

Dynamical Superconductivity in a Frustrated Many-Body System

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In triangular lattice structures, the spatial anisotropy can lead to rich equilibrium phase diagrams with regions containing frustrated, highly entangled states of matter. In this work we study the driven two-rung triangular Hubbard model and evolve these states out of equilibrium, observing how this initial frustration leads to an unexpected phase where, despite a lack of bi-partite structure to the underlying lattice, particle-hole SU(2) symmetry is approximately conserved. This conservation dictates the transient dynamics of the system, causing it to relax towards states with uniform off-diagonal order in both the spin-exchange and particle-hole degrees of freedom. We discuss possible implications of our results for a recent experiment on photo-induced superconductivity in $\kappa - (\text{BEDT} - \text{TTF})_2\text{Cu}[\text{N}(\text{CN})_2]\text{Br}$ molecules.

Introduction – Identifying and understanding the processes which prevent thermalization and decoherence in driven-dissipative quantum systems [1] is a unifying theme in ultracold atoms and condensed matter research [2, 3]. This comes with the potential to realize and functionalize exotic out-of-equilibrium quantum phases, both for the continued progress of fundamental research and for wider technological purposes. In ultrafast materials science, the counterintuitive experimental observation of light-induced superconductivity [4–12] has stimulated the field. In these experiments intense laser pulses have been reported to induce superconducting-like features, such as an inverse-frequency divergence of the imaginary part of the optical conductivity and vanishing resistivity, well above the materials’ equilibrium critical temperatures.

In a very recent experiment, specific vibrational modes of the charge-transfer salt $\kappa - (\text{BEDT} - \text{TTF})_2\text{Cu}[\text{N}(\text{CN})_2]\text{Br}$ were resonantly excited with mid-infrared radiation and the above-mentioned optical features were induced at temperatures several times higher than the equilibrium critical temperature T_c [11]. Moreover, following excitation, a large gap in the real part of the optical conductivity opened up – a feature not seen when cooling the molecular crystal below T_c . These results suggest a different mechanism for superconductivity compared to that when cooling the material. Within Ref. [11], a minimal microscopic two-rung triangular Hubbard lattice, with time-dependent parameters under resonant driving of specific phonon modes, was proposed as a model for the experiment.

A number of theoretical studies have explored the effects of carefully-tuned coherent driving on the prethermal dynamics of one and two dimensional bi-partite fermionic lattice models [13–20]. These studies are motivated by the opportunities arising from having dynamical

time-dependent Hubbard parameters, which have been experimentally realised in various contexts ranging from quantum simulators [21] to strongly-correlated materials via electronic [22–24] as well as vibrational excitations [25]. Their relevance, however, to organic materials such as the $\kappa - (\text{BEDT} - \text{TTF})_2X$ compounds is unclear, due to the dimerized BEDT – TTF molecules forming a half-filled triangular, non bi-partite lattice [26–29]. The triangular Hubbard model does not possess the same symmetries as its bi-partite counterpart and has a distinct, rich equilibrium phase diagram due to frustration induced by the hopping anisotropy and lattice geometry [30, 31].

In this paper we demonstrate how the interplay of heating through generic periodic driving and lattice frustration leads to complex nonequilibrium behavior in a triangular Hubbard model. Motivated by the results of Ref. 11, and the opportunity to explore the many-body dynamics of a driven frustrated system, we consider the two-rung triangular Hubbard model with time-dependent parameters and identify two distinct phases of the system when driving the ground state out of equilibrium. Beneath a critical value of the vertical hopping integral $\tau' < \tau'_c$ the system resides in a quasi-symmetry phase where there is an unexpected particle-hole SU(2) symmetry. This symmetry guides the dynamics of the system towards robust steady states which have uniform off-diagonal long-range order in both the spin-exchange and particle-hole channels. The relaxation also leads to a colossal increase in the long-range doublon correlations. Outside of this phase, particle-hole SU(2) symmetry is truly broken and the driven system cannot dynamically sustain order in the particle-hole channel.

We proceed to identify the origin of these distinct regimes - a rich phase diagram in the ground state of the system, with a large magnetically frustrated region. We show how this initial frustration leads to the unex-

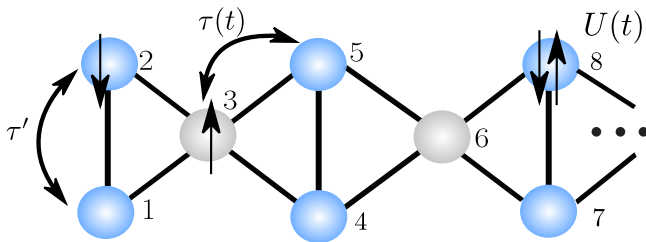


FIG. 1. First 8 sites of the two-rung triangular Hubbard model, see Eq. (1), with time-dependent nearest-neighbor hopping $\tau(t)$, static vertical hopping τ' , and a time-dependent local Hubbard interaction $U(t)$. When $\tau' = 0$, the blue versus grey sites represent a bi-partite splitting of the lattice.

pected dynamical conservation of particle-hole symmetry for a range of driving parameters, despite the lack of a bi-partite structure to the lattice. Finally we show how, even for a small amplitude pulse, driving the system near resonance within the quasi-symmetry phase causes rapid relaxation towards a doublon-ordered state. The optically induced lattice vibrations in the experiment in Ref. [11] occur close to this resonant point. We thus offer a possible explanation for the physical mechanism that might underly the transient onset of superconductivity observed in this experiment. More broadly, our results provide an understanding of how geometrical effects such as lattice frustration can significantly alter the non-equilibrium behavior of driven systems.

