A sequential sampling account of semantic relatedness decisions

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Abstract
We compare three sequential sampling models, the Race model, the leaky competing accumulator model (LCA) and the drift diffusion model (DDM), as novel computational accounts of choices and response times in semantic relatedness decisions. We focus on two empirical benchmarks, the relatedness effect, denoting faster "related" than "unrelated" decisions when judging the relatedness of word pairs, and an inverted-U shaped relationship between response time and the relatedness strength of word pairs. Using simulations, we show that the LCA and DDM, but not the Race model, can reproduce both effects. Furthermore, we show that the LCA and DDM differ in their account of the relatedness effect, producing it under different circumstances and using different mechanisms. This observation offers a critical test, involving a novel, inverted relatedness effect for low-relatedness word pairs. Re-analyzing a publicly available data set, we obtained credible evidence of such an inverted relatedness effect, which is consistent with the LCA, but not the DDM. These results provide strong support for the LCA as an accurate computational account of semantic relatedness decisions and suggest an important role for decision-related processes in (semantic) memory tasks.

Keywords: Semantic memory; Memory Retrieval; Decision-Making; Cognitive Modeling; Sequential Sampling Models

Introduction
As part of human declarative memory, semantic memory contains our general knowledge about the world. Most theoretical perspectives (Jones, Willits, & Dennis, 2015) view semantic memory as some kind of space or structure, such as a network that carries conceptual representations of objects in the world and the relationships between them. In this article, we investigate the cognitive processes involved in retrieving and acting on the strength of relationship between concepts, commonly referred to as semantic relatedness. Specifically, we evaluate three sequential sampling models as novel theoretical accounts for key empirical benchmarks in a frequently used semantic relatedness task.

Two benchmarks of semantic relatedness decisions
The literature has proposed many tasks to behaviorally measure people’s representations of semantic relatedness (Kumar, 2020; Wulff et al., 2019). Here we focus on the semantic relatedness decision task (SRDT), a two-alternative, forced-choice task that requires participants to decide whether two words are semantically related or not. Using this task, previous research has established two key benchmarks that models of the underlying processes should be able to account for.

The first benchmark is the so-called relatedness effect, describing that "related" responses tend to be given faster than "unrelated" responses (Balota & Black, 1997). The relatedness effect has been demonstrated repeatedly using the SRDT (Karwoski & Schachter, 1948; Balota & Paul, 1996; Balota & Black, 1997). The second benchmark is an inverted-U shaped relationship between semantic relatedness and response times (RTs), with strongly and weakly related words resulting in shorter RTs than moderately related words. The inverted-U shaped relationship was demonstrated by two recent studies (Kenett, Levi, Anaki, & Faust, 2017; Kumar, Balota, & Steyvers, 2019) using a similar SRDT, but different approaches to semantic relatedness.

Both benchmarks have been linked to theoretical accounts based on spreading activation in a semantic network (Collins & Loftus, 1975). According to such accounts, RTs are proportional to the distance between two concepts within the network, as distance determines the amount of activation required to spread from one concept to the other. Spreading activation, thus, produces faster responses to strongly-related compared to moderately-related word pairs (see Kenett et al., 2017), explaining half of the inverted U-shaped relationship and the relatedness effect as long as "related" responses are given to more strongly related word pairs than "unrelated" responses (Balota & Black, 1997)

It remains unclear, however, how spreading activation would explain faster responses to weakly than to moderately related pairs, and whether spreading activation alone is sufficient to explain the relatedness effect, or if other decision-relevant processes such as response caution or response competition play a role.

Present study
We propose that sequential sampling models, a popular modeling framework in the field of judgment and decision making (Busemeyer, Gluth, Rieskamp, & Turner, 2019), can potentially account for the benchmarks described above and help to illuminate the cognitive processes underlying semantic relatedness decisions. Sequential sampling models assume an accumulation process that in essence is similar to the notion of spreading activation and embed it within a decision process. We will first describe three such models and how they give rise to "related" and "unrelated" decisions in the SRDT. Then, we present a simulation study evaluating the model
predictions with respect to the inverted-U shaped relationship and the relatedness effect. Finally, we present an analysis of existing data to test a novel prediction derived from our simulation analysis.

