THE TOLERABLE WINDOWS APPROACH: THEORETICAL AND METHODOLOGICAL FOUNDATIONS

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Abstract. The tolerable windows (TW) approach is presented as a novel scheme for integrated assessment of climate change. The TW approach is based on the specification of a set of guardrails for climate evolution which refer to various climate-related attributes. These constraints, which define what we call *tolerable windows*, can be purely systemic in nature – like critical thresholds for the North Atlantic Deep Water formation – or of a normative type – like minimum standards for per-capita food production worldwide. Starting from this catalogue of knock-out criteria and using appropriate modeling techniques, those policy strategies which are compatible with all the constraints specified are sought to be identified. In addition to the discussion of the basic elements and the general theory of the TW approach, a modeling exercise is carried out, based on simple models and assumptions adopted from the German Advisory Council on Global Change (WBGU). The analysis shows that if the global mean temperature is restricted to 2 °C beyond the preindustrial level, the cumulative emissions of CO_2 are asymptotically limited to about 1550 Gt C. Yet the temporal distribution of these emissions is also determined by the climate and socio-economic constraints: using, for example, a maximal tolerable rate of temperature change of 0.2 °C/dec and a smoothly varying emissions profile, we obtain the maximal cumulative emissions, amounting to 370 Gt C in 2050 and 585 Gt C in 2100.

1. Introduction

Article 2 of the Framework Convention on Climate Change (FCCC) calls for the stabilization of greenhouse gas concentrations in the atmosphere at levels that ‘prevent dangerous anthropogenic interference with the climate system’ (United Nations, 1995). Besides this statically defined goal of climate policy, the FCCC requires that the goal of stable concentrations has to be achieved within a ‘time frame sufficient to allow ecosystems to adapt naturally to climate change, to ensure that food production is not threatened, and to enable economic development to proceed in a sustainable manner’ (United Nations, 1995). Although Article 2 fixes an overall goal for world-wide climate policy, the details of the goal remain unclear: What is the concentration which ensures the prevention of dangerous interference? What is a sufficient time frame? What does natural adaptation mean? How can food production be ensured?

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There exists a large number of studies, models, and investigations on these topics which try to further specify and elaborate on the objectives of the FCCC, and Article 2 in particular. The most comprehensive review is the latest 1995 assessment report of the Intergovernmental Panel on Climate Change (IPCC, 1996). Within this report, the contributions of Working Group II and III give detailed descriptions of the expected ecological and socio-economic impacts of climate change. The scope of the assessment is to relate these expectations to Article 2 and to assess proposed mitigation and adaptation measures to achieve the goals of the FCCC (Watson et al., 1996). Unfortunately, the IPCC report can be summarized as: *much is known but there is even more ignorance*.

Even if we could master all the ‘scientific aspects’ of Art. 2 (How do ecosystems adapt to any type of climate change? What is the food production potential at a given climate? What determines economic development, etc.), essential ‘ethical and normative’ questions remain: What does *natural* adaptation mean? What is *secure* food production? What is *sustainable* economic development? So far there are no definite and generally accepted answers to these questions. Inter alia, this gives rise to the emotionalized discussion on this highly sensitive subject and urgently calls for approaches with a clear-cut interface between scientifically analyzable interdependencies and the norms and values involved. To achieve this is one main objective of the approach presented in this paper.

Article 2 of the FCCC embraces the entire range of issues involved, mentioning potential climate impacts, e.g., ecosystem adaptation and anthropogenic interferences, as well as problems of emissions reduction, in particular the problem of sustainable economic development. This suggests one possible and, we believe, promising investigation strategy: the inclusion of *all* of these issues. A number of such *integrated assessments* (IA) have been conducted (see, e.g., Nakićenović et al., 1996; Weyant et al., 1996) that try to include the impacts *and* the socio-economic driving forces of climate change. A strong emphasis is placed on models as the central tool for IAs, although there are other, equally appropriate methods (e.g., Cohen, 1997; Strzepek et al., 1996). Weyant et al. (1996) distinguish between policy optimization and policy evaluation models. Well-known examples of the first class are the DICE/RICE models by Nordhaus (Nordhaus, 1992; Nordhaus and Yang, 1995) or the MERGE model by Manne et al. (1995). Within these models, the costs (due to mitigation of greenhouse gas (GHG) emissions) and benefits (avoided damages of climate change) are expressed in a homogeneous metric, usually as a ratio of gross world product (GWP) or in constant year US $. By applying optimization methods, the models are used to identify the least cost ‘optimal’ policy. For a recent analysis of the first generation of integrated assessment models and a ‘wish list’ for the second generation, see Schellnhuber and Yohe (1997).

The second class of models is represented by the IMAGE2 models (Alcamo, 1994) or the AIM project (AIM, 1997). In contrast to the policy optimization models, a much higher geographical resolution for a more detailed modeling of the involved processes is used. Rather than integrating the impacts and mitigations into one single measure, the models produce detailed pictures of climate impacts (biosphere, agricultural land use, etc.) based on predescribed scenarios in terms of economic parameters (prices, efficiencies, demographic and life-style projections, etc.). It is then up to the user of the model to decide whether he/she can accept the projected impacts under the assumed economic development.

The tolerable windows approach (TWA or TW approach) presented here is intermediate between these approaches. It was originally proposed by the German Advisory Council on Global Change (WBGU, 1995) in its statement prepared for the 1st Conference of the Parties in Berlin by one of the authors of the present paper. The main idea is to follow an inverse path, starting from a set of hypothetical climate evolutions considered tolerable with respect to their ‘anthropogenic interferences’. The model has to compute backwards to obtain the necessary economic, social, and political conditions which are consistent with this corridor of tolerable climates. There are, however, further normative constraints to be taken into account, some of them explicitly mentioned in the FCCC, others not. Examples concern the question of burden sharing (Annex I vs. Non-Annex I), the variation of measures and instruments according to the capabilities of individual parties, or the cost-effectiveness imperative of Art. 3. The basic question is therefore: What are the policies commensurable with *all* these and possible further constraints? In this paper we outline a modeling approach which eventually should be able to answer this question by supporting the specification of the constraints in terms of physical (i.e., not necessarily monetized) units.

In the next section, we elaborate on the basic principles and background of the approach. Then we present its original realization in more detail than in the actual WBGU statement (Section 3) for whose solution we present the mathematical argument in the Appendix (Section 6). Finally, we discuss the general formalism of the TW approach again using the model and the assumptions used in the WBGU statement (Section 4).

2. The Tolerable Windows Approach

In this section we want to discuss the conceptual basis of the tolerable windows approach. Besides describing the essentials, we especially focus on the position of the approach within familiar concepts of integrated assessments.

2.1. THE INVERSE METHOD

Many of the integrated assessment models dealing with climate change start from a predescribed policy scenario which is used as a principal input to the model. Examples range from scenarios for the emissions themselves, e.g., the IS92 scenarios from the IPCC as input to various impact studies (for a review see Watson et al. (1996); a classic example can be found in Rosenzweig and Parry (1994)), to more comprehensive assumptions on the future of the energy-economy system in general, e.g., prices or efficiencies in IMAGE. These models follow the actual sequence of causes and effects. For example, in a greenhouse warming simulation the emissions are the causes and the climate is the effect. But if we assume the mapping to be invertible, we can specify effects first and then compute backwards to obtain the corresponding causes. In our example this means that we infer an emission profile from a given climate evolution. The inversion can either be unique, i.e., only a single cause is related to the predefined effect, or ambiguous where more than one cause is related to the specified effect. This type of inverse calculation is the first cornerstone of the tolerable windows approach.

The introductory discussion on the objectives of the FCCC has shown that one aspect of the final goal is to avoid a dangerous interference with the climate system. If we use this idea together with our first cornerstone, we can add another stone to the construction of the TW approach: if we identify a climate which is acceptable with respect to Article 2 and its normative specification, then we can compute the corresponding emissions profile. This emissions path therefore represents an admissible policy with respect to human climate interference. The profile then has to be evaluated with respect to its social, economic, and political implications.

If we consider not only a single climate evolution but rather an entire set of acceptable climate evolutions, the *tolerable climate window*, then we obtain instead of an individual emission profile a set of corresponding admissible emission profiles. These profiles represent the option set for a climate policy: any one of these emission functions yields an acceptable climate evolution. Note that this window might be particularly large if the mappings between effects and causes which have been discussed above are multi-valued.

Now the following question arises: which of these ‘impact-tolerable’ emission profiles are tolerable also with respect to their socio-economic and political implications? Thus a number of further criteria have to be specified. One possibility, which is the natural continuation of the approach, is to specify constraints concerning these criteria: negotiable vs. nonacceptable allowances of emissions reductions, basic needs of energy services, social acceptability of carbon tax systems, etc. These constraints can be used as knock-out criteria, i.e., any climate policy which might be tolerable with respect to the implied climate impacts has to be abandoned when it violates one or more of these socio-economic knock-out criteria.

