

An Embedded and Embodied Cognition Review of Instructional Manipulatives

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Abstract Recent literature on learning with instructional manipulatives seems to call for a moderate view on the effects of perceptual and interactive richness of instructional manipulatives on learning. This “moderate view” holds that manipulatives’ perceptual and interactive richness may compromise learning in two ways: (1) by imposing a very high cognitive load on the learner, and (2) by hindering drawing of symbolic inferences that are supposed to play a key role in transfer (i.e., application of knowledge to new situations in the absence of instructional manipulatives). This paper presents a contrasting view. Drawing on recent insights from Embedded Embodied perspectives on cognition, it is argued that (1) perceptual and interactive richness may provide opportunities for *alleviating* cognitive load (Embedded Cognition), and (2) transfer of learning is not reliant on decontextualized knowledge but may draw on previous sensorimotor experiences of the kind afforded by perceptual and interactive richness of manipulatives (Embodied Cognition). By negotiating the Embedded Embodied Cognition view with the moderate view, implications for research are derived.

Keywords Instructional manipulatives · Embedded cognition · Embodied cognition

Introduction

In a seminal but critical paper on instructional manipulatives and their applications for the classroom, Ball (1992) stated “Understanding does not travel through the fingertips and up the arm” (p. 3). This statement was meant to go against an overly simplistic view (or “magical hope”; Ball 1992) prevalent in the literature concerning the effectiveness of learning with *physical* manipulatives. Many scholars today have followed suit (Brown et al. 2009; Kaminski et al. 2009a, b; McNeil and Jarvin 2007; Sarama and Clements 2009; Sherman and Bisanz 2009; Uttal et al. 1997), suggesting that “physicality is not important” and rather “their *manipulability* and *meaningfulness* make them [manipulatives] educationally effective”

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(original emphasis; Sarama and Clements 2009, p. 148). Indeed, it has been suggested that previously identified virtues of *physical* manipulatives—learning through concrete and perceptually rich physical practices—are not the drivers of learning (e.g., Triona and Klahr 2003; Zacharia and Olympiou 2011) and can even be detrimental to learning (e.g., DeLoache 2000, 2004; Finkelstein et al. 2005; Sloutsky et al. 2005). This “moderate view” has led to a trend towards minimizing perceptual and interactive richness of manipulatives, as evidenced by the upsurge of *mouse-based virtual* manipulatives (e.g., Clements 2000; Moyer et al. 2002) that compared to their physical counterparts differ in perceptual aspects (e.g., information gained from touching objects vs. manipulating them with a mouse) as well as interactive aspects (e.g., mouse-based interaction is constrained to one hand).

Drawing on insights from embedded embodied perspectives on cognition (Barsalou 1999, 2008; Clark 2005, 2008; de Vega et al. 2008; Hutchins 1995; Kiefer and Trumpp 2012; Lindgren and Johnson-Glenberg 2013; Shapiro 2011; Wilson 2002; Winn 2003) and state-of-the-art research on physical and virtual manipulatives and including tangible user interfaces¹ (Manches and O'Malley 2012), we suggest that some of the assumptions that underlie the moderate view are to some extent misguided, most centrally the following assumptions:

1. Higher perceptual and interactive richness of manipulatives imposes a high cognitive load on the learner, resulting in lower learning outcomes (Brown et al. 2009; McNeil and Jarvin 2007; Sarama and Clements 2009; Uttal et al. 1997)².
2. Transfer of learning from manipulatives involves a change in representation from concrete to symbolic, which is hindered by higher perceptual and interactive richness (Kaminski et al. 2009a, b; Uttal et al. 1997; for an overview of this popular view in education, see Nathan 2012).

Without dismissing the empirical evidence upon which the above assumptions are based, we suggest that a viable case can be made for a more embedded embodied perspective on learning with manipulatives, namely the following:

1. Under certain conditions, perceptual and interactive richness can alleviate cognitive load imposed on working memory by effectively embedding the learner's cognitive activity in the environment (Embedded Cognition claim).
2. Transfer of learning from manipulatives does not necessarily involve a change in representation from concrete to symbolic. Rather, learning from manipulatives often involves internalizing sensorimotor routines that draw on the perceptual and interactive richness of manipulatives (Embodied Cognition claim).

We hasten to note that we will not argue for an exclusively Embedded Embodied approach to learning from manipulatives; rather, this review attempts to negotiate the findings from the Embedded Embodied perspective with findings associated more with the moderate view. For example, while researchers holding the moderate view may suggest that when physicality is not important this is evidence against the Embedded Embodied view (e.g., Triona et al. 2005), our review will show that this would be an overly simplistic understanding of the relevance of

¹ Tangible user interfaces go beyond classical user interfaces (e.g., mouse, keyboard) and are designed to provide more natural or functional *physical* manipulation of virtual objects (Manches and O'Malley 2012; O'Malley and Stanton-Fraser 2004; Shaer and Homecker 2010).

² For example, “When children interact with manipulatives, their cognitive resources may be committed to representing and manipulating the objects and may be largely unavailable for other processes, such as accessing relevant concepts or implementing appropriate procedures” (McNeil and Jarvin 2007, p. 313).

Embedded Embodied Cognition to manipulatives. By combining findings from both streams of research, we aim to develop a more balanced view of how Embedded Embodied Cognition might guide the design of instructional manipulatives.

In the next section, we focus on *Embedded Cognition*, which suggests that effective learning depends on how learners coordinate their cognitive activity in concert with bodily and environmental resources. Embedded Cognition amounts to the idea that cognition is afforded and constrained by ongoing interactions between body and environment, emphasizing an intimate relationship between external artifacts and cognitive processes (Clark 2008; Hutchins 1995; Kirsh 1995, 2010; Wilson 2002). In the subsequent section, we focus on *Embodied Cognition*, which amounts to the claim that knowledge is grounded in sensorimotor routines and experiences (Barsalou 1999, 2008; Lakoff and Johnson 1999; Lakoff and Núñez 2000). In this section, we discuss empirical evidence that suggests that transfer of learning does not necessarily involve a concrete-to-abstract shift.

Importantly, Embedded Cognition and Embodied Cognition are complementary in their analysis of the role of the body in cognition (Shapiro 2011; Wilson 2002). Whereas Embedded Cognition focuses on the continuous coupling or “on-line” interaction with the environment, Embodied Cognition focuses on the role of these previously acquired sensorimotor experiences in “off-line” cognitive activity (i.e., disembedded from the environment).

For this review, we provide a selective overview of state-of-the-art research in cognitive and educational psychology. At the end of both the Embedded and Embodied Cognition sections, we provide a short intermediate discussion, making a connection with empirical evidence that aligns with a more moderate view. In the conclusion, we provide a brief overview of our main conclusions, research challenges, and educational implications in relation to learning with manipulatives.

Embedded Cognition

According to theories of Embedded Cognition, cognitive activity is not something that simply happens internally, but involves a continuous transaction between current states of the brain, body, and the environment (Clark 2008). As such, understanding cognition requires a broader level of analysis that considers how we use our body and the world during the unfolding of cognitive processes (Clark 2008; Hutchins 1995; Kirsh 2010; Wheeler 2007). Examples of Embedded Cognition are readily apparent: reducing working memory load by making notes during a conversation, using fingers to keep track of counting, asking another person to remind you of something, or using a tall building for navigating your way home, which alleviates the need to retain street names or spatial maps. As these examples show, Embedded Cognition refers to the adaptive flexibility of cognitive processes during interaction with the environment.

Although we can make the case that cognition might at times be disembedded during activities such as mental arithmetic, thinking about tomorrow’s chores, talking about something absent, etc., learning from manipulatives pertains to an embedded cognitive situation. That is, learning with manipulatives involves a tight coupling of external artifacts with perceptual and cognitive processes, in which the artifacts structure the learner’s cognitive states (Clark 2005). As such, learning from manipulatives does not differ in kind from examples we have provided above, such as finger-counting. As the finger-counting example also makes clear, however, manipulatives may in some cases become ill-suited for supporting cognitive states, just as arithmetic with large numbers might become difficult to perform through finger-counting. Thus, learning from manipulatives is always an embedded phenomenon in which some cognitive processes are more easily maintained than others.

A theoretical implication of Embedded Cognition is that the states of the body and the environment can be considered extra-neural contributors to (Hutchins 1995; Norman 1988), and in a more radical reading, external vehicles of cognition (Clark 2008; Clark and Chalmers 1998). Not only does Embedded Cognition hold the more widely accepted claim that the external environment may serve as external working memory, the transactions between the environment and the learner might dramatically change the way in which cognitive processes unfold. For example, thinking with or without a notepad may have dramatically different cognitive profiles. Bredo (1994, p. 28) provides an example of this dynamic coupling of cognition with the environment: “One draws, responds to what one has drawn, draws more, and so on. The goals for the drawing change as the drawing evolves and different effects become possible, making the whole development a mutual affair rather than a matter of one-way determinism.” As such, in Embedded Cognition, the external environment has an important status for understanding the way cognitive processes unfold.

That we use aspects of our environment in order to reduce cognitive load was demonstrated in an influential study by (Ballard et al. 1995; see also Haselen et al. 2000; Hayhoe et al. 1997). In this study, participants were asked to reproduce a pattern of colored blocks from a model as quickly as possible by clicking-and-dragging randomly ordered colored blocks from a *resource space* and order them in a *workplace*. Eye movements were monitored to provide insight into the strategies that are involved in solving this problem. It was found that participants opted for a “minimal memory strategy” as indicated by the many switches of eye fixations between the model, resource, and workplace area. That is, to minimize the components to be retained in memory, participants tended to gather information incrementally by first attending to the color and then the position, all just in time, instead of memorizing information all at once. Note that the use of a minimal memory strategy also emerged when participants physically manipulated real blocks (Ballard et al. 1995).

