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Error Analysis and System Calibration of a Dual-rotating-retarder Mueller Matrix Polarimeter

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Abstract. We developed a Mueller matrix polarimeter in the laboratory based on a dual-rotating-retarder configuraion. The Mueller matrix measurement accuracy can reach 2×10^{-3} after calibration by a 'clear' measurement. We use this system to measure the polarization properties of a wave-plate and a Liquid Crystal Variable Retarder (LCVR) sample after calibration. And we get the intrinsic properties of the LCVR and the wave plate, which are supposed to be carefully considered when using them.

1 Introduction

The 1 m New Vacuum Solar Telescope (NVST) is the largest ground-based solar telescope in China currently. One of its scientific goals is to diagnose the magnetic field on the solar surface accurately by spectro-polarimetry and magnetography (Liu et al. 2014; Yuan 2014; Hou et al. 2019). The main problem of these observations is instrumental polarization arising from the telescope, which should be accurately compensated by the polarization calibration of telescope. For this reason, accurate testing of the polarization elements and devices, which are used for calibration and polarimetric observation, is required in the laboratory, and we developed a Mueller matrix measurement system with a dual-rotating-retarder for the testing, and the desired polarimetric accuracy of the system is 0.005 or higher.

The dual-rotating-retarder configuration, which was originally proposed by Azzam (Azzam 1978), is commonly used in Mueller matrix polarimetry. The optimum retardance for the rotating retarder and optimum ratio between the two retarders' angular velocities have been derived in previous articles (Smith 2002; Ichimoto et al. 2006). And several papers have investigated the main systematic errors of this configuration, which can be classified into two categories (Goldstein 1992; Chenault et al. 1992; Broch et al. 2008, 2010). The first kind of systematic errors are caused by the retardance errors and orientation errors of polarization elements under the assumption of perfect polarization elements. Many researchers have done a lot of work to analyze the impact caused by retardance and orientation errors, and calibration methods have also been proposed to correct it (Goldstein 1992; Chenault et al. 1992; Cheng et al. 2017). The second kind of systematic errors are caused by the imperfections of the polarization elements, such as the non-uniformity and the interference of the

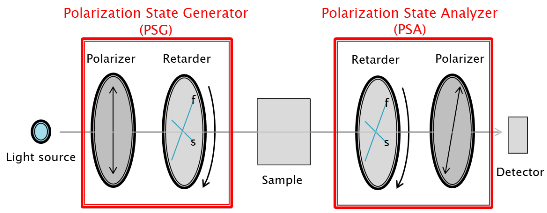

Figure 1. Basic configuration of the dual-rotating-retarder Mueller matrix polarimeter.

retarder, and other devices involved, such as the positioning error of the rotating stage. The influences of these imperfections can be calibrated by the Eigenvalue Calibration Method (ECM) (Compain et al. 1999).

The Mueller matrix measurement system we developed is calibrated by a 'clear' measurement without a sample in it, and the first kind of systematic errors can been corrected by the calibration. After the 'clear' measurement calibration, the accuracy of the Mueller matrix measurement system is about 2×10^{-3}, which meets the requirement. Thus, we use the measurement system to measure the quartz wave plate and a liquid crystal variable retarder, which are supposed to be used in the polarimetry of NVST, and we calculate the corresponding parameters of these samples.

In this paper, we discuss the Mueller matrix measurement system and the mathematical model we used for the data reduction in section 2. In section 3, we present the measurement results of polarization samples and derive the polarization model for these samples. In the last part of the paper, the main error sources of the measurement system are discussed.

2 Configuration and data model of the measurement system

2.1 Experimental setup

Fig. 1 shows the basic configuration of the dual-rotating-retarder Mueller matrix polarimeter. The main parts of this system are the Polarization State Generator (PSG) and the Polarization State Analyzer (PSA), both of which consist a fixed polarizer and a rotating retarder. The collimated light passes through the PSG, sample, PSA and is finally detected by an optical power meter. The two retarders we used are both quarter wave plates, which is practicable but not the optimum retardance for the configuration according to previous research (Smith 2002; Ichimoto et al. 2006). And we also choose a feasible angular scheme for the two rotating retarders, both of which rotate in 8 discrete positions by a step of 22.5° and 64 intensity measurements are made in total.

