

The missing link in gravitational-wave astronomy A summary of discoveries waiting in the decihertz range

Manuel Arca Sedda · Christopher P L Berry ·
Karan Jani · Pau Amaro-Seoane ·
Pierre Auclair · Jonathon Baird ·
Tessa Baker · Emanuele Berti ·
Katelyn Breivik · Chiara Caprini ·
Xian Chen · Daniela Doneva ·
Jose M Ezquiaga · K E Saavik Ford ·
Michael L Katz · Shimon Kolkowitz ·
Barry McKernan · Guido Mueller ·
Germano Nardini · Igor Pikovski ·
Surjeet Rajendran · Alberto Sesana ·
Lijing Shao · Nicola Tamanini ·
Niels Warburton · Helvi Witek · Kaze Wong ·
Michael Zevin

Received: 3 August 2020 / Accepted: 9 February 2021 / Published online: 29 April 2021

M Arca Sedda
Astronomisches Rechen-Institut, Zentrum für Astronomie, Universität Heidelberg,
Mönchhofstr. 12-14, Heidelberg, Germany

C P L Berry
Center for Interdisciplinary Exploration and Research in Astrophysics (CIERA), Department
of Physics and Astronomy, Northwestern University, 2145 Sheridan Road, Evanston,
IL 60208, USA
SUPA, School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, UK
E-mail: christopher.berry@northwestern.edu

K Jani
Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37212, USA

P Amaro-Seoane
Universitat Politècnica de València, IGIC, Spain
Kavli Institute for Astronomy and Astrophysics, Beijing 100871, China
Institute of Applied Mathematics, Academy of Mathematics and Systems Science, CAS,
Beijing 100190, China
Zentrum für Astronomie und Astrophysik, TU Berlin, Hardenbergstraße 36, 10623 Berlin,
Germany

P Auclair
Laboratoire Astroparticule et Cosmologie, CNRS UMR 7164, Université Paris-Diderot, 10
rue Alice Domon et Léonie Duquet, 75013 Paris, France

J Baird
High Energy Physics Group, Physics Department, Imperial College London, Blackett Lab-
oratory, Prince Consort Road, London, SW7 2BW, UK

T Baker

School of Physics and Astronomy, Queen Mary University of London, Mile End Road, London, E1 4NS, UK

E Berti

Department of Physics and Astronomy, Johns Hopkins University, 3400 N. Charles Street, Baltimore, MD 21218, USA

Katelyn Breivik

Canadian Institute for Theoretical Astrophysics, University of Toronto, 60 St. George Street, Toronto, Ontario, M5S 1A7, Canada

C Caprini

Laboratoire Astroparticule et Cosmologie, CNRS UMR 7164, Université Paris-Diderot, 10 rue Alice Domon et Léonie Duquet, 75013 Paris, France

X Chen

Astronomy Department, School of Physics, Peking University, Beijing 100871, China
Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing 100871, China

D Doneva

Theoretical Astrophysics, Eberhard Karls University of Tübingen, Tübingen 72076, Germany

J M Ezquiaga

Kavli Institute for Cosmological Physics, Enrico Fermi Institute, The University of Chicago, Chicago, IL 60637, USA

K E S Ford

City University of New York-BMCC, 199 Chambers St, New York, NY 10007, USA
Department of Astrophysics, American Museum of Natural History, New York, NY 10028, USA

M L Katz

Center for Interdisciplinary Exploration and Research in Astrophysics (CIERA), Department of Physics and Astronomy, Northwestern University, 2145 Sheridan Road, Evanston, IL 60208, USA

S Kolkowitz

Department of Physics, University of Wisconsin – Madison, Madison, WI 53706, USA

B McKernan

City University of New York-BMCC, 199 Chambers St, New York, NY 10007, USA
Department of Astrophysics, American Museum of Natural History, New York, NY 10028, USA

G Mueller

Department of Physics, University of Florida, PO Box 118440, Gainesville, Florida 32611, USA

G Nardini

Faculty of Science and Technology, University of Stavanger, 4036 Stavanger, Norway

Igor Pikovski

Department of Physics, Stevens Institute of Technology, Hoboken, NJ 07030, USA
Department of Physics, Stockholm University, SE-10691 Stockholm, Sweden

S Rajendran

Department of Physics and Astronomy, Johns Hopkins University, 3400 N. Charles Street, Baltimore, MD 21218, USA

A Sesana

Università di Milano Bicocca, Dipartimento di Fisica G. Occhialini, Piazza della Scienza 3, I-20126, Milano, Italy

Abstract Since 2015 the gravitational-wave observations of LIGO and Virgo have transformed our understanding of compact-object binaries. In the years to come, ground-based gravitational-wave observatories such as LIGO, Virgo, and their successors will increase in sensitivity, discovering thousands of stellar-mass binaries. In the 2030s, the space-based *LISA* will provide gravitational-wave observations of massive black holes binaries. Between the ~ 10 – 10^3 Hz band of ground-based observatories and the $\sim 10^{-4}$ – 10^{-1} Hz band of *LISA* lies the uncharted decihertz gravitational-wave band. We propose a *Decihertz Observatory* to study this frequency range, and to complement observations made by other detectors. Decihertz observatories are well suited to observation of intermediate-mass ($\sim 10^2$ – $10^4 M_\odot$) black holes; they will be able to detect stellar-mass binaries days to years before they merge, providing early warning of nearby binary neutron star mergers and measurements of the eccentricity of binary black holes, and they will enable new tests of general relativity and the Standard Model of particle physics. Here we summarise how a Decihertz Observatory could provide unique insights into how black holes form and evolve across cosmic time, improve prospects for both multimessenger astronomy and multiband gravitational-wave astronomy, and enable new probes of gravity, particle physics and cosmology.