To give another famous example, in a study by Kirsh and Maglio (1994), it was found that effective problem-solving behavior in the game Tetris does not solely rely on action that brings one physically closer to one’s goal, which they termed “pragmatic actions.” Rather, problem-solving also relies on “epistemic actions” that effectively structure the environment so as to uncover information that is hidden or cognitively demanding to compute. For instance, they found that advanced players, more often than less-advanced players, tended to rotate zoids physically instead of mentally when determining whether a zoid would fit the already-placed zoids at the bottom. This study as well as others (Gaschler et al. 2013; Goldstone and Sakamoto 2003) show that the environment does not only allow for offloading, but that efficient problem-solving evolves over time during interaction with the environment, and is dependent on how the agent learns to effectively negotiate internal and external resources.

In both of these experimental demonstrations, it seems that the cognitive system prefers to manage working memory load by making use of external resources when available. However, whether external resources are used also seems to depend on how readily information can be re-achieved from them. In a study by Gray and Fu (2004), participants were confronted with a task wherein subtle retrieval costs of attaining external task-relevant information in the context of programming a simulated VCR were manipulated. Lowering the ease of retrieval of information from external resources, from a single glimpse or an additional mouse click, changed the cognitive strategy of the subjects. When external information was directly accessible, participants leaned primarily on retrieving “perfect-knowledge-in-the-world.” However, when this external information was only indirectly available through a mouse click, participants were more likely to retrieve it from memory. Although the reliance on internal

memory lead to a higher number of mistakes, it was shown that this “imperfect-knowledge-in-the-head” was more quickly available compared to retrieving information externally. That is, based on computational modeling, Gray and Fu (2004) estimated the retrieval effort of relevant information expressed in the amount of milliseconds it takes to retrieve or recall information and showed that participants opt for the quickest problem-solving strategy with no a priori preference for internal or external resources. It seems therefore that the cognitive system “tends to recruit, on the spot, whatever mix of problem-solving resources will yield an acceptable result with a minimum of effort” (Clark 2008, p. 13; see also Borst et al. 2013; Fu 2011).

To date, Embedded Cognition research has been primarily focused on how and when the environment is used in terms of memory distribution (see also Droll and Hayhoe 2007; Gray et al. 2006). Although current research is extending its applications (e.g., Risko et al. 2013), it is still ill-understood how information is encoded during embedded cognitive situations, and whether different interactive possibilities for distributing internal and external resources result in different learning outcomes, that is, in different representations in long-term memory (Fu 2011). Especially the latter question seems to be of central importance for understanding how perceptual and interactive properties of manipulatives may affect learning. Since more research on that question is still clearly needed, we should be hesitant to accept any claims about effects of perceptual and interactive richness of manipulatives.

Embedded Cognition and Instructional Manipulatives

The theory of Physically Distributed Learning (Martin and Schwartz 2005; Schwartz and Martin 2006) suggests that the environment changes the way in which learning unfolds. According to this theory, the learning affordances of physical manipulation can be mapped onto four separate quadrants that roughly categorize physical learning in terms of the *stability* and the *adaptability* of the learner’s *ideas* and the *environment*, the quadrants *Repurposing* and *Mutual Adaptation* being important for present purposes. The quadrant called Repurposing pertains to a situation similar to the above-mentioned Tetris players who have learned to repurpose pragmatic actions that bring one closer to one’s goals for epistemic actions that reduce computational load (Kirsh and Maglio 1994). In this example, the environment is adaptable but ideas remain largely unchanged.

Most interesting for present purposes, however, are such situations in which new ideas arise through physical adaptation of the environment, called Mutual Adaptation. Martin and Schwartz give an example of a young child asked to come up with a one fourth share of eight candies. Children often focus on the *one* of one fourth, which leads them to adopt “one candy” to be the right answer. However, in physical interaction with eight candies, the child might push two candies apart, which increases the likelihood of reinterpreting the new arrangement of two candies as one group, putting the child on “a trajectory” to learn that one fourth of eight means attaining “four groups of two” first (Martin and Schwartz 2005, p. 590). Thus, mutual adaptation involves structuring the environment haphazardly without preconceived goals that affords new interpretations difficult to obtain by thought alone. As such, the theory of Physically Distributed Learning extends the current focus of Embedded Cognition, suggesting that the environment also changes the way learning unfolds.

Martin and Schwartz (2005) have empirically substantiated the theory of Physically Distributed Learning through multiple experiments (see also Martin et al. 2007). In the first two experiments reported by Martin and Schwartz (2005), children of 9 to 10 years old solved fraction operator problems (e.g., one fourth of eight) with physical pie or tile wedges using physical manipulation *and* pictorial line drawings of pie or tile pieces using a pen to highlight partitions. In the first two experiments, it was found that children using physical manipulatives

solved more problems correctly which was measured by the number of partitions created correctly and the number of correct answers that were provided verbally. More importantly, it was shown that physical self-guided partitioning was the driver of understanding rather than mere perception of the desirable end state, a correctly pre-partitioned organization. According to Martin and Schwartz (2005), physical open-ended interaction allows for exploration and search for new interpretations and structures, which benefits learning (see also Martin et al. 2007).

Complementary to these results, it has recently been shown that the beneficial role of physically manipulating the external environment enhances task performance in physics education (Stull et al. 2012). In a set of experiments, university-level physics students had to translate one type of diagram into another, called a diagrammatic translation task, which requires spatially translating the model into the other model's particular perspective. In all three experiments, it was found that students' translation accuracy of one 2D representation into another was promoted by active use of a concrete 3D model during the task (a classic ball and stick manipulative). Importantly, only the active physical use of the 3D model, as opposed to mere perception of the model, promoted task performance. In line with Kirsh and Maglio (1994), it was explained that the concrete model aids students in externalizing spatial rotation operations (Stull et al. 2012).

A critical note of concern, however, is that based on these data one cannot fully disentangle the role of self-guided physically manipulating objects from the visual input that this process also generates. Even though seeing the end state arrangement (Martin and Schwartz 2005) or the model (Stull et al. 2012) was not beneficial for learning, it is possible that watching someone else dynamically structuring the materials would produce the same learning benefits (for a review of the effectiveness of observational learning in educational contexts, see Van Gog and Rummel 2010; for the effectiveness of observing someone else exploring a problem space, see Osman 2010). Nevertheless, active skillful manipulation of these materials might in itself form the basis for performing similar cognitive tasks in the absence of the manipulatives, but this remains an open empirical question.

Martin and Schwartz (2005; Experiment 3) further explored whether there is an interaction between prior knowledge and environmental structure in instances of physically distributed learning. As a highly structured learning environment in the context of solving fraction problems, pie wedges were used since these already have a part of whole partition, whereas tiles were used as unstructured materials. It was found that children performed more correct partitions in solving fraction addition problems for which they had high prior knowledge when materials were structured compared to unstructured materials. In contrast, performance on multiplication problems for which children had low prior knowledge was unaffected by structure of the environment. They suggested that this finding indicates that a more mature understanding of the task allows for repurposing the environment more flexibly, with performance on low familiar tasks being more dependent on the environment's stability for action. However, they also raise a very interesting concern. That is, although a highly structured environment can aid problem solving, it might prevent learners from developing their own interpretation of how to solve a problem.

Indeed with children learning fraction additions in three sessions over a period of a week, it was found that those who had learned with pie wedges showed a lower ability to transfer skills to other manipulatives than children who had used tiles (experiments 4 and 5). Martin and Schwartz (2005) explain this finding in that pie wedges' structure gives the learner a part-of-wholes-interpretation "for free," presumably preventing children to learn how to make and interpret such groupings and whole structures by themselves. Simply put, externalized cognitive operations might in some instances reduce the necessity to understand its function (e.g., that pieces are part of a whole). Although research cited above already offers considerable

evidence that Embedded Cognition is an important factor for learning, a more recent example shows that possibilities for physical interaction indeed change the learning trajectory. In a set of experiments, Manches et al. (2010) sought to find out whether qualitative differences in manipulation predicted children's problem-solving strategies in a numerical partitioning task. In this task, children are asked to provide all the different ways in which a certain amount can be combined (e.g., the number of ways seven can be recombined [e.g., seven and zero, zero and seven, six and one, five and two, et cetera]). In the first study reported by Manches et al. (2010), children ranging from 5 to 7 years old were first asked to solve a partitioning problem without manipulation of any material (no material condition), and to subsequently solve two additional partitioning problems with paper and pencil (paper condition) and physical blocks (physical condition; order of physical and paper condition was counterbalanced). It was found that children provided significantly more unique solutions in the physical condition as opposed to the no material- and the paper condition. Qualitative observations were made that could explain this difference in terms of particular affordances that physical manipulatives have. For example, bimanual manipulation allowed for moving multiple blocks at a time and/or keeping track of block's locations through haptic sensation, which was not possible in the other conditions.

In the second experiment, it was investigated how the affordance of bimanual manipulation might have constrained particular use of strategies. It was predicted that when children ranging from 4 to 7 years old are instructed to manipulate only one object at a time (constraint condition), it would lead to different strategies as compared to children in a no constraint condition. Indeed, it was found that strategies differed dependent on whether manipulation was constrained. For example, reversing combinations (e.g., five and two into two and five) is much easier to perform when manipulating multiple objects at once than serial one-by-one manipulation. In the third study, this effect was replicated for a portion of the sample in a slightly different setup. The constraint condition was now set up as a virtual manipulative (children could click-and-drag only one virtual object on the screen). Taking the results together, this study suggested that with unconstrained physical manipulation come particular affordances that shape the trajectory of young children's learning of numerical partitioning.

