



Saving the Square of Opposition*

PIETER A. M. SEUREN

Max Planck Institute for Psycholinguistics, Nijmegen, the Netherlands

Received 19 April 2020 Accepted 15 December 2020

Contrary to received opinion, the Aristotelian Square of Opposition (SQUARE) is logically sound, differing from standard modern predicate logic (SMPL) only in that it restricts the universe U of cognitively constructible situations by banning null predicates, making it less unnatural than SMPL. U -restriction strengthens the logic without making it unsound. It also invites a cognitive approach to logic. Humans are endowed with a cognitive predicate logic (CPL), which checks the process of cognitive modelling (world construal) for consistency. The SQUARE is considered a first approximation to CPL, with a cognitive set-theoretic semantics. Not being cognitively real, the null set \emptyset is eliminated from the semantics of CPL. Still rudimentary in Aristotle's *On Interpretation* (*Int*), the SQUARE was implicitly completed in his *Prior Analytics* (*PrAn*), thereby introducing U -restriction. Abelard's reconstruction of the logic of *Int* is logically and historically correct; the LOCA (Leaking O-Corner Analysis) interpretation of the SQUARE, defended by some modern logicians, is logically faulty and historically untenable. Generally, U -restriction, not redefining the universal quantifier, as in Abelard and LOCA, is the correct path to a reconstruction of CPL. Valuation Space modelling is used to compute the effects of U -restriction.

1. Introduction

The present study forms part of a wider research programme, pursued by the author over the past half century and aimed *inter alia* at showing that human cognition and language are logically sound, and that the discrepancies between natural logical intuitions and standard logic are to be explained by restrictions imposed on the universe U of *cognitively constructible situations* (the term will be explained in a moment), without any appeal to Gricean pragmatic principles, which are considered too vague and too undisciplined to be acceptable as an explanatory device. The ultimate aim of the programme is to construct a cognitive processing system optimally combining logical power with empirical adequacy, without any concession to logical soundness or any appeal to pragmatic principles, and minimally differing from *standard modern predicate logic* (SMPL), considered the only solid point of departure for the reconstruction of *cognitive predicate logic* (CPL) but too contextually unrestricted and too exclusively extensional to enable consistent *hic et nunc*, context-dependent logical processing in interaction with thought processes. The present study is to be seen as a first step towards the reconstruction of CPL, taken to be innate in human cognition (Johnson-Laird 1983).

The leading hypothesis is that SMPL, if properly restricted, will be part of a logical system that is both logically sound and optimally functional for practical purposes. But in what ways can or must SMPL be restricted so as to make it cognitively manageable? The answer given is that restricting SMPL consists in restricting the U it operates in, U being defined in the Kantian terms of cognitive modelling (Seuren, *in prep.*) – a dimension that is absent from standard logic, where the notion of ‘Universe’ is normally equated with the *domain of quantification*, regardless of cognitive considerations. In CPL, U is based

*The author is much obliged to two referees, whose detailed comments have helped improve this study considerably.

on the notion of *proposition* and its ontological counterpart *fact*. A *proposition* is defined as the mental act of assigning a property to one or more (actual or virtual) entities. Each proposition creates, or gives rise to, a *virtual fact*, which becomes a *real fact* just in case it is in accordance with our world construal – that is, true. A real fact is thus whatever it is in reality that makes a proposition true. Facts have no independent being, yet are part of reality in that they are the reference values of propositions (Seuren 2009, 220–24). A set of related (virtual or real) facts forms a (virtual or real) *situation*, specified by a coherent set of propositions, that is, a *model*. Propositions become formally treatable in cognition when the properties are named as predicates and the entities are represented as such or serve as values of variables. *An unrestricted U is the set of all virtual or real situations created by the propositions that can be formed by means of the available predicates – the cognitively constructible situations mentioned above.* Since facts allow for reification as entities, they can in turn be assigned properties, giving rise to propositional embeddings under a higher predicate. Cognition is thus seen as an infinitely fertile creator of cognitively constructible situations created by propositions. *U-restriction now consists in leaving certain (classes of) cognitively constructible situations out of consideration* (Seuren 2009, 55–132).

That SMPL fails to reflect cognitive reality has been known from the start but has been attributed to the logical weakness of the human mind: ‘ordinary’ humans are taken not to abide by proper logic in their thinking or speaking but to slip into vagueness and ambiguity, giving in to practical convenience. Since the 1940s, it has been assumed (Russell 1940; Grice 1975; Levinson 1983, 2000; Horn 1984, 2004) that humans follow certain ‘pragmatic’ principles taken to overrule logical soundness. Yet to uphold that claim one must specify both what system one considers humans to violate and what logical system one attributes to humans. Pragmatics, however, has remained unclear on both counts, hovering between SMPL and the SQUARE for either, without any initiative to explore further possibilities. In fact, it has remained stuck in intuitive guesses that may have an immediate appeal but fail to show how precisely the pragmatically amended logic is left to function. The present study calls for the Gricean pragmatic principles to be replaced with an innate functionality-driven system of cognitive modelling controlled by CPL as a multiply restricted version of SMPL – a much greater challenge. The cognitive models are open to updating, adapting to the information state of the moment in interaction with cognition as a whole.

Although it is assumed that the human cognitive machinery is grounded in innate rational principles, these require both growth and nurture to come to fruition. Piaget has shown that Western children do not become logically ‘operative’ until after age 7 (Piaget 1954). Hallpike has shown that a lack of scholastic training leaves humans logically deficient at an elementary level (Hallpike 1979), which shows that nurture is necessary for natural logical competence: the more school the better the logic. One may assume that, at the most basic level, the focus is on the *interpretation* of propositions as expressed in language, rather than on their *logical consequences*, while at higher levels there is greater awareness of logical consequences, until, at the level of pure logic, the logical consequences overrule the interpretative properties, as in SMPL (Seuren 2010, 108–14).

As part of our wider programme, the present study deals specifically with the natural-language quantifiers ALL and SOME and the negation NOT. They are spelled in small caps, meaning that, for the time being, the finer but real and often language-specific distinctions between their linguistic variants, such as English *all*, *every*, *each*, *any* for ALL, or *a(n)*, *some*, *any*, *there is/are* for SOME, are ignored. NOT is taken into account in its standard bivalent form, even though a more detailed linguistic analysis shows this to be a gross oversimplification (Horn 1989; Seuren 2010, 334–42). Also ignored are possible

ambiguities due to the grammars of specific languages: all intended readings are clear from the text.

This study traces the ways these quantifiers have been dealt with by Aristotle, Boethius, Abelard, Ockham, a group of twentieth-century American logicians, and, last but not least, by the founders of SMPL. The corresponding systems are looked at from the point of view of logical soundness, the density of the logical relations holding in them (their ‘logical power’) in relation to their range of applicability, and their empirical standing as an account of natural logico-semantic intuitions. Restricting ourselves to the first stages of the reconstruction of CPL, we only take overall and obvious empirical criteria into account, focusing as yet more on overall principles than on precise empirical adequacy or immediate falsification criteria. Only at more advanced stages will it be possible, and necessary, to test the theory against more precise actual data.

For this hypothesis to hold, CPL must be traceable in human language, taken to express logically regimented propositions – an ancient and widely accepted but not fully explored view. Natural-language sentences are thus taken to be derived from underlying predicate-logical structures according to a well-defined system of correspondence or translation rules, which form the *grammar* of the language at issue (*Sechehaye 1908, 1926; McCawley 1972; Seuren 2018*). This aspect is not touched upon in the present study, which centres on the *semantic definitions in set-theoretical terms* of the quantifiers ALL and SOME.

It has been thought for over a century that Aristotle’s *Square of Opposition* (the SQUARE), is logically faulty as it fails for situations where the predicate in the subject term (here called the *R-predicate*) is uninstantiated, as in *All trolls eat geraniums* (which, by the way, is not true, as SMPL has it, but false, as the predicate *eat* requires actually existing entities for its subject term). But this failure is not a logical fault. All that is at issue is a restriction of the universe \mathbf{U} of all cognitively constructible situations (describable in terms of the available predicates): if SMPL is valid for the whole of \mathbf{U} , then it is necessarily also valid for any subset of \mathbf{U} . Universal applicability is a metarequirement, not a soundness condition, for a logical system. The question is *how* to restrict \mathbf{U} in such a way that cognitive reality is captured or at least approximated.

Aristotle’s predicate logic is treated as a first approximation to CPL, even if he may not have conceived of it in such terms himself. It exists in two versions, an earlier version presented in his *On Interpretation* (*Int*), where his predicate logic as developed up to that point is set out in explicit words, and a later version underlying his theory of syllogisms presented in *Prior Analytics* (*PrAn*), where the underlying predicate logic is not set out in explicit terms but must be culled from the available text. The former, less developed, version, called here *Aristotelian-Abelardian predicate logic* (AAPL), has hardly received any attention through the ages, whereas the latter reigned supreme in predicate logic for many centuries until it was ousted by SMPL in the early 1900s.

All varieties of predicate logic discussed in the present study are analysed and measured against the following three criteria:

- (i) logical soundness;
- (ii) universal applicability;
- (iii) empirical correctness with regard to logical and linguistic intuitions.

Thus far, it has proved impossible to devise a predicate-logical system that meets all three criteria to the full.¹ Criterion (i) not being negotiable, full satisfaction of criterion (ii)

¹ A variety of attempts have seen the light to obviate the negative features of SMPL, notably *Free Logics* (Nolt 2018) and *Paraconsistent Logics* (Priest et al. 2018). Neither, however, have been integrated into any wider framework; the latter in particular are, in addition, irrelevant to the present study, as they violate axiomatic principles of cognition.

appears to be possible only at the expense of criterion (iii), and vice versa. The present study investigates the conditions for full satisfaction of criterion (iii) while minimising the cost to criterion (ii). It is found that the greater the sacrifice to criterion (ii), the greater the logical power – that is, the density of logical relations – of the logical system: predicate logic gains in power as its universe of discourse is more restricted.

The SQUARE has the edge over AAPL and SMPL on criterion (iii), but it fails criterion (ii), as it is not valid for situations where the predicate in the subject term (the *R-predicate*) is uninstantiated. This fact has been presented as a fatal logical fault, falling under criterion (i) – an idea that has been instilled in logic students for over a century. But, as argued above, this is not a fault but a virtue, resulting from a restriction imposed on **U** for good functional reasons while leaving the system itself logically sound. AAPL and SMPL satisfy criterion (ii) but lose out on criterion (iii). Finally, an interpretation of the SQUARE has been gaining ground since the 1950s, here called the *Leaking O-Corner Analysis* or LOCA, which claims that the SQUARE can be made to satisfy criterion (ii). In actual fact, however, as is shown below, LOCA fails on all three counts.

The conclusion is that, for CPL to be successfully reconstructed, the semantic definitions of ALL and SOME that are valid for SMPL are to be maintained in the SQUARE (and thus in CPL), while, on the other hand, **U** must be properly restricted. Those proposed varieties of predicate logic where the semantic definitions of the quantifiers are modified, as in AAPL or LOCA, turn out either to unduly impoverish and complicate the logical system, as is the case with AAPL, or to lead to disaster, as with LOCA. The correct strategy to follow in constructing CPL is **U**-restriction. The strategy of semantic redefinition of the quantifiers, though worth trying, has proved counterproductive.