Another extension, more in the tradition of cost-benefit analysis (CBA) and directly oriented along the cost-effectiveness imperative of Article 3, is to formulate a general cost function and to obtain the least cost solution by a conventional optimization scheme. Any of these concepts, however, require the specification of tolerable windows which is closely related to the trade-off problem to be discussed in the following section.

2.2. THE TRADE-OFF PROBLEM AND THE SPECIFICATION OF TOLERABLE WINDOWS

The problem of greenhouse warming is a global issue created by many actors with different contributions to greenhouse gas emissions and diverse vulnerabilities to global warming. A global climate protection strategy can therefore evolve only through negotiations between many actors with possibly conflicting interests. The goal of such negotiations is to establish a jointly acceptable balance between the negative impacts of climate change and the socio-economic costs of a reduction of greenhouse gas emissions. The final compromise will need to consider a wide variety of impacts, including so called non-monetary quality-of-life factors as well as complex ethical issues such as geographic and inter-generational equity. A scientific analysis of the interactions between the climate system and the global social and economic system is clearly essential to provide a rational basis for such trade-off negotiations. However, the interplay between scientific analysis and the negotiation process is intricate. The three different approaches to integrated assessment outlined above (scenario computations, cost-benefit analyses and the inverse approach) represent alternative ways of combining scientific analyses with negotiations against the background of different philosophical assumptions. Maybe the most prominent dispute in this context is the one on optimization, i.e., whether it is possible to formulate a generally agreed criterion of optimality or not (Schellnhuber and Kropp, 1998). We do not want to express a preference for one or the other position. In this section, instead, we highlight some important aspects in the relation of our approach to these two paradigms.

In the scenario approach, a series of climate evolution paths for some predefined set of possible greenhouse gas emission scenarios is computed, and the search for a generally acceptable climate protection strategy based on the subsequent assessment of the computed climate change impacts and mitigation costs for each scenario is pursued in a second independent negotiation process. A disadvantage of this approach is that the assumed initial set of greenhouse gas emission scenarios may well not contain generally acceptable emission paths. Furthermore, the subsequent attempts to identify an agreeable strategy through further scenario computations can be time consuming and ineffective.

Cost-benefit analysis starts from the other extreme, particularly if it is applied with a single-valued ‘global welfare’ function. Such kind of a function can only be assumed to be valid if subtle issues like burden sharing, inter-generational equity and the monetization of human life are resolved. This assigns relative weights to the distributed present and future climate change impacts and mitigation costs, including not only normal economic costs, but also values such as the maintenance of species and the natural environment, or a commitment to future generations in accordance with the principles of ‘sustainable development’. Once these values have been agreed upon, one can determine in a single numerical optimization exercise the greenhouse gas emission path that maximizes the global welfare. The shortcoming of this approach is that the assumption of a prior agreement on the global welfare function is rather unrealistic. However, one can study with this approach the consequences of particular assumptions regarding the structure of the welfare function, for example the impact of intertemporal costing factors (Hasselmann et al., 1997; Nordhaus, 1997). The approach replaces the time consuming trial-and-error iterative determination of the optimal solution from forward-integrated scenarios through an appropriate automatic numerical optimization procedure.

One important refinement of modern cost-benefit analysis to some extent circumventing the integration of subtle issues like the one mentioned above is *multi-criteria* analysis which make use of a vector-valued welfare function, i.e., there are independent components which cannot be further aggregated. This is not a theory-based assumption, but is intended to reflect the more realistic situation that no total agreement on a single-valued welfare, i.e., specification of *all* relative weights, can be expected. However, some decision rule is needed to reduce the policy options into a set of agreeable strategies which then might be subject to a second round of negotiations to pick out a single option. This decision rule is the essential element of a multi-criteria analysis where the attributes are given by the different components of the welfare vector (Munasinghe et al., 1996). The tolerable windows approach could provide a rather general decision rule in this context and thus stands to some extent in the tradition of CBA.

Within the TW approach, it is assumed that thresholds can be specified, above or below which the values of the attributes are considered to be nonsatisfying, i.e., the corresponding climate policy is not acceptable. Possible attributes concern categories of climate change impact, e.g., food security, water availability, direct health effects (heat stress, vector borne diseases, etc.), as well as categories of emissions reduction effects, e.g., mobility and transportation needs, room heating or cooling, availability of process energy for industrial production, etc., or of regulatory measures to realize the mitigation, i.e., carbon taxes, certificates, joint implementations, etc. In general, the thresholds have to be stipulated by *political or societal judgments on the basis of scientific insight* or as scenarios within a more general scientific analysis. Examples for rather ‘natural’ constraints are given by a possible breakdown of the thermohaline ocean circulation (Rahmstorf, 1994, 1995; Stocker and Schmittner, 1997) or a runaway greenhouse effect, i.e., thresholds which are related to discontinuities in the functionality of one or the other subsystem. These discontinuities might equally arise in anthropogenic systems, e.g., by violation of generally accepted rules and attributes like human rights. Therefore these constraints can be considered as knock-out criteria which are by no means allowed to be overridden. Other, already more subtle thresholds might be given by the minimal amount of calories needed per day or the generally used limit of 1000 m^3 per capita and year for freshwater scarcity (Falkenmark and Widstrand, 1992). Most difficult to specify are thresholds which hardly can be related to discontinuities in a subsystems functionality, like a certain increase of the epidemiological potential of vector borne diseases (e.g., schistosomiasis, malaria (Martens et al., 1995)) or the economic costs of the mitigation measures. Most difficult, perhaps, are thresholds on the regulatory measures to realize any emissions reduction or other variables with a rather high compensatory potential.

It is sometimes objected that variabilities and uncertainties prevent a reasonable formulation of tolerable windows in general (Dowlatabadi, 1998) as in these cases nature itself might easily violate the constraints. Therefore an important property of the constraints should be the usage of either variables with a low degree of variability or of probability concepts, e.g., risk levels (Bruckner et al., 1999a). In the latter case the necessary discourse between science and society has to focus on acceptable levels of probability for a failure of important contributions to societal welfare. An example for this kind of constraint can be found in the Dutch law: every citizen has the right to be prevented from a flood occurring once in a hundred years – but hardly from events of biblical dimensions like the Deluge.

If the ‘optimization’ point of view is taken, the set-up of thresholds might be justified by assuming that the evaluation method involved in setting the constraints is assumed to be insufficient to define a single-valued welfare function and therefore to compute a unique optimal path for greenhouse gas emissions and the resulting climate evolution. Rather, they serve only to define a (still infinite) set of permissible paths within a set of tolerable windows. The establishment of an optimal solution would require again a further negotiation stage to determine the final trade-offs through a resolution of the remaining open questions, which could be, for example, the issue of cost-effectiveness. These further negotiations have to address and amalgamate those interests touched by climate change which still remain unsatisfied after keeping the attributes below their thresholds.

Formally, hard constraints can be justified within the framework of a ‘homo economicus’ approach (i.e., decisions are made to maximize a single-valued and well-ordered utility function) only if the evaluated general costs of climate impacts are assumed to increase extremely rapidly beyond the tolerable window boundary, so that the boundary represents in some sense a catastrophic limit. Although catastrophic climate transitions cannot be ruled out and are still discussed, for example, in the context of a runaway greenhouse effect or of a melting of the West Antarctic ice-shield, the boundaries of such transitions cannot yet be established reliably and must therefore be represented as soft ‘increasing statistical risk’ transitions.

Thus, from a formal viewpoint, it appears that the introduction of fixed constraints on the permissible climate evolution paths is inconsistent with a ‘homo economicus’ approach to climate change, and thus to general cost-benefit analysis. Instead, the TW approach might be related to the ‘bounded rationality’ concept (Simon, 1987) where exactly the existence of generally satisfactory thresholds is assumed: decisions are chosen rationally only to ensure satisfaction in this sense. Any further choice of one or another satisfying strategy is kept open to ‘stochasticity’. Consequently, the TW approach cannot be considered as a complete substitute or alternative to the usual cost-benefit analysis; instead it avoids the problem of a quantitative comparison of impacts and mitigation measures and hopes to provide a sufficient stratification of the set of policy options. If necessary, the subsequent negotiation can then focus on the impact of variations of the set of constraints, or even on a further specification of the general welfare function and the definition of an optimal emissions path.

Looking back on some political negotiations on environmental issues, it seems to us, however, that the tolerable windows approach mirrors more closely the normal processes. In practice, agreement on environmental protection issues is seldom achieved through direct application of cost-benefit analyses, but rather through a political agreement on acceptable limits of environmental impacts. Typical examples are the Montreal ozone protocol on the curtailment of CFC production, agreements on SO_2 emission limits, and the targets for CO_2 emissions reductions proposed in Rio within the FCCC. Agreements on emission limits were achieved in these cases on the basis of rather *qualitative* scientific assessments of the environmental impacts of the emissions, without a detailed trade-off analysis of the costs of an emissions reduction compared with the environmental consequences if the proposed limits were exceeded. In effect, the tolerable windows approach accepts the fact that critical constraints on environmental impact factors are generally set without invoking an optimization formalism.

