EVALUATION OF MERCIER'S CRITERION IN 3D HELIAC EQUILIBRIA

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Helically symmetric equilibria with strong $\ell = 1$ helical curvature and bean-shaped cross-section have attracted attention for a long time /1/ because of their conjectured good finite-β MHD stability properties. This expectation was verified stepwise with increasing credibility of the theoretical analyses /2, 3, 4/ to the point that $\langle\beta\rangle$ values of up to 0.3 were demonstrated as completely ideal MHD stable, i.e. stable to high-n ballooning as well as low-n internal and external modes /5/. The revived interest in toroidal versions of these equilibria /6/ (HELIAC) has prompted 3D finite-β computational studies /7, 8/ which verified the good gross equilibrium properties as were expected on the basis of the standard estimate $\beta_e \approx \iota^2/A$ and the large rotational transform (twist) per field period of $\iota_p \sim 0.3$. In this paper, we continue the stability analyses /2, 3/ of HELIAC equilibria on the basis of the JMC (currents J and Mercier Criterion) code for the evaluation of 3D computational equilibria with respect to Mercier's necessary stability criterion /9/. First results concerning tests and the W VII-AS stellarator /10/ have already been obtained with JMC /11/. We recall /4, 5/ that in the helically symmetric case of the type of equilibria considered here Mercier's criterion turned out to be sufficient. This stresses the interest of the results described here, although the sufficiency of Mercier's criterion in toroidal stellarators /12/ has not yet been proved.

The JMC code, which is to be described in detail elsewhere, is constructed as a diagnostic package applicable to the results obtained with a 3D energy minimizing equilibrium code which uses flux surfaces as one of the independent variables. Currently, the published version of the BBG code /13/ is being used and the only output needed concerns the geometry of the flux surfaces. The linear problems as i) $\vec{j}\cdot\nabla s = 0$, where $\vec{j}$ is the current density and s the flux label, ii) the currents I and J and the one-dimensional part of the equilibrium equation $p'V' = I'F_T' + J'F_P'$ (F_T, F_P toroidal and poloidal fluxes, which are input functions for the BBG code, V volume, $' = d/ds$), iii) the equation for $X = j_{\parallel}/B$ are solved by the JMC code itself in a way which is appropriate for the evaluation of Mercier's criterion. It is written in the following form:

$$K_2{}^2 - K_1K_3 > 0 \text{ for Mercier stability,}$$

$$K_2 = \frac{1}{2}(F_P'F_T'' - F_T'F_P'') + \int XB^2\sqrt{g}\,|\nabla s|^{-2}dudv\ ,$$

$$K_1 = I'F_T'' + J'F_P'' - V''p' + p'^2\int B^{-2}\sqrt{g}\,dudv + \int X^2B^2|\nabla s|^{-2}\sqrt{g}\,dudv\ ,$$

$$K_3 = \int B^2\sqrt{g}\,|\nabla s|^{-2}dudv\ .$$

Here, s, u, v are the independent variables and $\sqrt{g}$ the Jacobian in the BBG code /13/.

The following type of helically symmetric and toroidal HELIAC equilibria is investigated. The plasma boundary is given by

$$r = \Sigma\Delta_{\ell m}\cos 2\pi[(\ell - 1)u - mv]$$

$$z = \Sigma\delta_{\ell m}\sin 2\pi[(\ell - 1)u - mv]$$

$$\Delta_{20} = 1,\ \Delta_{11} = 0.96,\ \Delta_{22} = -0.24 \text{ or } -0.29,\ \Delta_{31} = 0.38,\ \Delta_{33} = 0.11,\ \Delta_{44} = -0.04$$

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$$\Delta_{\ell m} = \delta_{\ell m} = 0 \text{ for all other indices.}$$

Here, $\Delta_{22} = -0.24$ for the "nonresonant" case (where $0.274 \leq \iota_p \leq 0.314$) and $\Delta_{22} = -0.29$ for the resonant case (where $0.32 \leq \iota_p \leq 0.36$). Thus, the resonance $\iota_p = 1/3$ is considered in the second case. The ι_p interval indicated gives the range occurring between the magnetic axis and the boundary. Figure 1 indicates the geometry of the nonresonant case; the boundary in the resonant case differs only slightly. The only difference between the helically symmetric and the toroidal case is that the major torus radius R_T is chosen in such a way that an infinite ($R_T = \infty$) or finite ($R_T < \infty$) number N of field periods may be considered. The total twist is $\iota = N\iota_p$; the aspect ratio of a period is $A_p \approx 2.5$, and the toroidal aspect ratio is $A = NA_p$. In the range of β values considered the equilibria are net current free in good approximation, $AdJ/dI \lesssim 0.1$.

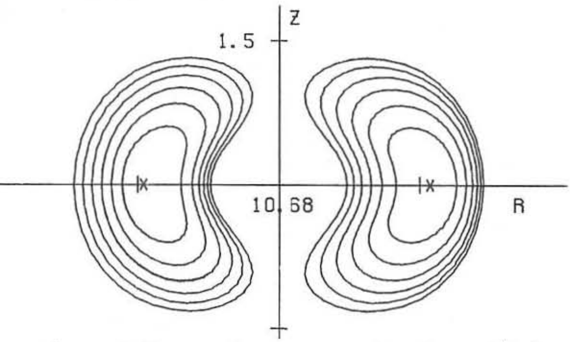

Fig.1: Cross-sections of flux surfaces at $v = 0$ and $v = 1/2$ for the nonresonant case with 4 periods and $\langle\beta\rangle = 0.075$.