Model and Method – The Hubbard model is a paradigmatic quantum lattice model which has relevance for high-temperature superconductivity [32], can be realized in ultracold atom experiments [33], and is solvable using the Bethe ansatz in one dimension [34–36].

The rich symmetry structure of the model is responsible for this solubility. On a bi-partite lattice, there are two $SU(2)$ symmetries, known as the ‘spin’ and ‘ η ’ symmetries [37]. These play a significant role in both the equilibrium and nonequilibrium physics of the model. For example, in a 1D chain, driving/ dissipative terms which break the spin symmetry have been shown to melt spin order and drive the system into long-time states with long-range correlations in the η channel. The preservation of the η $SU(2)$ symmetry under driving is responsible for this phenomenon, termed heating-induced order [38].

Here we focus on the role of heating in the dynamics of a driven non-integrable two-rung triangular Hubbard model, with the driving not explicitly breaking any underlying symmetries. The Hamiltonian is given by

$$H(t) = -\tau(t) \sum_{ij \in \langle \text{n.n.} \rangle, \sigma} (c_{\sigma,i}^\dagger c_{\sigma,j} + \text{h.c.}) - \tau' \sum_{ij \in \langle \text{vert.} \rangle, \sigma} (c_{\sigma,i}^\dagger c_{\sigma,j} + \text{h.c.}) + U(t) \sum_i n_{i,\uparrow} n_{i,\downarrow}, \quad (1)$$

where $n_{\sigma,i}$, $c_{\sigma,i}^\dagger$ and $c_{\sigma,i}$ are, respectively, number, creation and annihilation operators for fermions of spin

$\sigma \in \{\uparrow, \downarrow\}$ on site i . In Eq. (1), the first summation is a time-dependent hopping term, with strength $\tau(t)$, over the diagonal nearest-neighbor bonds pictured in Fig. 1 and the second is a hopping term, with strength τ' , over the vertical bonds. The last term is a time-dependent interaction term, with strength $U(t)$. We restrict the total number of particles to L , the number of lattice sites, and the total magnetisation to 0.

We describe the time-dependence of the nearest-neighbour hopping and interaction strengths via the parametrisation,

$$\begin{aligned} \tau(t) &= \bar{\tau} (1 + A_1 \sin^2(\Omega t) \exp(-(t - T_P)^2 / (2T_w^2))), \\ U(t) &= \bar{U} (1 + A_2 \sin^2(\Omega t) \exp(-(t - T_P)^2 / (2T_w^2))), \end{aligned} \quad (2)$$

where A_1 and A_2 are the amplitudes of the modulation of U and τ relative to their equilibrium values \bar{U} and $\bar{\tau}$. The frequency of the oscillations is characterised by Ω , whilst T_P and T_w describe the offset and width of the Gaussian envelope containing these oscillations. We emphasize that our observations in this paper are based on symmetry and frustration and, unlike typical Floquet engineering approaches, not specific to the parameters of the driving. In the Supplemental Material (SM) we demonstrate our results choosing different forms of the driving compared to those used in the main text.

The symmetry structure of the time-dependent Hamiltonian in Eq. (1) is key to our results. Firstly, we can show that there is a permanent spin $SU(2)$ symmetry $[H(t), S^{\pm, z}] \equiv 0$ where S^\pm and S^z are the total spin raising/lowering and counting operators respectively [35]. For $H(t)$ we can also show

$$[H(t), \eta^z] \equiv 0, \quad [H(t), \eta^+ \eta^-] \propto \tau', \quad (3)$$

where

$$\eta^+ = \sum_{i=1}^L f(i) c_{i,\uparrow}^\dagger c_{i,\downarrow}^\dagger, \quad \eta^z = \sum_i (n_{\uparrow,i} + n_{\downarrow,i} - 1), \quad (4)$$

and $\eta^- = (\eta^+)^\dagger$ are the total η operators and act on the doublons (locally paired fermions) and holons (empty sites) within the lattice. In Equation (4), $f(i)$ takes the value $+1(-1)$ for the blue (grey) lattice sites in Fig. 1. Equation (3) reveals that the system has an η $SU(2)$ symmetry in the limit $\tau' \rightarrow 0$, where the lattice becomes bi-partite. When $\tau' \neq 0$ this symmetry is broken to become a $U(1)$ symmetry in the total particle number.

With this knowledge of the symmetries in hand, we extend the methodology of [39–41] and propose that, in the long-time limit of $H(t)$, the system will reach a state of maximum entropy subject to the constraint that expectation values of conserved quantities must be preserved. In the case of an $SU(2)$ symmetry this constraint leads to heating-induced order, with the long-time state guaranteed to have uniform, long-range correlations in that

