

ESTIMATING TURBULENCE KINETIC ENERGY DISSIPATION RATES IN NUMERICALLY SIMULATED STRATOCUMULUS CLOUD-TOP AND CONVECTIVE BOUNDARY LAYER FLOW: EVALUATION OF DIFFERENT METHODS.

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ABSTRACT

We perform analysis of direct numerical simulation (DNS) data of two flow cases: stratocumulus cloud-top (SCT) and convective boundary layer (CBL). We test different methods for turbulence kinetic energy dissipation rate (EDR) retrieval. Among others we investigate performance of a new, iterative method, proposed recently in Waclawczyk *et al.* (2017), where an analytical model for energy spectra in the dissipative range is needed. We argue, the new method has some advantages over the standard spectral retrieval techniques. To apply it, only the information on the signals' cut-off wavelength is needed and it is not necessary to define the fitting range in the inertial part of the spectrum. With this, the new method could be a basis of a general algorithm for EDR retrieval, applicable to a wide range of different atmospheric data (e.g. from commercial aircrafts).

Moreover, we investigate how the presence of anisotropy due to shear, buoyancy and external intermittency in the flow affects the EDR retrieval based on the classical K41 for isotropic turbulence (Kolmogorov, 1941).

TEST CASE

Atmospheric turbulence is the key physical mechanism that is behind the occurrence of many atmospheric phenomena. An important quantity which characterises smallest scales of such flows is the mean turbulence kinetic energy dissipation rate ε . Still, the information on ε in atmospheric flows is scarce. The velocity time series obtained from airborne experiments are characterized by the presence of effective spectral cutoffs and are affected by measurement errors. Moreover, turbulence anisotropy, buoyancy and external intermittency affect the EDR estimates.

The available DNS data allow to test performance of different methods for ε retrieval in atmospheric configurations. In this work we consider two systems. The first one mimics the stratocumulus cloud-top region. Longwave radiative cooling of the upper region of the cloud leads to con-

vective instability, and this instability is a major source of cloud turbulence. These radiative properties imply a reference buoyancy flux $B_0 = 0.002 \text{ m}^2 \text{ s}^{-3}$ and a reference velocity scale $U_0 = (B_0 L_0)^{1/3} = 0.3 \text{ m s}^{-1}$, where L_0 is the radiative extinction length. The Reynolds number in the simulation is $U_0 L_0 / \nu = 800$, which is about 300 times smaller than that in the atmosphere. Further details of the settings and simulations can be found in Mellado (2017); Schulz & Mellado (2018).

The second system a dry, shear-free convective boundary layer (CBL) that grows into a linearly stratified atmosphere. The flow is driven by a constant and homogeneous surface buoyancy flux B_0 , and the buoyancy stratification of the free atmosphere is N^2 , where N is the buoyancy frequency. This configuration is representative of midday atmospheric conditions over land. Statistical properties are expressed as a function of the buoyancy Reynolds number $Re_0 = B_0 / (\nu N^2)$, the normalized vertical distance to the surface z/h , and the ratio h/L . Here, the variable $h(t)$ provides a measure of the CBL depth and is defined as $h \simeq (2B_0 N^{-2} t)^{1/2}$, the parameter $L = (B_0 / N^3)^{1/2}$ is the reference Ozmidov scale. The ratio h/L increases as the CBL grows into the linearly stratified atmosphere and attains a quasi steady regime beyond $h/L \simeq 10 - 15$. We consider data from a simulation with a buoyancy Reynolds number $Re_0 = 117$ and at a state of development $h/L \simeq 21.5$. Further details can be found in Mellado *et al.* (2016).