Keywords gravitational waves · decihertz observatories · multiband gravitational-wave astronomy · multimessenger astronomy · space-based detectors · black holes · neutron stars · white dwarfs · stochastic backgrounds · binary evolution · intermediate-mass black holes · tests of general relativity · Voyage 2050

L Shao

Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing 100871, China
National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China

N Tamanini

Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Am Mühlenberg 1,
14476 Potsdam-Golm, Germany

N Warburton

School of Mathematics and Statistics, University College Dublin, Belfield, Dublin 4, Ireland

H Witek

Department of Physics, King's College London, Strand, London, WC2R 2LS, UK

K Wong

Department of Physics and Astronomy, Johns Hopkins University, 3400 N. Charles Street,
Baltimore, MD 21218, USA

M Zevin

Center for Interdisciplinary Exploration and Research in Astrophysics (CIERA), Department of Physics and Astronomy, Northwestern University, 2145 Sheridan Road, Evanston, IL 60208, USA

1 The gravitational-wave spectrum

When new frequency ranges of the electromagnetic spectrum became open to astronomy, our understanding of the Universe expanded as we gained fresh insights and discovered new phenomena [1]. Equivalent breakthroughs are awaiting gravitational-wave (GW) astronomy [2,3]. Here, we summarise the *scientific potential of exploring the ~ 0.01 –1 Hz GW spectrum*.

The first observation of a GW signal was made in 2015 by the Laser Interferometer Gravitational-Wave Observatory (LIGO) [3]. Ground-based detectors such as LIGO [4], Virgo [5], and KAGRA [6] observe over a frequency spectrum ~ 10 – 10^3 Hz. This is well tailored to the detection of merging stellar-mass black hole (BH) and neutron star (NS) binaries [7]. Next-generation ground-based detectors, like Cosmic Explorer [8] or the Einstein Telescope [9, 10] may observe down to a few hertz. Only a small part of the GW spectrum can thus be observed by ground-based detectors, and extending to lower frequencies requires space-based observatories.

Lower frequency GW signals originate from coalescences of more massive binaries, and stellar-mass binaries earlier in their inspirals. Due for launch in 2034, the *Laser Interferometer Space Antenna (LISA)* will observe across frequencies $\sim 10^{-4}$ – 10^{-1} Hz [11], optimal for mergers of binaries with $\sim 10^6 M_\odot$ massive BHs [12, 13, 14]. *LISA* will be able to observe nearby stellar-mass binary BHs (BBHs) years–days prior to merger [15], when they could be observed by ground-based detectors. *Multiband* observations of BBHs would provide improved measurements of source properties [16, 17, 18], new constraints on their formation channels [19, 20], and enable precision tests of general relativity (GR) [16, 21].

Pulsar timing arrays are sensitive to even lower frequency GWs of $\sim 10^{-9}$ – 10^{-7} Hz [22], permitting observation of $\sim 10^9 M_\odot$ supermassive BHs [23]. Combining *LISA* and pulsar timing observations will produce new insights into the evolution of (super)massive BHs [24, 25].

The case for extending the accessible GW spectrum with an observatory that can observe in the ~ 0.01 –1 Hz *decihertz* range is explained in [26], based upon a White Paper that we submitted in response to ESA’s Voyage 2050 call, and here we summarise the highlights. Decihertz observations would:

1. Reveal the formation channels of stellar-mass binaries, complementing ground-based observations with deep multiband observations.
2. Complete our census of the population of BHs by enabling unrivaled measurements of intermediate-mass BHs (IMBHs), which may be the missing step in the evolution of (super)massive BHs.
3. Provide a new laboratory for tests of fundamental physics.

Decihertz observatories (DOs) have the capability to resolve outstanding questions about the intricate physics of binary stellar evolution, the formation of astrophysical BHs at all scales across cosmic time, and whether extensions to GR or the Standard Model of particle physics are required.

2 The potential of decihertz observatories

Decihertz observations would bridge space-based low-frequency detectors and ground-based detectors, giving us access to a wide variety of astrophysical systems:

1. *Stellar-mass binaries comprised of compact stellar objects—white dwarfs (WDs), NSs, and stellar-mass BHs.* Since BH and NS mergers are observable with ground-based detectors, a DO would allow multiband observations of these populations. WDs are inaccessible to ground-based detectors [27], and so can only be studied with space-based detectors. While the current-generation of ground-based detectors will detect stellar-mass BBHs to redshifts $z \sim 1\text{--}2$, next-generation detectors will discover them out to $z \sim 20$, enabling them to chart the evolution of the binary population across the history of the Universe [28]; a DO could match this range, far surpassing *LISA*. Furthermore, decihertz observations of compact-object mergers would provide valuable forewarning of multimessenger emission associated with merger events. If detected, multimessenger observations reveal details about the equation of state of nuclear density matter [29, 30, 31, 32, 33], the production of heavy elements [34, 35, 36, 37, 38], and provide a unique laboratory for testing gravity [39, 40, 41, 42], as well as potentially identifying the progenitors of Type Ia supernovae [43, 44, 45]. Even without finding a counterpart, correlation with galaxy catalogues can provide *standard siren* cosmological measurements [46, 47, 48, 49, 50, 51, 52, 53]. Following their detection by LIGO and Virgo, BHs and NSs are a *guaranteed* class of GW source [3, 7, 54]. With a large number of observations, we can infer the formation channels for compact-object binaries, and the physics that governs them [55, 56, 57, 58, 59, 60, 28, 61, 62]. Eccentricity is a strong indicator of formation mechanism [20, 19, 63, 64, 65, 66]; however, residual eccentricity is expected to be small in the regime observable with ground-based detectors [67, 68, 69, 70] while in some cases, BBHs formed with the highest eccentricities will emit GWs of frequencies too high for *LISA* [71, 72, 65, 73, 74, 75, 76]. Hence DOs could provide unique insights into binary evolution.
2. *IMBHs of $\sim 10^2\text{--}10^4 M_\odot$.* IMBHs could be formed via repeated mergers of stars and compact stellar remnants in dense star clusters [77, 78, 79, 80]. Using GWs, IMBHs could be observed in a binary with a compact stellar remnant as an intermediate mass-ratio inspiral (IMRI) [81, 82, 83, 84], or in a coalescing binary with another IMBH. A DO would enhance the prospects of IMRI detection to tens of events per year, with observations extending out to high redshift. Mergers involving a WD or a NS can lead to tidal disruption events with a bright electromagnetic counterpart [85, 86]. IMBHs binaries could be studied across the entire history of the Universe, charting the properties of this population and constraining the upper end of the pair-instability mass gap [87], while also providing a detailed picture of the connection (or lack thereof) between IMBHs and the seeds of massive black holes [88]. The connection between massive BHs and their