Importantly, however, unconstrained physical manipulation has also been shown to be suboptimal for learning (Stull et al. 2013). Stull et al. (2013) let students interact with a tangible user interface (TUI) that was designed to combine affordances of virtual and physical manipulatives. The TUI included sensorimotor features that are typically afforded by physical manipulatives, such as stereo-depth cues and a direct manipulation interface (see Stull et al. 2013, for details). The only features that differed from a physical model were (1) the shape of the tangible interface and its virtual representation (molecular model) were not the same, and (2) interactivity was constrained such that the students could only rotate the model around the axis of a single molecular bond. Note that physical manipulatives allows for rotations around an indefinite number of axes. In these experiments, learners had to perform a diagrammatic matching task, which involved manipulating the model to match the orientation of a particular 2D molecular diagram. Although accuracy levels were the same for both model types, the physical manipulative condition was significantly slower in completing the task (in comparison to the TUI). This higher efficiency in the TUI condition was ascribed to the constrained interactivity of the TUI which automatically focused students on the most task-relevant interactions. Indeed, additional analysis revealed that students who first worked with the TUI performed less irrelevant bond rotations in comparison to students who had worked with physical manipulatives first. As such, constrained interaction might aid in learning to efficiently solve problems in similar unconstrained situations.

A final example for the way in which interaction possibilities may change learning comes from a study reported by Antle (2012) and Antle et al. (2009). In this study, interaction styles

emerging from different manipulatives were investigated in the context of a Jigsaw Puzzle Task with dyads of children ranging from 7 to 10 years old, using either traditional physical manipulatives (PM), mouse-based virtual manipulatives, or a TUI. The TUI was a tabletop prototype with normal puzzle pieces; action was mapped through an infrared camera that allowed for audiovisual feedback when a piece was correctly placed. By calculating relative measures for interaction style to account for single versus multiple input differences (for details, see Antle 2012; Antle et al. 2009), it was shown that the PM and TUI conditions, which allowed for bimanual manipulation, resulted in more time spent performing epistemic actions, for example, grouping corner-, edge-, or same-color pieces into piles. Furthermore, it was found that more direct actions were taken in the PM and TUI condition as opposed to mouse-based virtual manipulatives. Although the design of this study does not allow for empirically rigorous conclusions about performance or learning (as Antle 2012 concurs), it does, together with findings from the previous studies, suggest that properties of the interaction may shape the way in which cognitive processes and learning might unfold. However, based on these studies, it is hard to derive clear design guidelines regarding unconstrained interactive richness.

Intermediate Discussion: Embedded Cognition and Manipulatives

In the previous section, it was shown that manipulatives afford possibilities for reducing internal computational load through interaction. Furthermore, such possibilities are quite easily and automatically incorporated into learning behaviors. Arguably the most important contribution of the Theory of Physically Distributed Learning and the empirical evidence that supports it is that although learning environments that are prestructured and thus constrained may reduce problem-solving steps and improve task performance, this reduction of task load does not necessarily benefit transfer of learning. Children who learned to solve fraction problems with pie wedges, in comparison to learning with tiles, were less able to transfer this knowledge to other materials that did not already have this part-of-wholes interpretation in its structure. Schwartz and Martin (2006) make the analogy with research on Dienes's (1973) base-10 blocks; children who become increasingly efficient to operate base-10 blocks for problem solving become dependent on (or "symbiotically tuned" to) these materials for its efficiency, underperforming in transferring this skill in the absence of these materials (e.g., isomorphic symbolic tasks; see Resnick and Omanson 1987). The tentative lesson we might draw from this is that design of manipulatives should at times allow for self-discovery rather than pre-constrained problem solving when transfer of learning is the goal. As such, embedded learning might at times unfold best when it is learner-centered as opposed to being completely accommodated by the environment.

A further implication is that specific perceptual and interactive properties of manipulatives that might afford embedded learning stand in relation to the kind of bodily actions the learner can perform (Gibson 1979). In the studies by Martin and Schwartz (2005) and Manches and colleagues (2010), it was shown that physicality of materials solicited specific patterns of interaction that led children to discover interpretations necessary for understanding the particular problem. "Solicited" in that children were simply drawn to the affordance of re-arranging the blocks and not driven by a preconceived end state in mind. This arguably shapes the learning trajectory in that it leads to what Martin and Schwartz (2005) call "mutual adaptation"; adaptations to the environment further influence adaptations to children's interpretation.

To elaborate on this, the role of perceptual properties in embedded learning might be indirectly related to possibilities for interaction. The research discussed above provides insights on how perceptual richness affects learners' perception of possibilities for action (e.g., objects being physical rather than virtual). This can be appreciated by a modified

interpretation of what Gray and Fu (2004) call *hard-* and *soft constraints*. That is, manipulatives have specific properties that make only certain actions possible (hard constraints). For example, consider a mouse-based virtual interface that only allows for uni-manual manipulation, or a pie-wedge that only allows for re-arranging parts in preset wholes. However, manipulative perceptual properties also determine which behavior given the possibilities is likely to be solicited (*soft constraints*). For example, Manches et al. (2010) reported that children who were instructed to manipulate physical blocks one at a time had difficulty not to use two hands or manipulate multiple blocks. This resonates with a host of behavioral and neurological evidence on motor affordances that has shown that perceptual properties of objects unreflectively solicit particular action repertoires (Gibson 1979; Snow et al. 2011; Symes et al. 2007; van Elk et al. 2014). In sum, whether an object is perceived to be easily manipulable impinges on the natural behavior it solicits from the learner.

Therefore, there is a case to be made that perceptual richness might impinge on how learners typically interact with the learning environment. As Dourish (2004) notes, “because we have highly developed skills for physical interaction with objects in the world—skills for exploring, sensing, assessing, manipulating, and navigating—we can make interaction easier by building interfaces that exploit these skills” (p. 206). Therefore, suggesting that “physicality is not important” in manipulatives and rather their “manipulability and meaningfulness make them educationally effective” (Sarama and Clements 2009, p. 148) might be at times misguided and involves an artificial distinction; perceptual richness may drive perceptions of manipulability.

The tentative conclusion we like to make up to this point is that contrary to the “moderate view” emphasis that perceptual and interactive richness of manipulatives can hinder learning, it should also be considered as an important source of learning. That is, perceptual and interactive richness may invite learners to interact in a certain way with the environment and therefore effectively embed learners’ cognitive activities. In the final discussion, we connect these insights with those from the upcoming review on Embodied Cognition and discuss implications and suggestions for future research.

Embodied Cognition

Embodied Cognition holds that the *format* of cognition is sensorimotor or modal-based instead of symbol-based (i.e., amodal; Barsalou 1999, 2008; for an overview see Svensson 2007). Furthermore, while the cognitive system might be disembedded and primarily dependent on internal cognitive processes in some cases, Embodied Cognition suggests that sensorimotor information made available during previous interactions is reused for internal cognitive processing. Thus, Embedded Cognition emphasizes an ongoing “on-line” interaction with the environment whereas Embodied Cognition primarily focuses on how the body shapes disembedded or “off-line” cognition.

Embodied Cognition is therefore especially suitable for explaining how learning with manipulatives might impinge on cognitive activity in the absence of manipulatives (e.g., mathematical notations, mental arithmetic). The classic perspective on cognition (Fodor 1975; Newell and Simon 1972) holds that transferring knowledge learned in one situation to another is dependent on establishing a set of complex symbolic rules. According to this view, knowledge resides in a rule-governed semantic system that needs to be decontextualized from immediate sensorimotor states and the environment. In contrast to this traditional approach, the Embodied Cognition framework attempts to provide a more continuous explanation of perception and action on the one hand, and cognition on the other, by suggesting that cognition is constituted in sensorimotor experiences. More specifically, knowledge is derived from

sensorimotor-coded routines stored within a generalized system that was originally developed to control an organism's motor behavior and perceive the world around it (Anderson 2008; Barsalou 1999, 2008; Svensson 2007). Currently, there is a great deal of interest from educational psychology in the notion of Embodied Cognition (Black 2011; Calvo and Gomila 2008, chapter 18; de Vega et al. 2008; Goldstone and Son 2005; Kiefer and Trumpp 2012; Lindgren and Johnson-Glenberg 2013). Often cited in this literature is Barsalou's (1999, 2008) perspective on Embodied Cognition, the Perceptual Symbol Systems Account. This perspective provides a fine-grained account of how knowledge might be embodied. In this account, concepts are *grounded* in the re-activation of specific neural patterns in multiple modalities (e.g., motor system, visual system, et cetera) that were activated during previous interactions with the environment. These activation patterns are suggested to be captured in a single multimodal representation: a Perceptual Symbol (Barsalou 1999).

Perceptual Symbols are not holistic or necessarily conscious vehicles of thought. Rather, Perceptual Symbols can selectively capture schematic aspects of sensorimotor regularities occurring in interaction that become stored in long-term memory (Goldstone and Son 2005). This allows for schematic extractions of perceptual but also introspective states that can be recombined in imagination. As such, concepts that are not readily available in the environment (e.g., a hammer made of pudding) might still be grounded in sensorimotor states by mashing the sensorimotor concept of hammer and pudding. Furthermore, it is held that perceptual symbols of very abstract concepts (e.g., truth, love) still rely on complex combinatorics of perceptual states (see also Lakoff and Johnson 1999; Lakoff and Núñez 2000). Importantly, particular sensorimotor states induced during interaction can trigger activation of Perceptual Symbols that activate stored sensorimotor information. Thus, one sensorimotor state induced by the environment can trigger a host of other sensorimotor states through activation spread. For example, seeing a hammer might induce modality specific simulations of the weight of the hammer.