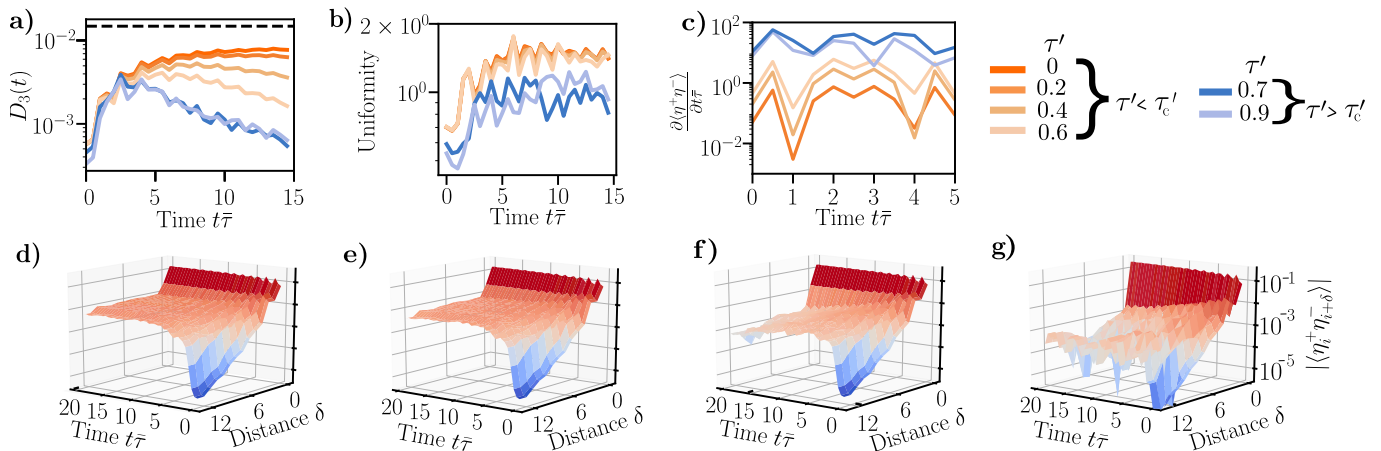


FIG. 2. Dynamics of the half-filled $L = 14$ -site two-rung triangular Hubbard model described by Eq. (1). The system is initialized in the ground state of $H(0)$, setting $\bar{U} = 4.8\bar{\tau}$ with the specified τ' and then time-evolved under $H(t)$, using the same \bar{U} and τ' , and $A_1 = -0.75$, $A_2 = A_1/2$, $\Omega = 2.25\bar{\tau}$, $T_p = 0$ and $T_w = \infty$. a) Off-diagonal doublon order $D_3(t)$ versus time, where $D_3(t)$ is defined in Eq (5). Black dotted line is the long-time analytical prediction for $\tau' = 0$. b) Uniformity of the particle-hole correlations, defined as $D_3(t)$ divided by the standard deviation of the correlations $|\langle \eta_i^+ \eta_j^- \rangle|$ for which $|i - j| \geq 3$. c) Rate of change of $\langle \eta^+ \eta^- \rangle$ in time. d-g) Full-time and distance dynamics of the particle-hole correlations for $\tau' = 0, 0.2\bar{\tau}, 0.6\bar{\tau}, 0.7\bar{\tau}$ respectively.

symmetry sector [38]. Hence when $\tau' = 0$ we expect that a sufficiently long and large amplitude driving from Eq. (2) should force the system to relax to a state where both long-range doublon and spin-exchange correlations coexist due to the conservation of $\eta^+ \eta^-$ and $S^+ S^-$. Meanwhile, when $\tau' \neq 0$ only $S^+ S^-$ is conserved and so all long-range correlations involving doublons should decay to 0 in the long-time limit.

Results – In the following calculations we demonstrate this explicitly, initializing the system in the ground state of $H(0)$ and time-evolving it under $H(t)$. We take advantage of the ladder structure of the Hamiltonian and reorder the lattice in Fig. 1 to realize an open boundary chain with nearest and next-nearest neighbor hopping. This allows us to perform our calculations using MPS methods, namely the Density Matrix Renormalization Group (DMRG) [42] and Time Evolving Block Decimation (TEBD) [43] algorithms. Using the Tensor Network Theory library [44], we have adapted these methods to include next-nearest neighbor interactions.

In order to quantify the correlations in the η SU(2) symmetry sector we calculate the expectation value of the particle-hole function $|\langle \eta_i^+ \eta_j^- \rangle(t)|$ which describes the mobility of a doublon between sites i and j at time t . We also introduce the doublon order parameter

$$D_\delta(t) = (1/N) \sum_{\substack{ij \\ |i-j| \geq \delta}} |\langle \eta_i^+ \eta_j^- \rangle(t)|, \quad (5)$$

where N is a normalisation parameter such that $D_\delta(t)$ is the average of the particle-hole correlations over distances greater than $\delta - 1$.

In Fig. 2 we drive the system with a long, large-amplitude pulse for different values of τ' . When $\tau' = 0$

we observe the formation of stable uniform long-range order in the particle-hole correlations. Meanwhile, for $\tau' \neq 0$, $\langle \eta^+ \eta^- \rangle$ is not conserved (Fig. 2c) and hence the doublon order is decaying away in the long-time limit. In Fig. 2a we provide an analytical prediction for the long-time doublon order when $\tau' = 0$ by simultaneously diagonalising the dual SU(2) symmetries. The discrepancy between the numerics and the analytical solution is due to the reflection symmetries of the lattice being unaccounted for, which are especially strong due to the two-rung nature of the lattice.