The transition from the TW approach to a traditional cost-benefit analysis can be achieved rather smoothly by replacing the hard-constraint windows by soft-shouldered windows in which the constraints determining the window limits are redefined as scale parameters characterizing a smoothly varying impact function. However, we consider in this paper the hard constraint form only, without expressing any preference for one approach or the other (for a more extensive discussion, see, e.g., Schellnhuber and Yohe, 1997).

3. A Simple Example: The WBGU Scenario

In this section we present a detailed analysis of the ‘WBGU Scenario’ used by the German Advisory Council on Global Change (WBGU) in its special report for the First Conference of the Parties to the FCCC in Berlin (WBGU, 1995). The idea of ‘inversely translating’ tolerable climate windows into admissible sets of GHG emissions was worked out by one of the authors of the present paper on behalf of the WBGU (see also Svirezhev et al., 1998). Though using only simple models and settings for a tolerable window of climate evolutions, the analysis is able to illustrate the major ingredients needed for a general tolerable windows analysis. In order to characterize the possible futures in terms of emissions it was asked: What is the maximal amount of fossil fuel which humankind can dispense within a given time horizon without encountering a dangerous climate change? We first introduce the models and then answer this question in Section 3.3.

3.1. A TENTATIVE TOLERABLE WINDOW FOR CLIMATE EVOLUTION

In this section we outline the Council's discussion which has brought forth a tolerable window formulated in terms of temperature and rate of temperature change. It has to be stressed that *this discussion cannot be a purely scientific one, but rather involves a number of normative inputs as well as ad-hoc assumptions and 'soft knowledge'*. Yet a body such as the WBGU* is exactly the type of forum where these necessary discussions can be successful, as they have to take place on the basis of scientific knowledge and in co-operation with politicians and stakeholders. Thus the following paragraphs provide the reader with a proto-typical illustration of how *a* tolerable climate window for global protection strategies might be constructed. The resulting domain can be seen as a first approximation to *the* tolerable climate window that may ultimately transpire from a more rigorous and comprehensive policy exercise based on state-of-the-art research results. Within such a policy exercise, a close co-operation between scientists and stakeholders is taking place using scientific results and/or models to obtain direct responses of the stakeholders on the systems outcomes of simulated political actions (Brewer, 1986; Tóth, 1986; see also Section 4).

The Council's window of tolerable climate evolutions is based on the following two principles (for a more complete discussion see Annex I in WBGU, 1995):

- 'Preservation of Creation', and
- prevention of excessive costs.

These two principles specify two independent thresholds for which it is assumed that no common quantification in terms of a single-valued utility function exists, i.e., we might perceive 'Creation' and 'excessive costs' as two *independent* components of a general 'welfare vector'.

For further scientific analysis, the principles need to be formulated in terms of mathematical variables which in the ideal case are embedded into a formal and detailed model. The first principle can be interpreted as a call for a limitation of ecological damages. For this to be achieved, a model would be required which relates any given climate change to the corresponding ecological impacts and, probably even more difficult, to identify an appropriate measure for what we might call ecological utility. As currently no generally accepted model nor such a measure exist, the global mean temperature T is used as a rough, but robust, indicator of climate change consequences. The first principle is therefore stipulated by requiring that the global climate must not deviate markedly from the interval of Quaternary

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* The German Advisory Council on Global Change (WBGU) is a body of natural and social scientists. Its major task is to submit annual reports on the state of the global environment to the German government, including recommendations for policy and research. This induces close discussion between the different sciences as well as between science and policy which is a necessary condition for the set-up of tolerable windows. According to its mandate, the Council is expected to also consider ethical aspects of global environmental change (WBGU, 1996).

fluctuations, which have brought forth the current distribution of vegetation and ecosystems. This means that

$$T_{\min} \leq T \leq T_{\max} \tag{1}$$

where

$$\begin{aligned} T_{\min} &= T_{\min}\ (\text{glacial}) - 0.5\,°\text{C} &&= \ \ 9.9\,°\text{C}, \\ T_{\max} &= T_{\max}\ (\text{interglacial}) + 0.5\,°\text{C} &&= 16.6\,°\text{C}. \end{aligned} \tag{2}$$

The figures used in (2) have been derived from Schönwiese (1987); $T_{\min}$ (glacial) corresponds to 10.4 °C (Wurm), $T_{\max}$ (interglacial) to 16.1 °C (Eem). Note that the interval of Quaternary fluctuations was extended by 0.5 °C at either end, i.e., the range of acceptable global mean temperatures is demarcated rather generously with respect to the principle formulated above.

The second principle, although it sounds more intuitive at first, is rather difficult to operationalize. Two more basic questions are involved: (i) what are the costs of a specific climate change and (ii) what is considered as *excessive* costs. Note that the costs to be taken into account in the present context are *damage* costs only. It seems reasonable to assume that the damage costs S depend on the global mean temperature *and* on the rate of its change, i.e., $S = S(T, \dot{T})$. This reflects the idea that the speed of a given climate change is a major determinant of the adaptive capacities of the systems exposed (Rijsberman and Swart, 1990; Tol, 1995; Pearce et al., 1996). Yet it is not clear what the functional relation for the damage function looks like. Some quite general considerations can be applied to obtain a simple form which reflects the basic requirements of nonlinearly increasing damages in temperature and its rate of change, as well as a decreasing adaptability at rising temperatures. Thus for $\dot{T} \geq 0$ we assume

$$S(T, \dot{T}) = \begin{cases} S_{\max} \left(\frac{\dot{T}}{\dot{T}_{\max}}\right)^2 (T - T_{\min})^{-1}, & T_{\min} \leq T \leq T_{\min} + 1 \\ S_{\max} \left(\frac{\dot{T}}{\dot{T}_{\max}}\right)^2, & T_{\min} + 1 \leq T \leq T_{\max} - 1 \\ S_{\max} \left(\frac{\dot{T}}{\dot{T}_{\max}}\right)^2 (T_{\max} - T)^{-1}, & T_{\max} - 1 \leq T \leq T_{\max} \end{cases} \tag{3}$$

where T, $T_{\min}$, and $T_{\max}$ are given in °C. A maximal damage cost $S_{\max}$ has been introduced which in the middle of the tolerable temperature regime corresponds to a maximum rate of temperature change, $\dot{T} = \dot{T}_{\max}$. Since in a first approximation adaptation to global cooling should lead to problems similar in severity to those caused by adaptation to global warming, the damage function is formulated as symmetric in $\dot{T}$. Note that this function differs slightly from the one used in WBGU (1995), yet it is more comprehensive for our purpose here.

Based on the damage function (3), the second principle is refined by requiring $S(T, \dot{T}) \leq S_{\max}$. For the first quadrant in $T - \dot{T}$ space ($\dot{T} \geq 0, T \geq T_1 =$

$T_{\text{preindustrial}}$), being the most interesting in the context of global warming, this condition yields the requirement:

$$T \leq T_{\max}, \tag{4a}$$

$$\dot{T} \leq \dot{T}_{\text{crit}} = \begin{cases} \dot{T}_{\max}, & T_1 \leq T \leq T_{\max} - 1 \\ \dot{T}_{\max}\sqrt{T_{\max} - T}, & T_{\max} - 1 \leq T \leq T_{\max}. \end{cases} \tag{4b}$$

Using $\dot{T}_{\max} = 0.2$ °C/decade the entire tolerable domain $\mathcal{D}$ takes the form as presented in Figure 1a.* The actual choice of $\dot{T}_{\max}$ is rather difficult, although some attempts do exist concerning the limits for natural systems (Rijsberman and Swart, 1990; Enquete–Kommission, 1990). Therefore the following discussion has to be seen as an *expert evaluation* of the economic implications of climatic change. Illustratively, it should be noted that the value of 0.2 °C/decade corresponds approximately to the mean rate of change of a CO_2 doubling scenario over the next century, if we assume a climate sensitivity of 2 °C. As this sensitivity is close to the lower end of the range given by the IPCC (Kattenberg et al., 1996), the mean rate of change can be expected to be higher. Since there are some assessments which calculate the damage costs of a CO_2 doubling to be about 2%, the value chosen for $\dot{T}_{\max}$ approximately corresponds to the specification of 'excessive' in the second principle, i.e., represents costs approaching 2% of the GDP. Yet the mentioned model calculations have neither taken into account extreme events (droughts, floods, tropical storms, etc.) nor possible synergies between the various trends of Global Change (WBGU, 1993) (e.g., interactions between anthropogenic greenhouse effect and soil degradation). Therefore it can be argued that the value of $\dot{T}_{\max}$ rather corresponds to a GDP loss of about 4–5% due to climatic change, which seems to be a reasonable value for the *upper limit* of bearable damage costs. In the early 90s, for example, this ratio of the GDP has been transferred from West to East Germany every year. This huge amount of transferred money has brought Germany to the brink of its financial capability (WBGU, 1996).