2. The SQUARE and SMPL

The first predicate-logical system in history was developed by the Macedonian-Greek philosopher Aristotle (384–322 BCE). This predicate logic – so called because its variables range over predicates – is set out in formally precise terms in Aristotle's *On Interpretation* (*Int*), followed shortly after by the longer treatise on syllogistic reasoning *Prior Analytics* (*PrAn*), which is formulated in the terms of the predicate logic set out in *Int*, but in a more developed and more crystallised form – a fact that seems to have escaped the notice of the majority of the historians of logic (a notable exception being *Kneale & Kneale 1962*). Aristotle himself never stated this Mark-2 version of his predicate logic explicitly: it looks as if he glossed over the differences for fear of getting mired in the complexities caused by the null set. The version underlying *PrAn* thus had to be distilled from it by analysis and reconstruction. This was not done until centuries later, when Aristotle's commentators, mainly Apuleius (ca 125–180), Ammonius (ca 440–520) and Boethius (ca 476–ca 525), laid it out in full. The result was the predicate logic known as the *Square of Opposition* (SQUARE). As such it entered the Middle Ages and became the standard doctrine of predicate logic until 1900.

In either form, Aristotle's predicate logic revolves around the two quantifiers ALL (\forall) and SOME (\exists), plus the standard bivalent logical negation NOT (\neg), which negates either a full proposition (*external negation*) or a propositional function (*predicate* or *internal negation*). For his logic, Aristotle considered the propositional schemata specified in (1a–h) below.

Before we proceed, however, one basic misunderstanding must be removed, to do with the relation between notation and system. One notation may be preferable to another in that it expresses the logical properties of the system more adequately, but the expressive qualities of a notation have, in principle, no bearing on the logical system it is meant to express. Yet generations of logicians have been told, over the past century, that when, around 1900, the SQUARE was given up in favour of SMPL, the main reason was that the

SQUARE does not allow for quantification over more than one term, as in *All boys admire some sportsmen*. But this is merely a defect of the notation used. The SQUARE as such does allow for those forms of quantification but its original notation could not express them.

The Russell notation repaired that defect, but in doing so it hit a snag that resulted from its *syntactic* notion of logic. This snag is removed when a *semantic* notion is adopted. The syntactic notion of logic implies that entailments are computable by means of algorithmical operations on the strings of symbols that are its formulae, the way operations are carried out in arithmetic: logic is reduced to transformational operations on strings of symbols. In the end, however, the Russell notation still fails in the extreme case that the totality **ENT** of entities (the domain of quantification) equals \emptyset .

This is shown as follows. In the Russell notation, as is well known, the positive universal and existential forms are written as $\forall x(Rx \rightarrow Mx)$ and $\exists x(Rx \wedge Mx)$, respectively. The quantifiers are defined by substitution: \forall yields truth iff for all values, and \exists iff for at least one value, in **ENT** of the bound variable x a true proposition results (if R is noninstantiated, Rx is false, and thus $Rx \rightarrow Mx$ true, for all values of x). However, when **ENT** = \emptyset , no value can be selected, which stalls the substitution process. Nowadays, the Russell notation has widely been replaced by a notation in terms of *Generalised Quantification Theory* (Mostowski 1957; Barwise & Cooper 1981), here called the *GenQ notation* – a step usually motivated by the fact that this theory provides a format for defining other quantifiers than just ALL and SOME, such as MOST, which the Russell notation cannot do (hence the term *generalised*).

But there is a deeper reason, which is that the GenQ notation *de facto* treats the quantifiers as binary higher-order predicates over pairs of sets, making it possible to define them *semantically*, by specifying their satisfaction conditions. SOME is now defined by the condition that the extensions of the R- and M-predicates have a nonnull intersection, while ALL requires that the extension of the R-predicate be a subset of the extension of the M-predicate. If the former equals \emptyset , the A-form comes out as true, since \emptyset is a subset of any set, also of **ENT**, which leaves SMPL intact as a logical system, while the problem of a null **ENT** is eliminated. But this takes us from a syntactic to a semantic concept of logic. The logical properties of a logical system are now no longer determined by meaningless algorithms applied to its formulae but by the semantic definitions, in terms of set-theoretical satisfaction conditions, of the quantifiers and other logical operators, together with any restrictions imposed on **U**.

The notion of **ENT** as the totality of all entities is not without complications. It is commonly used in logic, under the name of *Universe*, more or less as a synonym for *domain of quantification*, the set of independently given entities quantified over. For our purposes, however, it is necessary to distinguish between **U** and **ENT**, both part of reality, but, as per Kant, in the form reality is construed by the mind: **U**, as has been said, is the set of all mentally constructible real or virtual world situations created by true or false propositions; **ENT** consists of all real or virtual entities represented in, or created by, cognition at any given moment. Since, however, the inclusion of virtual entities will amount to a radical game change, as logic then becomes essentially intensional and presuppositional, with extensional logic as a derived product (much in the spirit of the nineteenth-century Austrian philosopher Alexius Meinong), we will, in the present study, keep things simple and transparent, exploiting the extensional overlap between SMPL and CPL by limiting the choice of predicates to those that can achieve truth only for those elements in **ENT** that have been marked in advance as actually existing, such as *be alive*, *run*, *laugh*, *kick*, *feed* and many others, while intensional predicates, which can achieve truth also, and sometimes only, for virtual entities, such as *frighten*, *delight*, *be worshipped*, *be dreamt of*, *be imaginary*, will be left out of consideration,

as has been the implicit tradition in logic anyway since Aristotle. **ENT** is thus, for the moment, limited to the collection of entities that are represented in cognition as having actual reality.²

Similar complications arise in a full-fledged cognitive version of set theory. We avoid those by leaving standard set theory as it is, except that the null set (\emptyset) is excluded from (the powerset of) **ENT** (see Seuren 2010, 71–79 for more discussion). This exclusion is justified because \emptyset plays no role in cognitive modelling. For cognition, \emptyset is not a set but the absence of actual entities satisfying an extensional predicate. (Teachers of set theory will recognise the difficulties beginning students have trying to master \emptyset .) So \emptyset must be eliminated from the semantics of CPL. We do this here by the following brute-force stipulation:

\emptyset is excluded from the power set of the totality **ENT** of all entities.³

(The real reason why \emptyset cannot occur in **ENT** is that if **ENT** is defined as consisting of entities that are represented as such in cognition, then some (extensional or intensional) predicate has been assigned to them: predicate-less entities cannot be members of **ENT**, as it is only the assignment of a predicate that creates an entity representation in **ENT**.)

If we thus keep **ENT** strictly extensional, ‘ $X = \emptyset$ ’ and ‘ $X \neq \emptyset$ ’ now serve as shorthand notations for the absence or presence, respectively, of an element in **ENT** satisfying the conditions for any *extensional* predicate denoting X . This does not affect the validity of natural set theory, as excising a class of sets with a given cardinality only trims but does not invalidate standard set theory. In fact, discarding \emptyset adds new theorems, namely those that are made nonvalid only by \emptyset . Thus, with \emptyset removed, it is now a theorem that, for all sets X and Y , if $X \subseteq Y$, then $X \not\subseteq \mathbf{ENT} - Y$ (the complement of Y in **ENT**), since it fails to hold only when $X = \emptyset$, \emptyset being the only set that is a subset of all sets.

Logical systems are formal not because they are algorithmic but because they take their semantics from set theory, *natural* set theory in the case of CPL. Furthermore, if *natural* set theory is the semantic underpinning of CPL, it is innately present in cognition, not in need of being taught (only of being nurtured), which is what gives rise to logical intuitions. This may be heresy to readers versed in modern logic, but it does make sense.

Now back to the nitty-gritty of the Aristotelian SQUARE. The traditional coding of the propositional forms by means of the capital letters A, I, E and O goes back to Boethius’ Latin commentary on Aristotle’s *Int*, written ca 510 CE. As is well-known, Boethius chose the codes A, I, E and O as mnemonic aids: A and I are the first two vowels of Latin *affirmo* (‘I affirm’), and E and O of *nego* (‘I deny’). An unfortunate feature of this Boethian coding, however, is that it reduces the eight forms to four, each representing two forms taken to be equivalent. This makes it impossible to distinguish the equivalent forms from each other when this is necessary for the analysis of those systems, such as AAPL, where the equivalences are torn apart and weakened to one-way entailments, and it leads to a fatal confusion in LOCA, as is shown below in Section 3.2. The coding used here repairs that defect in that only the capital letters A and I are used and the external and internal negations

² Predicates like *exist*, *be imaginary*, crucially show that a presuppositional logic is needed. For Russell (1937: 449), *The man does not exist* is necessarily false, as the predicate *exist* requires existence for truth and *the man* can only refer to a person p if p exists. In presuppositional logic, *The man exists* only asserts but does not presuppose the man’s actual existence (Seuren 2009, 217). So as not to introduce too many innovations at once, this complication is disregarded here.

³ Let $|X|$ stand for the cardinality of a set X and $P(X)$ for the power set of X . If $|X| = n$, $|P(X)| = 2^n$. When \emptyset is dropped from $P(X)$, resulting in the *restricted power set* $PR(X)$, $|PR(X)| = 2^n - 1$. Also, if $|Y| = |X| + 1$, $|PR(Y)| - |PR(X)| = 2^n$, which restores the old pattern once removed, so to speak.

are expressed as ‘ \neg ’ and ‘*’, respectively, the former being placed before, the latter after the capital letters A and I.

Here follow the eight forms, each with its new coding, its Russell notation, its GenQ notation, and its truth condition. R (*Restrictor*) and M (*Matrix*) range over predicates (both sigla taken from generalised quantification theory); $[[X]]$ stands for the extension of the predicate X in any ENT; $C[[X]]$ stands for the complement of $[[X]]$ in ENT:

(1) Form	Coding	Russell Notation	GenQ notation ⁴	Truth condition
a. All R is M	A	$\forall x(Rx \rightarrow Mx)$	$\forall x(Mx, Rx)$	$[[R]] \subseteq [[M]]$
b. Some R is M	I	$\exists x(Rx \wedge Mx)$	$\exists x(Mx, Rx)$	$[[R]] \cap [[M]] \neq \emptyset$
c. Not all R is M	$\neg A$	$\neg(\forall x(Rx \rightarrow Mx))$	$\neg\forall x(Mx, Rx)$	$[[R]] \not\subseteq [[M]]$
d. No (= not some) R is M	$\neg I$	$\neg(\exists x(Rx \wedge Mx))$	$\neg\exists x(Mx, Rx)$	$[[R]] \cap [[M]] = \emptyset$
e. All R is not M	A*	$\forall x(Rx \rightarrow \neg Mx)$	$\forall x(\neg Mx, Rx)$	$[[R]] \subseteq C[[M]]$
f. Some R is not M	I*	$\exists x(Rx \wedge \neg Mx)$	$\exists x(\neg Mx, Rx)$	$[[R]] \cap C[[M]] \neq \emptyset$
g. Not all R is not M	$\neg A^*$	$\neg(\forall x(Rx \rightarrow \neg Mx))$	$\neg\forall x(\neg Mx, Rx)$	$[[R]] \not\subseteq C[[M]]$
h. No R is not M	$\neg I^*$	$\neg(\exists x(Rx \wedge \neg Mx))$	$\neg\exists x(\neg Mx, Rx)$	$[[R]] \cap C[[M]] = \emptyset$

The following universal logical relations help define all systems concerned, the entailment relation being basic to all the others (P and Q range over propositions):

P **entails** Q ($P \vdash Q$): whenever P is true, Q is necessarily (in virtue of the meanings of P and Q) also true.

P and Q are **contraries** ($P \diamond Q$): P and Q cannot both be true, but may both be false, at the same time; that is: $P \vdash \neg Q$.