DISSIPATION RATE RETRIEVAL

The turbulence kinetic energy dissipation rate is defined as (Pope, 2000)

$$\varepsilon = 2\nu \langle s_{ij} s_{ij} \rangle, \quad (1)$$

where

$$s_{ij} = \frac{1}{2} \left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right),$$

$\langle \cdot \rangle$ is the ensemble average and $u'_i = u_i - \langle u_i \rangle$ denotes the i -th component of fluctuating velocity. The two common approaches for EDR retrieval use the inertial-range scaling form of the power spectra and structure functions. In the homogeneous and isotropic turbulence the following energy spectrum is assumed (Pope, 2000)

$$E(k) = C\varepsilon^{2/3}k^{-5/3}f_L(kL)f_\eta(k\eta) \quad (2)$$

where $C \approx 1.5$ derived from experimental data, f_L and f_η are non-dimensional functions, which specify the shape of energy-spectrum in the energy-containing range and the dissipation range, respectively. L is the length scale of large eddies and

$$\eta = \left(\frac{v^3}{\varepsilon} \right)^{1/4}$$

is the Kolmogorov length scale, which is connected with the dissipative scales. The function f_η tends to unity for small $k\eta$ while f_L tends to unity for large kL , such that in the inertial range, the formula $E(k) = C\varepsilon^{2/3}k^{-5/3}$ is recovered. In the homogeneous, isotropic turbulence the one-dimensional longitudinal and transverse energy spectra E_{11} and E_{22} , respectively, are related to the energy spectrum function $E(k)$, such that in the inertial range they follow formulae

$$E_{11}(k_1) = \alpha\varepsilon_{PS}^{2/3}k_1^{-5/3} \quad E_{22}(k_1) = \alpha'\varepsilon_{PS}^{2/3}k_1^{-5/3} \quad (3)$$

where $\alpha \approx 0.49$ and $\alpha' \approx 0.65$ and ε_{PS} should approximate ε . This allows to estimate the EDR from the 1D velocity signal with spectral cut-off. Alternatively, the profiles of the second and third order longitudinal structure functions in the inertial subrange can be used to calculate ε .

The new, iterative method to estimate ε , proposed in Waclawczyk *et al.* (2017) and Akinlabi *et al.* (2019) can be used for spectral cut-off's placed in the inertial, as well as intermediate or dissipative range. In this method we make use of Eq. (2) and assume certain form of the function f_η . If a velocity signal u'_{cut} is measured up to a cut-off wavenumber k_{cut} and we assume that the associated filter is rectangular in the wavenumber space, the following relation between $\langle (\partial u'_{cut} / \partial x)^2 \rangle$ and $\langle (\partial u' / \partial x)^2 \rangle$ is obtained

$$\left\langle \left(\frac{\partial u'}{\partial x} \right)^2 \right\rangle = \left\langle \left(\frac{\partial u'_{cut}}{\partial x} \right)^2 \right\rangle \underbrace{\frac{\int_0^\infty k_1^2 E_{11} dk_1}{\int_0^{k_{cut}} k_1^2 E_{11} dk_1}}_{\mathcal{C}_{\mathcal{F}}}, \quad (4)$$

where the fraction on the RHS is called the "correcting factor" $\mathcal{C}_{\mathcal{F}}$. Using relations between $E_{11}(k_1)$ and $E(k)$ in isotropic turbulence (Pope, 2000)

$$E_{11}(k_1) = \int_{k_1}^\infty \frac{E(k)}{k} \left(1 - \frac{k_1^2}{k^2} \right) dk, \quad (5)$$

and using Eq. (2), the following formula for $\mathcal{C}_{\mathcal{F}}$ was derived in Waclawczyk *et al.* (2017)

$$\mathcal{C}_{\mathcal{F}} = 1 + \frac{\int_{k_{cut}}^\infty \beta \eta \xi_1^2 \int_{k_1}^\infty \xi^{-8/3} f_\eta(\xi) \left(1 - \frac{\xi_1^2}{\xi^2} \right) d\xi d\xi_1}{\int_0^{k_{cut}} \beta \eta \xi_1^2 \int_{\xi_1}^\infty \xi^{-8/3} f_\eta(\xi) \left(1 - \frac{\xi_1^2}{\xi^2} \right) d\xi d\xi_1}. \quad (6)$$

If $\mathcal{C}_{\mathcal{F}}$ is known, the EDR can be estimated from the direct relation

$$\varepsilon_{NCR} = 15\nu \left\langle \left(\frac{\partial u'}{\partial x} \right)^2 \right\rangle = 15\nu \left\langle \left(\frac{\partial u'_{cut}}{\partial x} \right)^2 \right\rangle \mathcal{C}_{\mathcal{F}}. \quad (7)$$