There is increasing evidence that cognitions are intimately tied to the sensorimotor system (e.g., Kiefer and Trumpp 2012; Pecher and Zwaan 2005; Svensson 2007). Indeed, the sensorimotor system has been found to be implicated in thought processes as diverse as reading, mental arithmetic, problem-solving, and conversely, semantic areas are often implicated in sensor–motor interactions suggesting that both systems are intimately related (Barsalou 1999, 2008; Glenberg 2008; Martin 2007; Nathan 2008). To give an example, research shows that merely reading words that have olfactory, gustatory, or motor connotations (e.g., garlic, jasmine, salt, sour, kick, pick) as opposed to neutral words, activates brain regions that are involved in smelling, tasting, and moving (Barrós-Loscertales et al. 2011; Gonzalez et al. 2006; Hauk et al. 2004). Furthermore, when subjects are mentally processing numbers, activation of motor areas associated with finger movements is consistently found (Andres et al. 2007; Roux et al. 2003; Zago et al. 2001). In sum, the current state of the literature suggests that knowledge representations are intimately tied to the sensorimotor system, which raises the need to understand how the cognitive system draws from sensorimotor information that emerges during interaction with the environment.

Embodied Cognition and Manipulatives

In this section, we give a representative overview of research on manipulatives that specifically claims to be, or in our view seems to be, relevant to Embodied Cognition. We review three streams of research on transfer that provide varying degrees of support for either an Embodied Cognition perspective or the more moderate view mentioned in the introduction that seems to suggest that abstraction is hampered by perceptual and interactive richness.

Transfer by Internalizing Sensorimotor Information

The first line of research is well aligned with the Embodied Cognition perspective and shows that transfer of learning is simply internalization of sensorimotor information that is initially provided by the manipulative. To give a striking example: moderately advanced abacus users maintain high arithmetic capabilities during mental calculation in the absence of an abacus by “manipulating” what seems to be a mentally projected abacus. Such users often apply finger manipulations as if the abacus is physically accessible. Interestingly, expert abacus users even perform better in mentally manipulating as opposed to physically manipulating the abacus (Hatano et al. 1977; Hatano and Osawa 1983). This suggests that having had a very high number of sensorimotor experiences with the abacus can instantiate fully mental simulations without external support needed to maintain it. Importantly, the contention that nonverbal sensorimotor representations underlie mental calculation of abacus users has recently been strengthened; performance of mental calculation in normal subjects is inhibited by verbal interference, whereas for trained abacus users no interference effects are found (Frank and Barner 2012).

Furthermore, a recent study showed that participants who had learned with either a physical or virtual abacus performed equally well in recognizing number representations of an abacus presented on paper; however, virtual abacus-trained participants performed worse on a transfer task that required more complex arithmetic operations with a physical abacus in comparison to participants who had trained with a physical abacus (Flanagan 2013). Relatedly, in a study by Flusberg and Boroditsky (2011) on mental rotation, it has been found that sensorimotor experience with objects that are difficult to manipulate actually hindered effective mental rotation of those objects, whereas easily manipulable objects promoted mental rotation. These studies show that sensorimotor simulations that underlie cognition are very sensitive to the experiences afforded by manipulatives. As such, if we understand transfer of learning as learning to think without manipulatives, it does not necessarily involve decontextualization, but rather internalization of sensorimotor routines.

This development of internalized embodied knowledge seems to be a gradual process; that is, learners slowly dis-embed their mental activity from the environment. An obvious example is when children stop using finger gestures to count. Moreover, as mentioned earlier, abacus users that have an intermediate level of expertise often use gestures to support their thinking while experts do not need such support for their mental calculations, which suggest a kind of transition state between relying on purely external to internal recourses. In a similar vein, intermediate chess players perform better at thinking through moves (without manipulating the pieces) when a chessboard is present. In contrast, chess masters do not need external support in their mental chess-playing (Chase and Simon 1973). A relevant study that provides insight on when external support is of importance comes from Kirsh (2009) in which subjects played a mental tic-tac-toe game with the experimenter. It was found that external perceptual support of a sheet with a matrix depicted on it as opposed to providing no support, a blank sheet, aided performance. However, this external support was only beneficial when the tic-tac-toe game was complex (4×4 matrix), and especially for subjects who scored low on spatial ability. Thus, this study suggests that external support is especially helpful when computational load is high, and this depends on whether the subject is effective at performing those computations internally (e.g., spatial ability; Kirsh 2009). This might characterize how novices become experts. External structures are gradually internalized, and internalization being dependent on the learners’ “representational stability” (Hutchins 2005), that is, the ability to mentally stand in for external structures. For example, having low spatial cognitive ability—signifying difficulty in producing a stable representation—leads to a higher need to lean on external support (Kirsh 2009). Interestingly, the use of hand gestures can also be seen as an instance of

external support to maintain representational stability (Chu and Kita 2011; Chu et al. 2013; Radman 2013). For example, it has been found that frequency of spontaneous use of gestures is correlated with having low ability in spatial imagery and rotation (Chu et al. 2013).

Taking these results together, from an Embodied Cognition perspective, it can be argued that actively learning with manipulatives can establish sensorimotor routines that are internalized (i.e., embodied); without the need to invoke symbolic rules as expertise develops. Thus in these specific cases, interaction vs. thinking with a manipulative does not rely on a concrete to abstract shift; both modes of cognitive performance rely on the same representational format (sensorimotor routines), wherein increasing computational load is put on the brain as the learner is required to dis-embed cognitive activity (e.g., mental calculations).

Transfer by Actually or Mentally Simulating Text or Science Scenarios

The second line of research focuses on attaining conceptual and narrative understanding of texts and science materials through manipulatives. In these cases, the role of *grounding* a concept in sensorimotor experiences has been studied (Glenberg et al. 2004, 2011a, b; for a review see De Koning and Van der Schoot 2013). For example, in several experiments by Glenberg et al. (2004), first- and second-grade children read a text and manipulated toy figures that referred to, and offered a way to enact, the scenario of the text. It was found that children enacting the text scenarios (compared to only reading them) were better at story recall, making inferences from the story, and in their understanding of spatial relations mentioned in the story. Furthermore, having had practice with physical manipulation of toys, children who had to re-enact the scenario mentally through imagination showed similar improvements. Importantly however, it has been found that positive effect of manipulatives for text comprehension can be attained by virtual manipulatives as well (Glenberg et al. 2011a, b) and simply watching someone else enact the story can equally benefit learning (Marley et al. 2007, 2011).

As Glenberg and Kaschak (2002) argue, these results suggest that understanding of text arises through simulating the scenario's content. Manipulatives offer a way to ground the scenario's content directly, as such promoting simulation processes that underlie text comprehension (Glenberg et al. 2004). Similar findings are obtained in science education, in which the role of physical versus virtual manipulatives has been studied extensively (De Jong et al. 2013; Olympiou and Zacharia 2012; Olympiou et al. 2013; Triona and Klahr 2003; Triona et al. 2005; Zacharia and Constantinou 2008; Zacharia et al. 2012; Zacharia and Olympiou 2011).

For example, Zacharia and Olympiou (2011) investigated experimentation with heat and temperature by undergraduate students who interacted with either physical or virtual materials or both and were tested for conceptual learning through assessment of pre-test and post-test. In the physical condition, the materials consisted of normal beakers, waters, hotplate, et cetera, whereas virtual materials consisted of 2D approximations of those materials that could be manipulated with a mouse. It was found that participants learned equally across conditions (for an earlier study obtaining similar results, see Zacharia and Constantinou 2008). In another study, undergraduate physics students learned the workings and conceptual underpinnings of simple electrical circuits, such as voltage, and parallel vs. series circuits. In the critical conditions, during a 15-week physics course, students either learned through concrete physical materials or interactive computer simulations thereof (Finkelstein et al. 2005). It was found that students learning with physical versus virtual materials performed worse on a test of conceptual understanding, as well in their evaluations of a setup with physical materials.

These and other studies (Klahr et al. 2007; Triona and Klahr 2003; Triona et al. 2005) consistently show that in many cases physical manipulatives are replaceable by virtual ones without learning costs. Based on this research, it has been suggested that null (and negative;

e.g., Finkelstein et al. 2005) results concerning physicality seem to contradict “the embodied nature of cognition [that] would seem to suggest that type of materials [i.e., whether they are physical or virtual] would influence student’s learning” (Triona et al. 2005, p. 1). In similar vein, in a recent review on the role of physical and virtual manipulatives in laboratory education, it was suggested that physical laboratories should promote learning by offering “...tactile information that, according to theories of Embodied Cognition, fosters development of conceptual knowledge” (De Jong et al. 2013, p. 305).

Importantly, however, if one indeed takes the position that previous findings (including research on text comprehension) contradict with Embodied Cognition³, one must have a clear understanding of what Embodied Cognition would predict in a particular context. Unfortunately, in all the previously reported studies, it is not clearly explained why physical as opposed to virtual beakers, short springs, or toys would aid conceptual understanding, besides the broad but simplistic claim that tactile or multimodal experiences should aid conceptual learning (De Jong et al. 2013; Triona and Klahr 2003; Zacharia and Olympiou 2011).

What would a proper or more moderate reading of Embodied Cognition predict in these research contexts? Firstly, it cannot be denied that learners understand a physical beaker differently if one has haptic vs. no haptic experience with it, perhaps even producing richer multimodal simulations when thinking about it. However, there is no reason to assume that this information aids “learning” of the sort assessed in these experiments. In a recent study in physics learning, it was shown that Embodied Cognition does allow one to make more fine-grained predictions concerning the role of physicality. The researchers predicted based on Embodied Cognition that physical rather than virtual manipulatives would positively affect kindergartners’ learning, but only for those who had an incorrect preconception of mass before the learning phase. An incorrect conception being when a student incorrectly predicts a heavier object would go up on a beam balance. This was further explained that if a concept of mass and its effect on a balance beam is incorrectly or simply not instantiated in experience, additional physical experience becomes more important. Children were pre-assigned on the basis of whether they knew what a beam balance does to either the incorrect or correct preconception group, and were then further subdivided in a physical manipulative condition with real weights and balance beam vs. virtual manipulative condition in which children learned with a computer simulation of weights and beam balance. In line with the predictions, it was found that only children with an incorrect conception of mass in relation to the beam balance showed learning gains from physical materials. This can be explained by the fact that children with correct preconceptions already had a good understanding of mass (grounded in previous haptic experiences), and therefore had no additional relevant information to gain from learning from physical rather than virtual manipulatives. These and other results seem to show that sensorimotor information can indeed be important for learning (Black 2011; Han and Black 2011; Morris et al. 2007; for an overview see Sigrist et al. 2013).