P and Q are **subcontraries** ($P \diamond Q$): P and Q cannot both be false, but may both be true, at the same time; that is: $\neg P \vdash Q$.

P and Q are **equivalent** ($P \equiv Q$): P and Q are always true or false at the same time; that is: $P \vdash Q$ and $Q \vdash P$.

P and Q are **contradictories**: P and Q are both contraries and subcontraries; that is: $P \vdash \neg Q$ and $\neg P \vdash Q$.

It is now known that *Int* does not present the Aristotelian SQUARE but that the latter must be extracted from *PrAn*. Aristotle’s commentators apparently based themselves on *PrAn*, which, for most logicians, forms the centrepiece of Aristotle’s logic, and taught logic from there. The predicate logic underlying *PrAn* is especially attractive as it can be presented as a nicely regular quadrilateral diagram, the SQUARE, where all vertices are connected to each other by some logical relation. This diagram is only described verbally by Apuleius and Ammonius but first published, as far as is known, as an actual diagram in Boethius’ commentary (though it must have been in common use in logic classes long before). In the form it was presented by Boethius in the early sixth century, the SQUARE became the standard form of predicate logic throughout the Middle Ages and later, up until the twentieth century, when it was dethroned by SMPL. It is shown in Figure 1, but with the new coding for the proposition forms (‘C’ stands for ‘contraries’, ‘SC’ for ‘subcontraries’, ‘=’ for equivalence, arrows for entailments, ‘x’ for ‘contradictories’):

⁴ Multiple quantification is treated as a form of propositional embedding with bound variables as terms. In the GenQ notation, the sentence *All boys admire some sportsmen* is written as $\forall x(\exists y(\text{Adm}(x,y), \text{Sy}), \text{Bx})$, where ‘Adm’ stands for *admire*, ‘S’ for *sportsman* and ‘B’ for *boy*.

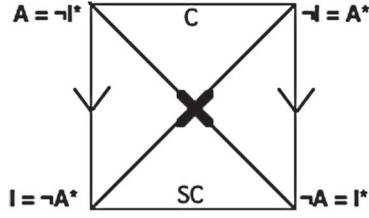


Figure 1. The Aristotelian-Boethian Square of Opposition (with the new coding).

During the period the SQUARE reigned supreme it was not generally realised that, with U unrestricted, it collapses in cases where the R-predicate is uninstantiated. That this is so is easily shown: if $[[R]] = \emptyset$, both the I-form and the I*-form are false, which eliminates the relation of subcontrariety (unless both I and I* are necessarily false); if both I and I* can be false at the same time, their contradictories $\neg I$ and $\neg I^*$ can be true at the same time, which eliminates the relation of contrariety; if contrariety is gone, so are the subalterns. The SQUARE is thus stripped down to just the dual equivalences – that is, to SMPL. The only way to rescue the SQUARE in an unrestricted U is to assume that $[[R]] \neq \emptyset$ is a necessary truth, entailed by any proposition in the system. This being an impossible condition in an unrestricted U , the nonnullness of $[[R]]$ must be imposed as an overall external constraint, so that the SQUARE as a whole is predicated on the condition that $[[R]] \neq \emptyset$ for each lexical choice of the R-predicate. This is how the SQUARE is saved here. The fact that a null R-predicate throws a wrench in the works of the SQUARE is known as *undue existential import*, or UEI.

Close reading of *Int* reveals that the predicate-logical system set out there is only part of the SQUARE, and that the missing parts are precisely those that, if inserted, would lead to the exclusion of situations where $[[R]] = \emptyset$. *Int* thus does not suffer from UEI, which is very much to Aristotle's credit, as it shows that he did not simply overlook such cases.

In *Int*, Aristotle explicitly distinguished the following logical relations between the forms listed in (1a-h), and only these:

- (2) a. A and $\neg A$ are contradictories;
- b. I and $\neg I$ are contradictories;
- c. $A \diamond \neg I$;
- d. $A \vdash \neg I^*$ but not vice versa;
- e. $I \vdash \neg A^*$ but not vice versa.

Aristotle thus took the external negation (\neg) to be an operator that inverts truth values. He also stated that A and $\neg I$ cannot both be true but can be false at the same time, or: $A \diamond \neg I$ ($= 2c$). He failed, however, in *Int*, to mention that if (2c) holds, it follows that $A \vdash I$, because, by (2c), if A is true, $\neg I$ must be false and therefore I must be true, so that, indeed, $A \vdash I$. This entailment, known as the *positive subaltern*, is not stated in *Int* (nor is its negative counterpart $A^* \vdash I^*$), but it is stated early on in *PrAn*, at 25a16–21. Moreover, (2d) and (2e) show that he also failed to state the duality of A and I : $A \equiv \neg I^*$ and $I \equiv \neg A^*$, saying instead only that $A \vdash \neg I^*$ ($= 2d$) and that $I \vdash \neg A^*$ ($= 2e$). By contrast, the dual equivalences are taken for granted in the Mark-2 version of his predicate logic as culled from *PrAn*. He failed, moreover, in *Int*, to state the relation of subcontrariety between I and I^* ($I \diamond I^*$), though he would have agreed that I and $\neg A$ are subcontraries, since, if $A \vdash I$, then it is not possible for A to be true (and $\neg A$ to be false) while I is false.

These gaps in the *Int* version of Aristotle's predicate logic suggest that, when writing *Int*, he was still unclear in his own mind about what to do with cases where $[[R]] = \emptyset$. But he was careful enough to avoid any commitment on that count. In *PrAn*, by contrast, he simply cut the Gordian knot and decided to rule out, by stipulation, all situations where $[[R]] = \emptyset$.

UEI has been a bone of contention in predicate logic over the past century and occasionally also earlier. This is due to the fact that if the SQUARE, with the quantifiers defined as in (3) below, is made to be applicable also to situations where $[[R]] = \emptyset$, it is perforce whittled down to a system with less logical power, resulting in SMPL, which mirrors standard set theory to a tee. The SQUARE is thus reduced to just the equivalences $A \equiv \neg I^*$ and $I \equiv \neg A^*$ (and thus also the contradictions between A and I^* , and I and A^*), all other logical relations being lost.

In the semantic interpretation of logic adopted here, the quantifiers are treated as higher-order predicates, semantically defined in terms of their satisfaction conditions captured in the following set-theoretic specifications of their extensions (X and Y range over sets of entities in **ENT**):

- (3) a. $[[ALL]] = \{ \langle X, Y \rangle \mid X \subseteq Y \}$
 b. $[[SOME]] = \{ \langle X, Y \rangle \mid X \cap Y \neq \emptyset \}$

The A-form 'All R is M' now turns out true when $[[R]] = \emptyset$, since \emptyset is a subset of any set.

While working on *Int*, Aristotle may well have seen that the crux of the UEI problem lies in the quantifier ALL: if ALL is assigned existential import, it is *de facto* redefined as in (4) instead of (3a), yielding precisely the system set out in *Int* (but not in *PrAn*):

- (4) $[[ALL]] = \{ \langle X, Y \rangle \mid X \subseteq Y \text{ AND } X \neq \emptyset \}$

With this definition, Aristotle's predicate logic *de facto* takes the form made explicit almost fifteen centuries later by the French philosopher Peter Abelard (1079–1142) (see Section 3.1 below). Aristotle himself never discusses this point explicitly, but in *Int* we see him take the precaution not to venture beyond those logical relations that are consistent with (4), with the result that the intuitively valid dual equivalences are kept at bay, being reduced to one-way entailments, as is shown below. In *PrAn*, however, he bans uninstantiated predicates altogether from his logical system, thus letting UEI in but adding the intuitively attractive dual equivalences to his system.

Aristotle's reason for this concession to natural intuitions was no doubt that he was primarily concerned with the question of whether natural human logical intuitions form a consistent logical system, precisely the question that occupies us here. He wanted *PrAn* to reflect natural logical intuitions, or else he would not have taken the four primary syllogistic schemata or, as they are known, 'moods' as 'perfect' (we would say 'axiomatic') on the grounds that they are intuitively transparent, while the remaining moods are derived from those by a formal proof method. He wrote (*PrAn* 24b22–26):

I call a syllogism perfect when it requires nothing beyond what is understood as such for the conclusion to follow necessarily; imperfect when more is required, which, though necessary on the grounds given, is not grasped immediately on the basis of the premisses. [translation mine; PAMS]

Aristotle may well have realised that his syllogistic was at risk for cases where uninstantiated predicates are involved. He therefore imposed on his entire syllogistic system the condition that *all* predicates occurring in the eight forms, not just those in the subject position of the A- or A*-form, must have a nonnull extension. It is also quite likely that his

uncertainty about what to do with uninstantiated predicates still persisted. After six days of very hard work, he would have needed a rest.

This has been recognised by modern scholars. Smith, for one, states (*Smith 2017*, 5.2): ‘Aristotle in effect supposes that *all* terms in syllogisms are non-empty’. The Kneales likewise state (*Kneale & Kneale 1962*, 60): ‘In order to justify Aristotle’s doctrine as a whole it is necessary [...] to suppose that he assumed application [that is, instantiation; PAMS] for *all* the general terms [that is, predicates; PAMS] with which he dealt’. For Aristotle’s syllogistic to be valid, *all* situations described by means of an uninstantiated predicate must be banned from **U**.

To show this, let us consider cases of Aristotelian syllogistic moods that are valid in terms of the SQUARE but not in terms of SMPL. There are at least three such moods: DARAPTI, FELAPTON, and BRAMANTIP.⁵ They become nonvalid in SMPL when $[[R]] = \emptyset$.

DARAPTI (3rd figure)	All R is S		
	All R is T	Ergo:	Some T is S
FELAPTON (3rd figure)	No R is S		
	All R is T	Ergo:	Some T is not S
BRAMANTIP (4th figure)	All R is S		
	All S is T	Ergo:	Some R is T

DARAPTI fails in SMPL because, when $[[R]] = \emptyset$, the two premisses are automatically true but their conjunction allows for a situation where *No T is S* is true, which contradicts the conclusion. FELAPTON fails in SMPL because, when $[[R]] = \emptyset$, the two premisses are again true but their conjunction allows for a situation where *All T is S* is true, which contradicts the conclusion. BRAMANTIP fails in SMPL because, when $[[R]] = \emptyset$, the major is vacuously true, and when the minor is also true, the conclusion is false. These three moods thus fail in SMPL precisely because of cases where $[[R]] = \emptyset$. The fact that Aristotle included them in his syllogistic shows that he did not reckon with situations where $[[R]] = \emptyset$. Had he taken these into account, he could not have considered these three moods valid.

From uninstantiated R-predicates we can extrapolate to uninstantiated M-predicates. Predicate variables in the syllogistic system are then allowed to denote only nonnull sets in **ENT**, as postulated by *Smith 2017* and *Kneale & Kneale 1962* mentioned above. This generalisation makes the system more uniform in that it is no longer necessary for the system to check in each case if the R-predicate is or is not instantiated. All it requires is that the power set of **ENT** be computed without taking \emptyset into account, which is not only cognitively realistic but also a perfectly valid way of defining extensionality, since all predicates that are uninstantiated in this world are the fruit of human imagination or memory, which, in the perspective of CPL, defines intensionality.⁶ The across-the-board banning of

⁵ Medieval logicians developed a mnemonic nomenclature for the various syllogistic forms (or ‘moods’) of *PrAn*. All names in this nomenclature consist of three syllables, the vowel of each syllable being A, I, E or O, referring to the Boethian names for the four categorial forms in Aristotle’s predicate logic. The first vowel refers to the major premiss of the syllogism, the second to the minor premiss, and the third to the conclusion. The prototypical syllogism BARBARA, for example, has an A-form major (e.g. *All felines are animals*), an A-form minor (e.g. *All cats are felines*), and an A-form conclusion (*All cats are animals*). Aristotle also distinguished four syllogistic *figures* according to the position of the *middle term* in the syllogistic mood (the middle term is the term that is common to both premisses). In the first figure, the middle term stands in subject position in the major and in predicate position in the minor (in Latin); in the second figure the middle term stands in predicate position in both premisses; in the third figure the middle term stands in subject position in both premisses; in the fourth figure the middle term stands in predicate position in the major and in subject position in the minor. In principle, the combination of name and figure should identify each syllogistic mood.