In order to calculate $\mathcal{C}_{\mathcal{F}}$ from Eq. (6) a value of η should first be specified, hence, an iterative procedure was proposed in Waclawczyk *et al.* (2017). It starts with an initial guess of ε^0 . With this, the corresponding value of the Kolmogorov length $\eta_0 = (v^3/\varepsilon^0)^{1/4}$ is calculated and introduced into Eq. (6) for $\mathcal{C}_{\mathcal{F}}$. The EDR after the first iteration, ε^1 is found from Eq. (7). The procedure can be repeated, i.e. the next approximation of η^1 can be calculated and substituted into Eq. (6). After several iterations the procedure converges to the final value of ε_{NCR} which should approximate ε with an error defined by a prescribed form $\Delta\eta = |\eta^{n+1} - \eta^n|$.

Moreover, in order to calculate $\mathcal{C}_{\mathcal{F}}$ from Eq. (6), certain form of f_η should be assumed. In this work we investigated three different forms of f_η (see Pope (2000)), the exponential spectrum

$$f_\eta = \exp(-\beta k\eta), \quad (8)$$

where $\beta = 2.1$, the Pao spectrum

$$f_\eta = \exp(-\beta(k\eta)^{4/3}), \quad (9)$$

with $\beta = 2.25$ and the Pope model

$$f_\eta = \exp(-\beta\{[(k\eta)^4 + c_\eta^4]^{1/4} - c_\eta\}), \quad (10)$$

where $\beta = 5.2$, $c_\eta = 0.4$.

RESULTS

For the startocumulus-top simulations we investigated 1D spectra of three velocity components at four horizontal planes

$$z = -5.2L_0, -3.5L_0, -1.7L_0, z = 0.1L_0.$$

The plane $z = -3.5L_0$ indicates the height of maximum buoyancy flux and the plane $z = 0.1L_0$ is placed in the upper part of stratocumulus cloud and indicates the position of the minimum buoyancy flux. This region of the flow is affected by the presence of shear, stable stratification as well as external intermittency and the spectra calculated at this plane deviate from K41 theory.

For the CBL case we perform analogous analysis for five horizontal planes, namely,

$$z = 0.25h, \quad 0.5h, \quad 0.75h, \quad 1.0h, \quad 1.14h.$$

Apart from the layer $z = 0.1L_0$ in the SCT we found close similarities between spectra in both flow cases. The inertial range scaling $\sim k_1^{-5/3}$ can be best recognised for the longitudinal spectra of horizontal velocity components u (see Fig. 1) and v and for the transverse spectra of the vertical velocity component w (see Fig. 2). In this latter case, however, the proportionality constant exceeds the isotropic value $\alpha' = 0.65$. We can conclude that anisotropy due to buoyancy will affect EDR estimates based on the vertical velocity component. They will be overpredicted in comparison to the true ε_{DNS} values inside the cloud and in the CBL, where buoyancy is a source of turbulence generation and underpredicted in the STC case at plane $z = 0.1L_0$ where the negative buoyancy damps the vertical velocity fluctuations. Hence, in the further part of the work the EDR estimates were based on the horizontal components of velocity u and v .

Deviations from the K41 scaling were observed for the transverse spectra of u and v , where $\sim k_1^{-a}$ or $\sim k_1^{-b}$ scaling, with a and b smaller than $5/3$, was found, see Figs 3 and 4. As we expect, the EDR estimates based on atmospheric measurements may also be biased due to these effects.

To test the performance of the new, iterative method for ε retrieval given by Eqs. (6) and (7) we compared different analytical forms of f_η , given in Eqs. (8–10) for the SCT case in Fig. (5) with $k_{cut} = 5.3m^{-1}$ which is within the dissipative range. In this plot, ε_{DNS} is the exact EDR value calculated from the formula (1) from the DNS data. As it is seen, the Pope formulation provides better fit with DNS than the Pao or exponential models. All results are underpredicted in comparison to ε_{DNS} at plane $z = 0.1L_0$.