Fig. 2 shows the experimental setup in the laboratory. The light source we used is a laser diode with a center wavelength of 532 nm. The polarizers in the PSG and PSA are both polarization prisms with extinction ratios higher than 10000:1. The two rotating waveplates are both compound zero-order quarter waveplates at 532 nm. The two rotating stages can rotate to target position with an accuracy of 0.03°. The intensity noise detected by the optical power meter is less than 0.1%. The PSG and PSA are assembled in separated arms, thus we can measure the Mueller matrix of both transmission and reflection.

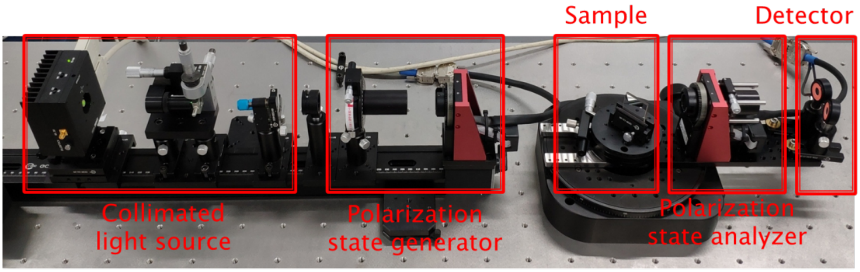

Figure 2. Experimental setup of Mueller matrix polarimeter in the laboratory.

2.2 Data model

The original data processing method is based on Fourier analysis and an equivalent data reduction method based on linear algebra has been implemented later(Chipman 1995; Smith 2002). And the linear algebra method is more generalized because it doesn't require evenly spaced measurements and can also be used in other polarimeters, such as polarimeters based on liquid crystal variable retarders. The linear algebra method is also less sensitive to error sources in system calibration(Cheng et al. 2017). Therefore, we derive our data model according to linear algebra, the method of matrix calculation.

The state of the polarization light in this paper is described by a Stokes vector, while the polarization properties of optical components are characterized by a Mueller matrix. We define the Q direction of the system by the polarization orientation of the prism in the PSG. The initial orientation of the two rotating retarders and the orientation of the polarization prism in the PSA can be defined as θ_{10}, θ_{20} and θ_P corresponding to Q direction. And the retardences of the two rotating retarders are defined as δ_1 and δ_2, which may deviate from 90° because of the polishing error. The following equation shows the relationship between the measured intensity and the system parameters by calculation of Stokes vector and Mueller matrix.

$$I_{i,j} = [1, 0, 0, 0] M_{P2}(\theta_P) M_{R2}(\theta_{2,j}, \delta_2) M_S M_{R1}(\theta_{1,i}, \delta_1) M_{P1}(0) [I_0, 0, 0, 0]^T , \tag{1}$$

Where $[I_0, 0, 0, 0]^T$ is the Stokes vector of the light source, $M_{P1}(0)$, $M_{P2}(\theta_P)$, $M_{R1}(\theta_{1,i}, \delta_1)$ and $M_{R2}(\theta_{2,j}, \delta_2)$ are the Mueller matrix of polarizers and retarders in the PSG and PSA, $\theta_{1,i}$ and $\theta_{2,j}$ are the orientation angles of the two rotating retarders. M_S is the Mueller matrix of the sample, and $[1, 0, 0, 0]$ vector is the intensity response vector of the power meter. The Mueller matrix of polarizers and retarders are as follows.

$$M_P(\theta) = \frac{1}{2} \times \begin{bmatrix} 1 & cos2\theta & sin2\theta & 0 \\ cos2\theta & cos^2 2\theta & cos2\theta sin2\theta & 0 \\ sin2\theta & cos2\theta sin2\theta & sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 , \end{bmatrix} \tag{2}$$

$$M_R(\theta,\delta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos^2 2\theta + sin^2 2\theta cos\delta & cos2\theta sin2\theta(1-cos\delta) & -sin2\theta sin\delta \\ 0 & cos2\theta sin2\theta(1-cos\delta) & cos^2 2\theta cos\delta + sin^2 2\theta & cos2\theta sin\delta \\ 0 & sin2\theta sin\delta & -cos2\theta sin\delta & cos\delta\,, \end{bmatrix} \quad (3)$$