As such, these examples show that Embodied Cognition would only predict that learning a particular concept through sensorimotor experience is important for those concepts that draw on that information for understanding of a particular concept. For example, mass must be grounded multimodally simply because mass cannot be easily determined by the visual modality alone (e.g., two objects might look the same but vary in weight). This directly aligns with the finding that text comprehension is promoted by virtual manipulatives to the same extent as by physical ones (Glenberg et al. 2011a, b). For example, we would only predict an effect of physical and interactive richness when the text involves information not readily

³ It is important to note this is a *possible* position that can be drawn from the results, not necessarily a position that all the authors of the previously reported studies take.

attainable through the visual modality alone. An example of such a scenario could be a protagonist that has to choose between two treasure cases that look the same, but weigh differently or one that is locked and the other can be opened. Furthermore, when increasing the quantity or complexity of these visually unattainable features (providing that they are not already grounded in previous experiences; Zacharia et al. 2012), one would predict that physical manipulation becomes beneficial to text comprehension.

Transfer by Replacement

The final stream of research we present here seems to entail greater problems for Embodied Cognition. This research involves manipulatives that hold the “task of figuring out that one thing is intended to represent another, that the meaning and importance of the symbol lie in its relation to its referent” (Uttal et al. 2009, p. 157; see also Uttal et al. 1997).

This line of research has shown that perceptually rich physical objects can actually hinder performances in cases where the manipulative stands-in-for something else (DeLoache 1987, 1991, 2000). In these studies, children ranging from 2 to 3 years old have to obtain a toy hidden in a room. Children must do this by watching the experimenter hide a toy in a 3D reconstructed model of the room accompanied with the instruction that the real toy is hidden at the same place as in the model. It has consistently been found that children perform worse with 2D representations rather than perceptually rich and realistic 3D mock-ups at retrieving the toy in the real room (DeLoache 1987, 1991). Furthermore, a glass plate put in front of the child—which prevents solicitations of acting on the model—actually improves inferential performance in contrast to a model that can be interacted with (DeLoache 2000).

Although not about manipulatives directly, but often presented as relevant to the domain of manipulatives, are findings from studies that show that learning abstract (mathematical) relations and extending them onto novel but principally isomorphic situations is promoted when it is instantiated in a more abstract form as opposed to a concrete, or perceptually rich form (De Bock et al. 2011; Goldstone and Sakamoto 2003; Goldstone and Son 2005; Johnson et al. 2014; Kaminski et al. 2008, 2009a, b, 2013; Sloutsky et al. 2005). For example, although concrete (cupcakes) in comparison to abstract (circles) instantiations were better for learning a mathematical relation (fractions), transfer of learning was higher for kindergartners who learned with the arbitrary symbolic instantiations (Kaminski et al. 2009a, b). In another well-known study of Kaminski et al. (2008), similar results were found showing that although concrete instantiation resulted in the highest performance on problem-solving, transfer of learning in which the same mathematical relation had to be deduced was hampered by concrete instantiations (also see Kaminski et al. 2013). Importantly, although concrete-to-abstract might prove to be a leap too far when there is too much emphasis on the concrete, it has recently been shown that such a symbolic leap may sometimes best unfold in steps, fading concreteness into abstract forms (Fyfe et al. 2014; Goldstone and Son 2005; cf. Johnson et al. 2014; McNeil and Fyfe 2012; Scheiter et al. 2010). For example, in a problem-solving task in which the proportion of different trees had to be discovered, it was found that by gradually morphing realistically visualized trees into less detailed green squares led to reliable learning benefits as compared to a generic text-based format (Scheiter et al. 2010).

According to the Dual Representation hypothesis (DeLoache 2000), inhibited performance with concrete objects can be explained by the fact that subjects have to attain a dual representation: the concrete object in its own right, and its referent. Perceptual richness, therefore, may simply incline participants towards treating the situation as one single concrete instance, as one representation. Indeed for some researchers, this explains why learning from some manipulatives (e.g., Dienes base-10 blocks; Resnick and Omanson 1987) is notoriously

difficult to translate into formalized forms (Uttal et al. 1997). Thus, it is suggested that manipulatives should be designed to be like symbols when they refer to some higher order else, avoiding perceptually rich and real world characteristics (Uttal et al. 1997).

Intermediate Discussion: Embodied Cognition

In this section, we made the case that transfer of learning does not necessarily rely on a concrete-to-abstract shift. We have presented three lines of research on transfer with manipulatives that seem to lead to different results on whether such a claim can be maintained.

Firstly, there are those situations in which thinking in the absence of manipulatives remains true to a sensorimotor format, which seems to be the case with mental calculation with abacus-trained users. We suggest that learning in this respect depends on the gradual internalization of sensorimotor routines. It is often gradual in that the learner slowly loses its dependence on external props (from full dependence of the environment, to projection with bodily resources [gestures], solely visual projection et cetera). As such, in lieu of the “concreteness fading” (Goldstone and Son 2005), it can be argued that transfer of learning from manipulatives often involves gradual fading of the interaction with the environment, making place for internal sensorimotor simulations, the speed of internalization being dependent on the learners capability of having “representational stability” to stand in for external goings-on (Hutchins 2005; Kirsh 2009).

The second line of research, with evidence from science education and reading comprehension, showed that the tenet of Embodied Cognition that concepts are grounded in sensorimotor experiences has been implicitly and unduly interpreted as learning should benefit from grounding concepts and procedures in the kind of perceptual richness that unfolds in real-world practices (cf. De Jong et al. 2013; Klahr et al. 2007; Triona and Klahr 2003; Triona et al. 2005). Moreover, the kind of assessment of learning in studies that we have cited above are not, and perhaps should not be, sensitive to the kind of perceptual richness-differences between physical and virtual environments that Embodied Cognition does acknowledge. Indeed, for Embodied Cognition, learning from a physical laboratory or virtual laboratory would result in different lived experiences and as such would have different multimodal associations when thinking about the learned context. However, for Embodied Cognition-driven experimental educational psychologists, the challenge is to be sensitive to sensorimotor information that allows the Perceptual Symbol (i.e., concept) “to do its work” in the context of a task. Needless to say, physical and virtual manipulative’s properties vary deeply in the sensorimotor information they can provide. Important to note is that much of the research on science education as discussed above has been agnostic to studying the affordances that come with Embedded Cognition which are undoubtedly relevant for the science education domain (e.g., ordering objects in 3D space as to determine which procedure comes first; for example, see Kastens et al. 2008). In sum, this line of research seems to suggest that a more moderate reading of Embodied Cognition would be appropriate, wherein perceptual and interactive richness in and of itself is not something that promotes learning, but is contextually dependent on the learning content being constituted on multimodal information.

The third stream of research seems to be on par with the moderate view that transfer of learning is hampered by perceptual richness (De Bock et al. 2011; Goldstone and Sakamoto 2003; Goldstone and Son 2005; Kaminski et al. 2008; 2009a, b; 2013; Sloutsky et al. 2005). Although such research might not fall in the domain of manipulatives (e.g., Kaminski et al. 2008), it has been argued that manipulatives have similar disadvantages if the adage of maximal perceptual richness is maintained (e.g., Uttal et al. 1997). We think this line of research cannot easily be dismissed and weakens in these particular cases the more simplistic reading of Embodied Cognition wherein *more* sensorimotor information *is better*.

Indeed, we would suggest that with abstract learning goals we should treat manipulatives as what Andy Clark calls *Surrogate Situations* (Clark 2005, 2008). In *Surrogate Situations*, cognition is to some extent decontextualized from the environment since it goes beyond the immediate environment, but not disembedded⁴, since the environment still provides a concrete surface that allows for deploying sensorimotor routines (e.g., just-in-time sensing; Ballard et al. 1997). Indeed, Nathan (2012) recently suggested that the research by Kaminski et al. (2008) does not show that interaction with materials hampers symbolic inferences (see however Deloache 1991). Clark (2005) similarly argues that it is important to retain possibilities for interaction, but keep non-essential detail low as to avoid “gravitational pull” of sensorimotor distractions (e.g., automatic visual attention cues). For example, it has recently been shown with children who have to judge relations of sameness and difference are best able to do this when labels and objects are used that have an “optimal vagueness” (Son et al. 2012). Optimally vague to be recognized as something familiar but not too perceptually rich to avoid what Andy Clark might call the “gravitational pull of perception-action routines” (see also Markman and Gentner 1993). That is, a vague or schematic as opposed to a concrete instantiation of objects that have a sameness relation are more easily generalized to other objects that share this sameness relation.

Thus, we might speculate that manipulatives for abstract thinking should be considered as “manipulable symbols” that still allows for the affordances that are related to Embedded Cognition but are minimally rich in perceptual detail. Indeed, it has been found that even in highly symbolic environments learners draw on perceptual features, such as (self-induced) spacing in algebraic expression, that guide their problem-solving strategies (see Landy and Goldstone 2007). For example, in an expression of $8 \times 4 + 6$, it is found in line with the syntactic structure that “ 8×4 ” is often written with less space between the symbols in comparison to $4 + 6$ as to denote a grouping order. As such, Landy and Goldstone (2007, p. 2038) suggest that spatial relations in the algebraic expression serve “to ground the abstract relationships they express in more immediately available sensorimotor relationships.” Interestingly, this use of space is highly similar to epistemic actions performed by Tetris players (Kirsh and Maglio 1994) as spacing allows the task to be structured as to reduce computational load.