Notably, the plots in Fig. 2 reveal that there is a critical value of τ' where the behavior of the system changes significantly under driving. For $\tau' < \tau'_c \approx 0.7\bar{\tau}$ uniform, doublon order forms on transient time-scales and follows closely the evolution for $\tau' = 0$. Meanwhile, for $\tau' > \tau'_c$ the system's response in the particle-hole sector is much less ordered and the corresponding off-diagonal correlations quickly decay away. This distinct change in the system's behaviour is underpinned by the degree to which $\langle \eta^+ \eta^- \rangle$ is conserved. In Fig. 2c we see that for $\tau' < \tau'_c$ the system appears to be in a 'quasi-symmetry' regime where $\langle \eta^+ \eta^- \rangle$ is not changing significantly in time. As τ' increases above the critical value τ'_c the rate of change of $\langle \eta^+ \eta^- \rangle$ jumps by over an order of magnitude.

We now probe the origin of these two distinct phases by calculating the properties of the ground state of the system - which we drove out of equilibrium in Fig. 2. In Fig. 3 we observe a rich phase diagram for the ground state of $H(0)$ in terms of the quantities $\langle \eta^+ \eta^- \rangle$ and $\langle S^+ S^- \rangle$. In the SM we provide additional plots of the two-point magnetic and spin-exchange correlations for various states within these diagrams. Within Fig. 3a, there is a large region where the ground state of the sys-

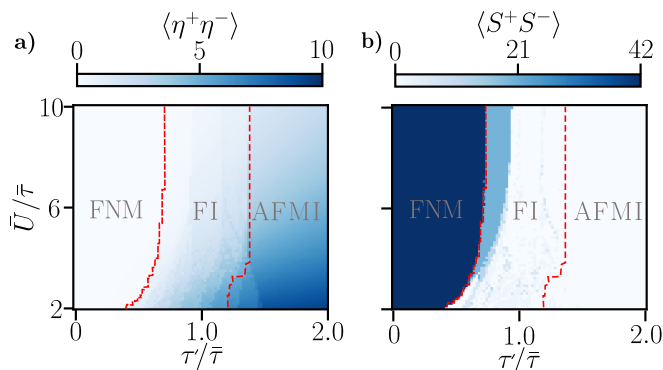


FIG. 3. a-b) Map of $\langle \eta^+ \eta^- \rangle$ and $\langle S^+ S^- \rangle$ versus \bar{U} and τ' for the ground state of the $L = 32$ -site triangular Hubbard model described in Eq. (1). The red dotted lines separate the different phases - Frustrated Non-Magnetic (FNM); Frustrated-Insulating (FI) and Anti-ferromagnetic Insulator (AFMI)

tem resides in the lowest eigenspace of $\eta^+ \eta^-$ despite it no longer being a true symmetry of the Hamiltonian. Here, the system demonstrates frustration through long-range spin-exchange order which alternates in sign. There is also an absence of any significant magnetism. We refer to this as a Frustrated Non-Magnetic (FNM) phase and within this phase, for a given U , the ground state for finite τ' has an exceptionally high overlap (almost unity) with the ground state for $\tau' = 0$. Hence, τ' acts like a perturbation which does not significantly change the ground state - resulting in a consistent value of $\langle \eta^+ \eta^- \rangle \approx 0$.

As τ' increases, the system transitions into the Frustrated Insulating (FI) phase, where some vertically-bonded sites localise and form anti-ferromagnetic pairs separate from the rest of the system, in which long-range order and an absence of magnetism still persists. For even higher τ' the system moves into an Anti-Ferromagnetic Insulating (AFMI) phase where all vertically-bonded sites form localised anti-ferromagnetic pairs. In this region, $\langle \eta^+ \eta^- \rangle$ and $\langle S^+ S^- \rangle$ become continuous versus \bar{U} and τ' , indicating an absence of frustration.

Fig. 2 shows that there is a clear distinction between driving the system in the FNM phase and in the FI phase. This arises from the effective conservation of $\langle \eta^+ \eta^- \rangle$ in the former, which leads to the build up of doublon order. In the SM we plot the change in $\langle \eta^+ \eta^- \rangle$ after evolving the states in the phase diagram of Fig. 3 out of equilibrium. We find that, as well as this quantity clearly distinguishing the different phases of the system, $\langle \eta^+ \eta^- \rangle$ is approximately conserved throughout the FNM phase and for various driving parameters - not just those considered in Fig. 2.

We understand this unexpected conservation of $\langle \eta^+ \eta^- \rangle$ as arising from several factors. Firstly, Eq. (3) dictates that the rate of change of $\langle \eta^+ \eta^- \rangle$ is proportional to τ' , which is lowest in the FNM phase. Secondly, $\langle S^+ S^- \rangle$ is conserved for all time and in the FNM phase this value is fairly large, restricting the driving from ever exciting