lower-mass counterparts could be further explored through observations of binaries orbiting massive BHs in galactic centres, or around IMBHs in smaller clusters [89, 90, 91, 92, 76, 93, 94, 95, 96, 97]. BBH–IMBH systems are a target for DO–ground-based multiband observation because they emit both $1\text{--}10^2$ Hz GWs and *simultaneously* $0.01\text{--}1$ Hz GWs.

3. *Cosmological sources as part of a stochastic GW background (SGWB)*. Both this and the other astrophysical sources serve as probes of new physics, enabling tests of deviations from GR and the Standard Model. A first-order phase transition in the early Universe can generate a SGWB [98, 99, 100, 101, 102, 103]; a DO would be sensitive to first-order phase transitions occurring at higher temperature, or with a shorter duration, compared to *LISA*. A DO would be sensitive to a SGWB from a source at ~ 1 TeV and beyond: TeV-scale phenomena have been considered to resolve with the hierarchy problem or the question of dark matter [104, 105, 106, 107, 108, 109], while 100 TeV-scale phenomena appear in new solutions to the hierarchy problem such as the relaxion [110, 111]. Furthermore, SGWB (non-)detection could constrain the properties of cosmic strings [112, 113] down to string tensions of $\sim 10^{-19}$, while *LISA* would reach $\sim 10^{-17}$ [114] and pulsar timing array observations currently constrain tensions to be $\lesssim 10^{-11}$ [115, 116].

Decihertz observations provide a unique insight into the physics of each of these sources, and observations would answer questions on diverse topics ranging from the dynamics of globular clusters to the nature of dark matter.

3 Decihertz mission concepts

The scientific return of a DO will depend upon its design. There are multiple potential technologies and mission concepts for observing the $0.01\text{--}1$ Hz GW spectrum. The *Advanced Laser Interferometer Antenna (ALIA)* [117, 118] is a heliocentric mission concept more sensitive than *LISA* in the $0.1\text{--}1$ Hz range. Other heliocentric DO concepts are *Taiji* [119, 120], most sensitive around 0.01 Hz, and *TianGo* [121], most sensitive in the $0.1\text{--}10$ Hz range. *TianQin* [122] is a Chinese geocentric mission concept. The *DECi-hertz Interferometer Gravitational-wave Observatory (DECIGO)* [123, 124] is a more ambitious concept with 1000 km Fabry–Perot cavity arms in heliocentric orbit; its precursor *B-DECIGO* would be a 100 km triangular interferometer in a geocentric orbit. The *Big Bang Observer (BBO)* is a concept consisting of four *LISA* detectors in heliocentric orbits with combined peak sensitivity over $0.1\text{--}1$ Hz range [125]. More modest designs are the *Geostationary Antenna for Disturbance-Free Laser Interferometry (GADFLI)* [126] and *geosynchronous Laser Interferometer Space Antenna (gLISA)* [127, 128] which are geocentric concepts. The *Sagnac interferometer for Gravitational wavE (SAGE)* [129, 130] consists of three identical CubeSats in geosynchronous orbit. These concepts are mostly variations on the classic *LISA* design of a laser interferometer.

