

# Tachyonic Kaluza-Klein modes and the AdS swampland conjecture

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## Abstract

We compute the Kaluza-Klein spectrum of the non-supersymmetric  $SO(3) \times SO(3)$ -invariant  $AdS_4$  vacuum of 11-dimensional supergravity, whose lowest-lying Kaluza-Klein modes belong to a consistent truncation to 4-dimensional  $\mathcal{N} = 8$  supergravity and are stable. We show that, nonetheless, the higher Kaluza-Klein modes become tachyonic so that this non-supersymmetric  $AdS_4$  vacuum is perturbatively unstable within 11-dimensional supergravity. This represents the first example of unstable higher Kaluza-Klein modes and provides further evidence for the AdS swampland conjecture, which states that there are no stable non-supersymmetric AdS vacua within string theory. We also find 27 moduli amongst the Kaluza-Klein modes, which hints at the existence of a family of non-supersymmetric  $AdS_4$  vacua.

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# 1 Introduction

The stability of anti-de Sitter (AdS) spacetimes has been a long-standing question in theoretical physics. The question is particularly interesting in the case of non-supersymmetric AdS spacetimes, which are not protected by supersymmetry arguments and corresponding positive mass theorems [1]. In the context of string theory, the fate of non-supersymmetric AdS vacua is especially important. For example, non-supersymmetric AdS vacua provide one of the most explicit ways to apply the AdS/CFT correspondence to QCD or condensed matter systems [2]. Moreover, non-supersymmetric AdS compactifications provide a simpler class of non-supersymmetric string solutions than de Sitter vacua, which are time-dependent. Therefore, non-supersymmetric AdS solutions can be seen as a natural stepping stone to understanding de Sitter vacua in string theory.

However, so far no fully-fledged examples of non-supersymmetric but stable AdS vacua in string theory have been constructed. One of the most efficient ways of constructing non-supersymmetric AdS vacua in string theory is by uplifting non-supersymmetric solutions of lower-dimensional gauged supergravities. While many such AdS solutions are known, for example in  $\mathcal{N} = 8$  supergravities in four [3–10] and five dimensions [11–13], all but a handful of such vacua are already perturbatively unstable within the lower-dimensional supergravity [1,8,9,13–20]. That is, typically some of the scalar fields of the lower-dimensional supergravity have masses that violate the Breitenlohner-Freedman (BF) bound [21]. Therefore, these scalars are tachyonic and generate an instability.

This lack of non-supersymmetric but stable AdS vacua, together with arguments based on a sharpened version of the Weak Gravity Conjecture (WGC) [22], has recently led to the AdS Swampland Conjecture [23], which states that all non-supersymmetric AdS vacua in string theory are unstable. The most prominent possible counterexample is the  $\text{SO}(3) \times \text{SO}(3)$ -invariant non-supersymmetric  $\text{AdS}_4$  vacuum of the  $\mathcal{N} = 8$   $\text{SO}(8)$  gauged supergravity in four dimensions [3,4]. This vacuum is perturbatively stable within the full four-dimensional  $\mathcal{N} = 8$  supergravity [19], which has long prompted hope that it may also be stable when uplifted to the full 11-dimensional supergravity.

Indeed, a perturbative instability would require the masses of the higher Kaluza-Klein modes to drop below those of lowest-lying modes which make up the consistent truncation to  $\mathcal{N} = 8$  supergravity and are above the BF bound [19]. Such behaviour of Kaluza-Klein towers has never previously been observed. Nonetheless, such a *perturbative* instability, therefore, would provide very concrete evidence for the AdS Swampland Conjecture, whose original arguments suggest a non-perturbative instability mechanism. However, since calculating the Kaluza-Klein spectra of supergravity compactifications is a notoriously difficult problem, the question of the perturbative stability of the  $\text{SO}(3) \times \text{SO}(3)$ -invariant vacuum of 11-dimensional supergravity has remained an open problem thus far.

Recently, a “brane-jet instability” was found for the  $\text{SO}(3) \times \text{SO}(3)$  vacuum [24]. There it is argued that probe branes feel a net repulsive force in certain areas of the compactification

manifold due to the varying warp factor of the 11-dimensional solution. This causes the probe branes to be expelled, hence signaling an instability. In [24] it is argued that because this instability is localised in the compactification manifold and the Coulomb branch of the probe branes may be reflected in scalars of 4-dimensional supergravity, this brane-jet result might indicate an instability triggered by higher Kaluza-Klein modes.

In this paper, we show that indeed the higher Kaluza-Klein modes of the  $\text{SO}(3) \times \text{SO}(3)$ -invariant  $\text{AdS}_4$  vacuum are unstable. We do this by employing the recently developed method [25, 26], which allow us to compute the Kaluza-Klein spectrum of any vacuum of  $\mathcal{N} = 8$  gauged supergravity obtained by a consistent truncation from 10-/11-dimensional supergravity, by tracking how the spectrum changes as the vacuum is deformed from the maximally-symmetric one. Here this allows us to follow the spectrum as the round  $S^7$ , corresponding to the maximally supersymmetric  $\text{AdS}_4 \times S^7$  vacuum, is deformed to the  $\text{SO}(3) \times \text{SO}(3)$ -invariant compactification and to compute the Kaluza-Klein spectrum of the  $\text{SO}(3) \times \text{SO}(3)$ -invariant vacuum up to level 6 above the  $\mathcal{N} = 8$  truncation. We find that the higher Kaluza-Klein modes, beginning at level  $n = 2$ , of the  $\text{SO}(3) \times \text{SO}(3)$ -invariant  $\text{AdS}_4$  solution drop below the BF bound and become tachyonic. Therefore, the  $\text{AdS}_4$  solution is perturbatively unstable in 11 dimensions. Interestingly, this instability is already present at the level of 11-dimensional supergravity and does not require non-perturbative effects of M-theory.

## 2 Kaluza-Klein spectroscopy

We begin by reviewing the result of [25, 26], which we will use to compute the Kaluza-Klein spectrum of the  $\text{SO}(3) \times \text{SO}(3)$ -invariant  $\text{AdS}_4$  vacuum. In [25, 26], Exceptional Field Theory (ExFT) [27] was used to derive mass formulae for the Kaluza-Klein spectrum of any vacuum obtained by a consistent truncation to a lower-dimensional maximally supersymmetric supergravity.

ExFT provides a convenient reformulation of 10-/11-dimensional supergravity, which unifies gravitational and flux degrees of freedom in a way that makes a  $E_{7(7)}$  symmetry manifest. In particular, the bosonic sector consists of a four-dimensional metric  $g_{\mu\nu}$ , a generalised metric  $\mathcal{M}_{MN}$  parameterising the coset space  $E_{7(7)}/\text{SU}(8)$ , and a four-dimensional gauge field  $A_\mu^M$  transforming in the **56** of  $E_{7(7)}$ , with  $\mu = 0, \dots, 3$  and  $M = 1, \dots, 56$ . As shown in [25, 26], in ExFT the Kaluza-Klein fluctuations of any vacuum of a given  $\mathcal{N} = 8$  gauged supergravity, whose uplift to 10-/11-dimensional supergravity is known, can be conveniently expressed as a product of the lowest-lying modes, which make up the consistent truncation to the  $\mathcal{N} = 8$  supergravity, with the scalar harmonics of the compactification manifold,  $\mathcal{Y}^\Sigma$ , *at the maximally symmetric point*. For example, for all vacua within the four-dimensional  $\text{SO}(8)$  gauged supergravity,  $\mathcal{Y}^\Sigma$  would be the scalar harmonics of the round  $S^7$ .

In this manner, the scalar fluctuations are parameterised by a tensor product of an  $\mathfrak{e}_{7(7)} \oplus \mathfrak{su}(8)$ -valued matrix with the scalar harmonics, which we label as above by  $\Sigma$ . Thus we represent the scalar fluctuations by  $j_A^B{}_\Sigma$ , where  $A, B = 1, \dots, 56$ , and which, for fixed  $\Sigma$ , is valued in

$\mathfrak{e}_{7(7)} \ominus \mathfrak{su}(8)$ , i.e.

$$\begin{aligned}\mathbb{P}_A{}^B{}_C{}^D j_C{}^D{}_\Sigma &= j_A{}^B{}_\Sigma, \\ j_A{}^C{}_\Sigma \delta_{BC} &= j_B{}^C{}_\Sigma \delta_{AC},\end{aligned}\tag{2.1}$$

where  $\mathbb{P}_A{}^B{}_C{}^D$  is the projector onto the adjoint of  $E_{7(7)}$ . From here onwards, we will freely raise/lower all  $A, B$  and  $\Sigma, \Omega$  indices by  $\delta_{AB}$  and  $\delta_{\Sigma\Omega}$ , respectively.