⁶ Owing to the *horror mentis* that has pervaded both logic and the human sciences for over a century, this point of view is still anathema in circles of modern logicians. This anathema is fundamentally misguided. In a desperate effort to keep the mind at bay, intensionality is ‘extensionalised’ by means of possible worlds, curiously treated as if they were extensional objects (*Lewis 1969*, 171–2). It is widely known and acknowledged, however, that this approach falls foul of the nonsubstitutivity *salva*

uninstantiated predicates is, moreover, underwritten by the fact that it makes no difference to the SQUARE whether or not the condition that $[[M]] \neq \emptyset$ is added to the condition that $[[R]] \neq \emptyset$. There thus seem to be sufficient grounds to conclude that Aristotle systematically excluded uninstantiated predicates from the SQUARE. And one is tempted to assume that he did so in order not to stray too far from intuitive naturalness.

A further consideration is relevant here. In Aristotle's syllogistic, no premiss represented by an A, I or O can be true when $[[R]] = \emptyset$, which leaves only the moods with the vowel E in both the first and the second syllable operational under that condition. But there is no such mood. This means that, when $[[R]] = \emptyset$, all pairs of premisses in the entire system will be false when $[[R]] = \emptyset$. In virtue of the rule *Ex falso sequitur quodlibet* ('given the falsity of a premiss anything follows'), any proposition, whether true or false, will follow from a false premiss, which makes all syllogisms, and thereby Aristotle's entire syllogistic edifice, otiose when $[[R]] = \emptyset$. If this had been his intention, one would have expected Aristotle, always extremely meticulous, to have said something in this regard. But he did not. This *argumentum ex silentio* provides some further justification for the assumption that Aristotle was not interested in cases where $[[R]] = \emptyset$ and simply left them out of consideration – so as to satisfy natural logical intuitions.

When, during the late nineteenth century, logic was more and more considered a mathematical discipline with implications of metaphysical necessity, the exclusion of cases where $[[R]] = \emptyset$ was increasingly seen as a fatal logical defect.⁷ This, however, is erroneous: the SQUARE is perfectly sound from a logical point of view, resulting as it does from a restriction on the universe U of all cognitively constructible situations that bans \emptyset from the power set of ENT . Such a restriction does not by itself infringe on the consistency of a logical system. It only infringes on the metalogical requirement that a logical system must be valid regardless of any contingent state of affairs. This condition may be attractive from the point of view of metaphysical necessity, but it impedes any investigation into the consequences for a given logical system when implemented as part of a cognitive processing mechanism. In such an implementation it must be allowed to exclude a particular class of cognitively constructible situations from U for practical or functional reasons. The argument here is that the SQUARE is merely the result of the systematic exclusion of those constructible situations where $[[R]] = \emptyset$, without any consequence for the logical soundness of the resulting system. In fact, as is shown below, banning the class of situations where $[[R]] = \emptyset$ into SMPL at a mathematically well-defined joint. This measure is, moreover, well-motivated from a cognitive point of view since, as has been shown, treating \emptyset as a set is highly problematic in any theory of human cognition. But the SQUARE itself, as a situationally curtailed version of SMPL, remains a sound and fully reliable system of predicate logic for those situations for which it is intended, without any change in the definitions of the quantifiers as given in (3a,b) above.

That the U -restriction at hand cuts into SMPL at a mathematically well-defined joint is shown as follows. In SMPL, ALL and SOME are logically independent, in that their conditions can be satisfied independently of each other. By contrast, the SQUARE excludes cases

veritate under an intensional operator of propositions denoting either the set of all possible worlds or the null set (Dowty *et al.* 1981, 175). Although no solution to this crucial problem has been, or is likely to be, found, logicians and formal semanticists proceed as if nothing were amiss. The *horror mentis* is so deeply entrenched that one simply turns away when it is proposed or suggested that intensionality can only be dealt with adequately in the context of a theory of cognitive models. Yet intensionality is an inherently cognitive phenomenon, to do with cognitive modelling (see Seuren 2013, Ch. 9, for extensive discussion).

⁷ Jaspers & Seuren 2016 shows that the extraordinary antagonism of the 'new' logicians, led by Bertrand Russell, with regard to the SQUARE was in large part due to Russell's extreme animosity with regard to the Catholic Church, which for a long time refused to accept SMPL and kept championing the SQUARE as the only correct predicate-logical system until the mid-twentieth century, considering it an integral part of Catholic doctrine. This infuriated Russell.

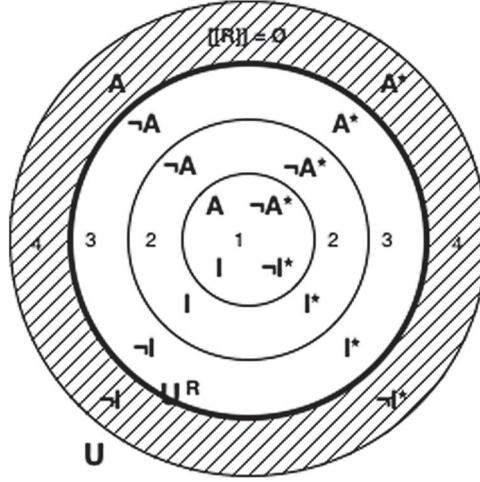


Figure 2. Valuation Space (VS) model of the SQUARE ($= \mathbf{U}^R$) and of SMPL ($= \mathbf{U}$).

where A is true but I false. This is illustrated by Figure 2, which reserves ‘spaces’ for situations where each of the four possible truth-conditional combinations in (5) are satisfied, and where space 4 is unique in having the condition that $[[R]] = \emptyset$, as opposed to the spaces 1–3, which require that $[[R]] \neq \emptyset$:

- (5) a. For space 1: $[[R]] \subseteq [[M]]$ and $[[R]] \cap [[M]] \neq \emptyset$; therefore: $[[R]] \neq \emptyset$
 b. For space 2: $[[R]] \not\subseteq [[M]]$ and $[[R]] \cap [[M]] \neq \emptyset$; therefore: $[[R]] \neq \emptyset$
 c. For space 3: $[[R]] \not\subseteq [[M]]$ and $[[R]] \cap [[M]] = \emptyset$; therefore: $[[R]] \neq \emptyset$
 d. For space 4: $[[R]] \subseteq [[M]]$ and $[[R]] \cap [[M]] = \emptyset$; therefore: $[[R]] = \emptyset$

The Valuation Space (VS) model in Figure 2 defines both the SQUARE and SMPL, and it shows that the SQUARE covers a proper subpart of SMPL, the only difference between the two systems being that situations where $[[R]] = \emptyset$ are excluded from the SQUARE but included in SMPL. While SMPL functions in an unrestricted \mathbf{U} , the SQUARE functions in the restricted universe \mathbf{U}^R of cognitively constructible situations where $[[R]] \neq \emptyset$.⁸

For any of the eight forms specified in (1) to be assigned to the spaces 1, 2 or 3 in Figure 2, it is necessary that $[[R]] \neq \emptyset$, since if either $[[R]] \cap [[M]] \neq \emptyset$ or $[[R]] \not\subseteq [[M]]$, it is impossible for $[[R]]$ to be null. By contrast, for any of the forms to be assigned to space 4 it is not only necessary but also sufficient that $[[R]] = \emptyset$, since if $[[R]] = \emptyset$, necessarily $[[R]] \subseteq [[M]]$ and if $[[R]] \not\subseteq [[M]]$, necessarily $[[R]] \neq \emptyset$. Hence, space 4 contains all and only those situations where the predicate R has a null extension. This crucial space 4 is excluded from the SQUARE but included in SMPL.

Now call the set of spaces in which a given proposition (or proposition type) P is true the *Valuation Space (VS)* of P , or $/P/$. This gives, for SMPL:

$$\begin{array}{llll} /A/ = \{1,4\} & /¬A/ = \{2,3\} & /A^*/ = \{3,4\} & /¬A^*/ = \{1,2\} \\ /I/ = \{1,2\} & /¬I/ = \{3,4\} & /I^*/ = \{2,3\} & /¬I^*/ = \{1,4\} \end{array}$$

For the SQUARE we get the following, which is the same as above but without Space 4:

$$\begin{array}{llll} /A/ = \{1\} & /¬A/ = \{2,3\} & /A^*/ = \{3\} & /¬A^*/ = \{1,2\} \\ /I/ = \{1,2\} & /¬I/ = \{3\} & /I^*/ = \{2,3\} & /¬I^*/ = \{1\} \end{array}$$

⁸ This method of visualising or modelling logical systems is called *Valuation Space (VS) Modelling* in Seuren 2010 and elsewhere. The notion and the term *Valuation Space* were introduced in Van Fraassen 1971, where, however, the philosophical and logical potential of that notion was left unexploited. VS-modelling, though a uniquely convenient way of defining and comparing logical systems, is, regrettably, not used in the theory or practice of standard logic.

In the SQUARE, $A \equiv \neg I^*$ since their VSs are identical, and similarly for I and $\neg A^*$. Then, $A/\neg I^*$ and $\neg A/I^*$ are contradictories because $A/\cap/I^*/ = \emptyset$ and $A/\cup/I^*/ = \mathbf{U}^R$. $A/\neg I^* \vdash I$, since $A/\subset I/$ and $I/\neg I^*/ \subset I/$; moreover, $A^*/\neg I \vdash I^*$, since $A^*/\subset I^*/$ and $I/\neg I \subset I^*/$. $A \diamond \neg I$ because $A/\cap/\neg I/ = \emptyset$ and $A/\cup/\neg I/ \neq \mathbf{U}^R$. Then, $I \diamond I^*$ because $I/\cup/I^*/ = \mathbf{U}^R$ and $I/$ and $I^*/$ are not each other's complement in \mathbf{U}^R . SMPL is a great deal poorer in logical relations than the SQUARE. Here, $A \equiv \neg I^*$ as their VSs are identical: {1,4}, and likewise for $I \equiv \neg A^*$, both I and $\neg A^*$ having the VS {1,2}. And, of course, the negation \neg still selects the complement of any VS in \mathbf{U} , but no other logical relations are detectable.

There are various reasons for maintaining that the SQUARE is a closer approximation to CPL than SMPL. One is, as we have seen, that it is natural to assume that \emptyset does not count as a set in cognitive modelling, since all predicates actually denoting the null set are a product of human imagination, created by humans thinking up possible or impossible alternative world models. A second reason is that it is easily argued that human thinking does not take place in the abstract terms of a universe of all constructible situations but stays within the bounds of contextually and situationally restricted universes (see note 15 below). For these and other reasons, it makes sense to posit that \mathbf{U} -restriction is a better way to reconstruct CPL than the strategy of redefining the meaning of the ALL quantifier.