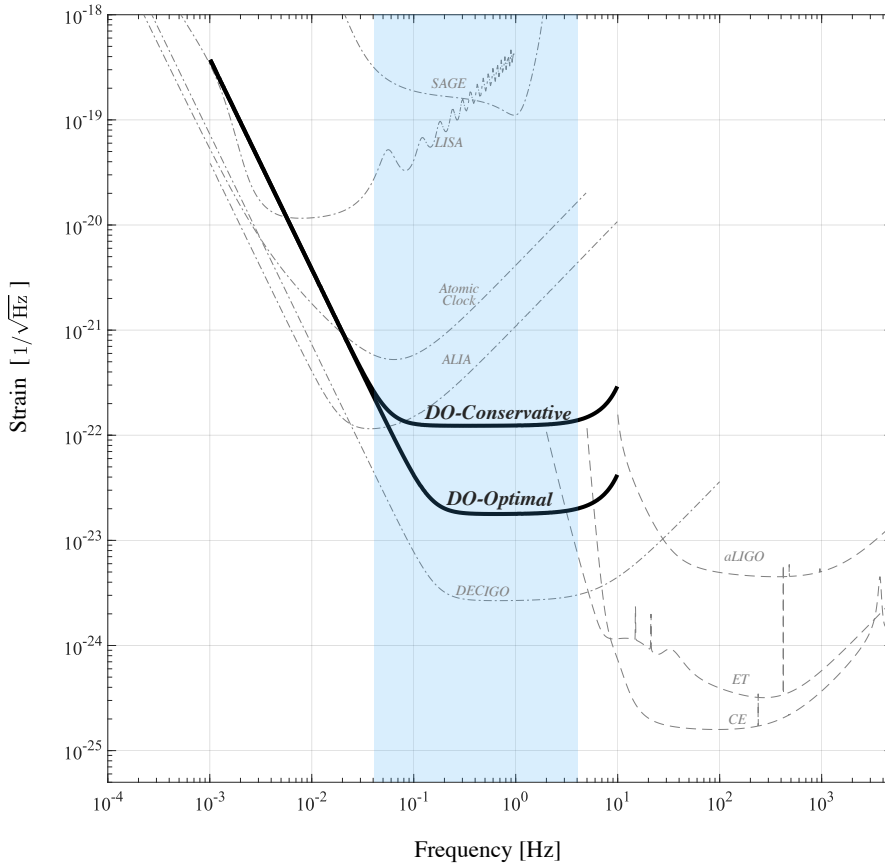


Fig. 1 Concept designs for Decihertz Observatories (DOs) fill the gap between *LISA* [11] and ground-based detectors like Advanced LIGO (aLIGO) [4], Cosmic Explorer (CE) [8] and the Einstein Telescope (ET) [10]. The example DO concepts *SAGE* [129,130], Atomic Clock [131,26], *ALIA* [117,118], DO-Conservative, DO-Optimal [135,26] and *DECIGO* [123,124] span a diverse set of technologies and illustrate the potential range in sensitivities.

In addition to laser interferometry, atomic-clock-based [131,132] and atom interferometer concepts are in development; for example, the *Mid-band Atomic Gravitational Wave Interferometric Sensor (MAGIS)* [133] and the *Atomic Experiment for Dark Matter and Gravity Exploration in Space (AEDGE)* [134] designs use atom interferometry. The range of technologies available mean that there are multiple possibilities for obtaining the necessary sensitivity in the decihertz range. Two illustrative *LISA*-like designs, the more ambitious DO-Optimal and the less challenging DO-Conservative, are presented in [26] to assess the potential range of science achievable with DOs. Potential sensitivity of DOs are illustrated in Figure 1 in comparison to other gravitational-wave observatories.

4 Summary

Observing GWs in the decihertz range presents huge opportunities for advancing our understanding of both astrophysics and fundamental physics. The only prospect for decihertz observations is a space-based DO. Realising the rewards of these observations will require development of new detectors beyond *LISA*. *There are many challenges in meeting the requirements of DO concepts; however, there are also many promising technologies that could be developed to meet these goals.* A DO mission ready for launch in 2035–2050 is achievable, and the science payoff is worth the experimental effort.

Acknowledgements This summary is derived from a White Paper submitted 4 August 2019 to ESA’s Voyage 2050 planning cycle on behalf of the *LISA* Consortium 2050 Task Force [135]. Further space-based GW observatories considered by the *LISA* Consortium 2050 Task Force include a microhertz observatory μ *Ares* [136]; a more sensitive millihertz observatory, the *Advanced Millihertz Gravitational-wave Observatory (AMIGO)* [137], and a high angular-resolution observatory consisting of multiple DOs [138].

The authors thanks Pete Bender for insightful comments, and Adam Burrows and David Vartanyan for further suggestions. MAS acknowledges financial support from the Alexander von Humboldt Foundation and the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – Project-ID 138713538 – SFB 881 (“The Milky Way System”). CPLB is supported by the CIERA Board of Visitors Research Professorship. PAS acknowledges support from the Ramón y Cajal Programme of the Ministry of Economy, Industry and Competitiveness of Spain, as well as the COST Action GWverse CA16104. This work was supported by the National Key R&D Program of China (2016YFA0400702) and the National Science Foundation of China (11721303). TB is supported by The Royal Society (grant URF\R1\180009). EB is supported by National Science Foundation (NSF) Grants No. PHY-1912550 and AST-1841358, NASA ATP Grants No. 17-ATP17-0225 and 19-ATP19-0051, NSF-XSEDE Grant No. PHY-090003, and by the Amaldi Research Center, funded by the MIUR program “Dipartimento di Eccellenza” (CUP: B81I18001170001). This work has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 690904. DD acknowledges financial support via the Emmy Noether Research Group funded by the German Research Foundation (DFG) under grant no. DO 1771/1-1 and the Eliteprogramme for Postdocs funded by the Baden-Württemberg Stiftung. JME is supported by NASA through the NASA Hubble Fellowship grant HST-HF2-51435.001-A awarded by the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., for NASA, under contract NAS5-26555. MLK acknowledges support from the NSF under grant DGE-0948017 and the Chateaubriand Fellowship from the Office for Science & Technology of the Embassy of France in the United States. GN is partly supported by the ROM-FORSK grant Project No. 302640 “Gravitational Wave Signals From Early Universe Phase Transitions”. IP acknowledges funding by Society in Science, The Branco Weiss Fellowship, administered by the ETH Zurich. AS is supported by the European Union’s H2020 ERC Consolidator Grant “Binary massive black hole astrophysics” (grant agreement no. 818691 – B Massive). LS was supported by the National Natural Science Foundation of China (11975027, 11991053, 11721303), the Young Elite Scientists Sponsorship Program by the China Association for Science and Technology (2018QNRC001), and the Max Planck Partner Group Program funded by the Max Planck Society. NW is supported by a Royal Society–Science Foundation Ireland University Research Fellowship (grant UF160093).