**Candidate Sequential Sampling Models**

**Race Model**

The Race model formulates a decision process where independent accumulators $I_1$ and $I_2$ accumulate noisy evidence over time until one of them reaches a decision threshold $Z$ (see Figure 1). In this context, evidence is an abstract unit of preference for either choice option.

To account for behavior in the SRDT, one accumulator $I_{related}$ is set to an externally derived estimate of semantic relatedness of a given word pair. Conceptually, this accumulator can be regarded as analogue to the strength of memory trace activation within a spreading-activation account. The accumulator, $I_{unrelated}$, is estimated from the data and serves as a reference value against which $I_{related}$ is compared. Thus, $I_{unrelated}$ acts as decision criterion comparable to the criterion in signal detection theory (Green & Swets, 1966). Specifically, when $I_{related}$ is larger than $I_{unrelated}$, the agent likely responds "related" and vice versa. It is assumed that the two accumulators are independent of each other.

Figure 1: Sequential sampling models. The left panel illustrates two accumulators in the race model representing "related" (orange) and "unrelated" (blue) decisions in the SRDT. The middle panel illustrates two decisions under the LCA, subject to different levels of lateral inhibition. The right panel illustrates the relative decision variable (gray) in the DDM between two decision boundaries.

**Leaky Competing Accumulator Model**

The Leaky Competing Accumulator model (LCA, Usher and McClelland, 2001) also models the decision as a competition between two accumulators$^1$. In contrast to the Race model, however, the accumulators in the LCA are not independent, but influence each other through lateral inhibition ($w$). This process inhibits the rate of one accumulator as a function of the evidence state of the competing accumulator. Figure 1 illustrates the effect of lateral inhibition, which can both speed up (solid lines) and slow down (dashed lines) the divergence of the accumulators, depending on how similar the accumulation rates are. Finally, the LCA is equipped with a leakage parameter $k$ that results in a decay of evidence over time.

To account for behavior in the SRDT, $I_{related}$ is set equal to the semantic relatedness of two words. The leakage $k$ and the within-trial noise $c$ are fixed. All other parameters, including $I_{unrelated}$, lateral inhibition $w$, and threshold $Z$ are estimated from the data.

**Drift Diffusion Model**

In contrast to the other two models, the DDM assumes a process of relative evidence accumulation. Specifically, the accumulation drifts according to a Wiener process in-between two decision boundaries, one for each response option, until one of them is reached. The direction and rate of accumulation is determined by the drift-rate $v$. Additional model elements include the boundary separation $a$, often interpreted as response caution, a starting point bias $z$, representing an a-priori preference towards either option, and a non-decision time $T_{re}$, reflecting sensory and motor preparatory processes.

To account for behavior in the SRDT, the drift-rate $v$ is equal to the difference between the semantic relatedness and the reference value, i.e., $v = I_{related} - I_{unrelated}$. Within-trial noise $c$ is fixed. All other parameters are estimated from the data.

**Simulation of semantic relatedness decisions**

To investigate how well the three models can account for the inverted-U shaped relationship of RTs and the relatedness effect, we generated data from each of the three models using a range of plausible parameter values and analyzed whether and how consistently they produce the two benchmark effects in question.

**Simulation details**

Parameter values for the two parameters, $I_{related}$ and $I_{unrelated}$, were chosen as follows. $I_{related} = (0.09, 0.20, 0.32, 0.43, 0.60)$ was set to five different levels representing low, low-medium, medium, medium-high, and high relatedness. The exact values were derived by calculating the cosine similarity values of a representative word-pair data set (Bruni, Tran, & Baroni, 2014), using the fastText word2vec model (Grave, Bojanowski, Gupta, Joulin, & Mikolov, 2018). $I_{unrelated} = (0.20, 0.26, 0.32, 0.38, 0.43)$ was set to five values ranging between "low-medium" and "medium-high" relatedness. This range was chosen to allow $I_{related}$ to be either smaller, equal or larger than $I_{unrelated}$.