3. Attempted 'Remedies' for the SQUARE: Abelard and LOCA

Attempts have been made to maintain as much as possible of the attractive and logically powerful SQUARE without giving up the situations where $[[R]] = \emptyset$. Two such attempts are discussed here, both in the (misguided) tradition of redefining ALL. The first was made by the twelfth-century French philosopher Peter Abelard. Since Abelard knew no Greek (knowledge of Greek was highly exceptional in twelfth-century Western Europe) and since most of Aristotle's works were not yet available in Latin, Abelard most probably had no access to the actual text of *PrAn*, though he had been brought up with the SQUARE as the standard system of predicate logic. Having only Boethius' Latin translation of, and commentary on, *Int* at his disposal when he sought to discover the roots of Aristotle's logic (Marenbon 2016), he found that Boethius' commentary, based as it was on the latter's knowledge of *PrAn*, did not match Aristotle's text in *Int*: Boethius had, in fact, misread *Int*, projecting the logic of *PrAn* on it. Since Abelard faithfully followed the text of *Int* while reconstructing the logic set out in it, he found that the result was not identical to the SQUARE. We thus speak of *Aristotelian-Abelardian Predicate Logic*, or AAPL, when referring to the logic set out in *Int*. AAPL is discussed in detail in Section 3.1.

The second attempt at saving the SQUARE as a valid system for all cognitively constructible situations was made, or rather, revived, by a group of twentieth-century philosophers, notably Moody 1953, Thompson 1953 and Parsons 2008, 2014, 2017. It resulted in LOCA (the name 'Leaking O-Corner Analysis' is due to the jarringly counterintuitive interpretation of the O-corner – in particular the I^* -form – resulting from it). Although its logical relations match those of the SQUARE, LOCA, intending to save the SQUARE from logical damnation, still claims that it is valid for all situations, including those where $[[R]] = \emptyset$. Section 3.2 below shows that this attempt has resulted in failure, as one might expect. In the following two subsections, AAPL and LOCA are compared with each other from the point of view of the three adequacy criteria mentioned in Section 1.

3.1. Aristotelian-Abelardian Predicate Logic (AAPL)

Abelard's answer to the UEI problem is found in his *Dialectica*, written ca 1117 and edited and published in *De Rijk* 1956. It consists in a *de facto* redefinition of ALL as in (4) above, repeated here as (6a), so that the A-form carries existential import and the subaltern

entailments are preserved. It is shown below that this results in the dual equivalences being weakened to the one-way entailments $A \vdash \neg I^*$ and $I \vdash \neg A^*$ (Seuren 2010, 136–38). The redefinition of ALL as in (6a) results in the reformulation, in both the Russell and the GenQ notation, of the A- and A*-forms as in (6b,c), respectively (ENT is a predicate; $[[ENT]] = ENT$; read ‘ $\exists x(Rx, ENTx)$ ’ as ‘something is R’, for extensional R):

- (6) a. $[[ALL]] = \{ \langle X, Y \rangle \mid X \subseteq Y \text{ AND } X \neq \emptyset \}$
 b. A: $\forall x(Rx \rightarrow Mx) \wedge \exists x(Rx)$ or: $\forall x(Mx, Rx) \wedge \exists x(Rx, ENTx)$
 c. A*: $\forall x(Rx \rightarrow \neg Mx) \wedge \exists x(Rx)$ or: $\forall x(\neg Mx, Rx) \wedge \exists x(Rx, ENTx)$

No other adjustments are made. In particular, SOME remains defined as before in (3b):

- (3) b. $[[SOME]] = \{ \langle X, Y \rangle \mid X \cap Y \neq \emptyset \}$

The equivalences $A \equiv \neg I^*$ and $I \equiv \neg A^*$ are not found in Aristotle’s *Int*, only the one-way entailments $A^* \vdash \neg I$ and its equivalent $I \vdash \neg A^*$ (Kneale & Kneale 1962, 57; Seuren 2010, 150). What one finds is (*Int* 20a20–23): ‘Then we have the following entailments: *All men are not just* entails *No man is just* and *Some man is just* entails the former’s contradictory *Not all men are not just*’ and (*Int* 18a9): ‘We have said that one affirmation has one contradictory and have specified which these are’. This he did at *Int* 17b17: ‘I call an affirmation and a negation contradictory opposites when what one signifies universally the other signifies not universally, e.g. *every man is white* – *not every man is white*, *no man is white* – *some man is white*’ (also quoted in Parsons 2017). But then Parsons makes Aristotle say that A and I* are contradictories, which is precisely what Aristotle avoids saying: all he says is that A and $\neg A$, and $\neg I$ and I, are contradictories. Aristotle’s systematic restraint in this regard justifies the inference that, being an angel rather than a fool, he feared to tread on ground that would commit him to the condition that the R-predicate must be instantiated.

AAPL may thus be taken to be a faithful reconstruction of Aristotle’s original concept of predicate logic as set out in *Int*, which, one understands, was Abelard’s intention. Abelard himself wrote (*De Rijk* 1956, 176; quoted in Seuren 2010, 175–176):

Similarly for categorical propositions, where the only real truth-value-inverting [‘dividens’] contradiction of any arbitrary positive proposition appears to be the one that has the negation proposed to it so that all its entailments are lost [‘totam eius sententiam destruit’]. For example, the contradictory of *Every human is human* is *Not every human is human*, and not *Some human is not human*, since there are situations where the first and the third are simultaneously false. For when not a single human exists, both of these two propositions are false: *Every human is human* and *Some human is not human*. [translation mine; PAMS]

This says in effect: $\neg A$ is, but I^* is not, the contradictory of A, since A and I^* may be false at the same time, namely when $[[R]] = \emptyset$. Abelard thus splits up the old O-corner into two corners, one for $\neg A$ and one for I^* , and, consequently, the old E-corner into one for $\neg I$ and one for A^* . Figure 3(c) shows that AAPL is defined by the following VSs:

$$\begin{array}{llll} /A/ = \{1\} & / \neg A/ = \{2,3,4\} & /A^*/ = \{3\} & / \neg A^*/ = \{1,2,4\} \\ /I/ = \{1,2\} & / \neg I/ = \{3,4\} & /I^*/ = \{2,3\} & / \neg I^*/ = \{1,4\} \end{array}$$

Two variants of the SQUARE are thus required, a trivial one with vertices for A, I, $\neg I$ and $\neg A$ (Figure 3(a)), and a nontrivial one with vertices for A, I, A^* and I^* (Figure 3(b)) (the

VSs read from Figure 3(c) are added to the forms for convenience). AAPL and SMPL now differ only in that, in space 4, A and A* count as true in SMPL but as false in AAPL.

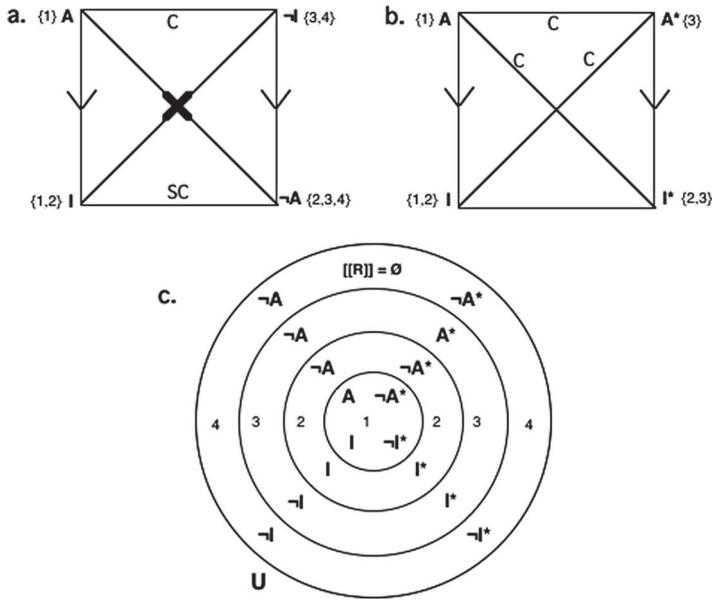


Figure 3. The two variants of the SQUARE for AAPL plus the VS model for AAPL.

Figure 3(a) trivially shows the same logical relations as the SQUARE, as it follows directly from the way negation and entailment are standardly defined. Figure 3(b) brings the internal negation into play and shows where AAPL deviates from the SQUARE. Now $/\neg I/$ and $/\neg A/$ differ from $/A^*/$ and $/I^*/$, respectively, in that Space 4 no longer figures in the VSs of the latter two, so that, in AAPL, $A^* \vdash \neg I$ and $I^* \vdash \neg A$ but not vice versa. I and I* are now no longer subcontraries, since they are now logically independent in that they can be both true and both false at the same time, or one can be true while the other is false (both are false when $[[R]] = \emptyset$). Yet, although I and I* are no longer subcontraries in AAPL, I and $\neg A$, I* and $\neg A^*$, $\neg A$ and $\neg I^*$, $\neg I$ and $\neg A^*$ still are, as can be read from the VS-model in Figure 3(c). But, apart from the pair I and $\neg A$, these subcontrarities cannot be expressed in Figure 3(a,b), as neither contains vertices for the forms $\neg A^*$ or $\neg I^*$. AAPL thus has to sacrifice a good part of the SQUARE, though less than SMPL, which only has the dual equivalences left. AAPL is both logically sound and universally applicable, but it is far from ideal from an empirical (intuitive) point of view, as it runs counter to the strong natural intuition that A and $\neg I^*$, as well as I and $\neg A^*$, form pairs of equivalents. On the other hand, it saves the natural intuition that ALL has existential import.

To prove the logical soundness of AAPL we must show that it is supported by standard, unrestricted set theory, as both AAPL and LOCA claim unrestricted applicability. The set-theoretic definitions of ALL and SOME as given in (4) and (3b) above are repeated here as (7a,b), respectively.

$$(7) \quad \begin{array}{l} \text{a.} \quad [[\text{ALL}]] = \{ \langle X, Y \rangle \mid X \subseteq Y \text{ AND } X \neq \emptyset \} \\ \text{b.} \quad [[\text{SOME}]] = \{ \langle X, Y \rangle \mid X \cap Y \neq \emptyset \} \end{array}$$

On the basis of these definitions we prove the following two theorems (in terms of standard set theory, since AAPL includes cases where $[[R]] = \emptyset$):

- (8) a. $A \vdash \neg I^*$ but not vice versa.
 b. $I \vdash \neg A^*$ but not vice versa.

Proof of (8a):

The truth conditions of A and $\neg I^*$ as specified for AAPL are as in (9a,b), respectively:

- (9) a. $A: \llbracket R \rrbracket \subseteq \llbracket M \rrbracket$ **AND** $\llbracket R \rrbracket \neq \emptyset$
 b. $\neg I^*: \llbracket R \rrbracket \cap C\llbracket M \rrbracket = \emptyset$

The condition $\llbracket R \rrbracket \subseteq \llbracket M \rrbracket$ in (9a) is equivalent to the condition $\llbracket R \rrbracket \cap C\llbracket M \rrbracket = \emptyset$ in (9b) but (9a) adds the extra requirement that $\llbracket R \rrbracket \neq \emptyset$. Therefore, if the more restricted (9a) holds, the less restricted (9b) necessarily also holds, but not vice versa.