3.2. THE COUPLED CLIMATE-CARBON CYCLE MODEL

Our simple model is formulated in terms of global mean temperature T, the carbon concentration C, and the cumulative anthropogenic CO_2 emissions F measured in Gt C. The model describes only the climate response to anthropogenic forcing and is furthermore restricted to CO_2 emissions alone. It is expressed in terms of differential equations approximating a pulse-response model that has been calibrated against three-dimensional carbon cycle and coupled ocean-atmosphere GCM simulations (Maier-Reimer and Hasselmann, 1987; Hasselmann et al., 1997). We

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[*] Note that according to the necessity of low variabilities in the entities used for the window (Section 2.2) the current formulation in terms of temperature and its rate of change is of limited use.

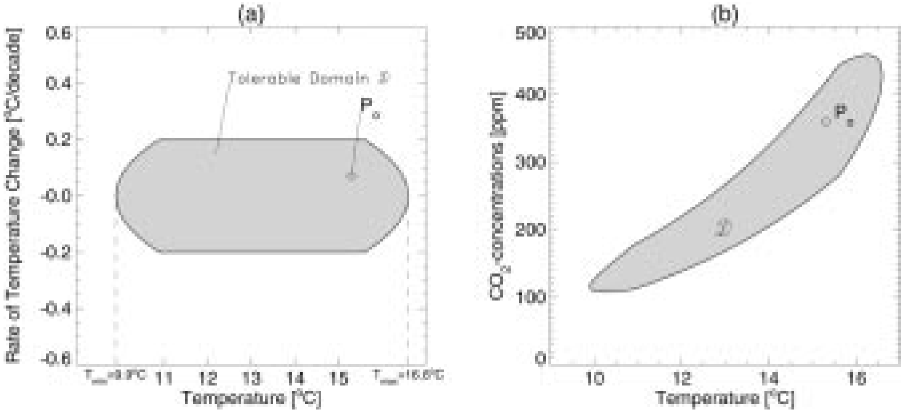

Figure 1. The window of tolerable climate evolutions formulated in terms of temperature and rate of temperature change (a) respectively carbon concentration (b). The maximal tolerable temperature is derived from historical records, whereas the maximal rate of temperature change is obtained by an expert evaluation.

have

$$\dot{F} = E \tag{5a}$$

$$\dot{C} = BF + \beta E - \sigma(C - C_1) \tag{5b}$$

$$\dot{T} = \mu \ln\left(\frac{C}{C_1}\right) - \alpha(T - T_1) \tag{5c}$$

where $C_1 = 290$ ppm and $T_1 = 14.6\,^{\circ}\mathrm{C}$ denote the preindustrial levels and E is the annual CO_2 emission measured in Gt C/a (the parameters and initial conditions are listed in Table I). The term BF in the equation for the carbon concentration determines the equilibrium limit of the ocean uptake of CO_2. For a given finite input F of CO_2 into the atmosphere, with vanishing emissions $E \equiv 0$ for some finite time, the asymptotic equilibrium solution of (5b) is given by $C = C_1 + \frac{B}{\sigma}F$. Thus the asymptotic airborne fraction is $B/\beta\sigma$, while the ocean uptake fraction is $1 - B/\beta\sigma$. The dissolution chemistry of carbon in the ocean yields $(B/\beta\sigma)$: $(1 - B/\beta\sigma)$ in the range from 0.15 : 0.85 to 0.08 : 0.92 (Maier-Reimer and Hasselmann, 1987).

Note that the model (5) consists of standard cause-effect-oriented differential equations, i.e., they are not directly applicable for computing the emission set admissible with the domain specified by (4). However, the inversion of the relation can be achieved by familiar techniques of control theory.

3.3. MAXIMIZATION OF CUMULATIVE EMISSIONS

Control theory provides a wide range of mathematical tools and methods to solve many different dynamical optimization problems like the one we are confronted

TABLE I

Parameters and initial conditions for the carbon-cycle climate system of Equation (5)

Parameter	Value	Initial condition	Value
B	$1.51 \cdot 10^{-3}$ ppm/(Gt C $\cdot$ a)	E_0	7.9 Gt C/a
β	0.47 ppm/(Gt C)	F_0	426 Gt C
σ	$2.15 \cdot 10^{-2} 1/a$	C_0	360 ppm
μ	$8.7 \cdot 10^{-2}$ °C/a	T_0	15.3 °C
α	$1.7 \cdot 10^{-2} 1/a$		

with here (Pontryagin et al., 1964; Papageorgiou, 1991). The first step is, of course, to specify and formulate the problem precisely. The system is given by (5) where the CO_2 emission $E(t)$ acts as the control variable. As we want to find the maximal amount of CO_2 which can be emitted without jeopardizing the climate, the objective function to be maximized is $F(\hat{t}) = \int_{t_0}^{\hat{t}} E(t)\,dt$. The constraints are specified by the tolerable window, Equation (4), in terms of T and $\dot{T}$ which by use of Equation (5c) can be expressed by the state variables T and C (see Figure 1b).

For $\hat{t} \to \infty$ the system has to approach an equilibrium with $\dot{X} = 0$ where X is any of the state variables T, C, and F. Insertion of this condition into Equation (5) yields

$$C_\infty = C_1 e^{\alpha(T_\infty - T_1)/\mu}, \tag{6a}$$

$$F_\infty = \frac{\sigma}{B}(C_\infty - C_1), \tag{6b}$$

$$E_\infty = 0. \tag{6c}$$

Thus, emissions have to decline to 0 in the long term. The corresponding maximal cumulative emission is achieved by the realization of the maximal possible temperature, i.e., $T_\infty = T_{\max} = 16.6\,°C$. This implies $C_\infty = 429$ ppm and $F_\infty = 1975$ Gt C. This value of F represents the absolute upper bound for the cumulative emissions as any larger F automatically implies a climate wandering outside of the tolerable window. Substracting $F_0 = 426$ Gt C emitted up to 1995, we obtain the remaining capacity, which is about 1550 Gt C. Note that this value is significantly lower than the estimated resource base of fossil fuel which according to Nakićenović (1996) amounts to about 3500 Gt C. Therefore the burning of fossil fuels is limited by its ‘sink end’ rather than by its ‘source end’.

In order to find the optimal solutions for finite times, we consider the time horizon $\hat{t}$ as a variable and determine the function $\hat{F}(\hat{t})$ of maximal admissible cumulative emissions. This function is not necessarily a controllable function, i.e., it might not be realized by a single emission profile. Yet it is monotonously increasing, as we want to assume the emissions to be larger than or equal to 0. Therefore, if we choose an arbitrary F and look for the minimal time $t_{\min}$ to reach that F, we get a function $t_{\min}(F)$ which is the inverse of $\hat{F}(\hat{t})$. This means that our problem is equivalent to a time-optimal problem – a class which is extensively dealt with in the literature (Pontryagin et al., 1964). The actual solution, however, strongly depends on the class of control functions $E(t)$, i.e., on the type of allowed emission profiles.

The most general specification is the implementation of an upper and a lower bound for the emissions. Setting the latter to zero, we get

$$E_{\min} = 0 \leq E \leq E_{\max}. \tag{7}$$

Further constraints might take into account properties of continuity or differentiability of the profiles $E(t)$ – some of them will be employed at the end of this section.

Consider now an arbitrary time horizon $\hat{t}$. Any solution of the problem stated so far stays within the window for all times $t \leq \hat{t}$. The behavior for times $t' > \hat{t}$ is not further specified or restricted. Yet it is obvious that the climate evolution should not violate the constraints in the future as well. We therefore have to require that there exists at least one control function $E(t')$ which keeps the evolution within the domain $\mathcal{D}$. In order to allow only those states (F, C, T) which have *acceptable futures*, we have used the minimal control profile $E(t') \equiv 0$ for times beyond the initial time horizon and checked whether the corresponding climate evolution stays within the window. This yields some minor modifications of the original domain, which, however, by use of numerical computations, are shown to be irrelevant in the interesting regime of F. Therefore we henceforth consider the original domain $\mathcal{D}$ alone.

This settles the problem we want to address here. Now it is the mere task of mathematics to find the solution. Therefore, the rest of this section might sound rather technical or obscure to some readers – yet the presentation of the treatment is necessary from our point of view. If the solution is correct with respect to the problem conditions, any 'uncomfortable peculiarities' of the solution can only be removed by changing the assumptions, which is – as cannot be stressed often enough – actually *not* the task of science alone.