The method of [25, 26] gives the mass matrices of the Kaluza-Klein modes in terms of four-dimensional  $\mathcal{N} = 8$  supergravity data and information about the uplift to 10-/11-dimensions *at the maximally symmetric point* of the four-dimensional supergravity. In particular, the effect of the deformation away from the maximally symmetric solution on the Kaluza-Klein masses can be deduced entirely from the four-dimensional information. This four-dimensional information consists of

- the embedding tensor of the  $\mathcal{N} = 8$  gauged supergravity,  $X_{MN}{}^P$  with  $M, N = 1, \dots, 56$ ,
- the four-dimensional scalar matrix  $\mathcal{V}_A{}^M \in E_{7(7)}/\text{SU}(8)$  of the vacuum we want to study.

The conventions for the four-dimensional scalar manifold are such that  $\mathcal{V}_A{}^M = \delta_A{}^M$  corresponds to the maximally symmetric point, e.g. the round  $S^7$  for the  $\text{SO}(8)$  gauged supergravity.

We also need the following higher-dimensional data:

- the scalar harmonics,  $\mathcal{Y}^\Sigma$ , of the compactification,
- the linear action of the Killing vector fields of the compactification on the scalar harmonics, which we denote by  $\mathcal{T}_M{}^\Sigma{}_\Omega$ .<sup>4</sup>

To be explicit,  $\mathcal{T}_M{}^\Sigma{}_\Omega$  is defined as

$$L_{\mathcal{K}_M} \mathcal{Y}^\Sigma = -\mathcal{T}_M{}^\Sigma{}_\Omega \mathcal{Y}^\Omega,\tag{2.2}$$

where  $\mathcal{K}_M$  are the Killing vectors of the compactification<sup>5</sup> and  $L$  denotes the Lie derivative. Therefore, the matrices  $\mathcal{T}_M{}^\Sigma{}_\Omega$  correspond to the generators of the gauge group (generated by the Killing vectors  $\mathcal{K}_M$ ) in the representation of the scalar harmonics  $\mathcal{Y}_\Sigma$ . These are normalised relative to the four-dimensional embedding tensor,  $X_{MN}{}^P$ , such that

$$[\mathcal{T}_M, \mathcal{T}_N] = -X_{[MN]}{}^P \mathcal{T}_P.\tag{2.3}$$

We emphasise once more that we only require the higher-dimensional information for the maximally-symmetric compactification, e.g. the round  $S^7$  for the  $\text{SO}(8)$  gauged supergravity,

<sup>4</sup>For non-compact gaugings, we need to know the action of a different set of vector fields, which are not all Killing vectors. The full details are given in [25, 26] but are not important here since we are working with the  $\text{SO}(8)$  gauged supergravity.

<sup>5</sup>With appropriate modification for non-compact gaugings as discussed in [25, 26].

irrespective of the four-dimensional vacuum that we are studying. The power of the method developed in [25, 26] is that the effect of the deformation away from the maximally symmetric point can be captured simply by dressing the four-dimensional embedding tensor  $X_{MN}{}^P$  and the generators  $\mathcal{T}_M{}^\Sigma{}_\Omega$  by the four-dimensional scalar matrix  $\mathcal{V}_A{}^M$ .

In particular, the scalar mass matrix,  $M^{AB\Sigma,CD\Omega}$  for the scalar fluctuations,  $j_{AB,\Sigma}$ , is given by

$$\begin{aligned}
M^{AB\Sigma,CD\Omega} j_{AB,\Sigma} j_{CD,\Omega} = & \frac{1}{7} [-6 X^{ACD} X^B{}_{CD} + 2 X^{CAD} X_C{}^B{}_D] j_A{}^{E,\Sigma} j_{BE,\Sigma} \\
& + \frac{2}{7} [2 X^{ACE} X^{BD}{}_{E} - X^{EAC} X_E{}^{BD}] j_{AB}{}^\Sigma j_{CD,\Sigma} \\
& - 4 X^{ACD} \mathcal{T}^{B,\Omega\Sigma} j_{AB,\Sigma} j_{CD,\Omega} - 4 X^{CAB} \mathcal{T}_C{}^\Omega{}_\Sigma j_A{}^{D,\Sigma} j_{BD,\Omega} \\
& + 24 \mathcal{T}_A{}^\Omega{}_\Lambda \mathcal{T}^{B,\Lambda}{}_\Sigma j^{AC,\Sigma} j_{BC,\Omega} - \mathcal{T}^{C,\Omega}{}_\Lambda \mathcal{T}_C{}^\Lambda{}_\Sigma j^{AB,\Sigma} j_{AB,\Omega},
\end{aligned} \tag{2.4}$$

where

$$\begin{aligned}
X_{AB}{}^C &= \mathcal{V}_A{}^M \mathcal{V}_B{}^N (\mathcal{V}^{-1})_P{}^C X_{MN}{}^P, \\
\mathcal{T}_A{}^\Sigma{}_\Omega &= \mathcal{V}_A{}^M \mathcal{T}_M{}^\Sigma{}_\Omega,
\end{aligned} \tag{2.5}$$

are the embedding tensor and linear action of Killing vectors dressed by the scalar matrix,  $\mathcal{V}_A{}^M$ , corresponding to the vacuum we are considering. Similar mass matrices for the spin-1 and spin-2 Kaluza-Klein modes can be found in [25, 26].

Note that not all the fluctuations in  $j_{AB,\Sigma}$  are physical. Some of these modes are Goldstone bosons eaten by massive vector fields, and some are eaten by the massive gravitons. We can remove these unphysical modes by fixing the gauge appropriately, as usual when computing Kaluza-Klein spectra.

## 2.1 SU(8) mass matrix

In the mass matrix (2.4), the  $A, B = 1, \dots, 56$  indices are raised/lowered by  $\delta_{AB}$ , which is not an invariant tensor of  $E_{7(7)}$ . Instead,  $\delta_{AB}$  breaks  $E_{7(7)}$  to its maximal compact subgroup, SU(8) and, correspondingly, the  $A, B$  indices should really be thought of as SU(8) indices.

Since the dressed embedding tensor  $X_{AB}{}^C$  is frequently given in SU(8)-covariant notation, it is worthwhile rewriting the mass matrix (2.4) in SU(8)-covariant notation. Let  $i = 1, \dots, 8$  denote fundamental SU(8) indices and upstairs/downstairs indices be related by complex conjugation. Under the decomposition  $E_{7(7)} \rightarrow \text{SU}(8)$ , the **56** and embedding tensor representation, **912**, decompose as

$$\begin{aligned}
\mathbf{56} &\longrightarrow \mathbf{28} \oplus \overline{\mathbf{28}}, \\
\mathbf{912} &\longrightarrow \mathbf{36} \oplus \overline{\mathbf{36}} \oplus \mathbf{420} \oplus \overline{\mathbf{420}}.
\end{aligned} \tag{2.6}$$

The **36** and **420** representations of the embedding tensor are known as the fermion shift matrices,

$A_1^{ij}$  and  $A_{2,i}{}^{jkl}$ . These satisfy

$$A_1^{[ij]} = 0, \quad A_{2,i}{}^{jkl} = A_{2,i}{}^{[jkl]}, \quad A_{2,i}{}^{ijk} = 0. \quad (2.7)$$

Explicitly, the relationship between the embedding tensor and the fermion shift matrices is [28]

$$\begin{aligned} X_{ij,kl}{}^{pq} &= -X_{ij}{}^{pq}{}_{kl} = \frac{1}{2}\delta_{[k}^{[p}A_{2,l]}{}^{q]ij} + \delta_{[k}^{[p}A_{1,l]}{}^{q]i}{}_{[j} \delta_{j]}^q, \\ X_{ij}{}^{kl,pq} &= \delta_{[i}^{[k}A_{2,j]}{}^{l]pq}, \quad X_{ij,kl,pq} = -\frac{1}{4!}\epsilon_{klpq}{}^{tuv}{}_{[i}A_{2,j]}{}^{tuv}, \end{aligned} \quad (2.8)$$

with the other components related by complex conjugation to the above.

Similarly, the dressed  $\mathcal{T}_A$  contain the SU(8) representations  $\mathbf{28} \oplus \overline{\mathbf{28}}$  as follows

$$\mathcal{T}_A{}^\Sigma{}_\Omega = (\mathcal{T}_{ij}{}^\Sigma{}_\Omega, \mathcal{T}^{ij,\Sigma}{}_\Omega), \quad (2.9)$$

with  $\mathcal{T}^{ij,\Sigma}{}_\Omega$  the complex conjugate of  $\mathcal{T}_{ij}{}^\Sigma{}_\Omega$ . Finally, since the scalar  $j_{AB,\Sigma}$ , for fixed  $\Sigma$ , parameterise the coset  $\mathfrak{e}_{7(7)} - \mathfrak{su}(8)$ , their only non-zero components are given by

$$j^{ijkl}{}_\Sigma = \bar{j}_{ijkl,\Sigma} = \frac{1}{4!}\epsilon^{ijklmnpq}j_{mnpq,\Sigma}, \quad (2.10)$$

where  $\bar{j}_{ijkl,\Sigma}$  is the complex conjugate of  $j_{ijkl,\Sigma}$ , and  $\epsilon^{ijklmnpq}$  is the eight-dimensional alternating symbol.