Next, we compared results with standard, power spectrum EDR retrieval method, as given in Eq. (3). The value of ε_{NCR} from Eq. (7) was calculated with the Pope's model (10) with $k_{cut} = 5.3m^{-1}$ in the STC case and $k_{cut} = 614m^{-1}$ in the CBL case. As it is seen in Fig. 6, predictions with the new, iterative model are comparable with ε_{PS} calculated from the power spectrum. 10 iterations were sufficient to calculate ε_{NCR} with the error $\Delta\eta = 10^{-6}$.

We argue, the iterative method have certain advantages over the standard spectral retrieval estimates. In the latter case, it is necessary identify the inertial-range part of the spectrum and choose the optimal fitting range to find ε_{PS} from Eq. (3). This is not easy when large sets of atmospheric data (e.g. from commercial aircrafts) are to be analysed. To apply the iterative method only the information on the cut-off wavelength are required. With this, writing an algorithm for EDR retrieval applicable for a wide range of data is easier.

Some regions of the considered flows are affected by the presence of external intermittency, i.e. spots of non-turbulent fluid within cloud or boundary layer. The intermittency ratio γ is the volume fraction of turbulent fluid, that is, $\gamma = 0$ in purely laminar and $\gamma = 1$ in purely turbulent flow. The presence of external intermittency may affect EDR retrieval, especially in the top region of the cloud (at $z = 0.1L_0$) and the top of the CBL. As it was argued in Akinlabi *et al.* (2019), the external intermittency can qualitatively be estimated by the ratio of the Liepmann and Tay-

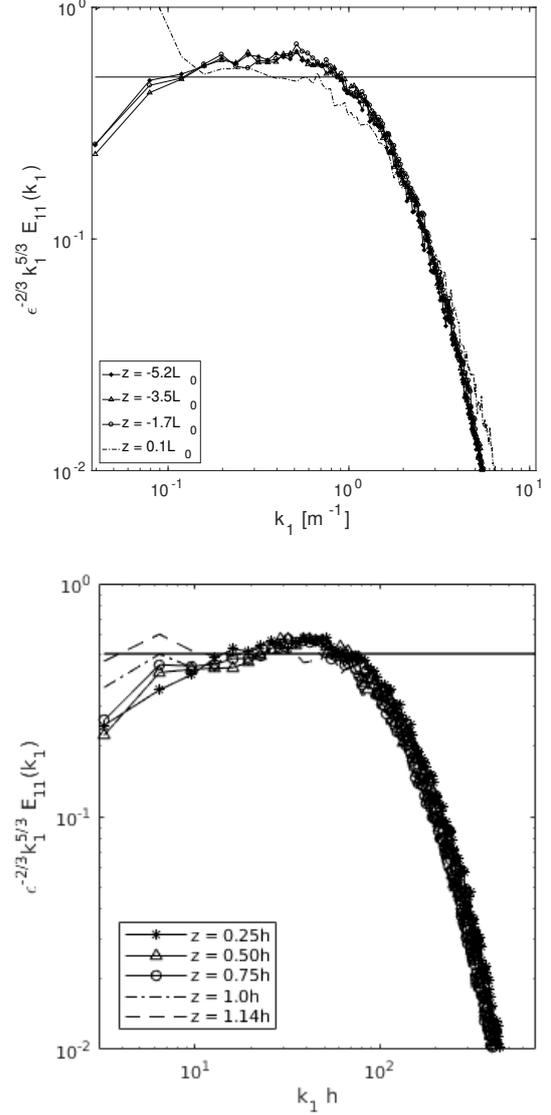


Figure 1. Compensated longitudinal spectra $E_{11}(k_1)$ for SCT (top figure), CBL (bottom figure)

lor microscales. The Liepmann scale is defined as

$$\Lambda = \frac{1}{\pi N_L},$$

where N_L is the number of times per unit length the fluctuating signal crosses the threshold 0. The longitudinal Taylor microscale equals (Pope, 2000)

$$\lambda_l = \left[\frac{2\langle u_l'^2 \rangle}{\langle (\partial u_l' / \partial x)^2 \rangle} \right]^{1/2} \quad (11)$$

and the transverse microscale is $\lambda_n = \lambda_l / \sqrt{2}$. It was shown by Sreenivasan *et al.* (1983) that in fully turbulent signals $\lambda_n / \Lambda = 1$.