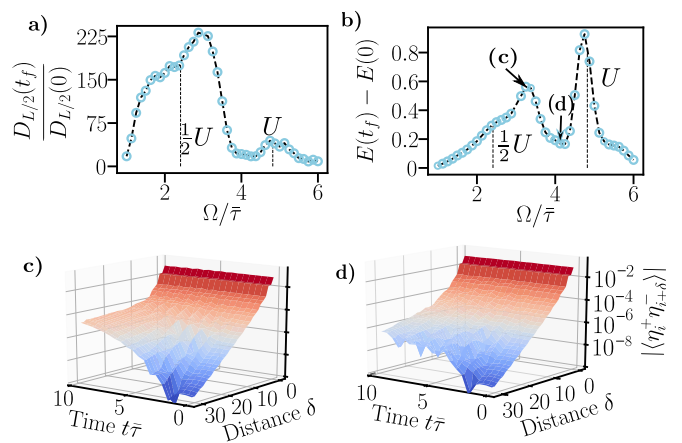


FIG. 4. Dynamical properties of the half-filled $L = 32$ -site two-rung triangular Hubbard model described by Eq. (1). The system is initialized in the ground state of $H(0)$, with $\bar{U} = 4.8\bar{\tau}$ and $\tau' = 0.25\tau$. This state is then evolved under $H(t)$ with the same \bar{U} and τ' and setting $A_1 = -0.15$, $A_2 = A_1/2$, $T_p = 5.0\bar{\tau}$, $T_w = 2.5\bar{\tau}$ with the specified Ω . a) Ratio of the long-range doublon order at times $t_f\bar{\tau} = 10$ and $t = 0$ versus Ω . The parameter $D_{L/2}(t)$ is defined in Eq (5). b) Energy difference between the initial state and the state at time $t_f\bar{\tau} = 10.0$. Energy is defined as $E(t) = \langle H(0) \rangle(t)$. c-d) Full time and distance dynamics of $|\langle \eta_i^+ \eta_{i+\delta}^- \rangle|$ for $\Omega = 3.25\bar{\tau}$ and $\Omega = 4.25\bar{\tau}$, these frequencies are also marked in b).

transitions which create an overlap with the higher eigenstates of $\eta^+ \eta^-$ (see SM for a brief proof). Lastly, the part of the propagator which breaks $\eta^+ \eta^-$ symmetry acts over the vertically bonded sites of the lattice. In the FNM phase frustration in the system means these pairs of sites are in a bound, entangled state with the remainder of the lattice and thus robust to this term in the propagator. As we increase τ' and move across each phase boundary this frustration diminishes, causing discontinuous jumps in the extent to which $\eta^+ \eta^-$ is preserved.

In Figure 2 we drove the system with a long, large amplitude pulse, demonstrating the formation of stable doublon order for $\tau' < \tau'_c$. In Fig. 4 we consider a larger system and use shorter-lived, smaller amplitude pulses to drive the system out of equilibrium for $\tau' < \tau'_c$, showing that the unexpected particle-hole symmetry still dictates the dynamics of the system. We focus on the role of the driving frequency Ω , plotting the long-range order and absorbed energy as a function of Ω in Figs. 4a-b. These quantities display doubly peaked profiles, with the peaks in the doublon order coinciding with those of the absorbed energy. This is indicative of heating-induced order, with the system relaxing to an ordered state due to the approximate conservation of $\langle \eta^+ \eta^- \rangle$ as it absorbs energy [38].

We emphasize this by showing the explicit evolution of the particle-hole correlations for two different frequencies in Figs. 4c-d. There is a slower induction of order when the system absorbs less energy from the driving field. Moreover, despite the finite value of τ' we see the sys-

tem is transiently relaxing towards a state with uniform off-diagonal correlations - indicated by the significant amplification and reordering of the long-range correlations. The short-range correlations are much less effected due to the short timescales and small pulse amplitude.

Interestingly, in Figs. 4a-b, for $\Omega \approx U$ the system absorbs the most energy from the pulse and yet less long-range doublon order is induced compared to the first peak. This is a result of the time dynamics becoming diabatic, the driving frequency is now the dominant timescale (the pulse form is $\sin^2(\Omega t)$) and is too rapid for the system to be able to significantly adapt its spatial configuration [18, 45] (see SM for further plots).

Conclusion - In this paper we have studied the dynamics of the anisotropic driven two-rung triangular Hubbard model. We have shown how frustration in the ground state leads to an unexpected phase where particle-hole symmetry is effectively conserved when the system is driven out of equilibrium. This conservation causes the system to relax towards a state with amplified, uniform, doublon correlations, despite the absence of a bi-partite structure to the underlying lattice.

The choice of a triangular Hubbard model was partly motivated by the role it has played in the modelling of the $\kappa - (\text{BEDT} - \text{TTF})_2X$ compounds. The rich dynamical behaviour and unexpected symmetry which occurs under driving may therefore be observable in these materials. Along these lines, the Hamiltonian in Eq. (1) was proposed as a model for a recent experiment optically exciting the vibrational modes of $\kappa - (\text{BEDT} - \text{TTF})_2\text{Cu}[\text{N}(\text{CN})_2]\text{Br}$ [11]. The quasi-symmetry phase and heating-induced order we have witnessed here provide a potential explanation for the observed onset of light-induced superconductivity in this experiment - the experimental parameters are consistent with the system being driven from within the FNM phase, and close to the first peak of Fig. 4a.

More generally, the geometry of the two-rung triangular Hubbard model allowed us to study the dynamics of a non-equilibrium frustrated lattice structure. We anticipate that other systems which possess complex, non-hypercubic geometries - such as Kagomé lattices [46], optical quasi-crystalline structures [47], and doped cuprates [48] - will display similarly rich physics.

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versity of Oxford Advanced Research Computing (ARC) facility <http://dx.doi.org/10.5281/zenodo.22558>.

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**SUPPLEMENTARY MATERIAL TO
DYNAMICAL SUPERCONDUCTIVITY IN A
FRUSTRATED MANY-BODY SYSTEM**

**Equilibrium and Non-Equilibrium Phase diagram of
the two-rung triangular Hubbard Hamiltonian**

Here we identify the key properties of the different phases of the Hamiltonian

$$H(t) = -\tau(t) \sum_{ij \in \langle n.n \rangle, \sigma} (c_{\sigma,i}^\dagger c_{\sigma,j} + \text{h.c.}) - \tau' \sum_{ij \in \langle \text{vert} \rangle, \sigma} (c_{\sigma,i}^\dagger c_{\sigma,j} + \text{h.c.}) + U(t) \sum_i n_{i,\uparrow} n_{i,\downarrow}. \quad (\text{S1})$$

A description of the various terms of this Hamiltonian and the lattice geometry are provided in the main text.