With the above conventions, the mass matrix (2.4) in SU(8)-covariant notation, is given by

$$\begin{aligned} M^{ijkl\Sigma}{}_{mnpq\Omega} j_{ijkl,\Sigma} j^{mnpq,\Omega} &= \left( -\frac{1}{2}A_1^{ij}A_{1,ij} + \frac{5}{24}A_{2,i}{}^{jkl}A_2^i{}_{jkl} \right) j_{klmn,\Sigma} j^{klmn,\Sigma} \\ &+ \left( 6A_{2,i}{}^{kmn}A_2^l{}_{jmn} - \frac{3}{2}A_2^m{}_{nij}A_{2,m}{}^{nkl} \right) j^{ijpq,\Sigma} j_{klpq,\Sigma} \\ &- \frac{2}{3}A_{2,i}{}^{mnp}A_2^q{}_{jkl}j^{ijkl,\Sigma} j_{mnpq,\Sigma} \\ &+ 4 \left( A_{2,l}{}^{ijk} \mathcal{T}_{mn,\Omega}{}^\Sigma + A_2^i{}_{lmn} \mathcal{T}^{jk,\Sigma}{}_\Omega \right) j_{ijkp,\Sigma} j^{lmnp,\Omega} \\ &- 4 \left( A_{2,i}{}^{jkl} \mathcal{T}_{kl,\Omega}{}^\Sigma + A_2^j{}_{ikl} \mathcal{T}^{kl,\Sigma}{}_\Omega \right) j_{jmnp,\Sigma} j^{imnp,\Omega} \\ &- 8 \left( A_{1,ik} \mathcal{T}^{kj,\Sigma}{}_\Omega + A_1^{jk} \mathcal{T}_{ki,\Omega}{}^\Sigma \right) j_{jmnp,\Sigma} j^{imnp,\Omega} \\ &+ 48 j_{ijpq,\Sigma} j^{klpq,\Omega} \mathcal{T}^{ij}{}_\Omega{}^\Lambda \mathcal{T}_{kl,\Lambda}{}^\Sigma - 2 j_{ijkl,\Sigma} j^{ijkl,\Omega} \mathcal{T}_{mn,\Omega}{}^\Lambda \mathcal{T}^{mn}{}_\Lambda{}^\Sigma. \end{aligned} \quad (2.11)$$

The first three lines above correspond to the mass matrix of the four-dimensional  $\mathcal{N} = 8$  supergravity [29], while the last four lines provide corrections of the mass for the higher Kaluza-Klein levels.

### 3 The $\text{SO}(3) \times \text{SO}(3)$ -invariant $\text{AdS}_4$ vacuum

The  $\text{SO}(3) \times \text{SO}(3)$ -invariant  $\text{AdS}_4$  vacuum was first found as a solution of four-dimensional  $\mathcal{N} = 8$   $\text{SO}(8)$ -gauged SUGRA [3, 4]. Since the  $\text{SO}(8)$ -gauged SUGRA arises as a consistent truncation of 11-dimensional supergravity on  $S^7$  [30], this non-supersymmetric  $\text{AdS}_4$  vacuum can be uplifted to a solution of 11-dimensional supergravity. Concretely, this is done by making use of the known uplift formulas for the internal metric [31] and the internal components of the three-form [32]. For the  $\text{SO}(3) \times \text{SO}(3)$ -invariant vacuum the resulting 11-dimensional solution has been worked out in [33]. As the relevant expressions are rather complicated and not needed here, we refer readers there for further details and explicit formulas.

Indeed, using the methods of [25, 26], we only need the explicit form of the four-dimensional scalar matrix at the  $\text{SO}(3) \times \text{SO}(3)$  stationary point [3] in order to compute the Kaluza-Klein spectrum. In four-dimensional  $\mathcal{N} = 8$  supergravity, the scalar matrix is an  $E_{7(7)}/\text{SU}(8)$  coset element,  $\mathcal{V}_A^M$ , in the fundamental representation **56** of  $E_{7(7)}$  [34]

$$\mathcal{V}_A^M(x) = \begin{pmatrix} u_{ij}{}^{IJ}(x) & v^{ijIJ}(x) \\ v_{ijIJ}(x) & u^{ij}{}_{IJ}(x) \end{pmatrix}. \quad (3.1)$$

Here  $i, j = 1, \dots, 8$  denote  $\text{SU}(8)$  indices, while  $I, J = 1, \dots, 8$  denote  $\text{SL}(8)$  indices; complex conjugation is realised by raising/lowering of all indices, for example  $u^{ij}{}_{IJ} = (u_{ij}{}^{IJ})^*$ . By exploiting the local  $\text{SU}(8)$  invariance we can bring  $\mathcal{V}$  into a unitary gauge, where it takes the form

$$\mathcal{V} = \exp \begin{pmatrix} 0 & \phi_{IJKL} \\ \phi^{IJKL} & 0 \end{pmatrix} \equiv \begin{pmatrix} u_{IJ}{}^{KL} & v^{IJKL} \\ v_{IJKL} & u^{IJ}{}_{KL} \end{pmatrix}, \quad (3.2)$$

where after gauge fixing we no longer need to distinguish between  $\text{SO}(8)$  and  $\text{SU}(8)$  indices. In this gauge, the  $u$  and  $v$  matrices are expressed as infinite sums, *viz.*

$$u_{IJ}{}^{KL} = \sum_{n=0}^{\infty} \frac{1}{(2n)!} [(\phi\phi^*)^n]_{IJKL}, \quad v^{IJKL} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} [\phi^*(\phi\phi^*)^n]_{IJKL}. \quad (3.3)$$

To give the scalar matrix at the  $\text{SO}(3) \times \text{SO}(3)$  invariant point, we introduce the four  $\text{SO}(3) \times \text{SO}(3)$ -invariant tensors

$$\begin{aligned} Y_{IJKL}^+ &= 4! (\delta_{IJKL}^{1234} + \delta_{IJKL}^{5678}), & Y_{IJKL}^- &= 4! (\delta_{IJKL}^{1235} + \delta_{IJKL}^{4678}), \\ Z_{IJKL}^- &= 4! (\delta_{IJKL}^{1234} - \delta_{IJKL}^{5678}), & Z_{IJKL}^+ &= 4! (\delta_{IJKL}^{1235} - \delta_{IJKL}^{4678}), \end{aligned} \quad (3.4)$$

where the two  $\text{SO}(3)$  subgroups act on the subspaces defined by  $I = 1, 2, 3$  and  $I = 6, 7, 8$ , respectively. Note that tensors  $Y^+$  and  $Z^+$  are self-dual, while  $Y^-$  and  $Z^-$  are antiself-dual (another  $\text{SO}(3) \times \text{SO}(3)$  invariant is  $F_{IJ} = \delta_{IJ}^{45}$ ). These objects satisfy a number of elementary identities which have been listed in [33].

With these definitions, we can parametrize the  $\text{SO}(3) \times \text{SO}(3)$  invariant scalar field config-

urations as

$$\phi_{IJKL}(\lambda, \omega) = \frac{\lambda}{2} \left[ \cos \omega (Y_{IJKL}^+ + iY_{IJKL}^-) - \sin \omega (Z_{IJKL}^+ - iZ_{IJKL}^-) \right], \quad (3.5)$$

with two independent parameters  $\lambda$  and  $\omega$ . To exponentiate the scalar expectation value it is, furthermore, useful to define the Hermitian projector <sup>6</sup>

$$\Pi = \frac{1}{8} (Y^+ + iY^-) (Y^+ - iY^-) = \frac{1}{8} (Z^+ - iZ^-) (Z^+ + iZ^-), \quad (3.6)$$

which satisfies

$$\Pi^2 = \Pi, \quad \Pi_{IJKL}^* = \Pi_{KLIJ}, \quad (Y^+ - iY^-) \Pi = Y^+ - iY^-, \quad (Z^+ + iZ^-) \Pi = Z^+ + iZ^-. \quad (3.7)$$

so  $\Pi$  is a Hermitian projector. In particular, using identities from [33], we find that

$$\phi \phi^* = 2\lambda^2 \Pi, \quad \phi^* \Pi = \phi^*. \quad (3.8)$$

After these preparations it is straightforward to determine the  $u$  and  $v$  matrices

$$\begin{aligned} u_{IJ}{}^{KL} &= \delta_{IJ}{}^{KL} + (\cosh(\sqrt{2}\lambda) - 1) \Pi_{IJKL}, \\ v^{IJKL} &= \frac{1}{2\sqrt{2}} \sinh(\sqrt{2}\lambda) \left[ \cos \omega (Y^+ - iY^-) - \sin \omega (Z^+ + iZ^-) \right]_{IJKL}, \end{aligned} \quad (3.9)$$

With these explicit expressions for the scalar matrix (3.9), we can now compute the dressed embedding tensor, or equivalently via (2.8) the fermion shift matrices,  $A_1^{ij}$  and  $A_{2i}{}^{jkl}$ , for any value of  $\lambda$  and  $\omega$ , which includes the  $\text{SO}(3) \times \text{SO}(3)$ -invariant vacuum. For the  $\text{SO}(8)$ -gauged supergravity we are considering, the fermion shift matrices can be computed via

$$(u^{kl}{}_{IJ} + v^{klIJ})(u_{im}{}^{JK} u^{jM}{}_{KI} - v_{imJK} v^{jmKI}) = -\frac{3}{4} A_{2i}{}^{jkl} + \frac{3}{2} \delta_i^{[k} A_1^{l]j}. \quad (3.10)$$

For the above scalar field configuration, we get [2, 3]

$$A_1^{ij}(\lambda) = \text{diag}(a, a, a, b, b, a, a, a), \quad (3.11)$$

with

$$a(\lambda) = \cosh(\sqrt{2}\lambda), \quad b(\lambda) = \frac{1}{2} \left( 1 + \cosh^2(\sqrt{2}\lambda) \right). \quad (3.12)$$

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<sup>6</sup>With the short-hand notation  $AB \equiv (AB)_{IJKL} \equiv A_{IJMN} B_{MNKL}$ .