The case of externally intermittent flow was considered by Akinlabi *et al.* (2019). Therein, we assumed as a first approximation, that in the externally intermittent flow, the

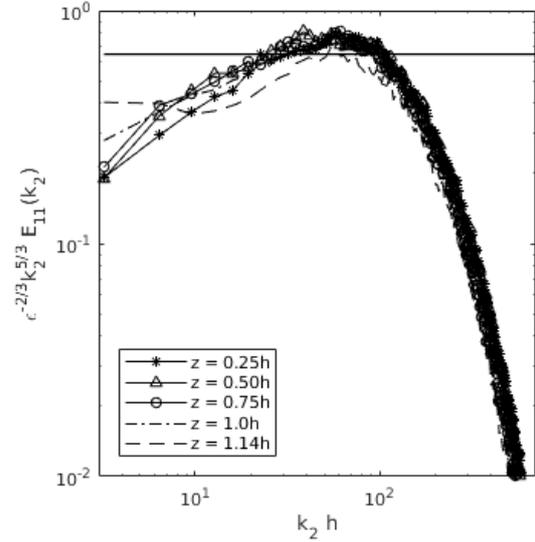
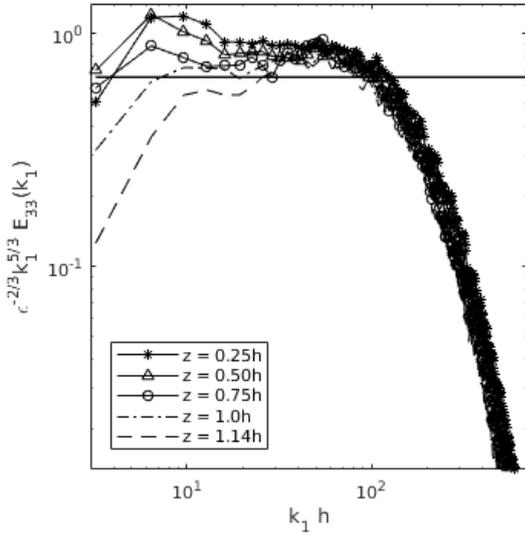
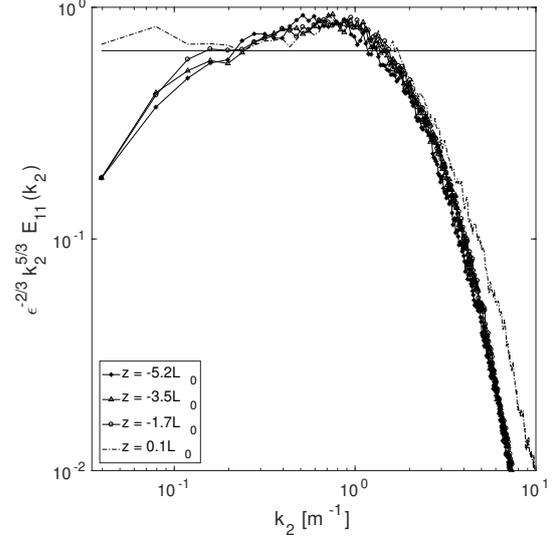
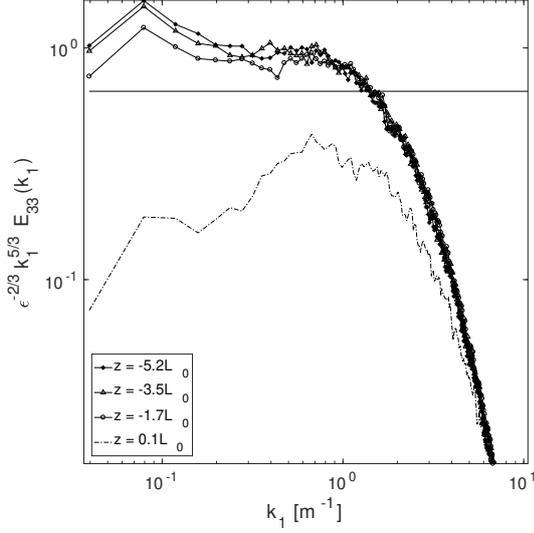