Shifting the detailed mathematical argumentation to the Appendix (Section 6), the general strategy for a time-optimal trajectory can be formulated as follows:

1. Emit as much as possible to reach the boundary as soon as is feasible.
2. Stay on the boundary afterwards.

For $E_{\max}$ large enough and without further constraints the fastest way to the boundary is achieved by a δ-peak of 124.8 Gt C (cmp. the dashed-dotted peak in Figure 2a; the dashed line indicates a more realistic, close to optimal emissions profile to be described below). Then the optimal emission profile slowly increases to about 8.5 Gt C$/a$ in the year 2010 (cmp. the solid line in Figure 2a) corresponding to the climate evolution along the horizontal branch of the climate window (cmp. the solid

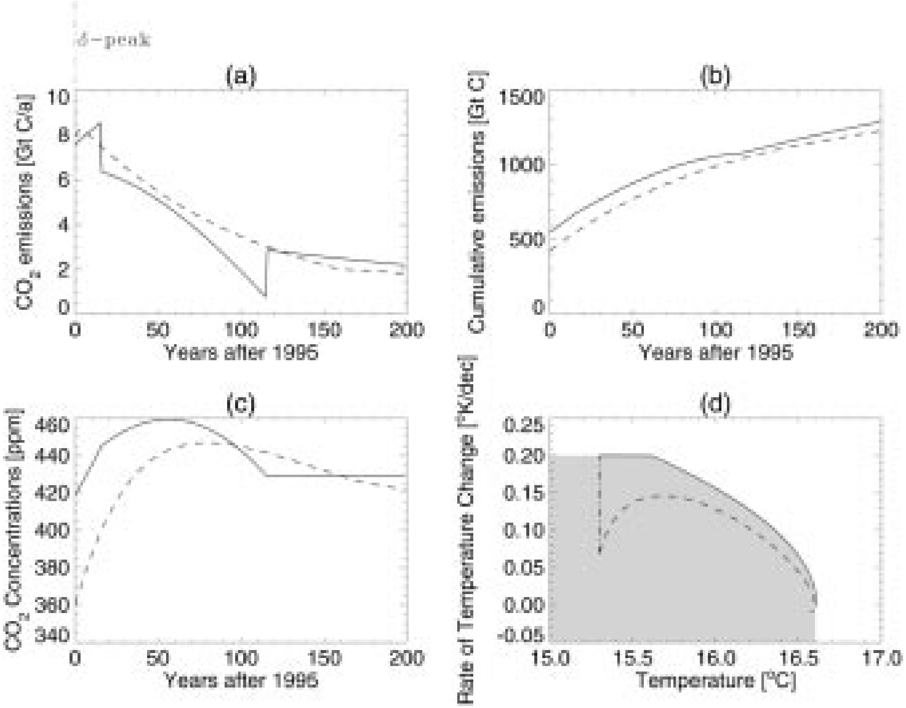

Figure 2. Optimal solutions with respect to maximization of cumulative emissions. Figures (a), (b), and (c) show the CO_2-emissions, the cumulative emissions, and the CO_2 concentrations, respectively, as a function of time. Figure (d) depicts the phase space evolution of the corresponding climates $(T, \dot{T})$ overlaid on the tolerable window. The solid lines (in connection with the dashed-dotted lines) represent the exact solutions if no restrictions are made for the emissions profiles which gives rise to the δ-peak and the two kinks in the emissions. The dashed lines are obtained for a restricted set of emissions which are characterized by a high degree of planning certainty and feasibility.

line in Figure 2d). The crinkle at $T = 15.6$ °C results in a sharp drop in emissions from 8.5 Gt C/a in the year 2010 to 6.3 Gt C/a in the year 2011, i.e., 25%. In order to keep the climate on the boundary of the window, emissions have to decrease smoothly till the climate reaches the singular endpoint of the domain ($T = 16.6$ °C, $\dot{T} = 0$). However, as the carbon cycle is not in equilibrium at this point, i.e., the ocean takes up more carbon than is released by human activities, emissions have to be increased from 0.8 Gt C/a to 2.9 GT C/a. Afterwards the exponential decay with rate B/β corresponds to an immediate uptake by the ocean, i.e., $\dot{C} = \dot{T} = 0$. Note that although the climate stays constant, there is still a positive emission and therefore $\dot{F} > 0$.

The emission profile as depicted in Figure 2a gives rise to the time evolution of the system as presented by the solid lines in Figures 2b through d for the years 1995 to 2195 (solid lines). Figure 2b actually shows the function $\hat{F}(\hat{t})$ we have looked for. It can be realized that $\hat{F}(\hat{t})$ is achieved by a single control function $\hat{E}(t)$ for all time horizons $\hat{t}$. Note that due to the initial δ-peak the graph starts at about

550 Gt C rather than at its initial value of 426 Gt C. The amount to be available from now on increases from 310 Gt C in 2020 to 475 Gt C, 640 Gt C, and 865 Gt C in 2050, 2100, and 2195, respectively. This means that in 2195 it has only used up about 55% of the total capacity of 1550 Gt C.

Focusing on the concentration profile (Figure 2c) one can see that the final equilibrium value of 429 ppm is exceeded by intermediate values with a maximum of 460 ppm. This is in contrast with the calculations by Wigley et al. (1996) and similar inverse calculations, which start from the concentration rather than from climate targets. In these studies the concentration target acts as a ceiling which must not be overridden at any time. The different time scales of the climate and the carbon-cycle system, however, do allow for some higher concentrations in between if the target is given in climate space alone.

The solution just discussed is characterized by different cusps and discontinuities which are not feasible in the real world. Yet, it is the correct solution under the assumptions made so far. The only way to improve the feasibility is to introduce further constraints or to modify the current ones. One reasonable possibility is to restrict the emissions profiles to functions with a smooth transition between business as usual and an exponential decay. This exponential tail might be built up by two successive functions with different rates. Taking a parabolic spline for the transition (cmp. Svirezhev et al., 1998) transforms the dynamic optimization problem treated so far into a conventional optimization task in a four-dimensional parameter space (one parameter for the time when the transition starts, one for the endpoint of the transition period, and two for the exponential rates). The result, for which the time evolution of the different state variables is shown as the dashed lines in Figure 2, has been recommended by the Global Change Council (WBGU) as the preferable scenario in terms of planning-security and feasibility. The transition starts in the year 2000 and takes just 3 years. Then an annual reduction of 0.91% is employed till 2155 and of 0.25% thereafter. The total amount of CO_2 emitted by this strategy from now through 2195 is ca. 800 Gt C which is about 93% of the much more unrealistic δ-peak solution (for 2050 we obtain about 370 Gt C and 585 Gt C for 2100). Note that the absolute asymptotic amount of admissible carbon release is independent of the actual path (Svirezhev et al., 1998).

The problem we have treated in this section covers only a small fraction of the entire climate problem by translating a tolerable climate window in a function of maximal admissible cumulative emissions. Yet a more complete treatment has to cover explicitly climate impacts and the socio-economics of emissions reductions. Also, the representation of admissible policies by one single function is not satisfying. This brings us to the more general questions of how the tolerable windows approach can be generalized and formalized, and how admissible futures can be characterized. We briefly address these questions in the next section.

4. Necessary and Sufficient Emissions Corridors

In the previous section, we have characterized the admissible set of GHG emissions merely by the function $\hat{F}(\hat{t})$ of maximal cumulative emissions. Yet we might ask: what are the admissible *emissions* in the year X? What is the feasible *concentration* in that year? Is it possible to realize a *temperature* of Y °C in year X? It is obvious that not all of these questions can simultaneously be answered by Figure 2. In order to clarify these points, a mathematically profound concept of the TW approach is needed which tells us how to compute the corresponding values and/or intervals. In this paper we only want to sketch this concept; for further details we refer the interested reader to an upcoming technical paper (Bruckner and Petschel-Held, 1999).