For  $A_{2i}{}^{jkl}(\lambda)$  we have [2]

$$\begin{aligned}
(A_2)_1{}^{234} &= (A_2)_2{}^{314} = (A_2)_3{}^{124} = -e^{-i\omega} f, \\
(A_2)_1{}^{235} &= (A_2)_2{}^{315} = (A_2)_3{}^{125} = ie^{-i\omega} f, \\
(A_2)_6{}^{784} &= (A_2)_7{}^{864} = (A_2)_8{}^{674} = -ie^{i\omega} f, \\
(A_2)_6{}^{785} &= (A_2)_7{}^{865} = (A_2)_8{}^{675} = e^{i\omega} f, \\
(A_2)_4{}^{678} &= ie^{i\omega} g, \quad (A_2)_5{}^{123} = -ie^{-i\omega} g, \\
(A_2)_4{}^{123} &= e^{-i\omega} g, \quad (A_2)_5{}^{678} = -e^{i\omega} g, \\
(A_2)_1{}^{145} &= (A_2)_2{}^{245} = (A_2)_3{}^{345} = -(A_2)_6{}^{645} = -(A_2)_7{}^{745} = -(A_2)_8{}^{845} = -if^2,
\end{aligned} \tag{3.13}$$

with

$$f(\lambda) = \frac{1}{\sqrt{2}} \sinh(\sqrt{2}\lambda), \quad g(\lambda) = \frac{1}{2\sqrt{2}} \sinh(2\sqrt{2}\lambda). \tag{3.14}$$

The potential of the four-dimensional supergravity depends only on  $A_1^{ij}$  and  $A_{2i}{}^{jkl}$  and for the field configurations considered here is given by

$$\begin{aligned}
\mathcal{P}(\lambda, \omega) &= -\frac{3}{4}(6a^2 + 2b^2) + \frac{1}{4}(12f^2 + 6f^4 + 4g^2) \\
&= \frac{1}{16} \left[ -36 \cosh(2\sqrt{2}\lambda) + \cosh(4\sqrt{2}\lambda) - 61 \right],
\end{aligned} \tag{3.15}$$

and does not depend on  $\omega$ . As shown in [33], a rotation by an angle  $\omega$  corresponds to a diffeomorphism in the internal dimensions, hence does not change the physical solution. The extremum is attained at

$$\lambda = \frac{1}{\sqrt{2}} \operatorname{arcosh}(\sqrt{5}) \quad \Leftrightarrow \quad \cosh(\sqrt{2}\lambda) = \sqrt{5}, \tag{3.16}$$

so that

$$a = \sqrt{5}, \quad b = 3, \quad f = \sqrt{2}, \quad g = \sqrt{10}, \tag{3.17}$$

at the  $\text{SO}(3) \times \text{SO}(3)$  stationary point, with  $\mathcal{P} = -14$ . The point  $\lambda = 0$  corresponds to the maximally supersymmetric  $\text{AdS}_4$  vacuum with  $\text{SO}(8)$  symmetry.

As mentioned previously, this stationary point is the only known stable non-supersymmetric stationary point of  $D = 4$   $\text{SO}(8)$  gauged supergravity [3, 19]: the scalar mass eigenvalues for all 48 physical scalar fields are above the BF bound (22 scalars are Goldstone bosons, and thus are absorbed by the 22 massive gauge fields remaining after symmetry breaking to  $\text{SO}(3) \times \text{SO}(3)$ ). Since for maximal AdS supergravities in other dimensions  $D > 4$  no stable non-supersymmetric AdS vacua are known, this vacuum has been one of the most promising candidates for a stable non-supersymmetric  $\text{AdS}_4$  solution of M theory. The critical question is, therefore, whether the Kaluza-Klein spectrum can produce a tachyonic instability, and this is the question that will be addressed in the following section. We note that by the mass formula (2.11), we only require the knowledge of  $A_1$  and  $A_2$  at the stationary point (3.11) and (3.13), and the dressed action of

Killing vectors on  $S^7$  to compute the Kaluza-Klein spectrum.

## 4 Scalar harmonics

To compute the Kaluza-Klein spectrum, we now only need the higher-dimensional information coming from the round  $S^7$ . The scalar harmonics on  $S^7$  can be expressed as symmetric traceless polynomials in the elementary harmonics  $\mathcal{Y}^a$ , with  $a = 1, \dots, 8$ , which satisfy  $\mathcal{Y}^a \mathcal{Y}^b \delta_{ab} = 1$ . Thus,

$$\{\mathcal{Y}^\Sigma\} = \{1, \mathcal{Y}^a, \mathcal{Y}^{a_1 a_2}, \dots, \mathcal{Y}^{a_1 \dots a_n}, \dots\}, \quad (4.1)$$

where  $\mathcal{Y}^{a_1 \dots a_n} \equiv \mathcal{Y}^{((a_1 \dots \mathcal{Y}^{a_n}))}$  denotes traceless symmetrisation. Hence the index  $\Sigma$  runs over the tower of symmetric traceless representations  $[n, 0, 0, 0]$  of  $\text{SO}(8)$ .

The action of the  $\text{SO}(8)$  Killing vectors of the round  $S^7$  on the harmonics is most easily described in the  $\text{SL}(8) \subset E_{7(7)}$  basis. Under this  $\text{SL}(8)$ , the **56** of  $E_{7(7)}$  decomposes as

$$\mathbf{56} \longrightarrow \mathbf{28} \oplus \mathbf{28}' . \quad (4.2)$$

Accordingly, the action of the  $\text{SO}(8)$  Killing vectors decomposes into

$$\mathcal{T}_M{}^{c_1 \dots c_n}{}_{d_1 \dots d_m} = \left( \mathcal{T}_{ab}, \mathcal{T}^{ab} \right)_{d_1 \dots d_m}{}^{c_1 \dots c_n} . \quad (4.3)$$

Our summation convention for the harmonic indices  $\Sigma, \Omega$  is such that

$$A^\Sigma B_\Omega = A B + A^a B_a + A^{a_1 a_2} B_{a_1 a_2} + \dots + A^{a_1 \dots a_n} B_{a_1 \dots a_n} + \dots \quad (4.4)$$

With this convention, we find

$$\begin{aligned} \mathcal{T}_{ab}{}^{c_1 \dots c_n}{}_{d_1 \dots d_n} &= c n \delta_{[a}^{(c_1} \delta_{b]}^{c_2} \dots \delta_{d_1}^{c_n)} , \\ \mathcal{T}^{ab, c_1 \dots c_n}{}_{d_1 \dots d_n} &= 0 , \end{aligned} \quad (4.5)$$

where  $c$  is a coefficient that is determined by the normalisation (2.3), i.e.

$$[\mathcal{T}_M, \mathcal{T}_N] = -X_{[MN]}{}^P \mathcal{T}_P . \quad (4.6)$$

In order to match the notation of section 3, we must convert between the  $\text{SL}(8)$  basis used here and the  $\text{SU}(8)$  basis used there. We explain how to do this in appendix A. In the conventions of section 3, the round  $S^7$  has  $A_1^{ij} = \delta^{ij}$  and  $A_{2i}{}^{jkl} = 0$ . Therefore, the corresponding embedding

tensor in the  $\text{SL}(8)$  basis is given by

$$\begin{aligned} X_{abcd}{}^{ef} &= -X_{ab}{}^{ef}{}_{cd} = -\frac{1}{\sqrt{2}} \left( \delta_{b[c} \delta_{d]a}^{ef} - \delta_{a[c} \delta_{d]b}^{ef} \right), \\ X^{abcd}{}_{ef} &= X_{ab}{}^{cd}{}_{ef} = 0, \\ X^{ab}{}_M{}^N &= 0, \end{aligned} \tag{4.7}$$

and the normalisation coefficient for the  $\mathcal{T}_M$  is  $c = -\frac{1}{\sqrt{2}}$ . Thus, the only non-zero components of the  $\mathcal{T}_M$  are

$$\mathcal{T}_{ab}{}^{c_1 \dots c_n}{}_{d_1 \dots d_n} = -\frac{n}{\sqrt{2}} \delta_{[a}^{(c_1} \delta_{b] (d_1} \delta_{d_2}^{c_2} \dots \delta_{d_n}^{c_n)}. \tag{4.8}$$