Proof of (8b):

The truth conditions of I and $\neg A^*$ as specified for AAPL are as in (10a,b), respectively:

- (10) a. $I: \llbracket R \rrbracket \cap \llbracket M \rrbracket \neq \emptyset$
 b. $\neg A^*: \llbracket R \rrbracket \not\subseteq C\llbracket M \rrbracket$ **OR** $\llbracket R \rrbracket = \emptyset$

The condition $\llbracket R \rrbracket \cap \llbracket M \rrbracket \neq \emptyset$ in (10a) is equivalent to the condition $\llbracket R \rrbracket \not\subseteq C\llbracket M \rrbracket$ in (10b), but $\llbracket R \rrbracket = \emptyset$ in (10b) adds the possibility that $\llbracket R \rrbracket = \emptyset$. Therefore, if the more restricted (10a) holds, the less restricted (10b) necessarily also holds, but not vice versa.

The dual equivalences are restored when the condition ‘**AND** $X \neq \emptyset$ ’ in (7a) is removed.

3.2. LOCA

A different attempt at saving the SQUARE is the LOCA system, proposed during the 1950s, though the idea was discussed by some late medieval philosophers (see Section 4). *Parsons 2017* describes LOCA as follows (see also *Horn 1989, 27*) (the false claim that this is the way the SQUARE has traditionally been interpreted is addressed in Section 4):

In fact, the traditional doctrine of [SQUARE] is completely coherent in the presence of empty terms. This is because on the traditional [Parsons means ‘LOCA’; PAMS] interpretation the O form lacks existential import. The O form is (vacuously) true if its subject term is empty, not false, and thus the logical interrelations of [SQUARE] are unobjectionable.

It will be shown in a moment that this is true when the O-form is read as $\neg A$ (in Parsons’ interpretation), but false when it is read as I^* , under uniform semantic definitions of ALL and SOME. The equivalence $\neg A \equiv I^*$ is thus lost. It is worth noting that the proof given below is predicated on the *semantic* definitions of the quantifiers, as LOCA introduces a *semantic ambiguity* of the quantifiers ALL and SOME.

The LOCALists found that, when the A-form is written as $\forall x(Rx \rightarrow Mx) \wedge \exists x(Rx)$, with the truth condition $\llbracket R \rrbracket \subseteq \llbracket M \rrbracket$ **AND** $\llbracket R \rrbracket \neq \emptyset$, and the dual equivalences are protected by imposing the stipulation that both I^* and A^* are true when $\llbracket R \rrbracket = \emptyset$, all logical relations of the SQUARE remain intact, making it look as if the SQUARE is universally valid, even in cases where $\llbracket R \rrbracket = \emptyset$. This is shown in the VS-model for LOCA in Figure 4(a) (the VS-model for AAPL is repeated as Figure 4(b)).

Figure 4 shows that AAPL and LOCA are identical for the Spaces 1, 2 and 3 (as are the SQUARE and SMPL). The crucial differences lie in Space 4, characterised by the truth condition that $\llbracket R \rrbracket = \emptyset$. Under that condition, the forms considered true in Space 4 in the three systems (the SQUARE does not count as it lacks space 4) are listed in (11):

- (11) Space 4: SMPL A A^* $\neg I$ $\neg I^*$
 AAPL: $\neg A$ $\neg A^*$ $\neg I$ $\neg I^*$
 LOCA: $\neg A$ A^* $\neg I$ I^*

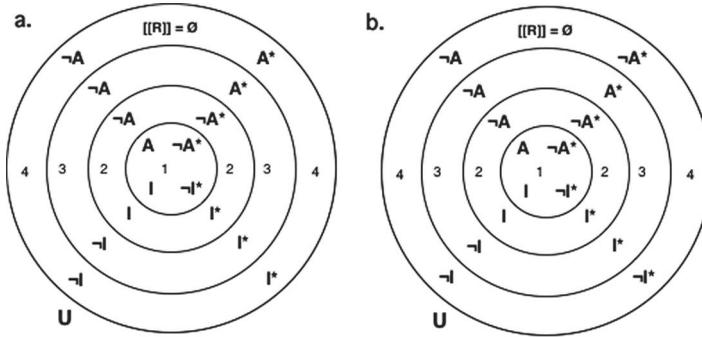


Figure 4. VS-model for (a) LOCA and (b) AAPL.

This makes it look as if the following logical relations hold in LOCA:

$$\begin{array}{ll}
 A \equiv \neg I^*: /A/ = /¬I^*/ = \{1\} & A^* \equiv \neg I: /A^*/ = /¬I/ = \{3,4\} \\
 I \equiv \neg A^*: /I/ = /¬A^*/ = \{1,2\} & I^* \equiv \neg A: /I^*/ = /¬A/ = \{2,3,4\} \\
 A \vdash I: /A/ \subset /I/ & A \diamond A^*: /A/ \cap /A^*/ = \emptyset \text{ and } /A/ \cup /A^*/ \neq U \\
 A^* \vdash I^*: /A^*/ \subset /I^*/ & I \diamond I^*: /¬I/ \cap /¬I^*/ = \emptyset, /I/ \cap /I^*/ \neq \emptyset \text{ and } /I/ \cup /I^*/ = U
 \end{array}$$

Since these are precisely the relations that define the SQUARE, LOCA appears to be logically equivalent to the SQUARE, yet, curiously, also valid for situations where $[[R]] = \emptyset$.

What strikes one in (11) is that, in both SMPL and AAPL, A and A*, and I and I*, are in step in space 4: when A or I is true, or false, so are their starred counterparts A* and I*. Not so in LOCA, which has A and I false in space 4 but A* and I* true (on pain of forfeiting the dual equivalences). This raises the alarm because it calls into question the semantic definitions of ALL and SOME. How can A and I produce falsity but A* and I* truth when $[[R]] = \emptyset$, that is, when the subject term predicate is uninstantiated? The only difference between the unstarred and the starred forms is that $[[M]]$ and $C[[M]]$ are different values for Y in (7) above (one value is the complement of the other in ENT), which should make no difference for the final truth value, as both fall in the range of Y.

And indeed, LOCA is plagued by a bug. It assigns existential import to ALL by defining the A-form as in (6a), but leaves the I-form as it was, just as AAPL does, which places LOCA and AAPL in the same starting block.⁹ But then LOCA slips up: it never

⁹ Parsons pays virtually no attention to Abelard. In *Parsons 2014*, 13, note 13, he writes: ‘Abelard (*D*, p. 170) thought that this [i.e. if $[[R]] = \emptyset$, I* is true; PAMS] is wrong; he held that the particular negative should be read ‘Not every A is B’ instead of ‘Some A isn’t B’. He blamed the latter “misreading” on Boethius, who wrote the latter form instead of the former, which Aristotle had used. But Aristotle (*PrAn* 27a36) uses both forms interchangeably.’ Here the following comments are in order. First, this misrepresents Abelard’s analysis (just as *Parsons 2017* misrepresents the logic of *Int*, as shown above): Abelard did not hold that I* should be read as $\neg A$; he held that the two have different truth conditions (see the passage quoted in Section 3.1, also quoted by Parsons himself, as shown a few lines down in the present footnote). Second, the reference to *PrAn* 27a36 is puzzling since Abelard’s reference point was *Int*, not *PrAn*. Third, it is not made clear what is meant by ‘wrong’ in the first sentence, as nothing is said about Abelard’s or Parsons’ criteria for correctness. Fourth, apart from Parsons’ footnote quoted above, Abelard is not mentioned at all in the entire 2014 book, which purports to ‘articulate’ medieval logic. *Parsons 2017* is a little more generous. There, in note 8, the passage in Abelard’s *Dialectica*, only referred to in the above quote, is given in full. This happens to be the same passage quoted in *Seuren 2010*, 175–76 and requoted above in Section 3.1 (though Parsons uses a different, unattributed translation), but Seuren’s extensive discussion of that passage is not mentioned. All that is said is that Abelard’s reconstruction of Aristotle’s predicate logic as set out in *Int* has not been ‘widely influential’. Parsons’ treatment of Abelard is curious, because Abelard’s system calls for a close comparison with Ockham’s, which receives abundant attention throughout Parsons’ 2014 book, and also because Abelard is known as an outstanding scholar (*King 2018*

specifies the A^* -form, tacitly implying, on pain of losing the dual equivalences, that the A^* -form should be transcribed as $\forall x(Rx \rightarrow \neg Mx)$ but this makes ALL ambiguous between a reading with and one without existential import, and SOME is dragged along with ALL.

Let us have a closer look, again in terms of standard set theory, since LOCA includes cases where $[[R]] = \emptyset$. What has gone wrong in LOCA can be read from (12), where the first column shows Parsons' definition of LOCA, the second column what should be added to LOCA to keep the logic sound, and the third column defines the SQUARE (the condition ' $\text{AND } [[R]] \neq \emptyset$ ' holds for the SQUARE as a whole, due to **U**-restriction).

	LOCA	SHOULD BE ADDED	SQUARE
(12) a.	A: $[[R]] \subseteq [[M]] \text{ AND } [[R]] \neq \emptyset$		$\text{AND } [[R]] \neq \emptyset$
b.	I: $[[R]] \cap [[M]] \neq \emptyset$		$\text{AND } [[R]] \neq \emptyset$
c.	$\neg A$: $[[R]] \not\subseteq [[M]] \text{ OR } [[R]] = \emptyset$		$\text{AND } [[R]] \neq \emptyset$
d.	$\neg I$: $[[R]] \cap [[M]] = \emptyset$		$\text{AND } [[R]] \neq \emptyset$
e.	A^* : $[[R]] \subseteq C[[M]]$	$\text{AND } [[R]] \neq \emptyset$	$\text{AND } [[R]] \neq \emptyset$
f.	I^* : $[[R]] \cap C[[M]] \neq \emptyset \text{ OR } [[R]] = \emptyset$		$\text{AND } [[R]] \neq \emptyset$
g.	$\neg A^*$: $[[R]] \not\subseteq C[[M]]$	$\text{OR } [[R]] = \emptyset$	$\text{AND } [[R]] \neq \emptyset$
h.	$\neg I^*$: $[[R]] \cap C[[M]] = \emptyset \text{ AND } [[R]] \neq \emptyset$		$\text{AND } [[R]] \neq \emptyset$

The LOCA column is made inconsistent by (12e): A^* should have the condition ' $\text{AND } [[R]] \neq \emptyset$ ' added for the definition of the quantifier \forall to remain uniform (the Y-variable in (3a) and (4) above ranges over both $[[M]]$ and $C[[M]]$), but that would kill the equivalence ' $A^* \equiv \neg I$ ' and thus the SQUARE, since, if ' $\text{AND } [[R]] \neq \emptyset$ ' is added to the truth condition of A^* , then, with $[[R]] = \emptyset$, A^* is false but $\neg I$ is true. But with the condition ' $\text{AND } [[R]] \neq \emptyset$ ' added, A^* still entails $\neg I$, as in AAPL, because in all cases where A^* is true, and thus $[[R]] \neq \emptyset$, $\neg I$ is necessarily also true. Now, given the definition of A^* in column 1, $\neg A^*$ must be defined as in (12g), without any further addition. Just as the condition ' $\text{AND } [[R]] \neq \emptyset$ ' had to be added to A^* to keep the definition of \forall uniform, the condition ' $\text{OR } [[R]] = \emptyset$ ' must be added to $\neg A^*$, for the same reason. But that addition would destroy the equivalence ' $\neg A^* \equiv I$ ', since, with the addition of ' $\text{OR } [[R]] = \emptyset$ ' and with $[[R]] = \emptyset$, $\neg A^*$ is true but I false. Similarly for I^* : adding ' $\text{OR } [[R]] = \emptyset$ ' to I^* destroys the uniform semantics of \exists , but leaving it out destroys LOCA. Clearly, A^* in (12e) turns out to be LOCA's Achilles heel.