The threshold parameter $Z = (0.05, 0.10, 0.15, 0.20, 0.25)$ in the Race model and the LCA, and the boundary separation $a = (0.05, 0.10, 0.15, 0.20, 0.25)$ in the DDM were set to five equally spaced levels in a range taken from a recent simulation study on the LCA (Miletić, Turner, Forstmann, & Maanen, 2017). Following the same study, the within-trial noise $c$ was set to .01, and the lateral inhibition parameter $w = (0.5, 1.2, 4)$ was set to four different values. The leakage parameter $k$ was fixed to 1. The non-decision time $T_{re}$ of the DDM was set to a

$^1$Note that we consider the implementation by Bogacz, Brown, Moehlis, Holmes, and Cohen (2006), which, in contrast to the original implementation by Usher and McClelland, permits the negative accumulator states.
plausible value of .2. We chose three levels of starting point bias \( z \), reflecting an a-priori bias towards "unrelated" \((z < .5)\) and "related" \((z > .5)\), as well as an unbiased DDM \((z = .5)\).

For each parameter combination, we simulated a total of 10,000 semantic relatedness decisions and associated RTs. For matters of efficiency, responses were simulated up to an response time of 7 seconds. Trials that would have taken longer were rejected.

**Inverted-U shaped relationship**

Figure 2 shows the relationships between relatedness and RTs produced by the three models under the various parameter combinations. The results for the Race model revealed a strictly monotone relationship, with higher levels of relatedness being associated with lower RTs. Hence, the Race model did not produce the inverted-U shaped relationship.

Under nearly all parameter combinations, the LCA exhibited an inverted-U shaped relationship between relatedness and RTs. Only for the smallest values of the criterion parameter \( I_{\text{unrelated}} \), the threshold parameter \( Z \), or the lateral inhibition parameter \( w \), did this pattern not emerge. Conversely, the inverted-U shaped relationship tended to be more pronounced for large criterion, threshold, and inhibition parameters. The pattern produced by the LCA exhibited a consistent right-skew, with moderately-low relatedness exhibiting the slowest RTs and very high relatedness being associated with faster RTs compared to very low ones. This pattern was particularly pronounced for low criterion \( I_{\text{unrelated}} \) values and attenuated for high criterion values.

The results for the DDM revealed equally consistent inverted-U shaped relationships. In contrast to the LCA, the DDM tended, on average, to produce symmetric relationships centered around \( I_{\text{related}} \approx I_{\text{unrelated}} \) where the drift is minimal (Ratcliff & Rouder, 1998). However, for \( I_{\text{related}} \neq I_{\text{unrelated}} \) the relationship also assumed substantially skewed to either side depending on the implied sign of the drift.

Taken together, the LCA and DDM but not the Race model produce the inverted-U shaped relationship of RTs. This is consistent with previous research showing that independent accumulator models fail to account for inverted-U shaped RTs associated with choice difficulty (Teodeorescu & Usher, 2013). It suggests that the lateral inhibition process in the LCA, which introduces accumulator dependency and distinguishes it from the Race model, and the relative accumulation in the DDM are chiefly responsible for model’s abilities to account for the inverted-U shaped relationship.

**Relatedness effect**

To assess how well the candidate models can account for the relatedness effect, we calculated for each simulation the standardized difference in RTs between "related" and "unrelated" measured as Cohen’s \( d \). Figure 3 shows these differences as a function of the difference between \( I_{\text{related}} \) and \( I_{\text{unrelated}} \), with positive values of Cohen’s \( d \) reflecting a positive relatedness effect.