The action of the  $\text{SO}(8)$  Killing vectors, dressed by the scalar matrix of the  $\text{SO}(3) \times \text{SO}(3)$ -invariant AdS vacuum (3.9), (in the  $\text{SU}(8)$  basis) is now given by

$$\begin{aligned} \mathcal{T}_{IJ}{}^{c_1 \dots c_n}{}_{d_1 \dots d_n} &= \frac{1}{4\sqrt{2}} \left[ \Gamma_{IJ}^{ab} + \left( \cosh(\sqrt{2}\lambda) - 1 \right) \Pi_{IJKL} \Gamma^{ab, KL} \right. \\ &\quad \left. + \frac{1}{2\sqrt{2}} \sinh(\sqrt{2}\lambda) \left( \cos \omega (Y^+ + iY^-) - \sin \omega (Z^+ - iZ^-) \right)_{IJKL} \Gamma^{ab, KL} \right] \mathcal{T}_{ab}{}^{c_1 \dots c_n}{}_{d_1 \dots d_n}. \end{aligned} \tag{4.9}$$

## 5 The Kaluza-Klein spectrum

We will now apply the method of [25, 26] and reviewed in section 2 to the  $\text{SO}(3) \times \text{SO}(3)$ -invariant  $\text{AdS}_4$  solution [3, 4] discussed in section 3. We have already discussed the fermion shift matrices  $A_1^{ij}$  and  $A_2{}_{i}{}^{jkl}$  at the  $\text{SO}(3) \times \text{SO}(3)$ -invariant point, eqs. (3.11), (3.13), and the action of the dressed Killing vectors on the scalar harmonics  $\mathcal{T}_A{}^\Sigma{}_\Omega$ , eq. (4.9), which we require to compute the Kaluza-Klein spectrum.

The Kaluza-Klein modes organise themselves into representations of the  $\text{SO}(3) \times \text{SO}(3)$  symmetry group. This is embedded into the  $\text{SO}(8)$  symmetry group of the round  $S^7$  according to the following branching of  $\text{SO}(8) \rightarrow \text{SO}(3) \times \text{SO}(3)$ :

$$\begin{aligned} \mathbf{8}_c &\longrightarrow (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}) \oplus 2 \cdot (\mathbf{1}, \mathbf{1}), \\ \mathbf{8}_{v,s} &\longrightarrow 2 \cdot (\mathbf{2}, \mathbf{2}), \end{aligned} \tag{5.1}$$

where the Kaluza-Klein scalar harmonics transform in the  $\mathbf{8}_v$ .

Using this branching, we can determine the  $\text{SO}(3) \times \text{SO}(3)$  representation of the Kaluza-Klein modes at level  $n$ . We do this by taking the tensor product of the  $\mathcal{N} = 8$  supergravity modes with the scalar harmonics at level  $n$  and decompose the result under  $\text{SO}(3) \times \text{SO}(3)$ . Indeed, the Kaluza-Klein fluctuation Ansatz of [25, 26] reflects this group-theoretic analysis at the level of the linearised dynamics by writing the fluctuation Ansatz as a product of the consistent truncation (which encapsulates the  $\mathcal{N} = 8$  supergravity modes) with the scalar harmonics.

For example, for the spin-2 Kaluza-Klein modes, we simply need to branch the  $[n, 0, 0, 0]$

representation of  $\text{SO}(8)$ , in which the scalar harmonics transform, to  $\text{SO}(3) \times \text{SO}(3)$  and find

$$\begin{aligned}
G_n &= \bigoplus_{l=0}^{n/2} (n-2l+1)(l+1) \cdot (\mathbf{n}-2\mathbf{l}+\mathbf{1}, \mathbf{n}-2\mathbf{l}+\mathbf{1}) \\
&\oplus \bigoplus_{l=1}^{n/2} \bigoplus_{k=0}^{l-1} (n-2l+1)(k+1) \cdot [(\mathbf{n}-2\mathbf{l}+\mathbf{1}, \mathbf{n}-2\mathbf{k}+\mathbf{1}) \oplus (\mathbf{n}-2\mathbf{k}+\mathbf{1}, \mathbf{n}-2\mathbf{l}+\mathbf{1})].
\end{aligned} \tag{5.2}$$

Similarly, we can determine the representations of the spin-1 Kaluza-Klein modes, which we denote by  $\tilde{V}_n$  at level  $n$ , by taking the tensor product of the  $\mathbf{28}$  of  $\text{SO}(8)$ , corresponding to the  $\mathcal{N} = 8$  vectors, with the  $[n, 0, 0, 0]$  representation of  $\text{SO}(8)$ . Finally, for the spin-0 KK modes, which we denote by  $\tilde{S}_n$  at level  $n$ , we take the tensor product of the  $\mathbf{35}_v \oplus \mathbf{35}_s$  of  $\text{SO}(8)$ , corresponding to the  $\mathcal{N} = 8$  scalars, with the  $[n, 0, 0, 0]$  representation of  $\text{SO}(8)$ . However, the resulting tensor products, just like the fluctuation Ansatz (2.1), contain also unphysical modes, in particular Goldstone scalars and vectors which are eaten by the massive vector fields and massive scalar fields. These can be removed by subtracting the massive Graviton representations from the massive vector representations and the massive vector representations,  $\tilde{V}_n$ , from the massive scalar representations. We denote the resulting physical Kaluza-Klein modes at level  $n$  by  $V_n$  and  $S_n$  and find

$$\begin{aligned}
V_n &= (n+1) \cdot (\mathbf{n}+\mathbf{3}, \mathbf{n}+\mathbf{3}) \oplus (5n+3) \cdot [(\mathbf{n}+\mathbf{3}, \mathbf{n}+\mathbf{1}) \oplus (\mathbf{n}+\mathbf{1}, \mathbf{n}+\mathbf{3})] \\
&\oplus \bigoplus_{l=0}^{(n-1)/2} ((27l-1)(n-2l) + 18n - 29l - 3) \cdot (\mathbf{n}+\mathbf{1}-2\mathbf{l}, \mathbf{n}+\mathbf{1}-2\mathbf{l}) \\
&\oplus \bigoplus_{l=1}^{n/2} 6(n+1-2l) \cdot [(\mathbf{n}+\mathbf{3}, \mathbf{n}+\mathbf{1}-2\mathbf{l}) \oplus (\mathbf{n}+\mathbf{1}-2\mathbf{l}, \mathbf{n}+\mathbf{3})] \\
&\oplus \bigoplus_{l=1}^{n/2} (27l(n+1-2l) - (n+3)) \cdot [(\mathbf{n}+\mathbf{3}-2\mathbf{l}, \mathbf{n}+\mathbf{1}-2\mathbf{l}) \oplus (\mathbf{n}+\mathbf{1}-2\mathbf{l}, \mathbf{n}+\mathbf{3}-2\mathbf{l})] \\
&\oplus \bigoplus_{l=2}^{n/2} \bigoplus_{k=0}^{l-2} 27(k+1)(n+1-2l) \cdot [(\mathbf{n}+\mathbf{1}-2\mathbf{k}, \mathbf{n}+\mathbf{1}-2\mathbf{l}) \oplus (\mathbf{n}+\mathbf{1}-2\mathbf{l}, \mathbf{n}+\mathbf{1}-2\mathbf{k})] \\
&\oplus \frac{9n}{4} (1+(-1)^n) \cdot (\mathbf{1}, \mathbf{1}),
\end{aligned} \tag{5.3}$$

$$\begin{aligned}
S_n = & 5(n+1) \cdot (\mathbf{n} + \mathbf{3}, \mathbf{n} + \mathbf{3}) \oplus (9n-1) \cdot [(\mathbf{n} + \mathbf{3}, \mathbf{n} + \mathbf{1}) \oplus (\mathbf{n} + \mathbf{1}, \mathbf{n} + \mathbf{3})] \\
& \oplus \bigoplus_{l=0}^{(n-1)/2} 2(21l(n-1-2l) + 12n-6) \cdot (\mathbf{n} + \mathbf{1} - \mathbf{2l}, \mathbf{n} + \mathbf{1} - \mathbf{2l}) \\
& \oplus \bigoplus_{l=1}^{n/2} 14(n+1-2l) \cdot [(\mathbf{n} + \mathbf{3}, \mathbf{n} + \mathbf{1} - \mathbf{2l}) \oplus (\mathbf{n} + \mathbf{1} - \mathbf{2l}, \mathbf{n} + \mathbf{3})] \\
& \oplus \bigoplus_{l=1}^{n/2} (42l(n+1-2l) - 5(n+3)) \cdot [(\mathbf{n} + \mathbf{3} - \mathbf{2l}, \mathbf{n} + \mathbf{1} - \mathbf{2l}) \oplus (\mathbf{n} + \mathbf{1} - \mathbf{2l}, \mathbf{n} + \mathbf{3} - \mathbf{2l})] \\
& \oplus \bigoplus_{l=2}^{n/2} \bigoplus_{k=0}^{l-2} 42(k+1)(n+1-2l) \cdot [(\mathbf{n} + \mathbf{1} - \mathbf{2k}, \mathbf{n} + \mathbf{1} - \mathbf{2l}) \oplus (\mathbf{n} + \mathbf{1} - \mathbf{2l}, \mathbf{n} + \mathbf{1} - \mathbf{2k})] \\
& \oplus \frac{1}{2} (1 + (-1)^n) (3 + 8n) \cdot (\mathbf{1}, \mathbf{1}).
\end{aligned} \tag{5.4}$$