The angular scheme of the two rotating retarders in this paper are as follows.

$$\begin{aligned} \theta_{1,i} &= \theta_{10} + (i-1)\cdot 22.5 \\ \theta_{2,j} &= \theta_{20} + (j-1)\cdot 22.5 \\ i, j &= 1, 2, \cdots, 8\,. \end{aligned} \quad (4)$$

Based on the Eq. (1), (2), (3) and (4), we can get the relation between measured intensity matrix $I_{8\times8}$ and system parameters I_0, θ_P, θ_{10}, θ_{20}, δ_1, δ_2, M_S.

$$I_{8\times8} = F(I_0, \theta_P, \theta_{10}, \theta_{20}, \delta_1, \delta_2, M_S)\,, \quad (5)$$

Thus, we can calibrate the system and calculate the Mueller matrix of sample based on Eq. (5). When we calibrate the measurement system with 'clear' measurement, we can get the measured intensity matrix $I_{8\times8}^{Clear}$ and the Mueller matrix of sample M_S is supposed to be identity matrix. By nonlinear least squares fitting of the measured intensity matrix, system parameters, $(\theta_P, \theta_{10}, \theta_{20}, \delta_1, \delta_2)$, can be derived. When we measure the samples after calibration, these system parameters are fixed and we calculate the Mueller matrix of the sample M_S by matrix operations, which is equivalent to the least squares solution.

3 Sample measurement

3.1 Clear measurement

As the first step, we measure the clear air to calibrate the instrument. The system parameters are obtained by nonlinear least squares fitting of 'clear' measurement, shown in Table 1. These parameters can be used for sample measurement as well as 'clear' measurement, we can calculate the Mueller matrix of clear air by the measured intensity matrix of clear air, which is supposed to be the identity matrix if all the optical components and devices are ideal ones. And the deviation of M_{Clear} from the identity matrix can characterize the system error of the Mueller matrix polarimeter, Table 2 shows the result of M_{Clear} and the standard deviation of multiple measurements. Thus, the accuracy of the Mueller matrix polarimeter is about 2×10^{-3} and the sensitivity of the system is better than 5×10^{-4}.

3.2 Liquid crystal variable retarder

Liquid Crystal Variable Retareds have been used more and more frequently in polarimetry and tunable filter recently. And NVST science group is developing a LCVR-based Lyot filters and trying to use LCVR in polarimetry as well. Before a LCVR can be used in these system, accurate polarization measurement of the LCVR should be implemented. Fig. 3

Table 1. System parameters calculated by the 'clear' measurement.

	θ_P	θ_{10}	θ_{20}	δ_1	δ_2	(Unit:°)
Fitting result	3.58	1.88	0.37	87.66	88.43	
95% confidence region	± 0.011	± 0.003	± 0.012	± 0.014	± 0.011	

Table 2. Results of the 'clear' measurement.

M_{Clear}				Standard deviation of M_{Clear}			
1	-0.0004	0.0008	0.0002	0.0	0.0002	0.0002	0.0002
-0.0003	1.0011	-0.0007	-0.0020	0.0004	0.0004	0.0004	0.0003
-0.0005	0.0010	1.0008	-0.0012	0.0003	0.0005	0.0004	0.0003
0.0002	-0.0011	0.0014	0.9991	0.0002	0.0002	0.0003	0.0001

shows the model of the LCVR we chose, which is compensated with a fixed retarder. And the polarization property of LVCR were measured by the Mueller matrix polarimeter we developed.

Fig. 4 shows the Mueller matrix of LCVR as a function of LCVR's retardance, which is obtained by the preliminary measurement. However, the polarization property cannot be characterized just by retardance and azimuth angle, the measured Mueller matrix deviate from Eq. 3. Since the LCVR is compensated with a fixed retarder, we build a corresponding model for it, as shown in Eq. 6.

$$M'_{LCVR}(\theta_0, \delta_0, \theta_{LCVR}, \delta_{LCVR}) = M_R(\theta_0, \delta_0) M_R(\theta_{LCVR}, \delta_{LCVR}) , \tag{6}$$

After applying the LCVR model $M'_{LCVR}(\theta_0, \delta_0, \theta_{LCVR}, \delta_{LCVR})$, the standard deviation of the fitting residual of the measured Mueller matrix decreases from 0.014 to 0.006. And we get $\theta_0 = -43.19°$, $\delta_0 = -93.19°$ and $\theta_{LCVR} = -40.44°$ after fitting, and there may be a small azimuth angle between the optical axis of the fixed retarder and LCVR. Thus, the corresponding LCVR model should be used when the LCVR is used in polarimetry and tunable filter.