As illustrated in Figure 3, the Race model simulation results demonstrate a consistently linear relationship across parameter combinations, where Cohen’s \( d \) values grow steadily from a substantial negative relatedness effect for negative differences between \( I_{\text{related}} \) and \( I_{\text{unrelated}} \) towards a substantial positive relatedness effect for positive differences. Threshold values moderated this relationship, with large thresholds resulting in a steeper relationship than lower ones. These results imply that the Race model predicts the relatedness effect for \( I_{\text{related}} > I_{\text{unrelated}} \) and an inverted relatedness effect for \( I_{\text{related}} < I_{\text{unrelated}} \). The LCA simulation results exhibit the same positive linear relationships as the Race model, with a similar moderation of the effect size by the threshold and lateral inhibition parameters. Overall the relationships emerged as somewhat more extreme for the LCA, with high threshold and high lateral inhibition values resulting in stronger effects.

In contrast to the Race and LCA models, the DDM simulation results showed only weak differences between RTs for "related" and "unrelated" responses. If anything, we found a slightly negative relationship between the difference in \( I_{\text{related}} \) and \( I_{\text{unrelated}} \) and effect size. This means that the DDM tends to predict an inverted relatedness effect for \( I_{\text{related}} < I_{\text{unrelated}} \) and a conventional relatedness effect for \( I_{\text{related}} > I_{\text{unrelated}} \). The boundary separation and starting point bias had no systematic effect on the direction of this relationship. However, changes in starting point resulted in additive shifts that implied either a consistent conventional \((z > .5)\) or inverted \((z < .5)\) relatedness effect irrespective of the difference between \( I_{\text{related}} \) and \( I_{\text{unrelated}} \).

In sum, all the three models are in principle able to predict the relatedness effect, but they do so to different degrees, under different circumstances, and using different mechanisms. Under all parameter combinations, the Race and LCA models predict a relatedness effect, but only when \( I_{\text{related}} > I_{\text{unrelated}} \). The DDM on the other hand predict a constant relatedness effect independent of the difference between \( I_{\text{related}} \) and \( I_{\text{unrelated}} \), but only in the presence of a starting point bias. These differences in model predictions suggest a novel, critical test for cognitive mechanisms underlying the SRDT: Does the relatedness effect emerge consistently across different levels of relatedness or could there be an inverted relatedness effect for less strongly related word pairs? In the next section, we will use empirical data to investigate this question.

**The Inverted Relatedness Effect**

To test the prediction of an inverted relatedness effect for weakly-related word pairs, we reanalyzed a publicly available data set by Kumar et al. (2019). In this study, \( N = 40 \) participants from Amazon Mechanical Turk performed 240 trials in a SRDT as described above. To be able to analyze RTs as a function of relatedness, we determined for each word pair

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2This finding is consistent with the property of (unbiased) diffusion processes that the expected RTs for both alternatives are equal (Ratcliff & McKoon, 2008).
cosine similarity scores using the `fastText word2vec` model (Grave et al., 2018). As done in previous analyses (Kumar et al., 2019; Kenett et al., 2017), we excluded trials with extremely short RTs (< 250 ms) since they are unlikely to have arisen from an evidence-accumulation process. In total, we analysed on average 220.7 (SD = 26.8) trials per participant, for which cosine values could be determined.

Figure 4 illustrates the relationship between relatedness (cosine) and response time for both "related" (orange) and "unrelated" (blue) decisions. For the upper range of cosine
Semantic relatedness values (cosine > .4) the data show the conventional relatedness effect, where "related" responses are on average faster than "unrelated" responses. For the remaining lower range, however, we observe the inverted relatedness effect, where "related" responses are actually slower than "unrelated" responses.

To confirm the above effect, we ran a Bayesian linear random effects model with random effects on the participant level. Consistent with the reversal of relatedness effects, we found a credible interaction effect of cosine similarity × response type on log RTs (95% highest posterior density interval, HDI: [−.68, −.42]). Also, there were main effects of cosine similarity (95% HDI: [.09, .28]) and response type (95% HDI: [.14, .25]), indicating slower RTs for more related pairs and for "related" responses, respectively. The former implies that, unlike previous studies, the current data set did not produce the conventional relatedness effect on aggregate.