Figure 2. Compensated transverse spectra $E_{33}(k_1)$ for SCT (top figure), CBL (bottom figure)

Figure 3. Compensated transverse spectra $E_{11}(k_2)$ for SCT (top figure), CBL (bottom figure)

statistics will change to $\gamma\langle u'^2 \rangle$ and $\gamma\langle (\partial u'/\partial x)^2 \rangle$. Moreover, the laminar part of the signal does not significantly contribute to the number of crossings, hence, we will detect γN_L crossings per unit length. With this, the Taylor microscale, the Liepmann scale and their ratio will change to

$$\lambda_{nl} = \left[\frac{\gamma\langle u'^2 \rangle}{\gamma\langle (\partial u'/\partial x)^2 \rangle} \right]^{1/2} = \lambda_n,$$

and

$$\Lambda_I = \frac{1}{\gamma\pi N_L} = \frac{1}{\gamma}\Lambda, \quad \text{hence} \quad \frac{\lambda_{nl}}{\Lambda_I} = \gamma \frac{\lambda_n}{\Lambda},$$

where the subscripts I are related to the statistics in the intermittent flow. If $\lambda_n/\Lambda \approx 1$, then in the intermittent flow, $\lambda_{nl}/\Lambda_I \approx \gamma$. We note, however, that both in the STC and CBL case λ_n/Λ was around 0.8 even in the core region

of the flow. The reason for this may be the strong non-Gaussianity of the velocity derivatives. Hence, in Akinlabi *et al.* (2019) the ratio

$$\frac{\lambda_{nl}/\Lambda_I}{(\lambda_n/\Lambda)_T} \quad (12)$$

(the subscripts T is used to denote mean value in the turbulent, core region of the flow) was compared with γ estimated from DNS based on the enstrophy values. In this work we show analogous results for the CBL case in Fig. 7. As it is observed, λ_{nl}/Λ_I decreases in the upper part of the CBL, similarly as γ .

CONCLUSIONS

With this study, we investigated how the anisotropy of turbulence in startocumulus clouds and convective boundary layer affects estimates of ε based on $1D$ velocity time series. Moreover, we tested the new, iterative method for

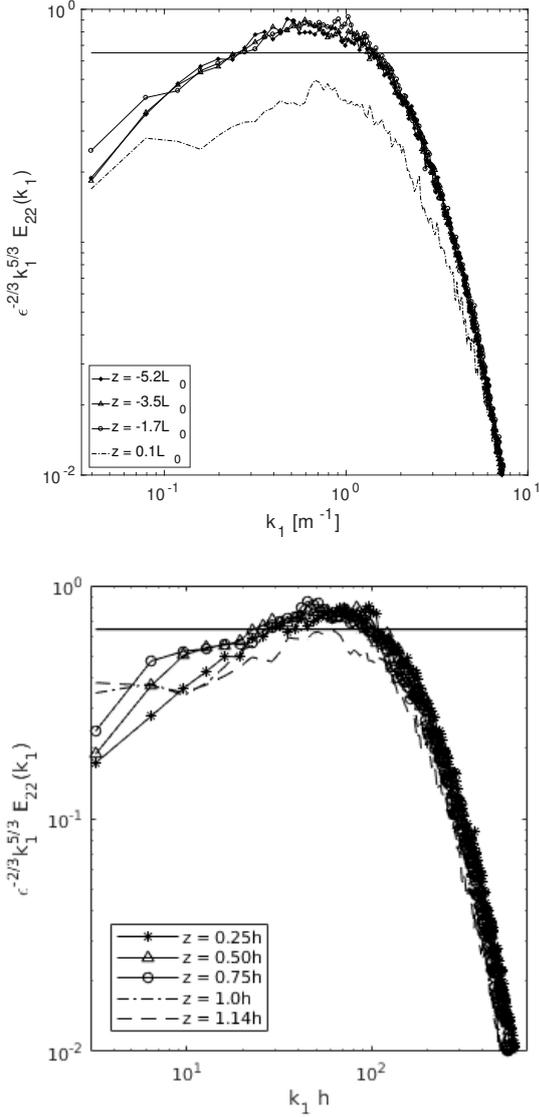


Figure 4. Compensated transverse spectra $E_{22}(k_1)$ for SCT (top figure), CBL (bottom figure)

the EDR retrieval. In the new method a universal formula for the dissipative part of the spectrum is employed. We show that this assumption works well, at least in the core region of the considered flow cases, not affected by the external intermittency. Hence, the new method can provide an interesting alternative to the standard approaches.