3.3 Quartz wave plate

Since a polarimeter based on a rotating wave plate is still used in NVST, we measured a quartz wave plate of NVST to get its polarization property. Fig. 5 shows the wave plate we measured (on the left) and measured results of rotating quartz wave plate (on the right). We measured the Mueller matrix of it in the center and on the edge as the wave plate rotates. Since the Mueller matrix of quartz wave plate we get fits with theoretical model Eq. 3 very well, we can use retardance and azimuth angle to represent the wave plate. As shown in the right part of Fig. 5, the retardance varies with azimuth angle, the red curve is the measured result on the edge and the blue curve is the measured result in the center area. The variation of retardance with azimuth angle on the edge is larger than that in the center, which may be caused by the nonuniformity of the wave plate. To mitigate the influence of retarder's

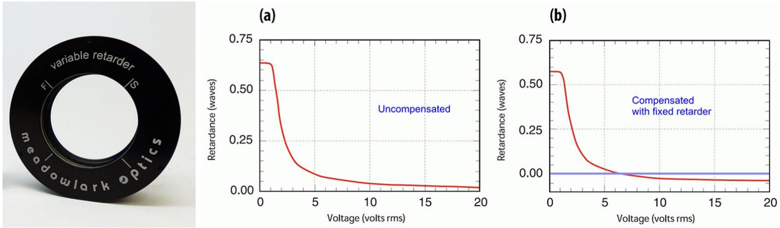

Figure 3. LCVR product model.

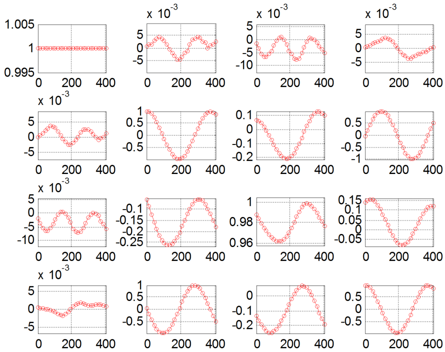

Figure 4. Measured Mueller matrix of LCVR.

nonuniformity, the light beam, which should be as uniform as possible, is supposed to pass through the center of the rotating wave plate.

4 Conclusion

We build up a Mueller matrix polarimeter based on the dual rotating retarder configuration, and the accuracy of the system is about 0.002 after clear measurement calibration. By measuring a rotating wave plate, the spatial distribution of the retardance is obtained. And the spatial nonuniformity may be the main reason limiting the measurement accuracy of Mueller matrix polarimeter, since the two rotating wave plates may also have such kind of nonuniformity. Thus, high precise adjustment of light source (uniform light source) and alignment (light pass through the rotating center of wave plate) could further increase the accuracy of the system. After measuring the LCVR, we have got the corresponding polarization model of LCVR after measurement, which is more suitable to characterize the LCVR we measured. And this model can be used when we use this LCVR for polarimetry or tunable filter.

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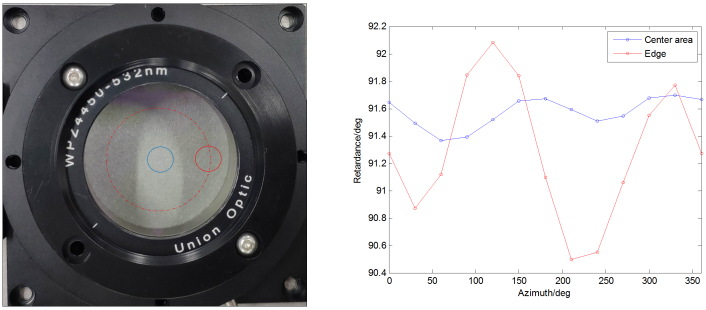

Figure 5. The rotating quartz wave plate we measured.

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