4.1. FORMAL ASPECTS OF THE TW APPROACH

Although the inverse method as sketched in Section 2.1 reflects the intrinsic logic of the TW approach, it is not very useful for its formalization as it would require rather complicated methods of nonlinear functional analysis. The formalization would even be more difficult or even impossible if uncertainties are taken into account. It is much easier to formulate the tolerable windows approach as a special subdiscipline of control theory which has attracted some attention within the last few years: the theory of *differential inclusions* or multivalued differential equations (Aubin and Cellina, 1984; Deimling, 1992; Kurzhanski and Vályi, 1997). We thus start from a deterministic, virtual model which we assume to be given by a set of differential equations in terms of the state variables $\mathbf{x} \in \mathbb{R}^{\mathbf{n}}$ and the control variables $\mathbf{u} \in \mathcal{U}(\mathbf{x}; \mathbf{t})$, i.e.,

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}; t) \tag{8}$$

with initial conditions $\mathbf{x}(t_0) = \mathbf{x}_0$. The set of allowed control functions is encoded by $\mathcal{U}(\mathbf{x}; t)$. We may assume that a set of knock-out criteria exists which refers to various pertinent, climate-sensitive attributes. These attributes can be purely systemic in nature – like critical thresholds for the North Atlantic Deep Water formation (Rahmstorf, 1994, 1995; Stocker and Schmittner, 1997) – or of the purely normative type – like minimum standards for the per-capita food production world wide. Let us denote the individual tolerable windows deduced from each of these attributes by $\mathcal{D}_i(\mathbf{x}; t)$. The resulting tolerable domain $\mathcal{D}(\mathbf{x}; t)$ is given by the intersection of all individual domains and we formally characterize it by introducing the evaluation function $B(\mathbf{x}; t)$ via

$$B(\mathbf{x}; t) \le 0, \text{iff } \mathbf{x} \in \mathcal{D}(\mathbf{x}; t) = \bigcap_i \mathcal{D}_i(\mathbf{x}; t). \tag{9}$$

Note that without loss of generality we have allowed the domain $\mathcal{D}$ to be a function of time and of the actual state $\mathbf{x}$. Inserting the constraint $\mathbf{u} \in \mathcal{U}(\mathbf{x}; t)$ of the control functions into the general model (8) we obtain the differential inclusion as

$$\dot{\mathbf{x}} \in \mathcal{F}(\mathbf{x}; t), \quad \mathbf{x} \in \mathcal{D}(\mathbf{x}; t) \tag{10}$$

where

$$\mathcal{F}(\mathbf{x}; t) = \{\mathbf{f}(\mathbf{x}, \mathbf{u}; t) | \mathbf{u} \in \mathcal{U}(\mathbf{x}; t)\} \tag{11}$$

is a set-valued function, sometimes simply called *multi*.

The set of all functions $\mathbf{x}(t)$ satisfying Equation (10) is equal to the set of admissible futures. Note that this set corresponds to an admissible set of control functions, i.e., emissions profiles or instruments. The reason for formalizing the tolerable windows approach in this rather complicated-looking manner is simply that there exists a number of powerful theorems regarding the existence of solutions and methods to obtain these solutions (Aubin and Cellina, 1984; Deimling, 1992; Kurzhanski and Vályi, 1997). We have to distinguish between a *necessary* and a *sufficient* representation of the solution set and we now briefly outline their characteristics. For a detailed discussion on the application of these theorems we refer the reader again to the technical paper (Bruckner and Petschel-Held, 1999).

In the case of the necessary representation, the so-called *funnel* is computed which includes *all* differentiable functions which fulfill the differential inclusion (10). Sometimes the funnel is called the admissible set with corresponding admissible functions. Under quite general assumptions, the boundary of the funnel can be represented by $G(\mathbf{x}; t) = 0$ where $G(\mathbf{x}; t) \leq 0$ denotes its interior. Then it can be shown (Panasyuk, 1990; Bruckner and Petschel-Held, 1999) that $G(\mathbf{x}; t)$ fulfills the partial differential equation

$$\frac{\partial G}{\partial t} + \max_u \left\{ \sum_k \frac{\partial G}{\partial x_k} f_k(\mathbf{x}, \mathbf{u}; t) + \lambda(t) B^{(j)}(\mathbf{x}; t) \right\} = 0, \tag{12}$$

where B denotes the tolerable window (see Equation (9)). Here j is the smallest integer value for which the control variable $\mathbf{u}$ appears explicitly in the partial derivative $B^j \equiv \frac{\partial^j B}{\partial t^j}$ of the window function B. The Lagrange multiplier λ is equal to 0 whenever $B^{(j)} < 0$. The numerical results presented in the next section have been obtained by an algorithm based on Equation (12). Note the similarity with the generalized Hamilton-Jacobi-Bellman equation which allows the use of similar techniques.

It has to be stressed that $G[\mathbf{x}(\mathbf{t}); t] \leq 0$ represents a necessary condition, i.e., any admissible function $x(t)$ which stays within the tolerable windows obeys $G[\mathbf{x}(\mathbf{t}); t] \leq 0$ with G satisfying (12). On the other hand, not any arbitrary function which fulfills these conditions is compatible with the tolerable window. In order to obtain a *sufficient* condition which allows to check the admissibility directly, we need to parameterize the control functions, i.e., $\mathbf{u}(\mathbf{p})$. Using one of the theorems on differential inclusions mentioned above and simple numerical integration methods, we can specify the regimes with admissible functions $\mathbf{x}[\mathbf{u}(\mathbf{p})]$ and, in particular, the boundary separating parameters with admissible evolution from those with forbidden ones. Then for any parameter set $\mathbf{p}$ we can immediately state whether it generates an admissible or a forbidden evolution.

4.2. SOLUTION SETS FOR THE WBGU SCENARIO

For the sake of illustration, we now want to apply the formal machinery presented in the last section to the WBGU Scenario introduced in Section 3. So far we have considered the CO_2 emissions as the direct control variable. As we have seen, however, this might yield rather strange and unrealistic looking results, e.g., discontinuities in the emissions profile. We therefore extend the model by

$$\dot{E} = uE, \tag{13}$$

where $|u| \leq u_{\max}$. Henceforth we want to consider u, i.e., the rate of emissions change, as the control variable which yields continuous emissions profiles $E(t)$. For $u_{\max}$ we use a value of 2% per year which is in correspondence with other, similar studies (Alcamo and Kreilemans, 1996; Matsuoka et al., 1996). Yet this value is used for illustrative reasons only, as the socio-economic feasibility of the emissions profile has to be decided on the basis of further analysis of its implications to civilization.

As the funnel of admissible evolutions is an object in a high-dimensional space (in the case of the WBGU Scenario it is actually five-dimensional: four state variables and time) we have to use some simple visualization. Figure 3 depicts the *projections* of the funnel of the system (Equations (5) and (13)) on the four independent state variables (T, C, F, E) and time. One might want to call these projections corridors, but *not* safe corridors. First of all we have to note that the lower edge of each projection corresponds to a permanent reduction of emissions, i.e., $u(t) = \text{const.} = -u_{\max}$. The upper edges, however, are more difficult to achieve and are in general *not realizable by a single control function.* For the corridor in the cumulative emissions F, however, the upper edge corresponds to the function $\hat{F}(t)$ defined in Section 3. In case of the emissions themselves, the boundary is build up by so-called bang-bang solutions (Pontryagin et al., 1964) depicted by the dashed lines, i.e., either $u(t') = u_{\max}$ or $u(t') = -u_{\max},\ \forall t' \leq t$. It can be seen that admissible emissions which increase for the near future are successively characterized by a long-lasting continuous reduction. In order to allow a maximum of emissions in the 22nd century, emissions have to be reduced within the next 20 years, followed by freezing for another 100 or so years.

Note that the maximal achievable concentrations within the next 200 years are not given by the asymptotic value of 429 ppm: it is possible to realize higher concentrations of up to 460 ppm for some points in time. This again illuminates the fact that it might be misleading to use concentration limits as exclusive future

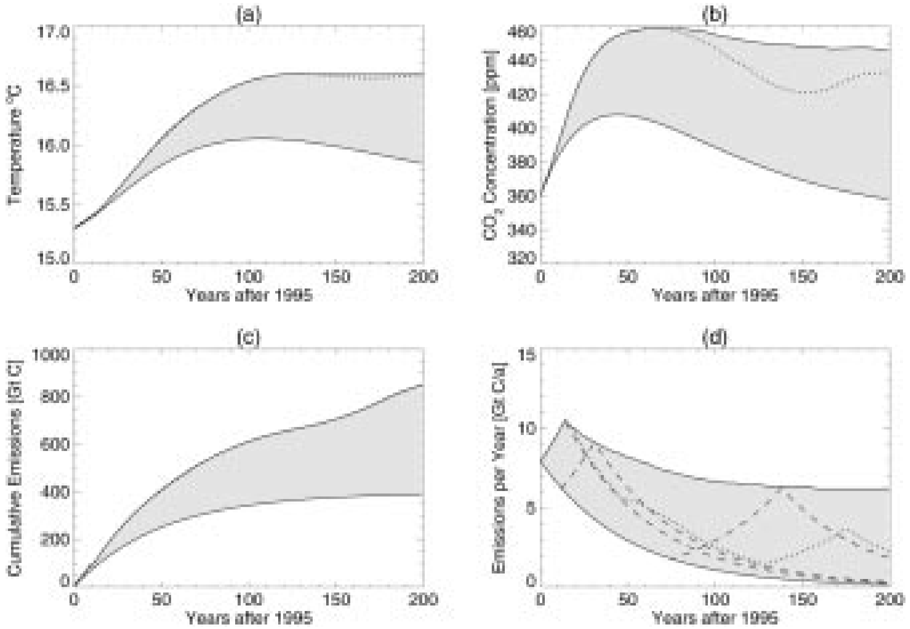

Figure 3. Necessary representation of the set of admissible climate evolutions for the simple model described in Section 3. Each plot represents a projection of the entire set onto the state variables plotted as a function of time, i.e., it represents all possible values for the specific time. Note, that not every arbitrary function completely lying within the domain necessarily corresponds to a tolerable climate evolution. The dashed lines in (d) are typical orbits building up the boundary of the domain. The dotted lines indicate solutions maximizing cumulative emissions.

constraints for the climate system. This result holds also if the emissions are not allowed to rise again (Toth et al., 1997) after being reduced for the first time.