If Parsons had added, as he should have, the extra condition ' $\text{AND } [[R]] \neq \emptyset$ ' to (12e), his system would have been perfectly sound but identical to AAPL: then the equivalence ' $A^* \equiv \neg I$ ' would be lost and A^* would entail $\neg I$ but not vice versa, etc. As it is, however, LOCA makes both \forall and \exists ambiguous, thus making both A and I true and false at the same time when $[[R]] = \emptyset$, which puts us back in square one, so to speak. Now, \forall and \exists are *de facto* defined as in (13a,b) and (14a,b), respectively, which is no good at all:

- (13) a. $[[\forall]] = \{ \langle X, Y \rangle \mid X \subseteq Y \text{ AND } X \neq \emptyset \}$ if Y is expressed as a noncomplement
 b. $[[\forall]] = \{ \langle X, Y \rangle \mid X \subseteq Y \}$ if Y is expressed as a complement
- (14) a. $[[\exists]] = \{ \langle X, Y \rangle \mid X \cap Y \neq \emptyset \}$ if Y is expressed as a noncomplement
 b. $[[\exists]] = \{ \langle X, Y \rangle \mid X \cap Y \neq \emptyset \text{ OR } X = \emptyset \}$ if Y is expressed as a complement

In sum, if A^* is given the truth condition $[[R]] \subseteq C[[M]] \text{ AND } [[R]] \neq \emptyset$, as it should be in light of (12a), the result is the logically sound and universally valid but empirically deficient system of AAPL. But if the truth condition of A^* is as in (12e), without ' $\text{AND } [[R]] \neq \emptyset$ ', as LOCA must have it on pain of losing the dual equivalences, the result is a

logical shambles. That is, AAPL, the SQUARE and SMPL are sound systems, but LOCA is not.¹⁰

Then, LOCA cannot embody CPL, as is claimed by Parsons: ‘People have always taken the square to embody principles by which one can reason’ (Parsons 2017). Given this author’s repeated claim that ‘the tradition’ has always taken LOCA to be the correct interpretation of the SQUARE, this amounts to saying that people have always taken LOCA to embody principles by which one can reason. Apart from the fact that this runs against the facts of history and would condemn the human race to perennial logical failure, LOCA is also empirically untenable, not only because it implies a fictitious ambiguity for SOME and ALL, but also, and more crassly, because it expects language users to consider sentences like (15a,b) to be true in the real world, which has no trolls in it:

- (15) a. Some trolls are not married.
b. Not all trolls are married.¹¹

Also, as has been observed by many, it would make a sentence like (16a) true but its close synonym (16b) false in the real world, which is unacceptable to the language-using public, no matter what language they speak:¹²

- (16) a. Some trolls are not married.
b. Some trolls are unmarried/single.

As has been known for a long time (see notes 12 and 15), (16a) and (16b) differ logically in that *not* in (16a) selects the complement of [[married]] in **ENT**, while *un-* in (16b) does so within the subset of **ENT** whose members satisfy the preconditions for being married (including the precondition of real existence) – a shrunk complement itself lexicalised in English as *single*. We have here the widely misunderstood distinction between ‘not P’ and ‘not-P’ for any predicate P. This difference cannot be handled in the logical systems at hand

¹⁰ The surface word *some* does have a real ambiguity in that it means either ‘some perhaps all’ or ‘some but not all’. The latter was posited as a separate quantifier SOME-BUT-NOT-ALL and incorporated into the SQUARE, among others, by the largely forgotten French logicians Augustin Sesmat (1885–1957) and Robert Blanché (1898–1975), and the American Paul Jacoby (1915–1993) (see *Jaspers & Seuren 2016*). These did not care at all about UEI (which, from the point of view of CPL, was just as well), but their work led to an extension of the SQUARE to a HEXAGON, with SOME-BUT-NOT-ALL and its contradictory ALL-OR-NONE added, which in turn opened new perspectives for a theory of CPL, as argued in *Seuren 2010, 2014* and *Seuren & Jaspers 2014*. Sesmat, Blanché and Jacoby, all three ardent Catholics, wished to save the SQUARE for religious or ideological reasons, defending it against the fiercely anti-Catholic Russell. As regards the LOCALISTS, Ernest Moody was also an ardent Catholic (his father John Moody, founder of Moody’s Rating Agency, was a convert to Catholicism). Manley Thompson felt that the SQUARE deserved to be saved on the grounds that, in his view, ordinary language is full of vagueness and ambiguity and should, therefore, be ‘cognitively reformed’ (*Thompson 1961*), much in the spirit of Victoria Lady Welby, Bertrand Russell, the early twentieth-century Dutch Significs movement (*Schmitz 1989*), or, during the 1910s, the short-lived Potsdam Forte-Kreis (*Holste 1992*), all of whom or which naïvely believed that the world would be a better place if people learned better logic and changed their thinking and their language accordingly. (The present author’s reason for defending the SQUARE is that it is a good, and historically important, starting point for an empirically adequate theory of CPL.)

¹¹ Intuitively, (15b) is equivalent to *Some trolls are and some are not married*, an equivalence none of the existing logical systems can produce. CPL produces this equivalence by making the question of whether *all R* is *M* conditional, in normal speech, upon a context where it is already given that *some R* is *M*. Given that context, the negation ‘Not all *R* is *M*’ necessarily leaves ‘Some *R* is *M*’ intact (see *Seuren & Jaspers 2014, 621–2; Seuren 2014, 522–4*).

¹² Anticipating the notion of U-restriction (but confusing U with what we call **ENT**), the London logician Augustus De Morgan (1806–1871) wrote (*De Morgan 1847, 37*): ‘If language were as perfect and as copious as we could imagine it to be, we should have, for every name which has a positive signification, another which merely implies all other things: thus, as we have a name for a tree, we should have another to signify every thing that is not a tree. As it is, we have sometimes a name for the positive, and none for the negative, as in *tree*: sometimes for the negative and none for the positive, as in *parallels*: sometimes for both, as in a frequent use of *person* and *thing*. [...] *But the contraries of common language usually embrace not the whole universe, but some one general idea* [italics mine; PAMS]. Thus, of men, *Briton* and *alien* are contraries: every man must be one of the two, no man can be both. *Not-Britain* and *alien* are identical names, and so are *not-alien* and *Briton*.’

as it needs a presuppositional logic. In any case one cannot say that \emptyset and whatever complement of P share a nonnull intersection. We thus have to conclude that LOCA is logically and empirically untenable. An appeal to Gricean pragmatics will be of no avail.

Both Moody and Thompson acknowledged the conflict with natural logical intuitions but both put logic before language and cognition. Moody writes (*Moody 1953*, 51–2):

The particular negative must, as negation of the universal affirmative, be analyzed as a disjunction of the negations of the two parts of the universal affirmative. Consequently the formula of the particular negative *is not properly represented by the word formula* ‘Some F is not a G’, but only by the formula, ‘Not every F is a G’, which is satisfied either because nothing is an F, or because something is an F which is not a G. [italics mine; PAMS]

Thompson likewise puts language and cognition in the back seat (*Thompson 1953*, 253):

Even if we agree with the new defenders of Aristotle that the decision which leads to the modern analysis is repugnant to ordinary speech, we can still argue that this is more desirable than a decision repugnant to logical analysis itself.

The same goes for Parsons (*Parsons 2014*, 12):

Suppose that the term ‘A’ is empty. Then [...] ‘No A is B’ is [...] true. So, by the principle of subalternation, ‘Some A isn’t B’ is also true. But to modern ears, ‘Some A isn’t B’ should be false if ‘A’ is empty. After all, ‘some A’ has scope over the rest of the proposition. What is going on?¹³

While acknowledging the objections coming from ‘modern ears’ – as if more ancient ears would have interpreted I* differently – he does not take these objections seriously. In Parsons (*Parsons 2008*, 5), logic is again ranked above language/cognition. Worse, objecting language users are told to swap one unnaturalness for another, much greater one:

When the modern position [i.e. SMPL] is defended, it is not defended as correct for natural language, because the purpose of logic is not to mirror all of the subtleties of natural language. What is important is that the logical notation be coherent and useful. If it does not perfectly match the usage of ordinary language, that is not on its own important for a system of logic. Indeed, if you are sure that ordinary language universal affirmatives should be false when their subject term is empty, then you may represent that fact by translating them into modern logical notation adding a conjunct. Instead of symbolizing ‘Every A is B’ by ‘ $\forall x(Ax \rightarrow Bx)$ ’, symbolize it as ‘ $\exists xAx \ \& \ \forall x(Ax \rightarrow Bx)$ ’.

Parsons thus appears to adopt the habit of most logicians and philosophers, who avail themselves of language-cum-cognition when it suits them, but when it does not, logic prevails and the clash with language-cum-cognition is dismissed as a trifle or glossed over by a hand-waiving appeal to pragmatics. That logic itself is open to multiple variations does not seem to have occurred to them. One wonders why cognitive scientists and theoretical linguists, barring lone exceptions, have let this pass for so long. Is it because logic has been felt to be too arcane to tackle?

In a general sense, it looks very much as if it is impossible to preserve all the logical relations of the SQUARE in a predicate-logical system valid in an unrestricted U, while at the

¹³ What is going on is clear enough: if, following Parsons, ALL is redefined as in (4) = (6a) above, then indeed $\neg I \vdash \neg A$, but $\neg I \not\vdash I^*$, since, as shown in Figure 3c or Figure 4b above, $I/\neg I/ = \{3,4\}$ and $I/\neg A/ = \{2,3,4\}$, so that $I/\neg I/ \subseteq I/\neg A/$, but $I^*/ = \{2,3\}$, so that $I/\neg I/ \not\subseteq I^*/$, precisely the way Abelard has it.

same time complying with normal criteria of empirical adequacy. The developers of SMPL knew this. They too started out trying to save the SQUARE as a valid logical system for all cognitively constructible situations, and they too found that this is impossible without concessions to natural intuitions. But they faced the consequences and made it clear that their new predicate logic had no implications for the psychological processing of logical arguments but was meant, purely and simply, to be valid for any domain or ‘universe’ consisting of discrete entities, each characterised by a nameable property. This tied their new system up closely with mathematics and metaphysical necessity.

Linguistic adequacy should thus be a welcome but inessential bonus for modern logicians. In fact, however, this has not led to a reluctance to engage with natural language.¹⁴ Perhaps they were carried away by the all too trendy ‘linguistic turn’ which infested philosophy and the humanities alike during much of the twentieth century. Perhaps they fell victim to hubris and romantic but naïve optimism, as did those who felt that ordinary people should learn and practise the new logic, which would lead to greater clarity of expression in daily life and hence to a better world where peace, trust and love would reign supreme and crime and conflict would be banned forever (see note 10 above). Whatever may have been the case, mainstream twentieth-century logic has not brought greater clarity to the vexed question of the relation between language-cum-cognition on the one hand and logic on the other, which is in large part due to the fact that the notion of **U**-restriction has not been taken into account.¹⁵

We thus conclude that no account of natural human logic will do unless interpretative space is reduced to practically manageable proportions. This means that, to save the SQUARE in its extensional integrity, **U** must at least be restricted to the situations where no predicate is uninstantiated, the null set being unfit for practical reasoning, no matter its role in mathematical set theory. Rational talk involving uninstantiated predicates is restricted to contexts where mental world or situation models have been thought up or remembered in which the predicates in question are (were) not uninstantiated. There is no reason why such **U** restriction should be considered logically unsound, since the resulting restricted logical system is congruent with what is found in SMPL for the situations concerned. All the SQUARE does is exclude situations described by means of uninstantiated predicates.