First, an equilibrium with $\langle\beta\rangle = 0.075$ and approximately parabolic (in the distance from the magnetic axis) pressure profile is considered. This case is stable if helically symmetric. Considering $N = 13$, 10, 6, and 4, we find decreasing stability, as is shown in Fig.2 for $s = 1/2$, with a stability boundary at $N \approx 7$. Discussion of the various stabilizing and destabilizing terms in the criterion shows that the contributions of the first and second terms in K_1 as well as the influence of K_2 are negligible, so that the stability behaviour is governed by the last three terms of K_1. Instability comes about by an increase of the fifth term, which contains the parallel current density. Unstable behaviour occurs first at the plasma boundary in keeping with the fact that the pressure gradient is strongest there for the profile chosen.

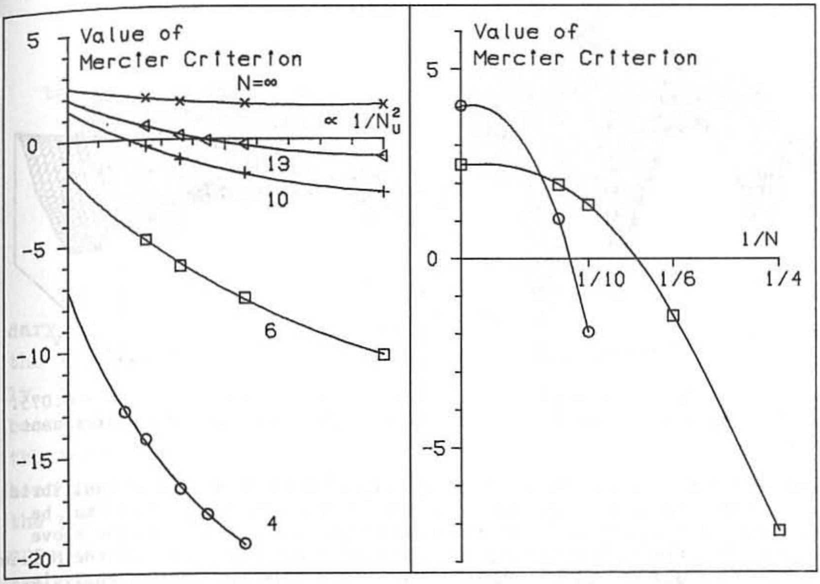

Fig.2: The left part of the figure shows the extrapolations of the values of the Mercier criterion at s = 1/2 to zero mesh size for the nonresonant case with <β> = 0.075 and N periods. The finest mesh used is 29/56/56, the coarsest 13/24/24. The right-hand part of the figure shows the extrapolated values for <β> = 0.075 (□) and 0.13 (o) as functions of 1/N.

The above result cannot, however, be considered to yield a reliable stability boundary for the following reasons. Considering N = 4 and the resonant case, the code result shows a large parallel current density with the 1/3-resonance structure (see Fig.3). Accordingly, Mercier's criterion is formally strongly violated. We do not stress the credibility of this result; more refined methods of investigating the actual structure which may occur at a low-order resonance already exist /14, 15/. Rather, we would advocate the conclusion that avoiding low-order resonances is advisable. In the nonresonant case (see Fig.3), some resonance structure is still visible in accordance with the choice of the twist ι = 0.294, which avoids the resonances 1/4 and 1/3. The main Fourier component of the current density here is the m = 1, n = 0 component which is driven by the torus effect. This component (which has the structure of the original PS current) likewise occurs in an axisymmetric torus. It here brings about the Mercier unstable behaviour (see Fig.2). On the other hand, the extrapolation to zero mesh size is difficult, because in the cases considered here the extrapolation yields a value which is approximately one order of magnitude smaller than the sum of the absolute values of the three constituents.

If the interpretation of the above results as Mercier instability at low N is correct, the unstable behaviour should be more obvious for a higher β value. Considering <β> = 0.13, we indeed find a stability boundary of N ≈ 12 at s = 1/2. Figure 2 shows the extrapolated results for <β> = 0.075 and 0.13.

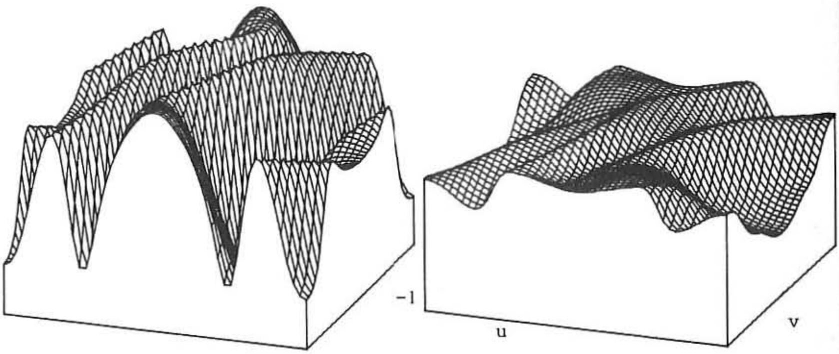

Fig.3: $X = j_{\shortparallel}/B$ as a function of u and v at s = 1/2 and N = 4, $<\beta> = 0.075$. The left-hand part shows the resonant, the right-hand part the nonresonant case.

Apart from the general comment that "strongly" three-dimensional equilibria (i.e. equilibria with large twist per period and only few periods) may be considered with reserve, there are two obvious conclusions from the above results: i) optimization may improve the stability behaviour, ii) the Mercier instability study has to be complemented by low node number mode analysis.

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