We can now compute the masses of these Kaluza-Klein modes using the mass matrix (2.4). We use *Mathematica* to compute the eigenvalues of this matrix and trust in its capacities for handling large numerical matrices. Projecting the mass matrices onto the different irreducible  $\text{SO}(3) \times \text{SO}(3)$  in (5.4), we were able to compute the eigenvalues up to and including level 6 above the four-dimensional  $\mathcal{N} = 8$  supergravity modes with sufficient confidence. As a first consistency check the numerical results precisely reproduce the correct massless Goldstone modes. As another check, we work with traceful harmonics  $\mathcal{Y}^{(a_1 \dots a_n)}$ , such that the computation at  $n = 6$  includes and reproduces all previous even levels. Our results are plotted in figures 1 – 3.

The Kaluza-Klein spectrum shows two interesting features: firstly, there are tachyonic modes amongst the higher Kaluza-Klein modes, and secondly, there are 27 moduli, i.e. massless scalar fields which are not eaten by the massive vector fields, at level 2.

**Tachyonic Kaluza-Klein modes** The level 0 scalar fields,  $S_0$ , i.e. those of  $\mathcal{N} = 8$  supergravity, have masses at and above the BF bound [19]. We find this is also true for  $S_1$ , the scalars at level 1. However, starting at level 2 above the  $\mathcal{N} = 8$  scalars, there are *tachyonic* scalar fields amongst the Kaluza-Klein modes, i.e. some spin-0 Kaluza-Klein modes have masses *below* the BF bound, corresponding to  $m_{BF}^2 = -2.25L^{-2}$ , where  $L$  is the AdS length. Therefore, the AdS<sub>4</sub> vacuum is perturbatively unstable within 11-dimensional supergravity. We list the tachyonic modes and their mass eigenvalues in table 1.

Interestingly, the tachyonic modes only appear in the symmetric representations  $(\mathbf{k}, \mathbf{k})$  of  $\text{SO}(3) \times \text{SO}(3)$ , and not in the representations  $(\mathbf{k}, \mathbf{l})$  with  $k \neq l$ . Moreover, while the overall Kaluza-Klein spectrum shows increasing masses with level  $n$ , see figure 1, the masses of the tachyonic modes seem to remain stable, neither decreasing nor increasing, with increasing level  $n$ , as shown in figure 2. This seems to generate a sort of “mass gap” just above the BF bound. However, this may be an artefact of only working with the first few Kaluza-Klein levels. Indeed, if we assign the states to Kaluza-Klein towers, as shown in figure 3, we find that each Kaluza-

Level 2		Level 3		Level 4		Level 5		Level 6	
$m^2 L^2$	Irrep								
-3.117	(1, 1)	-3.146	(2, 2)	-2.950	(1, 1)	-2.721	(2, 2)	-2.400	(3, 3)
-2.821	(1, 1)	-2.892	(2, 2)	-2.922	(1, 1)	-2.657	(2, 2)	-2.266	(3, 3)
-2.532	(3, 3)	-2.741	(4, 4)	-3.114	(3, 3)	-3.056	(4, 4)	-2.910	(5, 5)
-2.448	(3, 3)	-2.446	(4, 4)	-2.801	(3, 3)	-2.567	(4, 4)	-2.895	(7, 7)
-2.361	(5, 5)	-2.627	(6, 6)	-2.876	(5, 5)	-2.930	(6, 6)	-2.577	(9, 9)
				-2.752	(7, 7)	-2.736	(8, 8)		

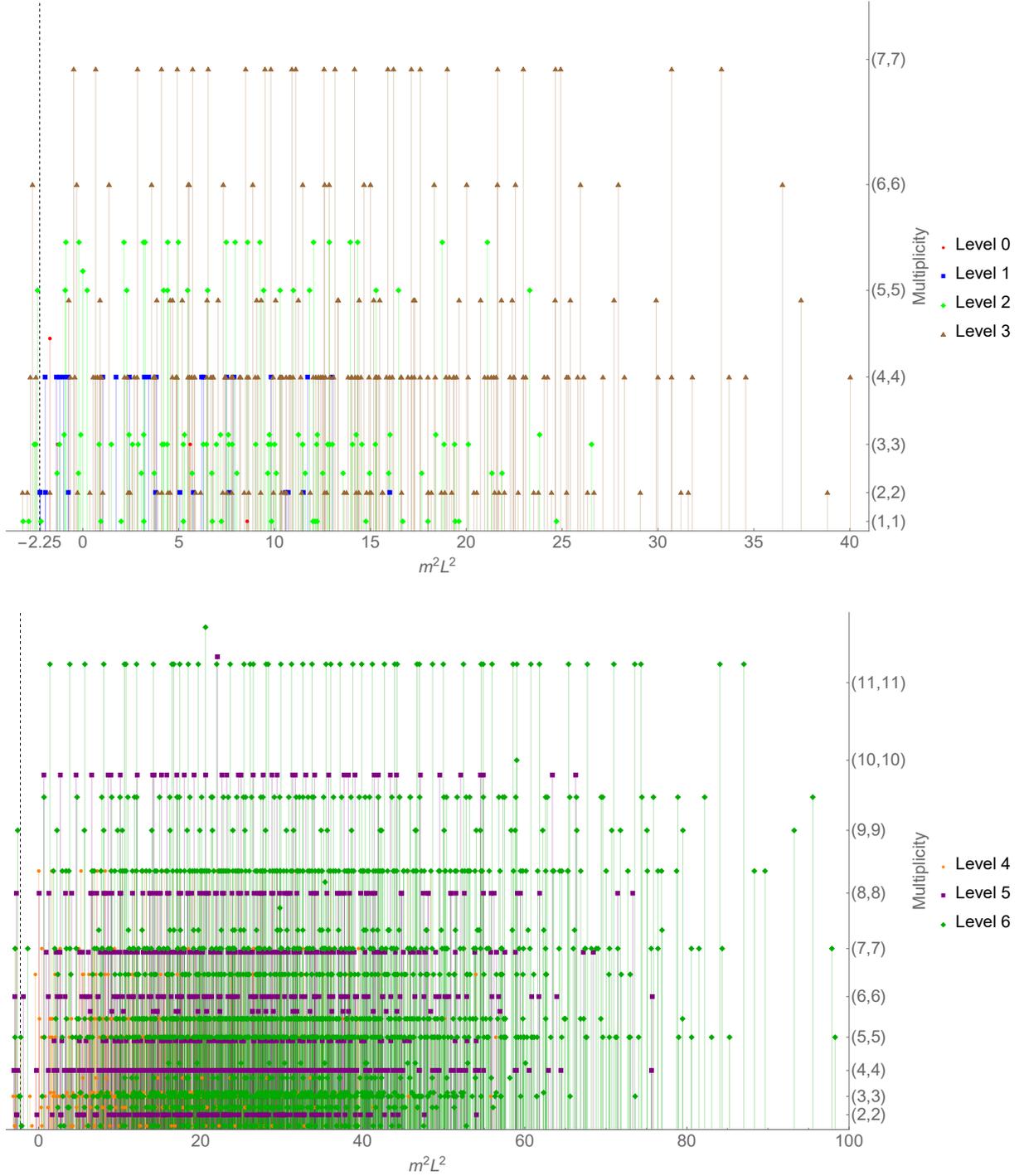
**Table 1:** Tachyonic Kaluza-Klein modes, their masses,  $m$ , in terms of the AdS length  $L$  and their representations under  $\text{SO}(3) \times \text{SO}(3)$ .

Klein tower eventually has increasing masses with increasing level  $n$ . Therefore, we expect that at a high enough level, there will be no more tachyonic states and the total number of tachyons in the Kaluza-Klein spectrum is finite.