4. LOCA’s Historical Claims

Special attention must be paid to the historical claims made by LOCALISTS, especially in *Parsons 2008, 2014, 2017* – claims that do not square with what the relevant texts tell us. Parsons has claimed on multiple occasions that the LOCA interpretation of the SQUARE

¹⁴ Bertrand Russell set a bad example with his botched analysis of the natural-language definite article *the* as the existential quantifier plus a uniqueness clause (*Russell 1905*, see *Seuren 2010*, 317–21). In fact, the definite article is a salient indicator that CPL depends on **U**-restriction.

¹⁵ The enormous importance for logical theory of the notion of **U**-restriction is forcefully brought out in *presupposition theory*, which accounts for the logical effects of presuppositions by appealing to the process of *incrementation*, progressively restricting the current **U** to the *discourse domain*, that is, a mental model of what has been established so far in current text. Aristotle gave the first impulse to this theory in *Categories* 13b20–25 (translation by E.M. Edghill): ‘*Socrates is ill* is the contrary of *Socrates is well*, but not even of such composite expressions is it true to say that one of the pair must always be true and the other false. For if Socrates exists, one will be true and the other false, but if he does not exist, both will be false; for neither *Socrates is ill* nor *Socrates is well* is true, if Socrates does not exist at all.’ De Morgan followed this up by writing (*De Morgan 1847*, 41): ‘But if we remember that in many, perhaps most, propositions, the range of thought is much less extensive than the whole universe, commonly so called, we begin to find that the whole extent of a subject of discussion is, for the purpose of discussion, what I have called a *universe*, that is to say a range of ideas which is either expressed or understood as containing the whole matter under consideration.’ Gottlob Frege then touched upon the topic in his classic *Frege 1892*, followed by Peter Strawson, who, through *Strawson 1950* and later writings, triggered the avalanche of present-day literature on the presupposition problem (see *Seuren 2010*, Ch. 10 for a survey).

represents ‘the tradition’ in the study of predicate logic. This claim is false. It is false when we look at Aristotle. Nowhere in *Int* or *PrAn* or anywhere else does Aristotle state, imply, suggest or intimate that the negative particular form I^* should be considered true when $[[R]] = \emptyset$ (for discussion, see Seuren 2010, 149–55).¹⁶ The same goes for the whole of Antiquity and the Middle Ages, up until the fifteenth century.¹⁷

Parsons has, over the past two decades, repeatedly tried to convince the academic community that LOCA was common fare during the Middle Ages and that William of Ockham (1285–1347), in particular, endorsed LOCA.¹⁸ As regards Ockham, Parsons (in *Parsons 2008*, 5) quoted selectively from Ockham’s *Summa Logicae* II.3. In a lengthy discussion, Seuren 2010, 163–166 presented the full passage, both in Latin and in English, showing that, in that passage, Ockham unsuccessfully attempts to integrate Aristotle’s ‘indefinite’ proposition type, as in *Trolls are mischievous* or *A troll is mischievous*, into the SQUARE (which Aristotle had also tried and likewise without success), and is thus not concerned at all with LOCA. Parsons’ reply may be distilled from *Parsons 2017*, where, without mentioning Seuren, he admitted that ‘the truth of indefinite propositions with empty subjects does not bear on the forms of propositions that occur in the square’. In *Parsons 2017*, note 6, he gave it another shot, this time quoting from Ockham’s *Summa Logicae*, I.72 (translation unattributed):

In affirmative propositions a term is always asserted to supposit for something. Thus, if it supposits for nothing the proposition is false. However, in negative propositions the assertion is either that the term does not supposit for something or that it supposits for something of which the predicate is truly denied. Thus a negative proposition has two causes of truth.

Again, however, Ockham is misquoted, this time right at the start. What Ockham says is ‘In *such* affirmative propositions’ (‘In propositionibus *talibus* affirmativis’), where ‘such’ refers to propositions that *supposit personally* (‘supponere personaliter’), that is, have a definite subject term. When Ockham’s analysis is applied to quantified propositions, as Parsons does (but Ockham did not do), then *Not all horses whinny* also has ‘two causes of truth’, namely when the predicate *horse* is uninstantiated or when it is instantiated but

¹⁶ Read reckons Aristotle to be a LOCAList, quoting from *AnPr* 26b15–17 and *AnPr* 27b21–23 (Read 2015, 541, 539). But all Aristotle says at either locus (though admittedly in somewhat convoluted wording) is that I^* is consistent with both $\neg I$ and $\neg A$, which is unspectacular because, in the SQUARE, $\neg I$ entails I^* and I^* is equivalent with $\neg A$. Read also misrepresents me, saying, *inter alia*, that I claim in Seuren 2010 that ‘it was the medievals who widened the scope of syllogistic to allow empty terms’ (Read 2015, 536). Yet Seuren 2010 says nothing of the kind. In fact, it says nothing at all about syllogistic: the terms *syllogistic* and *syllogism* occur only once each, and in contexts that are alien to the topic at hand. In addition, Read seems to be generally unclear about the difference between AAPL and the SQUARE.

¹⁷ Horn 1985, 28 makes the following extraordinary claim: ‘For Apuleius, Boethius, Abelard, and Buridan, any proposition has existential import if and only if it < has > affirmative < quality > hence A and I, but not E and O, statements can be true only if something exists which satisfies the subject term. The qualitative view of existential import, which was the received position for the ancients and medievals, has more recently been endorsed by Brentano [...], Peirce [...], and Thompson [...]’. This already impressive pedigree is made even more impressive in Horn 1997, 157, where the LOCA view is further attributed to Carroll 1896, Strawson 1952 and Kneale & Kneale 1962. The attributions to Apuleius, Boethius, and Abelard have already been shown to be spurious. Buridan simply restates, without argument, the equivalence of I^* and $\neg A$ (Seuren 2010, 162), just as Parsons does. Brentano held nothing of the kind: ‘The lack of an attribute b [...] is itself a property’ (Morscher 2018, 3.4), which implies existential import for I^* but not for $\neg A$ (which in itself is empirically problematic; see note 11). As for Peirce, Carroll, Strawson and the Kneales, Seuren 2010, 160–61, 167) presents actual quotes from these authors showing that they say the opposite. The only one remaining is Thompson, who indeed defended LOCA, as we have seen.

¹⁸ Ockham’s nominalism kept his views from becoming part of mainstream thinking, as Aquinean realism remained the doctrine of the all-powerful Church. In fact, Ockham had to flee from a Papal court in Avignon, was excommunicated and died in exile, more or less sharing the lot of Abelard, who was excommunicated twice and died in misery. No chance, therefore, that the writings of either could be part of mainstream medieval philosophy, as per *Parsons 2008*, 5).

not all of the instances satisfy the predicate *whinny*. So far so good (*pace* note 11). But I^* propositions are *positive*, not *negative*, propositions, though with a negated predicate. This makes Parsons' claim that Ockham embraced LOCA baseless. If Ockham had applied his argument to the SQUARE, he would have landed himself in the same trouble as Parsons, as $\neg A$ would no longer be equivalent to I^* , but he did not. Nothing suggests that Ockham even toyed with LOCA. The notion that Ockham subscribed to LOCA is plucked from the air, not from his texts. Why aren't authors better informed and more meticulous?

As for the fifteenth century, *Ashworth 1973* provides evidence that during that period a bevy of philosophers defended LOCA.¹⁹ After that, however, nothing was heard about it any more for a long time. Seuren shows, quoting representative authors (*Seuren 2010*, 166–7), that LOCA was not taken into consideration during the following centuries until there was another outbreak in the mid-twentieth century, which is still lasting, and spreading.

In the end, LOCA amounts to little. It is logically inconsistent, its historical claims do not stand up to scrutiny, it has not contributed anything to logic, and it has not taught us anything about CPL or about the relation between language and logic. But it has been a useful five-finger exercise in traditional predicate logic.

5. Conclusion and Further Prospects

The application of the three criteria of logical soundness, universal applicability and empirical correctness has partly been a rescue operation. We have rescued the SQUARE from allegations of logical unsoundness and from spurious salvation, and we have rescued the history of logic from an attempt at undue partisan appropriation. But positive results have also been achieved. The definition of the quantifiers ALL and SOME as higher-order predicates with set-theoretically defined satisfaction conditions, in conjunction with the technique of VS-modelling, have created greater clarity and enriched the available logical toolkit. This has helped not only to lay bare the faults of LOCA but also to gain a sharper insight into the other three systems considered, AAPL, the SQUARE and SMPL. It has been made clear that only the latter two stand the test of time and scrutiny, and that the only way to move forward into the realm of cognitive logic is to follow the path of **U**-restriction, as the alternative path of redefining the quantifiers leads nowhere.

The way further forward, however, has not been embarked on in the present study, as it seemed wise, for the purpose at hand, not to flood the reader with too many novelties. Yet, further away from the limelight, more daring sorties have been undertaken. Presupposition theory, which rests on **U**-restriction by incrementation through running discourse, has already yielded substantive results (*Seuren 2010*, Ch. 10). **U**-restriction has also been explored as part of cognitive development, the hypothesis being that SOME, for any predicate P , is cognitively prior to ALL, for the same P (*Jaspers 2005*; *Seuren 2014*), which explains why NOT-ALL intuitively implies SOME-BUT-NOT-ALL, an entailment that is otherwise unobtainable (see note 11). *Seuren & Jaspers 2014* has shown that natural-language lexicons contain many configurations of lexical items that share logical relations with the

¹⁹ These may well have taken their cue from Arabic interpretations of Aristotelian logic. The Tunisian historian of logic Saloua Chatti mentions the Uzbek (or Persian) philosopher Al-Fārābī (ca 870–950) as the originator of LOCA (*Chatti 2019*, 350): 'Al-Fārābī is [...] the first author to endorse and defend explicitly the view according to which the affirmative quantified propositions have an < existential > import, while the negative ones do not'—which defines LOCA provided the 'negative ones' are taken to be $\neg I \equiv A^*$ and $\neg A \equiv I^*$. This interesting claim calls for further investigation. Chatti herself remains unclear, only mentioning $\neg A$, not I^* (*ib.*: 34–5), whose equivalence with $\neg A$ is precisely what is at issue. The existing literature is likewise inconclusive. We know, however, that, as far back as the twelfth century, Arabic philosophers regularly had Christian Arabic-speaking West-Europeans among their audience (*Burnett 2004a*). Late medieval philosophers avidly read the Arabic philosophers, as many of the latter's works had meanwhile become available in Latin (*Chatti 2019*, 3–4, quoting *Burnett 2004b*).

SQUARE, and that all these configurations systematically avoid lexicalisations for the O-corner, a phenomenon well-known for the SQUARE but now also found to pervade the lexicon as a whole, and reducible to U-restriction. In conjunction with the various forms of U-restriction, other extensions of CPL are being worked at, such as the extension of the SQUARE with the SOME-BUT-NOT-ALL operator (see note 10), which makes the surface operator *some* linguistically, but not logically, ambiguous in a realistic and well-documented sense and allows for an integration of the SOME-BUT-NOT-ALL operator into the SQUARE, yielding a logical HEXAGON with highly promising implications for a cognitively plausible predicate logic (Seuren 2014; Seuren & Jaspers 2014). All of which indicates that U-restriction is essential for a proper understanding of how predicate logic functions in human cognition.

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