Conclusion

Most scientific discourses show cyclical and reactionary patterns of progress—continually recycling and tempering theories in light of new findings. While early promoters of manipulatives, such as Montessori or Pestalozzi, held that unconstrained, self-guided, manipulation of physical objects would automatically impress complex ideas upon the mind (Page 1990), in more recent literature such views are equated with “magical hopes” (Ball 1992) or “folk psychology or vague theory” (Triona and Klahr 2003, p. 171). Indeed, research seems to indicate that more moderate claims about the role of perceptual and interactive richness are warranted, which has been important for furthering our understanding of learning with manipulatives (Brown et al. 2009; Kaminski et al. 2009a, b; McNeil and Jarvin 2007; Sarama and Clements 2009; Sherman and Bisanz 2009; Uttal et al. 1997). However, in this paper we have in turn made the claim in light of Embedded Embodied Cognition that the current moderate view is also to some extent misguided if it is not negotiated with findings we have provided in this review. The research reviewed here from an Embedded Cognition perspective

⁴ Note that Clark (2005) uses “disembodied” here. We use disembedded as to consistently make a distinction between embeddedness and embodiment.

firstly suggests that learners quite naturally draw on external support from the environment to alleviate cognitive load (e.g., Ballard et al. 1997; Kirsh and Maglio 1994). Secondly, learners are affected by subtle changes in the environment that influence the ease of attaining information either internally or externally (e.g., Gray and Fu 2004; Risko et al. 2013). Thirdly, embedded learning can be constrained by manipulatives that impose a certain course of action (e.g., Martin and Schwartz 2005; Stull et al. 2012), whereas self-guided problem-solving strategies can be effective, but seem to be moderated by the perception of possibilities for action on manipulatives (e.g., Antle 2012; Manches et al. 2010; Stull et al. 2013).

However, it is not yet clear how manipulatives can be designed in such a way that these different processes are optimally supported, especially in relation to each other. In other words, based on the evidence reviewed here, it is not yet possible to derive clear instructional design guidelines. As such, one of the challenges for research on Embedded Cognition and manipulatives is to determine how perceptual and interactive properties alter both the way interaction can occur (hard constraints) as well as how these properties impinge on learners' likely course of action given the possibilities (soft constraints). Tangible user interfaces seem well-suited for addressing such questions, as they provide a plethora of possibilities in maintaining physical interaction that can be related to perceptual properties in digital learning environments (Manches and O'Malley 2012; O'Malley and Stanton-Fraser 2004; Shaer and Hornecker 2010). Another important research question is how differing numbers of affordances that elicit external as opposed to internal learning strategies relate to long-term memory representations that are the source of transferring knowledge in the absence of manipulatives.

Current research reviewed here from an Embodied Cognition perspective seems to indicate that successful transfer of learning, in which the goal of manipulatives is to structure thinking in the absence of those manipulatives, does not necessarily involve decontextualization from perceptual and interactive constraints of manipulatives (e.g., Frank and Barner 2012). In the research discussed here, it became clear that embedded interactions become embodied and aid in off-line thinking. We have further made the case that this often occurs gradually, wherein external support is faded out when expertise develops⁵ and is dependent on the *internal* representational stability that the learner can maintain (e.g., Hatano et al. 1977; Hatano and Osawa 1983; Kirsh 2009). As such, an interesting prediction to be tested in future research would be that it is important for transfer of learning that learners have enough sensorimotor experience with a manipulative to be able to think without it. Interestingly, research seems to indicate that internal representational stability is promoted when interaction is easy (Flanagan 2013), suggesting that ease of manipulability affects ease of internalization.

Nevertheless, it has also become clear that whether Embedded Embodied Cognition can help make relevant predictions also depends on the learning goals and the assessment of whether these have been attained. In line with a moderate view, perceptual richness is not beneficial to learning when the assessed learning outcomes do not depend on multimodal information (e.g., Glenberg et al. 2011a, b; Triona and Klahr 2003). In fact it can be argued that much of the research reviewed here actually shows that perceptual richness might hamper making abstract inferences (e.g., Kaminski et al. 2009a, b). On a speculative note, we have argued that manipulatives might still be important for learning abstract relations since they provide the learner with external support, and that current research should focus on how embedding learners in manipulable but not perceptually rich learning environments (i.e., surrogate situations).

Although theories of Embedded Embodied Cognition might be a suitable starting point for research on these open questions and enjoys empirical support to weaken moderate claims

⁵ Important to note, this depends on whether expertise is defined as a disembedded cognitive capability.

presented in the introduction, an important shortcoming of the current perspective in terms of educational implications is that given the current state of the literature it is difficult to provide guidelines for how manipulatives should be designed. Yet, an important educational implication we can take home from the Embedded Embodied perspective is that mouse-based virtual manipulatives, which reduce perceptual and interactive richness compared to physical manipulatives or tangible user interfaces, do not necessarily optimize the learner's cognitive load either. Furthermore, it has recently been argued by Nathan (2012) that research that shows that perceptually rich representations might not be suitable for bringing across abstract symbolic relations, should not lead educators to adopt the view that learning should go “without exposure to perceptually rich stimuli” since it “robs learners of opportunities to learn how to recognize deep structure and filter out irrelevancies” (Nathan 2012, p. 137). We would make a similar argument that educational design and research should focus on ways to expose learners to a range of interactive possibilities from which efficient externally mediated problem-solving strategies might arise.

To end with a theoretical note, the Embedded Embodied perspective, as opposed to a moderate view, attempts to provide an account of how the central aspect of manipulatives, that is, what sensorimotor information they provide, is beneficial for learning. Learning from manipulatives is always sensorimotor in nature—i.e., it always involves some degree of bodily interaction of the learner with the environment, if not, it ceases to be a manipulative. Indeed, when “subtracting” learning with manipulatives from learning with other materials such as texts or non-interactive instructional animations, we will always be left with perceptual and interactive richness as the key residual difference at the side of manipulatives. Thus, any perspective that seeks to guide instructional design of manipulatives should specify how the *body in action* affords processing of information not easily maintained with other learning materials and how this relates to long-term knowledge representations. In our opinion, the research reviewed here suggests that while more research is clearly necessary, the Embedded Embodied Cognition perspective provides a more promising starting point than a moderate view for furthering our understanding of how perceptual and interactive richness might aid learning.

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References

- Anderson, M. L. (2008). On the grounds of (X)-grounded cognition. In P. Calvo & T. Gomila (Eds.), *Handbook of cognitive science: an embodied approach* (pp. 423–435). New York: Elsevier.
- Andres, M., Seron, X., & Oliver, E. (2007). Contribution of hand motor circuits to counting. *Journal of Cognitive Neuroscience*, 19, 563–576.
- Antle, A. N. (2012). Exploring how children use their hands to think: an embodied interactional analysis. *Behaviour & Information Technology*, 32, 938–954.
- Antle, A. N., Droumeva, M., & Ha, D. (2009). Hands on what?: Comparing children's mouse-based and tangible-based interaction. In *Proceedings of the 8th International Conference on Interaction Design and Children* (pp. 80–88).
- Ball, D. L. (1992). Magical hopes: manipulatives and the reform of math education. *American Educator*, 16(2), 14–18.
- Ballard, D. H., Hayhoe, M. M., & Pelz, J. B. (1995). Memory representations in natural tasks. *Journal of Cognitive Neuroscience*, 7(1), 66–80.
- Ballard, D. H., Hayhoe, M. M., Pook, P. K., & Rao, R. P. N. (1997). Deictic codes for the embodiment of cognition. *Behavioral and Brain Sciences*, 20, 723–767.
- Barrós-Loscertales, A., González, J., Pulvermüller, F., VenturaCampos, N., Bustamante, J. C., Costumero, V., et al. (2011). Reading “salt” activates gustatory brain regions: fMRI evidence for semantic grounding in a novel sensory modality. *Cerebral Cortex*, 22, 2554–2563.