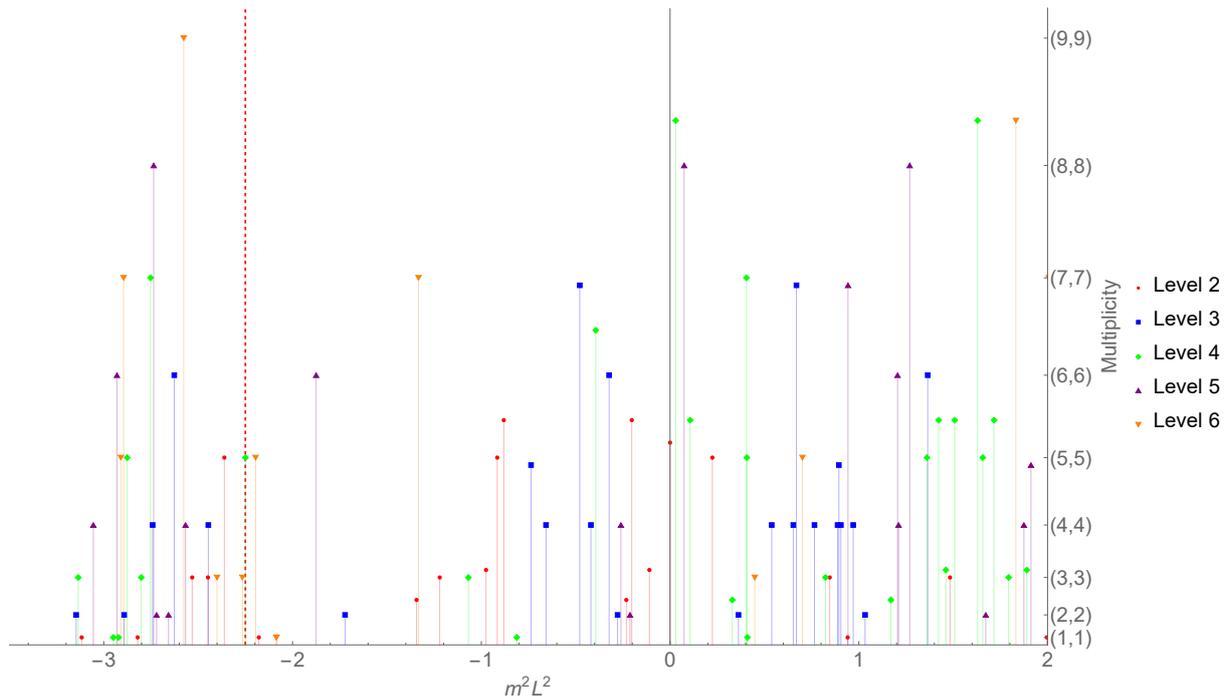
Indeed, we expect the total number of tachyonic modes to be finite due to the fact that the scalar mass operator on a compact manifold is a self-adjoint elliptic operator. More specifically, it is a generalised Lichnerowicz operator which in the case at hand is obtained by computing the second metric variation of the internal part of the bosonic action of  $D = 11$  supergravity around the vacuum corresponding to the uplift of the  $\text{SO}(3) \times \text{SO}(3)$  critical point. After choosing a gauge (see e.g. [35] for the corresponding analysis around the maximally supersymmetric  $\text{SO}(8)$  vacuum), this operator has the same principal symbol as the Laplace-Beltrami operator for the given metric, hence a discrete spectrum with no accumulation point [36]. What is remarkable, however, is that the mass spectrometer formula of [25,26] allows us to explicitly follow the “flow” of the low-lying eigenvalues as the vacuum is deformed from the maximally supersymmetric one to the  $\text{SO}(3) \times \text{SO}(3)$  vacuum, and that the eigenvalues are not uniformly lifted up, but exhibit a crossover behaviour.

**Moduli** At level 2 we find an additional 27 massless scalar fields on top of the Goldstone scalars. These moduli of the solution transform in the  $3 \cdot (\mathbf{3}, \mathbf{3})$  representation of  $\text{SO}(3) \times \text{SO}(3)$ . These modes, therefore, correspond to infinitesimal perturbations of the  $\text{AdS}_4$  vacuum which preserve the  $\text{AdS}_4$  part but deform the compactification in a way which breaks the  $\text{SO}(3) \times \text{SO}(3)$  symmetry. At levels 3 – 6, all massless scalar fields are Goldstone modes, suggesting that these 27 modes are the only moduli.

It would be interesting to determine if these moduli can be integrated into finite deformations. Since this is a non-supersymmetric vacuum, already the existence of infinitesimal moduli suggests some hidden structure. This same structure may also protect the moduli from obstructions at higher orders. If these modes can be integrated to finite moduli, the non-supersymmetric  $\text{AdS}_4$  vacuum studied here would belong to a family of  $\text{AdS}_4$  vacua with even fewer symmetries. It is unlikely that the other members of the family, if it exists, would admit a consistent truncation to 4-dimensional supergravity so that the other members can only be studied directly in 11



**Figure 1:** Spectrum of scalar Kaluza-Klein modes, plotted as the multiplicity of Kaluza-Klein mass eigenvalues for levels 0 – 6.  $m$  is the mass of the Kaluza-Klein mode,  $L$  is the AdS length and the multiplicity is determined by the  $SO(3) \times SO(3)$  representation,  $(\mathbf{k}, \mathbf{k})$ , of the mode. The dashed line denotes the BF bound  $m^2 L^2 = -2.25$ .



**Figure 2:** Spectrum of scalar Kaluza-Klein modes with masses in the range  $-3.5 \leq m^2 L^2 \leq 2$  with the BF bound  $m^2 L^2 = -2.25$  represented as a dashed red line. Here,  $m$  is the mass of the mode,  $L$  is the AdS length and the vertical axis shows the multiplicity which is determined by the  $\text{SO}(3) \times \text{SO}(3)$  representation of the modes.

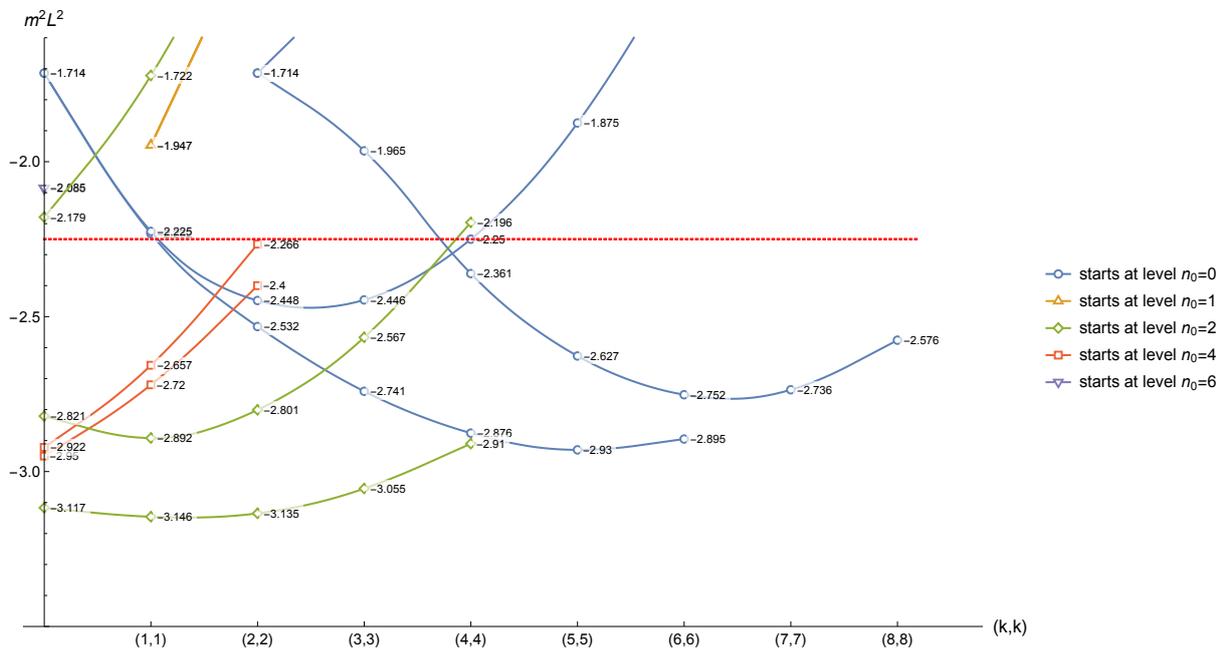
dimensions.

Using the fluctuation Ansatz of [25, 26] it is possible to determine the 11-dimensional fields corresponding to the tachyons and moduli we identify. This would allow us to study the possible endpoint of the instability and whether there are obstructions to integrating up the moduli to finite deformations.

## 6 Conclusions

In this paper, we computed the Kaluza-Klein spectrum of the non-supersymmetric  $\text{SO}(3) \times \text{SO}(3)$ -invariant  $\text{AdS}_4$  vacuum of 11-dimensional supergravity [33]. We showed that some of the higher Kaluza-Klein modes (starting at level 2 above the four-dimensional supergravity level) violate the BF bound and thus trigger an instability. Therefore, even though this AdS vacuum is stable within the maximal four-dimensional  $\text{SO}(8)$  gauged supergravity, it is not stable within 11-dimensional supergravity. Our result presents the first example of tachyonic Kaluza-Klein modes and dramatically confirms the AdS swampland conjecture [23] holds for this vacuum, which has long stood out as a possible counterexample.