References

1. M. Longair, *The Cosmic Century* (Cambridge University Press, Cambridge, 2006). DOI 10.1017/CBO9781139878319
2. B.S. Sathyaprakash, B.F. Schutz, *Living Rev. Rel.* **12**, 2 (2009). DOI 10.12942/lrr-2009-2
3. B.P. Abbott, et al., *Phys. Rev. Lett.* **116**(6), 061102 (2016). DOI 10.1103/PhysRevLett.116.061102
4. J. Aasi, et al., *Class. Quant. Grav.* **32**, 074001 (2015). DOI 10.1088/0264-9381/32/7/074001
5. F. Acernese, et al., *Class. Quant. Grav.* **32**(2), 024001 (2015). DOI 10.1088/0264-9381/32/2/024001
6. T. Akutsu, et al., *Nature Astron.* **3**(1), 35 (2019). DOI 10.1038/s41550-018-0658-y
7. B.P. Abbott, et al., *Phys. Rev.* **X9**(3), 031040 (2019). DOI 10.1103/PhysRevX.9.031040
8. B.P. Abbott, et al., *Class. Quant. Grav.* **34**(4), 044001 (2017). DOI 10.1088/1361-6382/aa51f4
9. B. Sathyaprakash, et al., *Class. Quant. Grav.* **29**, 124013 (2012). DOI 10.1088/0264-9381/29/12/124013, 10.1088/0264-9381/30/7/079501. [Erratum: *Class. Quant. Grav.* **30** 079501 (2013)]
10. S. Hild, et al., *Class. Quant. Grav.* **28**, 094013 (2011). DOI 10.1088/0264-9381/28/9/094013
11. P. Amaro-Seoane, et al., arXiv e-prints 1702.00786 (2017)
12. A. Klein, et al., *Phys. Rev.* **D93**(2), 024003 (2016). DOI 10.1103/PhysRevD.93.024003
13. S. Babak, J. Gair, A. Sesana, E. Barausse, C.F. Sopuerta, C.P.L. Berry, E. Berti, P. Amaro-Seoane, A. Petiteau, A. Klein, *Phys. Rev.* **D95**(10), 103012 (2017). DOI 10.1103/PhysRevD.95.103012
14. C.P.L. Berry, S.A. Hughes, C.F. Sopuerta, A.J.K. Chua, A. Heffernan, K. Holley-Bockelmann, D.P. Mihaylov, M.C. Miller, A. Sesana, *Bull. Am. Astron. Soc.* **51**(3), 42 (2019)
15. A. Sesana, *Phys. Rev. Lett.* **116**(23), 231102 (2016). DOI 10.1103/PhysRevLett.116.231102
16. S. Vitale, *Phys. Rev. Lett.* **117**(5), 051102 (2016). DOI 10.1103/PhysRevLett.117.051102
17. K. Jani, D. Shoemaker, C. Cutler, *Nature Astron.* **4**(3), 260 (2019). DOI 10.1038/s41550-019-0932-7
18. C. Liu, L. Shao, J. Zhao, Y. Gao, *Mon. Not. Roy. Astron. Soc.* **496**, 182 (2020). DOI 10.1093/mnras/staa1512
19. K. Breivik, C.L. Rodriguez, S.L. Larson, V. Kalogera, F.A. Rasio, *Astrophys. J.* **830**(1), L18 (2016). DOI 10.3847/2041-8205/830/1/L18
20. A. Nishizawa, E. Berti, A. Klein, A. Sesana, *Phys. Rev.* **D94**(6), 064020 (2016). DOI 10.1103/PhysRevD.94.064020
21. A. Toubiana, S. Marsat, S. Babak, E. Barausse, J. Baker, *Phys. Rev.* **D101**(10), 104038 (2020). DOI 10.1103/PhysRevD.101.104038
22. R.N. Manchester, *Class. Quant. Grav.* **30**, 224010 (2013). DOI 10.1088/0264-9381/30/22/224010
23. C.M.F. Mingarelli, T.J.W. Lazio, A. Sesana, J.E. Greene, J.A. Ellis, C.P. Ma, S. Croft, S. Burke-Spolaor, S.R. Taylor, *Nature Astron.* **1**(12), 886 (2017). DOI 10.1038/s41550-017-0299-6
24. M. Pitkin, J. Clark, M.A. Hendry, I.S. Heng, C. Messenger, J. Toher, G. Woan, J. *Phys. Conf. Ser.* **122**, 012004 (2008). DOI 10.1088/1742-6596/122/1/012004
25. M. Colpi, et al., *Bull. Am. Astron. Soc.* **51**(3), 432 (2019)
26. M.A. Sedda, et al., *Class. Quant. Grav.* **37**(21), 215011 (2020). DOI 10.1088/1361-6382/abb5c1
27. T.B. Littenberg, K. Breivik, W.R. Brown, M. Eracleous, J.J. Hermes, K. Holley-Bockelmann, K. Kremer, T. Kupfer, S.L. Larson, *Bull. Am. Astron. Soc.* **51**(3), 34 (2019)
28. V. Kalogera, et al., *Bull. Am. Astron. Soc.* **51**(3), 242 (2019)