Firstly, we explore the properties of the ground state of $H(0)$, with $\bar{U} = U(0)$ and $\bar{\tau} = \tau(0)$ being the equilibrium values of the nearest-neighbour hopping and interaction strengths. In the top row of Supplemental Fig 1, for a range of \bar{U} and τ' , we reproduce the phase diagram from the main text as a function of $\langle S^+ S^- \rangle$. In the remainder of the figure we plot the spin-exchange and magnetic order matrices for the 3 different phases. The relevant operators for computing these matrices are

$$S_i^z = n_{\uparrow,i} - n_{\downarrow,i}, \quad S_i^+ = c_{i,\uparrow}^\dagger c_{i,\downarrow}, \quad S_i^- = c_{i,\downarrow}^\dagger c_{i,\uparrow}. \quad (\text{S2})$$

Within the Frustrated Non-Magnetic (FNM) phase there is large, long-range spin-exchange order throughout the system, with the checkerboard pattern which alternates in sign reflecting the frustrated nature of this phase. There is also much less magnetic order compared to the other phases (indicated by the magnitude of the spin-spin correlations $\langle S_i^z S_j^z \rangle$). As τ' increases the system moves into a Frustrated Insulating (FI) phase, where the long-range spin-exchange order is broken up by vertically bonded pairs of sites which become antiferromagnetic and unbound from the rest of the lattice. Nonetheless the remaining sites are still entangled and have some long-range order. Finally, for large enough τ' the system resides in an Anti-Ferromagnetic Insulating phase (AFMI) dominated by pairs of sites (those connected by a vertical bond) which are anti-ferromagnetic singlets unentangled with the rest of the system. The central sites of the lattice retain some correlations between each other, although these decay away with distance.

In Supplemental Fig. 2 we show how these different phases also dictate the out-of-equilibrium behaviour of the system. Specifically, we consider the change in $\langle \eta^+ \eta^- \rangle$ after driving the ground state of $H(0)$ out of equilibrium with $H(t)$ for a short period of time. In Supplemental Fig. 2 we plot this change for the same range of \bar{U} and τ' as in Supplemental Fig. 1 and choose

several forms of $U(t)$ and $\tau(t)$. We see that the value of $\langle \eta^+ \eta^- \rangle$ is approximately conserved when starting in the FNM phase whilst this conservation is suddenly broken when moving over the phase boundary. Moreover, there is also a clear distinction between driving the system from the FI and the AFMI phase. The largest change in $\langle \eta^+ \eta^- \rangle$ occurs in the AFMI phase, where the system is no longer frustrated. These observations hold for all 3 choices of driving - indicating that frustration, as opposed to the specifics of the driving, is the relevant factor in our results.

**Bound on the η and spin values in the half-filled
Hubbard model**

Here, we demonstrate that for the half-filled Hubbard model with equal numbers of spin-up and spin-down particles

$$\sqrt{1 + 4\langle \eta^+ \eta^- \rangle} + \sqrt{1 + 4\langle S^+ S^- \rangle} \leq L + 2, \quad (\text{S3})$$

for all states. Our starting point is to consider a subspace of the Fock basis where all basis states have i doublons (sites occupied by two fermions of opposing spin). It then immediately follows that these basis states also have $L/2 - i$ singlons (sites occupied with a single fermion) with spin \uparrow and $L/2 - i$ singlons with spin \downarrow .

The eigenvalues of $\eta^+ \eta^-$ in this reduced basis are $\eta^+ \eta^- = k(k+1)$, $k = 0, 1, \dots, i$ and for $S^+ S^-$ the eigenvalues are $S^+ S^- = m(m+1)$, $m = 0, 1, \dots, L/2 - i$. As a result we have $k + m \leq L/2$, and we can substitute in $\langle \eta^+ \eta^- \rangle$ and $\langle S^+ S^- \rangle$ for k and m to get Supplemental Eq. (S3), which is independent of the number of doublons, i . Hence, this bound applies generally to the basis states of the half-filled Hubbard model - and therefore all wavefunctions or density matrices of this model. In the main text, this bound means that when the system starts in the FNM phase, where the value of $\langle S^+ S^- \rangle$ is largest, excitations which cause the wavefunction to have a finite overlap with any of the higher eigenstates of $\eta^+ \eta^-$ are disallowed, restricting the dynamics of $\eta^+ \eta^-$. Outside of this phase $S^+ S^-$ reduces, eventually reaching 0 where all eigenstates of $\eta^+ \eta^-$ become accessible under driving.