In particular, we found that higher Kaluza-Klein modes can have masses below the lowest-lying modes, contradicting the observation based on supersymmetric solutions. Another inter-



**Figure 3:** Lowest Kaluza-Klein modes in the representations  $(\mathbf{k}, \mathbf{k})$  of  $SO(3) \times SO(3)$  arranged into Kaluza-Klein towers, such that the  $n$ th dot on a line starting at level  $n_0$  with  $k = k_0$  sits at level  $n_0 + n$  with  $k = k_0 + n$ . The colour code indicates the starting level  $n_0$  for the various towers. The dashed red line corresponds to the BF bound.

esting feature of the Kaluza-Klein spectrum is that we found 27 massless scalar fields at level 2 which are not eaten by massive vector fields and, therefore, the  $SO(3) \times SO(3)$ -invariant  $AdS_4$  solution has infinitesimal moduli which break the  $SO(3) \times SO(3)$  symmetry. Finally, this paper demonstrates the power of the methodology developed in [25, 26] which allowed us to perform the first computation of the Kaluza-Klein spectrum of a supergravity solution whose cover is not spin.

There are several questions about the  $SO(3) \times SO(3)$   $AdS_4$  vacuum studied here that it would be interesting to investigate. Firstly, it would be worthwhile to better analytically understand the structure of the Kaluza-Klein spectrum. For example, why do tachyons only appear in the symmetric representations  $(\mathbf{k}, \mathbf{k})$  of  $SO(3) \times SO(3)$ ? Another question is to identify the  $S^7$  modes that correspond to the tachyons and moduli we uncovered. One way of doing this is by evaluating the mass matrix (2.4) as a function of the deformation parameter  $\lambda$  which links the round  $S^7$  to the  $SO(3) \times SO(3)$ -invariant deformation studied here, see (3.9). A second direction of inquiry is to use the fluctuation Ansatz in [25, 26] to uplift the Kaluza-Klein modes, whose masses we computed here, to fluctuations of the 11-dimensional solution. This would shed light on the possible end-point of the instability triggered by the Kaluza-Klein modes and also on the relationship to the brane-jet instability of this  $AdS_4$  vacuum [24]. Similarly, it would allow us to explore whether the infinitesimal moduli we found in the Kaluza-Klein spectrum can be integrated to finite moduli of the 11-dimensional  $AdS_4$  solution. This would imply that this non-supersymmetric  $AdS_4$  vacuum is part of a continuous family of non-supersymmetric  $AdS_4$  vacua in 11-dimensional supergravity, which break the  $SO(3) \times SO(3)$  symmetry.

To conclude we would like to emphasise that, despite the important progress reported in this paper, the question as to the existence or non-existence of stable non-supersymmetric  $AdS$  vacua of string and M-theory remains wide open. First of all, there are non-supersymmetric vacua in the  $ISO(7)$  gauged theory which are stable within the four-dimensional gauged supergravity [6, 7, 10, 37]. Moreover, using the consistent truncation of massive IIA supergravity on  $S^6$  [38, 39], these vacua can be uplifted to 10-dimensional solutions of string theory. Therefore, the method of [25, 26] can be readily applied to address the stability of these non-supersymmetric  $AdS_4$  vacua, which have also recently been shown to be brane-jet stable [37]. Moreover, there is a far richer variety of non-supersymmetric stationary points for maximal (and non-maximal) gauged supergravities in three dimensions than for higher dimensions  $D \geq 4$ , whose stability remains to be investigated. For instance, the maximal gauged  $SO(8) \times SO(8)$  gauged theory of [40] possesses more than 2700 critical points [41], among them at least one stable non-supersymmetric one [42]. However, a major unsolved problem here concerns the possible uplifts of these vacua to 10 or 11 dimensions, as these cannot correspond to standard Kaluza-Klein compactifications.

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## A SU(8) vs. SL(8, $\mathbb{R}$ ) Bases

We can express a vector  $\mathcal{T}_M$  in the **56** representation of  $E_{7(7)}$  in terms of antisymmetric tensors in the SL(8,  $\mathbb{R}$ ) and SU(8) bases as follows. Under SL(8,  $\mathbb{R}$ )  $\subset E_{7(7)}$ , the **56** of  $E_{7(7)}$  decomposes as **56**  $\longrightarrow$  **28**  $\oplus$  **28'** and accordingly we can write

$$\mathcal{T}_M = \left( \mathcal{T}_{ab}, \mathcal{T}^{ab} \right), \quad (\text{A.1})$$

where  $a, b = 1, \dots, 8$  are used for the SL(8,  $\mathbb{R}$ ) basis and  $\mathcal{T}_{ab} = -\mathcal{T}_{ba}$  and  $\mathcal{T}^{ab} = -\mathcal{T}^{ba}$  are real. On the other hand, under SU(8)  $\subset E_{7(7)}$ , **56**  $\longrightarrow$  **28**  $\oplus$  **28** and accordingly we have

$$\mathcal{T}_M = \left( \mathcal{T}_{ij}, \mathcal{T}^{ij} \right), \quad (\text{A.2})$$

where now  $i, j = 1, \dots, 8$  are used for the SU(8) basis and  $\mathcal{T}_{ij} = -\mathcal{T}_{ji}$  and  $\mathcal{T}^{ij} = -\mathcal{T}^{ji}$  are complex but related by complex conjugation, i.e.

$$\mathcal{T}^{ij} = \bar{\mathcal{T}}_{ij}. \quad (\text{A.3})$$

The relationship between these two bases is analogous to the relationship between SU(1, 1) and SL(2,  $\mathbb{R}$ ) [34]

$$\begin{pmatrix} \mathcal{T}_{ij} \\ \mathcal{T}^{ij} \end{pmatrix} \equiv \frac{1}{4\sqrt{2}} \Gamma^{ij} \begin{pmatrix} \mathcal{T}_{ab} + i\mathcal{T}^{ab} \\ \mathcal{T}_{ab} - i\mathcal{T}^{ab} \end{pmatrix}, \quad (\text{A.4})$$

using the SO(8)  $\Gamma$ -matrices  $\Gamma_{ij}^{ab}$  with the normalisation

$$\Gamma^{ab,ij} \Gamma_{cd,ij} = 16 \delta_{cd}^{ab} = 8 \left( \delta_c^a \delta_d^b - \delta_d^a \delta_c^b \right). \quad (\text{A.5})$$

Similar relations hold for other representations of  $E_{7(7)}$  by writing them as tensor products of the **56** representation and acting on each **56** as in (A.4).

For completeness, we will also give the relationship between the scalar field fluctuations in the SU(8) and SL(8,  $\mathbb{R}$ ) bases. In the SU(8) basis, the scalar Kaluza-Klein modes are parameterised, as in (2.10), by complex self-dual four-forms of SU(8), i.e.

$$j^{ijkl}_{\Sigma} = \bar{j}_{ijkl,\Sigma} = \frac{1}{4!} \epsilon^{ijklmnpq} j_{mnpq,\Sigma}, \quad (\text{A.6})$$

where  $\bar{j}_{ijkl,\Sigma}$  is the complex conjugate of  $j_{ijkl,\Sigma}$ , and  $\epsilon^{ijklmnpq}$  is the eight-dimensional alternating symbol. On the other hand, in the SL(8) basis the  $j_{AB,\Sigma}$  are parameterised by a real symmetric

traceless tensor,  $\phi_{ab,\Sigma}$ , and a real self-dual four-form  $\phi_{abcd,\Sigma}$  as follows

$$\begin{aligned} j^{ab}{}_{cd,\Sigma} &= -j_{cd}{}^{ab}{}_{\Sigma} = \delta^{[a}{}_{[c}\phi^{b]}{}_{d],\Sigma}, \\ j^{abcd}{}_{\Sigma} &= j_{abcd,\Sigma} = \phi_{abcd,\Sigma}, \end{aligned} \tag{A.7}$$

where

$$\phi_{ab,\Sigma} = \phi_{ba,\Sigma}, \quad \phi^a{}_{a,\Sigma} = 0, \quad \phi_{abcd,\Sigma} = \frac{1}{4!}\epsilon_{abcd}{}^{efgh}\phi^{efgh}{}_{\Sigma}, \tag{A.8}$$

and all indices are raised/lowered by  $\delta_{ab}$ .

The relation between the real scalar fields  $\phi_{ab,\Sigma}$  and  $\phi_{abcd,\Sigma}$  in the  $\text{SL}(8,\mathbb{R})$  basis and the complex self-dual field  $j_{ijkl,\Sigma}$  is given by

$$j_{ijkl,\Sigma} = \frac{1}{16}\Gamma_{ij}^{ab}\left(\phi_{ac,\Sigma}\delta_{bd} + i\phi_{abcd,\Sigma}\right)\Gamma_{kl}^{cd}, \tag{A.9}$$

so that for instance

$$j^{ijkl,\Sigma}j_{ijkl,\Sigma} = \frac{3}{2}\phi^{ab,\Sigma}\phi_{ab,\Sigma} + \phi^{abcd,\Sigma}\phi_{abcd,\Sigma}. \tag{A.10}$$

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