29. B.P. Abbott, et al., *Phys. Rev. Lett.* **121**(16), 161101 (2018). DOI 10.1103/PhysRevLett.121.161101
30. G. Montana, L. Tolos, M. Hanauske, L. Rezzolla, *Phys. Rev.* **D99**(10), 103009 (2019). DOI 10.1103/PhysRevD.99.103009
31. E.R. Most, L.R. Weih, L. Rezzolla, J. Schaffner-Bielich, *Phys. Rev. Lett.* **120**(26), 261103 (2018). DOI 10.1103/PhysRevLett.120.261103
32. M.W. Coughlin, T. Dietrich, B. Margalit, B.D. Metzger, *Mon. Not. Roy. Astron. Soc.* **489**(1), L91 (2019). DOI 10.1093/mnras/slz133
33. B. Margalit, B.D. Metzger, *Astrophys. J.* **880**(1), L15 (2019). DOI 10.3847/2041-8213/ab2ae2
34. B.P. Abbott, et al., *Astrophys. J.* **850**(2), L39 (2017). DOI 10.3847/2041-8213/aa9478
35. R. Chornock, et al., *Astrophys. J.* **848**(2), L19 (2017). DOI 10.3847/2041-8213/aa905c
36. N.R. Tanvir, et al., *Astrophys. J.* **848**(2), L27 (2017). DOI 10.3847/2041-8213/aa90b6
37. S. Wanajo, *Astrophys. J.* **868**(1), 65 (2018). DOI 10.3847/1538-4357/aae0f2
38. D.M. Siegel, J. Barnes, B.D. Metzger, *Nature* **569**(7755), 241 (2019). DOI 10.1038/s41586-019-1136-0
39. B.P. Abbott, et al., *Astrophys. J.* **848**(2), L13 (2017). DOI 10.3847/2041-8213/aa920c
40. B.P. Abbott, et al., *Phys. Rev. Lett.* **123**(1), 011102 (2019). DOI 10.1103/PhysRevLett.123.011102
41. E. Belgacem, Y. Dirian, S. Foffa, M. Maggiore, *Phys. Rev.* **D98**(2), 023510 (2018). DOI 10.1103/PhysRevD.98.023510
42. E. Belgacem, et al., *JCAP* **1907**(07), 024 (2019). DOI 10.1088/1475-7516/2019/07/024
43. W. Hillebrandt, M. Kromer, F.K. Röpkke, A.J. Ruiter, *Front. Phys.(Beijing)* **8**, 116 (2013). DOI 10.1007/s11467-013-0303-2
44. D. Maoz, F. Mannucci, G. Nelemans, *Ann. Rev. Astron. Astrophys.* **52**, 107 (2014). DOI 10.1146/annurev-astro-082812-141031
45. I. Mandel, A. Sesana, A. Vecchio, *Class. Quant. Grav.* **35**(5), 054004 (2018). DOI 10.1088/1361-6382/aaa7e0
46. B.F. Schutz, *Nature* **323**, 310 (1986). DOI 10.1038/323310a0
47. C.L. MacLeod, C.J. Hogan, *Phys. Rev.* **D77**, 043512 (2008). DOI 10.1103/PhysRevD.77.043512
48. H.Y. Chen, M. Fishbach, D.E. Holz, *Nature* **562**(7728), 545 (2018). DOI 10.1038/s41586-018-0606-0
49. B.P. Abbott, et al., *Astrophys. J.* **909**(2), 218 (2021). DOI 10.3847/1538-4357/abdc67
50. K. Kyutoku, N. Seto, *Phys. Rev.* **D95**(8), 083525 (2017). DOI 10.1103/PhysRevD.95.083525
51. W. Del Pozzo, A. Sesana, A. Klein, *Mon. Not. Roy. Astron. Soc.* **475**(3), 3485 (2018). DOI 10.1093/mnras/sty057
52. C. Cutler, D.E. Holz, *Phys. Rev.* **D80**, 104009 (2009). DOI 10.1103/PhysRevD.80.104009
53. A. Nishizawa, A. Taruya, S. Saito, *Phys. Rev.* **D83**, 084045 (2011). DOI 10.1103/PhysRevD.83.084045
54. R. Abbott, et al., arXiv e-prints (2020)
55. I. Mandel, R. O’Shaughnessy, *Class. Quant. Grav.* **27**, 114007 (2010). DOI 10.1088/0264-9381/27/11/114007
56. S. Stevenson, C.P.L. Berry, I. Mandel, *Mon. Not. Roy. Astron. Soc.* **471**(3), 2801 (2017). DOI 10.1093/mnras/stx1764
57. C. Talbot, E. Thrane, *Phys. Rev.* **D96**(2), 023012 (2017). DOI 10.1103/PhysRevD.96.023012
58. M. Zevin, C. Pankow, C.L. Rodriguez, L. Sampson, E. Chase, V. Kalogera, F.A. Rasio, *Astrophys. J.* **846**(1), 82 (2017). DOI 10.3847/1538-4357/aa8408
59. J.W. Barrett, S.M. Gaebel, C.J. Neijssel, A. Vigna-Gómez, S. Stevenson, C.P.L. Berry, W.M. Farr, I. Mandel, *Mon. Not. Roy. Astron. Soc.* **477**(4), 4685 (2018). DOI 10.1093/mnras/sty908
60. M. Arca Sedda, M. Benacquista, *Mon. Not. Roy. Astron. Soc.* **482**(3), 2991 (2019). DOI 10.1093/mnras/sty2764
61. M. Arca Sedda, M. Mapelli, M. Spera, M. Benacquista, N. Giacobbo, *Astrophys. J.* **894**(2), 133 (2020). DOI 10.3847/1538-4357/ab88b2