**Diabatic Behaviour When Driving at Large
Frequencies**

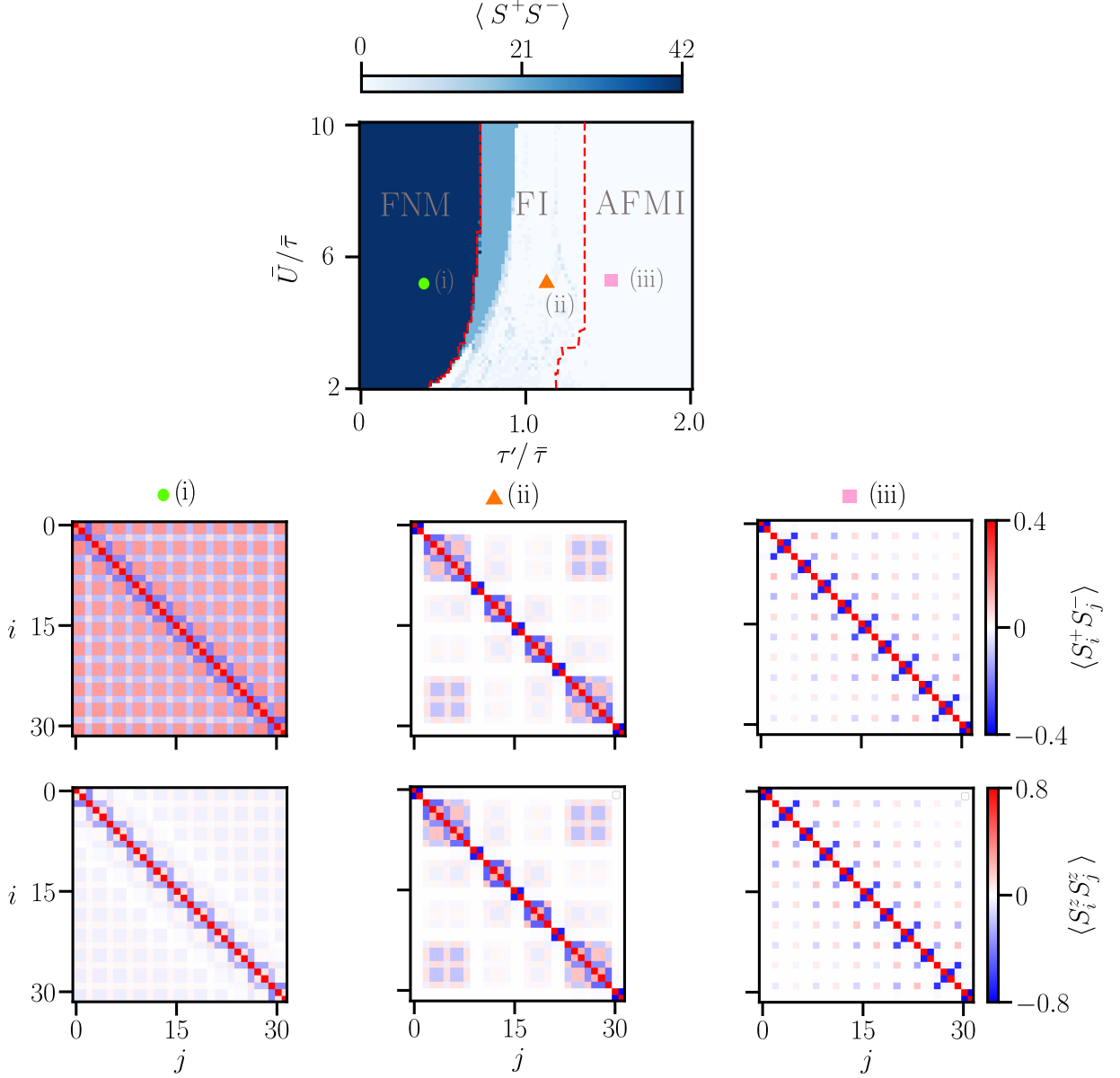
In Supplemental Fig. 3 we consider again the Hamiltonian in Supplemental Eq. (S1) and plot the dynamics of the particle-hole correlations at various distances for two different driving frequencies, with the form of the driving

now the same as in the main text

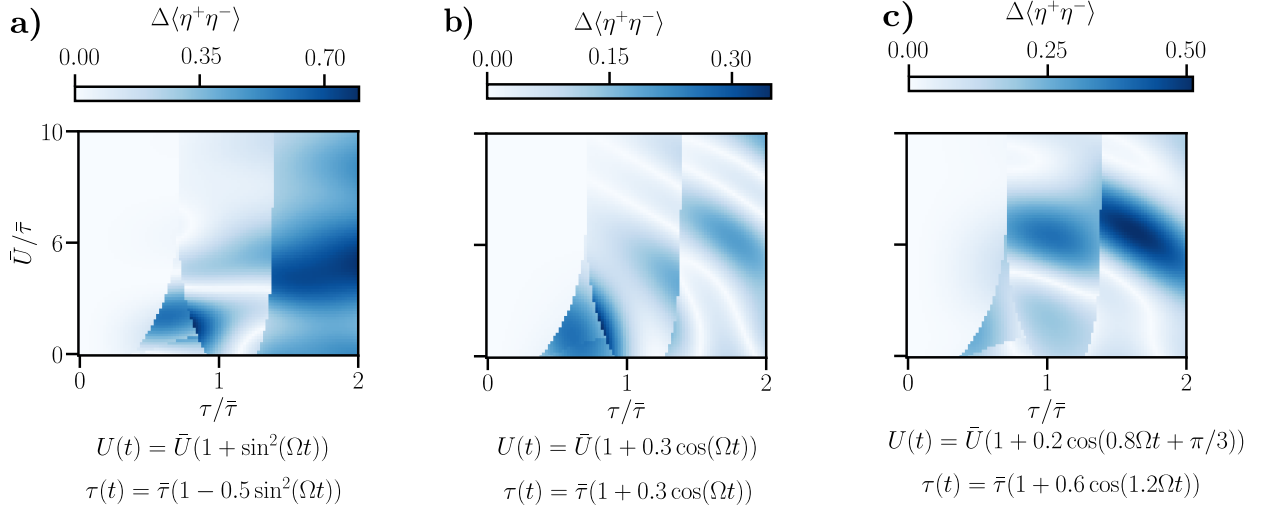
$$\begin{aligned}\tau(t) &= \bar{\tau}(1 + A_1 \sin^2(\Omega t) \exp(- (t - T_P)^2 / (2T_w^2))), \\ U(t) &= \bar{U}(1 + A_2 \sin^2(\Omega t) \exp(- (t - T_P)^2 / (2T_w^2))).\end{aligned}\tag{S4}$$