- Barsalou, L. W. (1999). Perceptual symbol systems. *Behavioral and Brain Sciences*, 22(4), 577–660.
- Barsalou, L. W. (2008). Grounded cognition. *Annual Review of Psychology*, 59, 617–645.
- Black, J. B. (2011). *Embodied cognition and learning environment design. Theoretical foundations of student-centered learning environments*. New York: Routledge.
- Borst, J. P., Buwalda, T. A., van Rijn, H., & Taatgen, N. A. (2013). Avoiding the problem state bottleneck by strategic use of the environment. *Acta Psychologica*, 144(2), 373–379.
- Bredo, E. (1994). Reconstructing educational psychology: situated cognition and Deweyan pragmatism. *Educational Psychologist*, 29(1), 23–35.
- Brown, M. C., McNeil, N. M., & Glenberg, A. M. (2009). Using concreteness in education: real problems, potential solutions. *Child Development Perspectives*, 3(3), 160–164.
- Calvo, P., & Gomila, T. (2008). *Handbook of cognitive science: an embodied approach*. San Diego: Elsevier.
- Chase, W. G., & Simon, H. A. (1973). Perception in chess. *Cognitive Psychology*, 4, 55–81.
- Chu, M., & Kita, S. (2011). The nature of gestures' beneficial role in spatial problem solving. *Journal of Experimental Psychology: General*, 140, 102–116.
- Chu, M., Meyer, A., Foulkes, L., & Kita, S. (2013). Individual differences in frequency and salience of speech-accompanying gestures: the role of cognitive abilities and empathy. *Journal of Experimental Psychology: General*. doi:10.1037/a0033861.
- Clark, A. (2005). Beyond the flesh: some lessons from a mole cricket. *Artificial Life*, 11(1–2), 233–244.
- Clark, A. (2008). *Supersizing the mind: embodiment, action, and cognitive extension*. New York: Oxford University Press.
- Clark, A., & Chalmers, D. (1998). The extended mind. *Analysis*, 58(1), 7–19.
- Clements, D. H. (2000). 'Concrete' manipulatives, concrete ideas. *Contemporary Issues in Early Childhood*, 1(1), 45–60.
- De Bock, D., Deprez, J., Van Dooren, W., Roelens, M., & Verschaffel, L. (2011). Abstract or concrete examples in learning mathematics? A replication and elaboration of Kaminski, Sloutsky, and Heckler's study. *Journal for Research in Mathematics Education*, 42(2), 109–126.
- De Jong, T., Linn, M. C., & Zacharia, Z. C. (2013). Physical and virtual laboratories in science and engineering education. *Science*, 340(6130), 305–308.
- De Koning, B. B., & Van der Schoot, M. (2013). Becoming part of the story! Refueling the interest in visualization strategies for reading comprehension. *Educational Psychology Review*, 25, 261–287.
- de Vega, M., Glenberg, A. M., & Graesser, A. C. (2008). *Symbols, embodiment and meaning*. Oxford: Oxford University Press.
- DeLoache, J. S. (1987). Rapid change in the symbolic functioning of very young children. *Science*, 238(4833), 1556–1557.
- DeLoache, J. S. (1991). Symbolic functioning in very young children: understanding of pictures and models. *Child Development*, 62(4), 736–752.
- DeLoache, J. S. (2000). Dual representation and young children's use of scale models. *Child Development*, 71(2), 329–338.
- DeLoache, J. S. (2004). Becoming symbol-minded. *Trends in Cognitive Sciences*, 8(2), 66–70.
- Dienes, Z. P. (1973). *The six stages in the process of learning mathematics*. Slough: National Foundation for Education Research/Nelson.
- Dourish, P. (2004). *Where the action is: the foundations of embodied interaction*. Massachusetts: MIT Press.
- Droll, J. A., & Hayhoe, M. M. (2007). Trade-offs between gaze and working memory use. *Journal of Experimental Psychology: Human Perception and Performance*, 33(6), 1352.
- Finkelstein, N. D., Adams, W. K., Keller, C. J., Kohl, P. B., Perkins, K. K., Podolefsky, N. S., et al. (2005). When learning about the real world is better done virtually: a study of substituting computer simulations for laboratory equipment. *Physical Review Special Topics-Physics Education Research*, 1(1), 1–8.
- Flanagan, R. (2013). Effects of learning from interaction with physical or mediated devices. *Cognitive Processing*, 14, 213–215.
- Flusberg, S. J., & Boroditsky, L. (2011). Are things that are hard to physically move also hard to imagine moving? *Psychonomic Bulletin & Review*, 18(1), 158–164.
- Fodor, J. A. (1975). *The language of thought*. Cambridge: Harvard University Press.
- Frank, M. C., & Barner, D. (2012). Representing exact number visually using mental abacus. *Journal of Experimental Psychology: General*, 141(1), 134–149.
- Fu, W. T. (2011). A dynamic context model of interactive behavior. *Cognitive Science*, 35(5), 874–904.
- Fyfe, E., McNeil, N., Son, J., & Goldstone, R. (2014/this issue). Concreteness fading offers the best of both concrete and abstract instruction. *Educational Psychology Review*.
- Gaschler, R., Vaterrodt, B., Frensch, P. A., Eichler, A., & Haider, H. (2013). Spontaneous usage of different shortcuts based on the commutativity principle. *PLoS ONE*, 8(9), e74972.
- Gibson, J. J. (1979). *The ecological approach to visual perception*. Boston: Houghton-Mifflin.
- Glenberg, A. M. (2008). Embodiment for education. In P. Calvo & T. Gomila (Eds.), *Handbook of cognitive science: an embodied approach* (pp. 355–372). New York: Elsevier.
- Glenberg, A. M., & Kaschak, M. P. (2002). Grounding language in action. *Psychonomic Bulletin & Review*, 9(3), 558–565.

- Glenberg, A. M., Gutierrez, T., Levin, J. R., Japuntich, S., & Kaschak, M. P. (2004). Activity and imagined activity can enhance young children's reading comprehension. *Journal of Educational Psychology*, 96(3), 424–436.
- Glenberg, A., Willford, J., Gibson, B., Goldberg, A., & Zhu, X. (2011a). Improving reading to improve math. *Scientific Studies of Reading*, 16, 316–340.
- Glenberg, A. M., Goldberg, A. B., & Zhu, X. (2011b). Improving early reading comprehension using embodied CAI. *Instructional Science*, 39(1), 27–39.
- Goldstone, R. L., & Sakamoto, Y. (2003). The transfer of abstract principles governing complex adaptive systems. *Cognitive Psychology*, 46(4), 414–466.
- Goldstone, R. L., & Son, J. Y. (2005). The transfer of scientific principles using concrete and idealized simulations. *The Journal of the Learning Sciences*, 14(1), 69–110.
- Gonzalez, J., Barros-Loscertales, A., Pulvermüller, F., Meseguer, V., & Sanjuán, A. (2006). Reading cinnamon activates olfactory brain regions. *NeuroImage*, 32, 906–912.
- Gray, W. D., & Fu, W. (2004). Soft constraints in interactive behavior: the case of ignoring perfect knowledge in-the-world for imperfect knowledge in-the-head. *Cognitive Science*, 28(3), 359–382.
- Gray, W. D., Sims, C. R., Fu, W.-T., & Schoelles, M. J. (2006). The soft constraints hypothesis: a rational analysis approach to resource allocation for interactive behavior. *Psychological Review*, 113, 461–482.
- Han, I., & Black, J. B. (2011). Incorporating haptic feedback in simulation for learning physics. *Computers & Education*, 57(4), 2281–2290.
- Haselen, G. L. V., van der Steen, J., & Frens, M. A. (2000). Copying strategies for patterns by children and adults. *Perceptual and Motor Skills*, 91(2), 603–615.
- Hatano, G., & Osawa, K. (1983). Digit memory of grand experts in abacus-derived mental calculation. *Cognition*, 15(1), 95–110.
- Hatano, G., Miyake, Y., & Binks, M. G. (1977). Performance of expert abacus operators. *Cognition*, 5(1), 47–55.
- Hauk, O., Johnsrude, I., & Pulvermüller, F. (2004). Somatotopic representation of action words in human motor and premotor cortex. *Neuron*, 41, 301–307.
- Hayhoe, M. M., Pook, P. K., & Rao, R. P. N. (1997). Deictic codes for the embodiment of cognition. *Behavioral and Brain Sciences*, 20, 723–767.
- Hutchins, E. (1995). *Cognition in the wild*. Cambridge: MIT Press.
- Hutchins, E. (2005). Material anchors for conceptual blends. *Journal of Pragmatics*, 37(10), 1555–1577.
- Johnson, A. M., Reisslein, J., & Reisslein, M. (2014). Representation sequencing in computer-based engineering education. *Computers & Education*, 72, 249–261.
- Kaminski, J. A., Sloutsky, V. M., & Heckler, A. F. (2008). Learning theory: the advantage of abstract examples in learning math. *Science*, 320(5875), 454–455.
- Kaminski, J. A., Sloutsky, V. M., & Heckler, A. F. (2009a). Transfer of mathematical knowledge: the portability of generic instantiations. *Child Development Perspectives*, 3(3), 151–155.
- Kaminski, J. A., Sloutsky, V. M., & Heckler, A. F. (2009b). The devil is in the superficial details: why generic instantiations promote portable mathematical knowledge. *Child Development Perspectives*, 3, 151–155.
- Kaminski, J. A., Sloutsky, V. M., & Heckler, A. F. (2013). The cost of concreteness: the effect of nonessential information on analogical transfer. *Journal of Experimental Psychology: Applied*, 19(1), 14–29.
- Kastens, K. A., Liben, L. S., & Agrawal, S. (2008). Epistemic actions in science education. In C. Freksa, N. S. Newcombe, P. Gardenfors, & S. W. Wöfl (Eds.), *Proceedings of the International Conference on Spatial Cognition VI: learning, reasoning, and talking about space* (pp. 205–215). Heidelberg: Springer.
- Kiefer, M., & Trumpp, N. M. (2012). Embodiment theory and education: the foundations of cognition in perception and action. *Trends in Neuroscience and Education*, 1, 15–20.
- Kirsh, D. (1995). The intelligent use of space. *Artificial Intelligence*, 73(1), 31–68.
- Kirsh, D. (2009). Projection, problem space and anchoring. In *Proceedings of the 31st annual conference of the cognitive science society* (pp. 2310–2315). Hillsdale: Lawrence Erlbaum Associates.
- Kirsh, D. (2010). Thinking with external representations. *AI & Society*, 25(4), 441–454.
- Kirsh, D., & Maglio, P. (1994). On distinguishing epistemic from pragmatic action. *Cognitive Science*, 18, 513–549.
- Klahr, D., Triona, L., Lara, M., & Williams, C. (2007). Hands on what? The relative effectiveness of physical versus virtual materials in an engineering design project by middle school children. *Journal of Research in Science Teaching*, 44(1), 183–203.
- Lakoff, G., & Johnson, M. (1999). *Philosophy in the flesh: the embodied mind and its challenge to Western thought*. New York: Basic Books.
- Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: how the embodied mind brings mathematics into being*. New York: Basic books.
- Landy, D., & Goldstone, R. L. (2007). How abstract is symbolic thought? *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 33, 720–733.
- Lindgren, R., & Johnson-Glenberg, M. (2013). Emboldened by embodiment six precepts for research on embodied learning and mixed reality. *Educational Researcher*, 42(8), 445–452.