62. R. Farmer, M. Renzo, S. de Mink, M. Fishbach, S. Justham, *Astrophys. J. Lett.* **902**(2), L36 (2020). DOI 10.3847/2041-8213/abbadd
63. A. Nishizawa, A. Sesana, E. Berti, A. Klein, *Mon. Not. Roy. Astron. Soc.* **465**(4), 4375 (2017). DOI 10.1093/mnras/stw2993
64. B. Canuel, et al., *Sci. Rep.* **8**(1), 14064 (2018). DOI 10.1038/s41598-018-32165-z
65. K. Kremer, S. Chatterjee, K. Breivik, C.L. Rodriguez, S.L. Larson, F.A. Rasio, *Phys. Rev. Lett.* **120**(19), 191103 (2018). DOI 10.1103/PhysRevLett.120.191103
66. L. Randall, Z.Z. Xianyu, arXiv e-prints 1907.02283 (2019)
67. P.C. Peters, *Phys. Rev.* **136**, B1224 (1964). DOI 10.1103/PhysRev.136.B1224
68. B.P. Abbott, et al., *Astrophys. J.* **818**(2), L22 (2016). DOI 10.3847/2041-8205/818/2/L22
69. J. Samsing, E. Ramirez-Ruiz, *Astrophys. J.* **840**(2), L14 (2017). DOI 10.3847/2041-8213/aa6f0b
70. C.L. Rodriguez, P. Amaro-Seoane, S. Chatterjee, K. Kremer, F.A. Rasio, J. Samsing, C.S. Ye, M. Zevin, *Phys. Rev.* **D98**(12), 123005 (2018). DOI 10.1103/PhysRevD.98.123005
71. L. Randall, Z.Z. Xianyu, *Astrophys. J.* **878**(2), 75 (2019). DOI 10.3847/1538-4357/ab20c6
72. D.J. D’Orazio, J. Samsing, *Mon. Not. Roy. Astron. Soc.* **481**(4), 4775 (2018). DOI 10.1093/mnras/sty2568
73. M. Arca-Sedda, G. Li, B. Kocsis, arXiv e-prints 1805.06458 (2018)
74. K. Kremer, et al., *Phys. Rev.* **D99**(6), 063003 (2019). DOI 10.1103/PhysRevD.99.063003
75. M. Zevin, J. Samsing, C. Rodriguez, C.J. Haster, E. Ramirez-Ruiz, *Astrophys. J.* **871**(1), 91 (2019). DOI 10.3847/1538-4357/aaf6ec
76. X. Chen, P. Amaro-Seoane, *Astrophys. J.* **842**(1), L2 (2017). DOI 10.3847/2041-8213/aa74ce
77. S.F. Portegies Zwart, S. McMillan, *Astrophys. J.* **528**, L17 (2000). DOI 10.1086/312422
78. M. Giersz, N. Leigh, A. Hypki, N. Lützgendorf, A. Askar, *Mon. Not. Roy. Astron. Soc.* **454**(3), 3150 (2015). DOI 10.1093/mnras/stv2162
79. M. Arca Sedda, A. Askar, M. Giersz, arXiv e-prints 1905.00902 (2019)
80. R. Abbott, et al., *Astrophys. J. Lett.* **900**(1), L13 (2020). DOI 10.3847/2041-8213/aba493
81. P. Amaro-Seoane, J.R. Gair, M. Freitag, M. Coleman Miller, I. Mandel, C.J. Cutler, S. Babak, *Class. Quant. Grav.* **24**, R113 (2007). DOI 10.1088/0264-9381/24/17/R01
82. D.A. Brown, H. Fang, J.R. Gair, C. Li, G. Lovelace, I. Mandel, K.S. Thorne, *Phys. Rev. Lett.* **99**, 201102 (2007). DOI 10.1103/PhysRevLett.99.201102
83. C.L. Rodriguez, I. Mandel, J.R. Gair, *Phys. Rev.* **D85**, 062002 (2012). DOI 10.1103/PhysRevD.85.062002
84. C.J. Haster, Z. Wang, C.P.L. Berry, S. Stevenson, J. Veitch, I. Mandel, *Mon. Not. Roy. Astron. Soc.* **457**(4), 4499 (2016). DOI 10.1093/mnras/stw233
85. J.H. Chen, R.F. Shen, *Astrophys. J.* **867**(1), 20 (2018). DOI 10.3847/1538-4357/aadfa
86. M. Eracleous, S. Gezari, A. Sesana, T. Bogdanovic, M. MacLeod, N. Roth, L. Dai, *Bull. Am. Astron. Soc.* **51**(3), 10 (2019)
87. J.M. Ezquiaga, D.E. Holz, *Astrophys. J. Lett.* **909**(2), L23 (2021). DOI 10.3847/2041-8213/abe638
88. M. Volonteri, P. Natarajan, *Mon. Not. Roy. Astron. Soc.* **400**, 1911 (2009). DOI 10.1111/j.1365-2966.2009.15577.x
89. B. McKernan, K.E.S. Ford, B. Kocsis, W. Lyra, L.M. Winter, *Mon. Not. Roy. Astron. Soc.* **441**(1), 900 (2014). DOI 10.1093/mnras/stu553
90. I. Bartos, B. Kocsis, Z. Haiman, S. Márka, *Astrophys. J.* **835**(2), 165 (2017). DOI 10.3847/1538-4357/835/2/165
91. N.C. Stone, B.D. Metzger, Z. Haiman, *Mon. Not. Roy. Astron. Soc.* **464**(1), 946 (2017). DOI 10.1093/mnras/stw2260
92. B. McKernan, et al., *Astrophys. J.* **866**(1), 66 (2018). DOI 10.3847/1538-4357/aadae5
93. X. Chen, S. Li, Z. Cao, *Mon. Not. Roy. Astron. Soc.* **485**(1), L141 (2019). DOI 10.1093/mnras/slz046
94. L. Gondán, B. Kocsis, P. Raffai, Z. Frei, *Astrophys. J.* **860**(1), 5 (2018). DOI 10.3847/1538-4357/aabfee