In order to obtain a sufficient conditioning of the funnel, we have used the following parametrization of possible emissions profiles:

$$E(t) = \begin{cases} E_0\,(1+\gamma_{\mathrm{BAU}}t), & 0 < t \le t_1, \\ \Phi(t), & t_1 < t \le t_2, \\ E(t_2)\,e^{-\gamma t} & t > t_2 \end{cases} \tag{14}$$

where $\mathbf{p} \equiv (t_1[a], t_2[a], \gamma[\%/a])$ is the set of basic parameters, $\Phi(t)$ denotes a quadratic spline interpolation between t_1 and t_2, and γ_{BAU} is the initial annual growth rate according to business as usual ($\gamma_{\mathrm{BAU}} = 2\%/a$) (Grubb et al., 1995). The three-dimensional parameter space is scanned by using a finite subset of $\mathbf{p}$ whose elements are checked individually with respect to their admissibility. The resulting phase diagram is shown in Figure 4: parameters above the depicted surface correspond to admissible, those below the surface to forbidden climate evolutions with respect to the tolerable window from Equation (4). In addition, values of γ

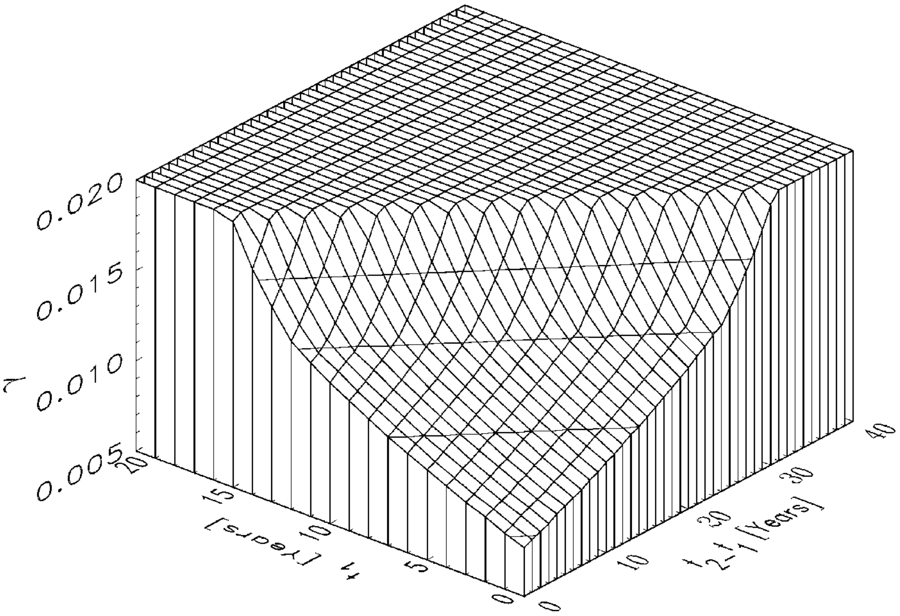

Figure 4. Surface of parameter values γ which separates admissible functions above from forbidden emission profiles below them. The Y- and X-axis denote the transitions times t_1 and $t_2 - t_1$ for leaving business as usual and for the transition to a constant emissions reduction with rate γ, respectively. For illustration contour lines at $\gamma = 0.8\%/a$, $1.1\%/a$, $1.4\%/a$ and $1.7\%/a$ are drawn as well. The regime of admissible profiles is characterized by a rather short-term transition to a moderate reduction of carbon emissions. The 'waiting time' to start with the transition from business as usual to an effective reduction is limited to 15 years.

higher than $2\%/a$ are not allowed due to the predescribed emission reduction limit. The main implications can be summarized as follows:

- The maximal allowed time span to follow business-as-usual is limited to about 15 years, again followed by a long-lasting reduction of $2\%/a$. Alternatively, if we leave the business-as-usual path right now, the maximal length of the transition period amounts to 30 years. Yet every increase in the delay results in a decrease for the allowed transition period.
- If we would like to keep the reduction rate as small as possible now and in the long run, it is necessary to reduce emissions by at least 0.7% per year (if we start reduction now).

In order to check the robustness of our results we have conducted a number of sensitivity analyses. In particular, we have changed the window size and the maximal rate $u_{\max}$ of emissions change. It turns out that for a maximal rate of temperature change of $\dot{T}_{\max} = 0.1$ °C/decade with $T_{\max}$ unchanged and $u_{\max} = 2\%/a$, the set of admissible emissions profiles is empty! This means that it is either necessary to accept a higher rate of temperature change or a stronger reduction rate of GHG emissions. It turns out that the funnel is more sensitive to a change in the rate of temperature change, i.e., a small increase of $\dot{T}_{\max}$ offers more control options than a corresponding increase of the admissible emissions rate. This implies that it is more helpful to focus the discussion on the actual value of $\dot{T}_{\max}$ than on the one of $u_{\max}$.

5. Summary and Perspectives

In this paper we have outlined a new concept for integrated assessments of climate change which we call the *tolerable windows (TW) approach*. This modeling approach stands in between the so-called policy evaluation and policy optimization models (Weyant et al., 1996). In the first class, a pre-defined scenario of socio-economic development is translated into climate impacts on various sectors and environmental compartments. In the second class, a single-valued global welfare functional is specified which includes costs and benefits of mitigation measures. Then that climate policy is computed which maximizes the global welfare function. In contrast, the TW approach deems it unrealistic to assume that a single-valued global welfare function can be formulated within the climate negotiation process. Instead, within the TW approach the reconciliation of a *multitude* of quality standards with respect to climate impacts and the driving forces of climate is pursued. Thus the basic normative input to the approach is the specification of *tolerable windows* concerning these standards. It has to be stated, however, that these inputs are ultimately not the task of science, but rather have to be specified within a negotiation process between politicians, scientists, economists, etc.

The identification of policy options which are compatible with all pre-specified windows is then the subject of a model-based scientific analysis. This analysis results in a tolerable corridor of future evolutions and policy options. Which policy option is selected is then either subject to further criteria, e.g., cost-effectiveness, social compatibility, etc., or the selection can be kept open, i.e., either politics or the ‘run of time’ decides on the particular path chosen from the corridor.

We have illustrated the approach using simple models and assumptions, the so-called WBGU Scenario. The basic normative setting was adopted from the German Advisory Council on Global Change (WBGU, 1995) in the form of a tolerable *climate* window expressed in terms of global mean temperature and its rate of change. A simple climate model for the global mean temperature has been used to characterize those emission profiles which are compatible with the climate window.

In a first step we have sought for the maximal cumulative emission within an arbitrary time horizon which does not jeopardize the global climate. It turns out that the total, asymptotic (i.e., $t \to \infty$) capacity amounts to about 1550 Gt C. Yet the use of about 45% of this capacity is restricted to the time after 2200! For the near future up to 2050 we can use only 30%, i.e., 475 Gt C. Yet these numbers have to be taken with care. Besides the simplicity of the model, the optimization yields a rather unrealistic emissions profile to realize these values. In particular, it would be necessary to emit 125 Gt C within one single year! This reflects the fact that the constraints set so far are clearly insufficient with respect to the feasibility of the emissions. A more realistic setting (smoothness, exponential reduction) basically requires a reduction of GHG emissions of about 1% for the next 160 years (WBGU, 1995).

In the second step, we have provided the general theory for the overall class of control problems of that type. It has been shown that *differential inclusions*, i.e., $\dot{\mathbf{x}} \in \mathcal{F}(\mathbf{x}; t), \quad \mathbf{x} \in \mathcal{D}(\mathbf{x}; t)$ are best suited for formalizing the general TW approach. Here, $\mathcal{F}$ and $\mathcal{D}$ are set-valued functions where $\mathcal{D}$ represents the intersection of all individual windows to be respected. It is important to note that so far no formalism to specify a general sufficient representation of the solution set exists. In the WBGU Scenario we therefore have presented two different delineations of the solution set: a general necessary and a restricted sufficient one. In the first case, a necessary condition is given which allows to directly check the non-admissibility of arbitrary emission functions. By restriction to a special set of emissions profiles a sufficient condition can be computed which enables a complete check of any emissions profile of that class. These methods have been used to compute – what we might call – *emissions corridors* (compare Alcamo and Kreilemans, 1996, Matsuoka et al., 1996). It turns out that the maximal CO_2 emission in 2200 amounts to about 6.1 Gt C. In our model these emissions can be achieved if an immediate emissions reduction of $2\%/a$ is realized till 2105 and emissions are increased by 2% per year afterwards. The main result of the analysis of the sufficient condition is that the transition time from business-as-usual to an effective, exponential reduction of CO_2 strongly depends on the delay till the transition is started. In particular, the delay time is restricted to 15 years. In that case, however, an immediate long lasting reduction of $2\%/a$ has to follow up. In contrast, if we start transition now the change of course might take up to 30 years.