- Manches, A. D., & O'Malley, C. (2012). Tangibles for learning: a representational analysis of physical manipulation. *Personal and Ubiquitous Computing*, 16(4), 405–419.
- Manches, A., O'Malley, C., & Benford, S. (2010). The role of physical representations in solving number problems: a comparison of young children's use of physical and virtual materials. *Computers & Education*, 54(3), 622–640.
- Markman, A., & Gentner, D. (1993). Structural alignment during similarity comparisons. *Cognitive Psychology*, 25, 431–467.
- Marley, S. C., Levin, J. R., & Glenberg, A. M. (2007). Improving Native American children's listening comprehension through concrete representations. *Contemporary Educational Psychology*, 32(3), 537–550.
- Marley, S. C., Szabo, Z., Leven, J. R., & Glenberg, A. M. (2011). Investigation of an activity-based text-processing strategy in mixed-age child dyads. *The Journal of Experimental Education*, 79(3), 340–360.
- Martin, A. (2007). The representation of object concepts in the brain. *Annual Review of Psychology*, 58, 25–45.
- Martin, T., & Schwartz, D. L. (2005). Physically distributed learning: adapting and reinterpreting physical environments in the development of fraction concepts. *Cognitive Science*, 29(4), 587–625.
- Martin, T., Lukong, A., & Reaves, R. (2007). The role of manipulatives in arithmetic and geometry tasks [Electronic version]. *Journal of Education and Human Development*, 1, 1–10.
- McNeil, N. M., & Fyfe, E. R. (2012). "Concreteness Fading" promotes transfer of mathematical knowledge. *Learning and Instruction*, 22, 440–448.
- McNeil, N. M., & Jarvin, L. (2007). When theories don't add up: disentangling the manipulatives debate. *Theory into Practice*, 46(4), 309–316.
- Morris, D., Tan, H., Barbagli, F., Chang, T., & Salisbury, K. (2007). Haptic feedback enhances force skill learning. In EuroHaptics Conference, 2007 and Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems. World Haptics 2007. Second Joint (pp. 21–26).
- Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching Children Mathematics*, 8, 372–377.
- Nathan, M. J. (2008). An Embodied Cognition perspective on symbols, gesture, and grounding instruction. In M. De Vega, A. M. Glenberg, & A. C. Graesser (Eds.), *Symbols and embodiment: debates on meaning and cognition* (pp. 375–396). New York: Oxford University Press.
- Nathan, M. J. (2012). Rethinking formalisms in formal education. *Educational Psychologist*, 47(2), 125–148.
- Newell, A., & Simon, H. A. (1972). *Human problem solving*. Englewood Cliffs: Prentice-Hall.
- Norman, D. A. (1988). *The psychology of everyday things*. New York: Basic books.
- O'Malley, C., & Stanton-Fraser, D. (2004). *Literature review in learning with tangible technologies (Report 12)*. Bristol: Nesta FutureLab Series.
- Olympiou, G., & Zacharia, Z. C. (2012). Blending physical and virtual manipulatives: an effort to improve students' conceptual understanding through science laboratory experimentation. *Science Education*, 96(1), 21–47.
- Olympiou, G., Zacharia, Z. C., de Jong, T. (2013). Making the invisible visible: Enhancing students' conceptual understanding by introducing representations of abstract objects in a simulation. *Instructional Science*, 41, 575–587.
- Osman, M. (2010). Observation can be as effective as action in problem solving. *Cognitive Science*, 32(1), 162–183.
- Page, M. (1990). *Active learning: historical and contemporary perspectives*. Unpublished manuscript, University of Massachusetts. Amherst: ERIC Document Reproduction Service, No. ED 338389.
- Pecher, D., & Zwaan, R. A. (2005). *Grounding cognition: the role of perception and action in memory, language, and thinking*. Cambridge: Cambridge University Press.
- Radman, Z. (2013). *The hand, an organ of the mind*. Massachusetts: MIT Press.
- Resnick, L., & Omanson, S. (1987). *Learning to understand arithmetic*. Hillsdale: Advances in instructional psychology.
- Risko, E. F., Medimorec, S., Chisholm, J. D., & Kingstone, A. (2013). Cognitive offloading in the identification of rotated objects: A natural behavior approach. *Cognitive Science*. doi:10.1111/cogs.12087.
- Roux, F.-E., Boetto, S., Sacko, O., Chollet, F., & Tremoulet, M. (2003). Writing, calculating, and finger recognition in the region of the angular gyrus: a cortical stimulation study of Gerstmann syndrome. *Journal of Neurosurgery*, 99, 716–727.
- Sarama, J., & Clements, D. H. (2009). 'Concrete' computer manipulatives in mathematics education. *Child Development Perspectives*, 3(3), 145–150.
- Scheiter, K., Gerjets, P., & Schuh, J. (2010). The acquisition of problem-solving skills in mathematics: how animations can aid understanding of structural problem features and solution procedures. *Instructional Science*, 38(5), 487–502.
- Schwartz, D. L., & Martin, T. (2006). Distributed learning and mutual adaptation. *Pragmatics & Cognition*, 14(2), 313–332.
- Shaer, O., & Hornecker, E. (2010). Tangible user interfaces: past, present, and future directions. *Foundations and Trends in Human-Computer Interaction*, 3(1–2), 1–137.

- Shapiro, L. A. (2011). *Embodied cognition*. New York: Routledge.
- Sherman, J., & Bisanz, M. (2009). Equivalence in symbolic and nonsymbolic contexts: benefits of solving problems with manipulatives. *Journal of Educational Psychology*, 101(1), 88–100.
- Sigrist, R., Rauter, G., Riener, R., & Wolf, P. (2013). Augmented visual, auditory, haptic, and multimodal feedback in motor learning: a review. *Psychonomic Bulletin & Review*, 20, 21–53.
- Sloutsky, V. M., Kaminski, J. A., & Heckler, A. F. (2005). The advantage of simple symbols for learning and transfer. *Psychonomic Bulletin & Review*, 12(3), 508–513.
- Snow, J. C., Pettypiece, C. E., McAdam, T. D., Mclean, A. D., Stroman, P. W., Goodale, M. A., et al. (2011). Bringing the real world into the fMRI scanner: repetition effects for pictures versus real objects. *Scientific Reports*, 1(130), 1–10.
- Son, J. Y., Smith, L. B., Goldstone, R. L., & Leslie, M. (2012). The importance of being interpreted: grounded words and children's relational reasoning. *Frontiers in Psychology*, 3(45), 1–12.
- Stull, A. T., Hegarty, M., Dixon, B., & Stieff, M. (2012). Representational translation with concrete models in organic chemistry. *Cognition & Instruction*, 30(4), 404–434.
- Stull, A. T., Barrett, T., & Hegarty, M. (2013). Usability of concrete and virtual models in chemistry instruction. *Computers in Human Behavior*, 29(6), 2546–2556.
- Svensson, H. (2007). Embodied simulation as off-line representation. Licentiate thesis, University of Linköping/University of Skövde, Sweden.
- Symes, E., Ellis, R., & Tucker, M. (2007). Visual object affordances: object orientation. *Acta Psychologica*, 124(2), 238–255.
- Triona, L. M., & Klahr, D. (2003). Point and click or grab and heft: comparing the influence of physical and virtual instructional materials on elementary school students' ability to design experiments. *Cognition and Instruction*, 21(2), 149–173.
- Triona, L. M., Klahr, D., & Williams, C. (2005). Point and click or build by hand: comparing the effects of physical vs. virtual materials on middle school students' ability to optimize an engineering Design. In *Proceedings of the 30th Annual Conference of the Cognitive Science Society* (pp. 2202–2205).
- Uttal, D. H., Scudder, K. V., & DeLoache, J. S. (1997). Manipulatives as symbols: a new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology*, 18(1), 37–54.
- Uttal, D. H., O'Doherty, K., Newland, R., Hand, L. L., & DeLoache, J. S. (2009). Dual representation and the linking of concrete and symbolic representations. *Child Development Perspectives*, 3(3), 156–159.
- van Elk, M., van Schie, H., & Bekkering, H. (2014). Action semantics: a unifying conceptual framework for the selective use of multimodal and modality-specific object knowledge. *Physics of Life Reviews*. doi:10.1016/j.plrev.2013.11.005.
- Van Gog, T., & Rummel, N. (2010). Example-based learning: integrating cognitive and social-cognitive research perspectives. *Educational Psychology Review*, 22, 155–174.
- Wheeler, M. (2007). *Reconstructing the cognitive world: the next step*. Cambridge: MIT Press.
- Wilson, M. (2002). Six views of Embodied Cognition. *Psychonomic Bulletin & Review*, 9(4), 625–636.
- Winn, W. (2003). Learning in artificial environments: embodiment, embeddedness and dynamic adaptation. *Technology, Instruction, Cognition and Learning*, 1(1), 87–114.
- Zacharia, Z. C., & Constantinou, C. P. (2008). Comparing the influence of physical and virtual manipulatives in the context of the Physics by Inquiry curriculum: the case of undergraduate students' conceptual understanding of heat and temperature. *American Journal of Physics*, 76, 425–430.
- Zacharia, Z. C., & Olympiou, G. (2011). Physical versus virtual manipulative experimentation in physics learning. *Learning and Instruction*, 21(3), 317–331.
- Zacharia, Z. C., Loizou, E., & Papaevripidou, M. (2012). Is physicality an important aspect of learning through science experimentation among kindergarten students? *Early Childhood Research Quarterly*, 27(3), 447–457.
- Zago, L., Pesenti, M., Mellet, E., Crivello, F., Mazoyer, B., & Tzourio-Mazoyer, N. (2001). Neural correlates of simple and complex mental calculation. *NeuroImage*, 13, 314–327.