The reversal of relatedness effects for low relative to high relatedness values is inconsistent with the DDM, which can only account for constant relatedness effects, but is predicted by the Race and LCA models. Another noteworthy observation from Figure 4, lending further support for the LCA, is the noticeable right-skew in distribution of RTs, which is consistent with the shape of LCA’s simulation results presented in Figure 2.

A key implication of our results is that semantic relatedness decisions are likely to be susceptible to typical decision-related phenomena such as sensitivity to differences in base rates or incentives, or to strategic tendencies, such as embodied by speed-accuracy trade-offs. This is fully consistent with resource-rational accounts of memory retrieval (Dougherty, Harbison, & Davelaar, 2014), according to which the continuation of retrieval processes is subject to an assessment of costs and benefits. We believe that formal accounts as presented by sequential sampling models can be instrumental in describing and predicting existing and novel phenomena in this direction, especially due to their ability to jointly account for choice and response times (Kraemer, Fontanesi, Spektor, & Gluth, 2020).

Our findings also tie into an ongoing discussion on disentangling representational (structures) and the cognitive processes that draw on it (Siew, Wulff, Beckage, & Kenett, 2019; Kenett, Beckage, Siew, & Wulff, 2020). It has been argued that computational modeling may be one route to potentially accomplishing this (Kumar, 2020; Wulff et al., 2019). Approaches in this direction typically consider a representational structure, such as a word-vector space or a free-association network, and retrieval processes, such as spreading activation or random walks (Siew et al., 2019). Some of these retrieval processed include elements of decision processes, such as Luce choice rule to transform memory activation to response probabilities (Jones, Gruenenfelder, & Recchia, 2018; Wulff, Hills, & Hertwig, 2013). However, with few exceptions (e.g., Jones et al., 2018), these elements are viewed as auxiliary

### General Discussion

Semantic relatedness is a key concept in cognitive science that underlies models of human memory, reasoning, and creativity. The behavioral output of these models is oftentimes thought to be linked directly to estimates of semantic relatedness as derived, for instance, using vector-space models. This has resulted in a simplified view of semantic relatedness decisions, where the probability of a "related" decision and associated response times are thought to be proportional to the strength of semantic relatedness. Our results suggest that this view needs correction. We demonstrated that the class of sequential sampling models, which is regularly employed for various kinds of decisions, such as old-new decisions in recognition memory (Ratcliff, 1978) or preferential choices between monetary lotteries (Busemeyer & Townsend, 1993), can fully account for existing and novel benchmarks of semantic relatedness decisions. The best performing model (see Table 1), the Leaky Competing Accumulator model, achieved this by not only considering the strength of semantic relatedness, but also a reference level against which semantic relatedness is evaluated, as well as competition between the responses. These results highlight the importance of decision-related processes in behavior pertaining to semantic relatedness and suggest interesting avenues for future scientific inquiry.

### Table 1: Phenomena explained.

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<thead>
<tr>
<th>Phenomenon</th>
<th>Race</th>
<th>LCA</th>
<th>DDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverted U Shape</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Relatedness effect</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>Interaction effect</td>
<td>✓</td>
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<td>✓</td>
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</tbody>
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3The regression analysis additionally controlled for two other known influences on RTs, word frequency (95% HDI: [−.01, .05]) and word length (95% HDI: [.01, .03]), of which the latter effect was credible (longer words led to higher RTs).
assumptions rather than an integral part of the cognitive processes underlying behavior. Based on our results, we would argue that there is much to gain by taking decision-related processes more seriously. One route to doing this could be to integrate features of sequential sampling models, especially those embodied by the LCA, into existing frameworks, with the goal of building joint models of representational structure, retrieval processes, and decision processes.

References