The discussion of the TW approach as presented here is intended to be exemplary and as transparent as possible. We therefore have tried to err on the side of comprehensibility instead of comprehensiveness, e.g., by applying a rather simple climate model and a highly stylized set of normative settings. This implies that all numerical results should be seen as provisional and that normative settings are not proposed to be accepted now and forever world-wide. Perpetual climate impact and socio-economic research is obviously required to allow an increasingly educated formulation of tolerable windows. Defining socio-economic guardrails in terms of inacceptable GDP losses (Tóth et al., 1998), for example, might be more appropriate than specifying a maximum rate of emission reduction solely. Furthermore, as the definition of tolerable windows involves value judgments (Helm et al., 1998), the windows may obviously differ depending on the decision-maker seeking for scientific advice (Bruckner et al., 1998).

All numerical results are obtained by a simple climate model delivering rather smooth climate evolution paths typical for reduced-form models. The actual climate evolution generated by an emission path will be superimposed by natural climate variability, oscillations, and anomalies not represented in the climate model. Taking into account this smoothing procedure therefore reveals that the climate window investigated is not as severe as it might seem if it were interpreted in terms of actual climate change. Consequently, we have not addressed the issue of how to stay within the climate window in a control-theoretical sense, i.e., by taking into account repetitive measurements of actual system conditions (cf. Dowlatabadi, 1998). We would only like to mention that staying within an emission corridor might be easier than strictly following a crisp emission path qualified as optimal on the grounds of some criteria set.

Despite these caveats, we conceive the TW approach as a promising new concept for integrating assessments as it enables us to pay attention simultaneously to all the different aspects of the general welfare. As has been stated elsewhere (e.g., Shukla, 1997, Weyant et al., 1996), there is a lack of such multivariate approaches, e.g., concerning different cultural values. Yet the TW approach is also open to traditional cost-benefit analysis, as the sharp edges of the windows can be softened. However, the approach is only as good as the models employed. Therefore, an international research network orchestrated by PIK has started the ICLIPS project in which the development of adequate models and their integration within the tolerable window philosophy is pursued.

6. Appendix: The Mathematics of Optimal Control

In this short appendix we want to sketch the mathematical argument guaranteeing that the radical strategy formulated in Section 3.3 actually maximizes the cumulative emissions. The control problem is given by the differential equations (5) with constraints specified by the tolerable window (i.e., $B(C, T) \leq 0$), and the task to find the time-optimal solution to realize a given cumulative emission F_1. We denote the shortest time to achieve F_1 by t_{opt}. Further, we want to assume that there exists a control profile $0 < E_B[\mathbf{x}_R(t)] < E_{\max}$ such that the corresponding trajectory $\mathbf{x}_R(t)$, runs exactly on the frame of the window, i.e., $B(C_R, T_R) = 0$ on the boundary. Of course, a rigorous proof would have to show the existence of this control function, rather than doing it numerically.

The basic mathematical theorem to be applied here is Theorem 25 in Pontryagin et al. (1964). It provides a *necessary* condition for the optimality of a solution of a general control problem. The basic element of the condition is the formulation of a Hamiltonian, whose explicit form depends on whether it is considered inside the domain or at its boundary. What we actually want to show here is that, besides the initial phase to reach the boundary, any segment of a trajectory inside the domain does *not* fulfill the necessary condition. To do so, we divide this trajectory into its initial part Γ_0 which ends when reaching the boundary for the first time and the successive part Γ_1.

We assume that Γ_1 has a finite part Γ_1^{int} in the interior of the domain. Then the Hamiltonian takes the form (Pontryagin et al., 1964)

$$\begin{aligned} H(E(t),\mathbf{x}(t),\boldsymbol{\psi}) &= \sum_{i=1}^{3} \psi_i f_i \\ &= \psi_1 E + \psi_2 \beta E + \psi_2 BF - \psi_2 \sigma (C - C_1) + \\ &\quad \psi_3 \mu \, ln \frac{C}{C_1} - \psi_3 \alpha (T - T_1) \\ &= (\psi_1 + \beta\psi_2) E + \Psi(C, T; \psi_2, \psi_3), \end{aligned} \tag{15}$$

where we have used $\mathbf{x} = (F, C, T)$ and the new conjugate variables ψ_i. According to Pontryagin's Principle the optimal solution is then characterized by $\hat{E}(t)$ with $H(\hat{E}(t), \mathbf{x}, \boldsymbol{\psi}) = \sup_{E(t)} H(E(t), \mathbf{x}, \boldsymbol{\psi}) \geq 0$ with

$$\frac{d\psi_i}{dt} = -\frac{\partial H}{\partial x_i}. \tag{16}$$

This implies

$$\hat{E}(t) = \begin{cases} E_{\max}, & \text{if} \psi_1 + \beta\psi_2 > 0 \\ 0, & \text{if } \psi_1 + \beta\psi_2 < 0 \\ \text{undetermined}, & \text{else}. \end{cases} \tag{17}$$

Using Equations (5), (15), and (16) we obtain

$$\dot{\psi}_2 = \sigma\psi_2 - \frac{\mu}{C}\psi_3 = \sigma\psi_2 - \frac{\mu\psi_3^0}{C} e^{\alpha t}. \tag{18}$$

If $\psi_1 + \beta\psi_2 = 0$ for some finite time interval, we get $\dot{\psi}_1 = -B\psi_2 = (B/\beta)\psi_1$ and thus $\psi_1 = \psi_1^0 \exp(Bt/\beta)$. This implies

$$\psi_2 = -\frac{\psi_1^0}{\beta} e^{Bt/\beta}. \tag{19}$$

As (18) and (19) cannot be fulfilled simultaneously (except for $\boldsymbol{\psi} \equiv 0$ which is not allowed by Pontryagin's Principle), the undetermined case is of measure zero, i.e. applies only during the switching times of length zero.

If we consider an internal point $\mathbf{x}^{\text{int}}$ at time t_1, Equation (17) implies that it either must be reached with $E(t') = 0$ or $E(t') = E_{\max}$ for $t_1 - \Delta t \leq t' \leq t_1 \leq t_{\text{opt}}$ and some finite time interval $\Delta t > 0$. Yet, for $E_{\max}$ large enough one would leave the window within Δt. Therefore any internal point can be achieved with $E(t') = 0$ only. Considering $\mathbf{x}_1^{\text{int}} = F_1$ this implies that F_1 would be realized already at time $t_1 - \Delta t$ which is in contradiction to the assumption that t_{opt} is the shortest possible time to achieve F_1. Thus the *final* point $\mathbf{x}_1$ cannot be in the interior of the domain, i.e., the fastest trajectory to realize F_1 has to end on the boundary.

Next, consider the time $\tau,\ 0 < \tau < t_{\mathrm{opt}}$ with a maximal cumulative emission $F(\tau) = F_\tau$. According to the argument given in the last paragraph, the corresponding state $\mathbf{x}_\tau$ is located on the edge of the domain, i.e., $B(C_\tau, T_\tau) = 0$. Thus, for every state $\mathbf{x}'(\tau)$ in the inner region of B, i.e. $B(C'(\tau), T'(\tau)) < 0$, $F'(\tau) < F_\tau$. Then at time $\tau + \epsilon \leq t_{\mathrm{opt}}, \epsilon > 0$ we have $F(\tau + \epsilon) = F(\tau) + \epsilon E_B(\mathbf{x}_\tau)$ and $F'(\tau + \epsilon) = F'(\tau) < F(\tau + \epsilon)$. This implies that the optimal solution for time $\tau + \epsilon$ is given by the continuation of the solution for time τ by the boundary control $E_B(\mathbf{x}_\tau)$. Therefore once being located on the boundary of the domain at time t, any later optimal solution is realized by the boundary control.

The first part of the strategy formulated in Section 3.3 states that the boundary should be reached as fast as possible; as a matter of fact it has to happen by a δ-peak like emission. From the argument given above, we learn that for E_{max} large enough within a finite time interval, the window would be left. For infinite emissions within a time interval of measure zero, but with a finite integral, however, one observes that the constraint is not violated and yields for the initial condition exactly the cumulative emission of 124.8 Gt C.

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