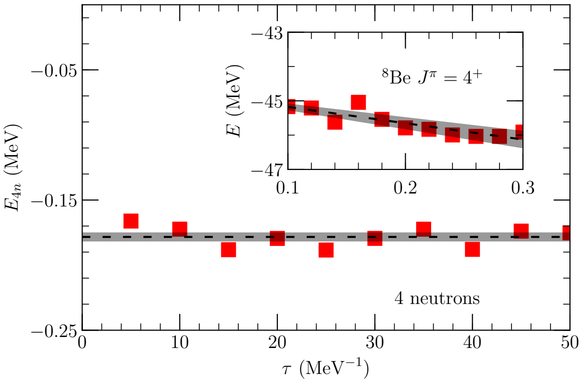

FIG. 1. The $4n$ energy in an external Woods-Saxon well with $V_0 = -1.25$ MeV and $R_{\text{WS}} = 6.0$ fm as a function of imaginary time for the local chiral N^{2}LO interactions used in Ref. [1]. The inset (extracted from Ref. [14]) shows that an unbound $J^\pi = 4^+$ state of ^{8}Be decays very rapidly as a function imaginary time evolution (note the axes are in the same units).

Regarding the extrapolation procedure itself, we are aware that some care is needed, which is why we sought to establish that our extrapolation works well in the two-body S-wave (two-Gaussian) potential case as discussed in Ref. [1]. To reinforce this point, we have performed additional calculations for two two-body resonances shown in Fig. 2. As can be seen in Fig. 2, our extrapolation procedure correctly identifies the locations of the two resonances within the uncertainties of the fit. Furthermore, as our Letter notes, using the current extrapolation, we cannot make a comment about the width. It is entirely possible that the width is very broad (see also Ref. [9]) and therefore the resonance would have little or no ef-

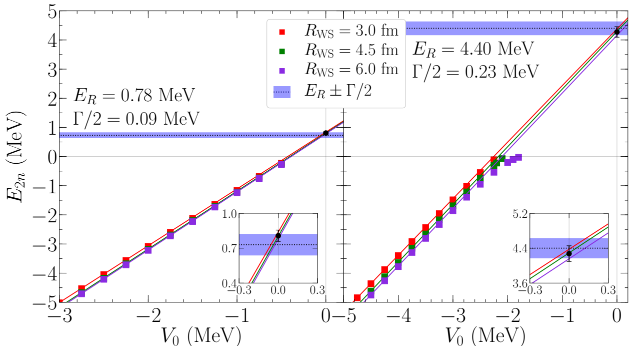

FIG. 2. The energy of two neutrons trapped in various Woods-Saxon wells interacting via an S-wave (two-Gaussian) potential as in Ref. [1] tuned to give two different resonances. The linear extrapolations to zero well depth correctly give the position of the resonance in both cases.

fect on the observable scattering dynamics. In fact, we acknowledge that our current extrapolation cannot distinguish between a resonance and a virtual state.

In conclusion, the existence of few-neutron structures is ultimately a question that experiments have to decide. It remains an intriguing open question whether these systems exhibit resonances, virtual states, or other localized features of the cross section unrelated to S-matrix poles.

We thank S. Dietz for useful discussions and benchmark calculations. This work was supported by the U.S. Department of Energy under Contract No. DE-AC52-06NA25396, the NUCLEI SciDAC project, the ERC Grant No. 307986 STRONGINT, and the Deutsche Forschungsgemeinschaft through Grant No. SFB 1245.

---

* stefano@lanl.gov
† hans-werner.hammer@physik.tu-darmstadt.de
‡ pklos@theorie.ikp.physik.tu-darmstadt.de
§ joel.lynn@physik.tu-darmstadt.de
¶ schwenk@physik.tu-darmstadt.de

[1] S. Gandolfi, H. W. Hammer, P. Klos, J. E. Lynn, and A. Schwenk, Phys. Rev. Lett. **118**, 232501 (2017).
[2] S. C. Pieper, Phys. Rev. Lett. **90**, 252501 (2003).
[3] K. Kisamori *et al.*, Phys. Rev. Lett. **116**, 052501 (2016).
[4] C. A. Bertulani and V. Zelevinsky, J. Phys. G **29**, 2431 (2003).
[5] N. K. Timofeyuk, J. Phys. G **29**, L9 (2003).
[6] R. Lazauskas and J. Carbonell, Phys. Rev. C **72**, 034003 (2005).
[7] E. Hiyama, R. Lazauskas, J. Carbonell, and M. Kamimura, Phys. Rev. C **93**, 044004 (2016).
[8] A. M. Shirokov, G. Papadimitriou, A. I. Mazur, I. A. Mazur, R. Roth, and J. P. Vary, Phys. Rev. Lett. **117**, 182502 (2016).
[9] K. Fossez, J. Rotureau, N. Michel, and M. Płoszajczak, Phys. Rev. Lett. **119**, 032501 (2017).
[10] C. Greene, "Adiabatic hyperspherical picture of $3n$ and $4n$ states," in *Proceedings of the XXII International Conference on Few-Body Problems in Physics, Caen, France* (to be published.).
[11] R. Lazauskas and J. Carbonell, Phys. Rev. C **71**, 044004 (2005).
[12] A. Deltuva, Phys. Lett. B **782**, 238 (2018).
[13] A. Deltuva and R. Lazauskas, Phys. Rev. Lett. **123**, 069201 (2019).
[14] S. Pastore, R. B. Wiringa, S. C. Pieper, and R. Schiavilla, Phys. Rev. C **90**, 024321 (2014).