The two frequencies we consider correspond to the two peaks of the distributions in Fig. 4 of the main text.

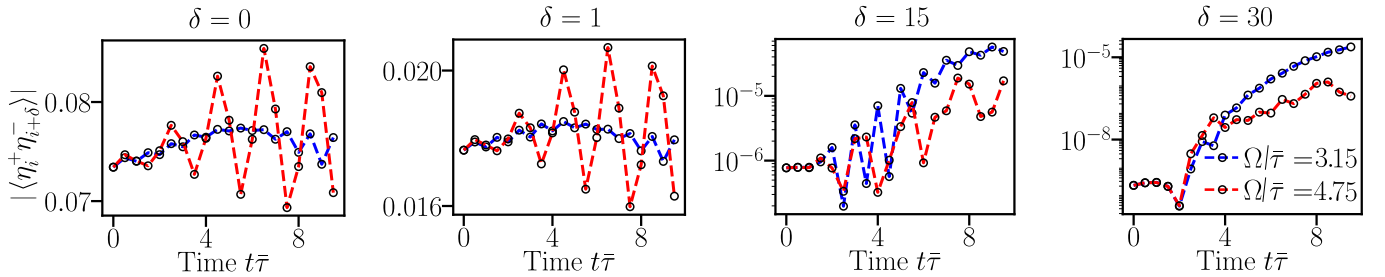
We see that at the higher frequency, the driving induces a much stronger change in the short-range correlations as opposed to the longer-range correlations. The opposite occurs at the lower frequency. This is a consequence of, at the higher frequency, the driving frequency being the largest timescale in the system. As a result a ‘freezing effect’ occurs where the system struggles to adapt its spatial configuration and the driving induces a more local response in the system’s dynamics [18, 45].



Supplemental Fig. 1. Top Row) Reproduced from Fig. 3b of the main text. Expectation value of $\langle S^+ S^- \rangle$ for the ground state of the $L = 32$ two-rung triangular Hubbard model, see Supplemental Eq. (S1), as a function of \bar{U} and τ' . The red dotted lines separate the different phases - a Frustrated Non-Magnetic Phase (FNM); a Frustrated Insulating Phase (FI) and an Anti-Ferromagnetic Insulating phase (AFMI). Second Row) Matrix of spin-exchange correlations $\langle S_i^+ S_j^- \rangle$ between sites i and j for the 3 different phases of system: i) FNM phase, $\bar{U}, \tau' = 5.0\bar{\tau}, 0.4\bar{\tau}$, ii) HFI Phase, $\bar{U}, \tau' = 5.0\bar{\tau}, 1.1\bar{\tau}$, iii) AFMI Phase $\bar{U}, \tau' = 5.0\bar{\tau}, 1.5\bar{\tau}$. Bottom row) Matrix of magnetic correlations $\langle S_i^z S_j^z \rangle$ between sites i and j for the same 3 systems as above.



Supplemental Fig. 2. Change in $\langle\eta^+\eta^-\rangle$ after starting in the ground state of the $L = 8$ -site two-rung triangular Hubbard model $H(0)$ from Supplemental Eq. S1 and time evolving the system under $H(t)$. Each plot corresponds to a different time-dependence of the nearest-neighbour hopping and interaction strengths (provided below each plot). We fix $\Omega = 2.25\bar{\tau}$ and calculate $\Delta\langle\eta^+\eta^-\rangle = |\langle\eta^+\eta^-\rangle(0) - \langle\eta^+\eta^-\rangle(t_f)|$ for the given range of \bar{U} and τ' , with $t_f\bar{\tau} = 2.0$.



Supplemental Fig. 3. Simulation of the half-filled driven $L = 32$ triangular Hubbard lattice in Supplemental Eq. (S1). The system is initialized in the ground state of $H(0)$, setting $\bar{U} = 4.82\bar{\tau}$ and $\tau' = 0.25\bar{\tau}$. The system is then time evolved under $H(t)$, using the same \bar{U} and τ' and setting $A_1 = -0.15, A_2 = -0.075, T_w = 2.5\tau(0), T_p = 5.0\tau(0)$ and Ω to the specified frequency (red, $\Omega/\bar{\tau} = 3.25$, blue $\Omega/\bar{\tau} = 4.75$). These frequencies corresponds to the peaks of Figs 4a and 4b in the main text. Here we plot the time evolution of the average magnitude of the particle-hole correlations at distances $\delta = 0, 1, 15, 30$ respectively. For $\delta = 0$ this corresponds to the on-site doublon density. For $\delta = 15$ and $\delta = 30$ the y-axis is plotted on a logarithmic scale.