95. A. Secunda, J. Bellovary, M.M. Mac Low, K.E. Saavik Ford, B. McKernan, N. Leigh, W. Lyra, Z. Sándor, *Astrophys. J.* **878**(2), 85 (2019). DOI 10.3847/1538-4357/ab20ca
96. A. Rasskazov, B. Kocsis, *Astrophys. J.* **881**, 20 (2019). DOI 10.3847/1538-4357/ab2c74
97. Y. Yang, I. Bartos, Z. Haiman, B. Kocsis, Z. Marka, N.C. Stone, S. Marka, *Astrophys. J.* **876**(2), 122 (2019). DOI 10.3847/1538-4357/ab16e3
98. A. Kosowsky, M.S. Turner, *Phys. Rev.* **D47**, 4372 (1993). DOI 10.1103/PhysRevD.47.4372
99. M. Kamionkowski, A. Kosowsky, M.S. Turner, *Phys. Rev.* **D49**, 2837 (1994). DOI 10.1103/PhysRevD.49.2837
100. G. Gogoberidze, T. Kahniashvili, A. Kosowsky, *Phys. Rev.* **D76**, 083002 (2007). DOI 10.1103/PhysRevD.76.083002
101. C. Caprini, R. Durrer, G. Servant, *JCAP* **0912**, 024 (2009). DOI 10.1088/1475-7516/2009/12/024
102. M. Hindmarsh, S.J. Huber, K. Rummukainen, D.J. Weir, *Phys. Rev. Lett.* **112**, 041301 (2014). DOI 10.1103/PhysRevLett.112.041301
103. M. Hindmarsh, S.J. Huber, K. Rummukainen, D.J. Weir, *Phys. Rev.* **D92**(12), 123009 (2015). DOI 10.1103/PhysRevD.92.123009
104. L. Randall, G. Servant, *JHEP* **05**, 054 (2007). DOI 10.1088/1126-6708/2007/05/054
105. G. Nardini, M. Quiros, A. Wulzer, *JHEP* **09**, 077 (2007). DOI 10.1088/1126-6708/2007/09/077
106. T. Konstandin, G. Nardini, M. Quiros, *Phys. Rev.* **D82**, 083513 (2010). DOI 10.1103/PhysRevD.82.083513
107. T. Konstandin, G. Servant, *JCAP* **1112**, 009 (2011). DOI 10.1088/1475-7516/2011/12/009
108. S. Bruggisser, B. Von Harling, O. Matsedonskyi, G. Servant, *Phys. Rev. Lett.* **121**(13), 131801 (2018). DOI 10.1103/PhysRevLett.121.131801
109. E. Megias, G. Nardini, M. Quirós, *JHEP* **09**, 095 (2018). DOI 10.1007/JHEP09(2018)095
110. N. Arkani-Hamed, T. Han, M. Mangano, L.T. Wang, *Phys. Rept.* **652**, 1 (2016). DOI 10.1016/j.physrep.2016.07.004
111. P.W. Graham, D.E. Kaplan, J. Mardon, S. Rajendran, W.A. Terrano, *Phys. Rev.* **D93**(7), 075029 (2016). DOI 10.1103/PhysRevD.93.075029
112. T. Vachaspati, A. Vilenkin, *Phys. Rev.* **D31**, 3052 (1985). DOI 10.1103/PhysRevD.31.3052
113. J.J. Blanco-Pillado, K.D. Olum, B. Shlaer, *Phys. Rev.* **D89**(2), 023512 (2014). DOI 10.1103/PhysRevD.89.023512
114. P. Auclair, et al., *JCAP* **2004**, 034 (2020). DOI 10.1088/1475-7516/2020/04/034
115. S.A. Sanidas, R.A. Battye, B.W. Stappers, *Phys. Rev.* **D85**, 122003 (2012). DOI 10.1103/PhysRevD.85.122003
116. J.J. Blanco-Pillado, K.D. Olum, X. Siemens, *Phys. Lett.* **B778**, 392 (2018). DOI 10.1016/j.physletb.2018.01.050
117. P.L. Bender, M.C. Begelman, J.R. Gair, *Class. Quant. Grav.* **30**, 165017 (2013). DOI 10.1088/0264-9381/30/16/165017
118. G. Mueller, J. Baker, et al., *Bull. Am. Astron. Soc.* **51**(7), 243 (2019)
119. W.R. Hu, Y.L. Wu, *Natl. Sci. Rev.* **4**(5), 685 (2017). DOI 10.1093/nsr/nwx116
120. W.H. Ruan, Z.K. Guo, R.G. Cai, Y.Z. Zhang, *Int. J. Mod. Phys.* **A35**(17), 2050075 (2020). DOI 10.1142/S0217751X2050075X
121. K.A. Kuns, H. Yu, Y. Chen, R.X. Adhikari, *Phys. Rev.* **D102**(4), 043001 (2020). DOI 10.1103/PhysRevD.102.043001
122. J. Luo, et al., *Class. Quant. Grav.* **33**(3), 035010 (2016). DOI 10.1088/0264-9381/33/3/035010
123. S. Sato, et al., *J. Phys. Conf. Ser.* **840**(1), 012010 (2017). DOI 10.1088/1742-6596/840/1/012010
124. S. Kawamura, et al., arXiv e-prints 2006.13545 (2020)
125. J. Crowder, N.J. Cornish, *Phys. Rev.* **D72**, 083005 (2005). DOI 10.1103/PhysRevD.72.083005
126. S.T. McWilliams, arXiv e-prints 1111.3708 (2011)
127. M. Tinto, J.C.N. de Araujo, O.D. Aguiar, M.E. da Silva Alves, arXiv e-prints 1111.2576 (2011)

-
128. M. Tinto, D. DeBra, S. Buchman, S. Tilley, *Rev. Sci. Instrum.* **86**, 014501 (2015). DOI 10.1063/1.4904862
 129. S. Lacour, et al., *Class. Quant. Grav.* **36**(19), 195005 (2019). DOI 10.1088/1361-6382/ab3583
 130. G.M. Tino, et al., *Eur. Phys. J.* **D73**(11), 228 (2019). DOI 10.1140/epjd/e2019-100324-6
 131. S. Kolkowitz, I. Pikovski, N. Langellier, M.D. Lukin, R.L. Walsworth, J. Ye, *Phys. Rev.* **D94**(12), 124043 (2016). DOI 10.1103/PhysRevD.94.124043
 132. J. Su, Q. Wang, Q. Wang, P. Jetzer, *Class. Quant. Grav.* **35**(8), 085010 (2018). DOI 10.1088/1361-6382/aaebd2,10.1088/1361-6382/aab2eb. [Erratum: *Class. Quant. Grav.* 35,no.24,249501(2018)]
 133. P.W. Graham, J.M. Hogan, M.A. Kasevich, S. Rajendran, R.W. Romani, arXiv e-prints 1711.02225 (2017)
 134. Y.A. El-Neaj, et al., *EPJ Quant. Technol.* **7**, 6 (2020). DOI 10.1140/epjqt/s40507-020-0080-0
 135. M. Arca Sedda, et al., arXiv e-prints 1908.11375v1 (2019)
 136. A. Sesana, et al., arXiv e-prints 1908.11391 (2019)
 137. V. Baibhav, et al., arXiv e-prints 1908.11390 (2019)
 138. J. Baker, et al., arXiv e-prints 1908.11410 (2019)