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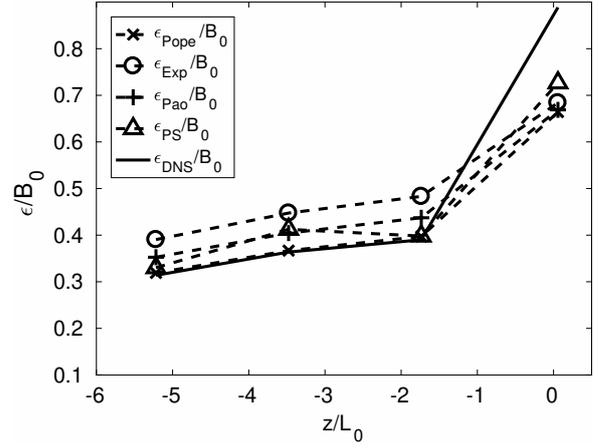


Figure 5. EDR estimates from Eq. (7) for different forms of f_η in Eq. (6) compared with ϵ_{DNS}

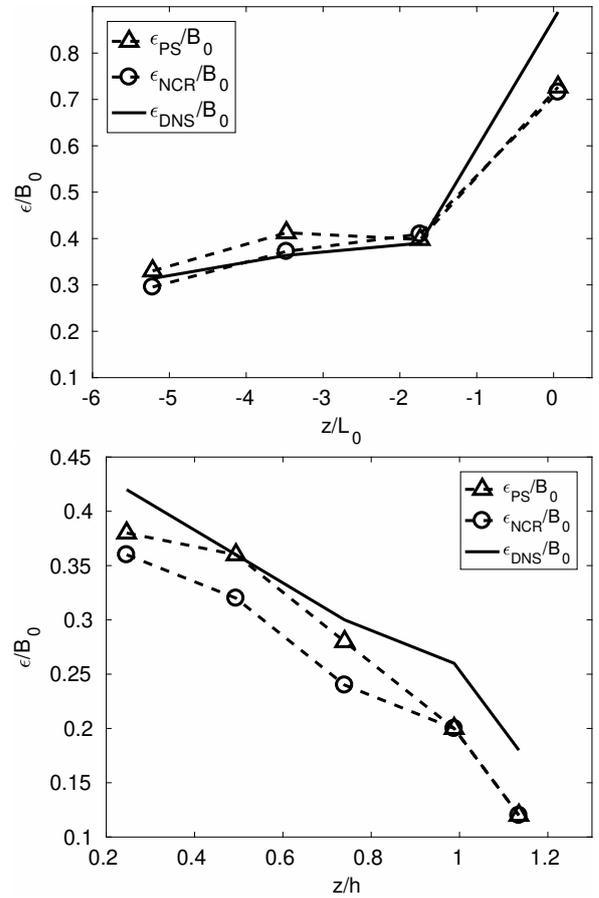


Figure 6. EDR estimates with the power spectrum and the iterative method, STC (top figure), CBL (bottom figure).

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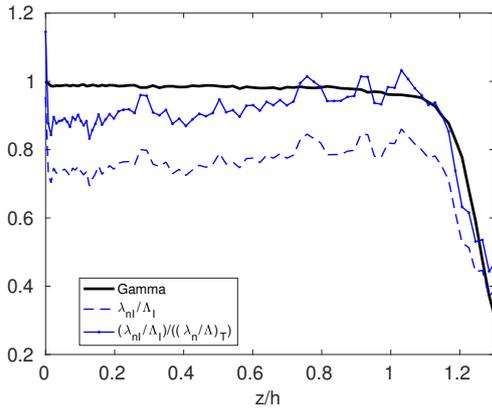


Figure 7. Values of the intermittency factor calculated from the enstrophy and λ_n/Λ as a function of z/h for the CBL case.

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