Letter

Continuum approach to real time dynamics of $(1+1)$D gauge field theory: Out of horizon correlations of the Schwinger model

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We develop a truncated Hamiltonian method to study nonequilibrium real time dynamics in the Schwinger model—the quantum electrodynamics in $D = 1 + 1$. This is a purely continuum method that captures reliably the invariance under local and global gauge transformations and does not require a discretization of space-time. We use it to study a phenomenon that is expected not to be tractable using lattice methods: we show that the $(1 + 1)$D quantum electrodynamics admits the dynamical horizon violation effect which was recently discovered in the case of the sine-Gordon model. Following a quench of the model, oscillatory long-range correlations develop, manifestly violating the horizon bound. We find that the oscillation frequencies of the out-of-horizon correlations correspond to twice the masses of the mesons of the model suggesting that the effect is mediated through correlated meson pairs. We also report on the cluster violation in the massive version of the model, previously known in the massless Schwinger model. The results presented here reveal a novel nonequilibrium phenomenon in $(1 + 1)$D quantum electrodynamics and make a first step towards establishing that the horizon violation effect is present in gauge field theory.

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I. INTRODUCTION

Computing real time dynamics of an interacting many-body quantum system is a notoriously difficult problem. It has been currently getting an overwhelming amount of attention due to the fast developing field of nonequilibrium physics both in high energy [1–9] and condensed matter physics [10–12] on one side and renewed interest in chaos and information scrambling on the other side [13–17]. It is also becoming a matter of increased experimental importance [18–21]. The set of tools to deal with the problem has been greatly enriched by developments and new insights in integrability theory [22–24], holography [25–29] and numerical algorithms such as density matrix renormalization group (DMRG) [30,31], tensor networks (TNS) [32–34] and lattice gauge theory [35,36]. Although in the present time, there is an abundance of excellent numerical methods available for discrete systems, the methods for the real time evolution directly in the continuum remain scarce and less developed.

A powerful class of algorithms are the *truncated Hamiltonian methods* (THM) [37–45]. They are numerical methods for *quantum field theories* (QFT) that work in the continuum and do not require a discretization of space-time. They can be applied to a wide set of tasks like computing spectra [37,39–47] and level spacing statistics [48,49], studying symmetry breaking [45], correlation functions [50,51], real time dynamics [50–54] and also gauge field theories [55,56]. The class of methods originates from the *truncated conformal space approach* (TCSA) introduced by Yurov and Zamolodchikov [37]. A QFT model on a compact domain is regarded as point along the renormalization group flow from the *ultraviolet* (UV) fixed point generated by a relevant perturbation. The conformal field theory (CFT) algebraic machinery is used to represent the Hamiltonian as a matrix in the basis of the UV fixed point CFT Hilbert space. Finally, an energy cutoff is introduced to obtain a finite matrix which enables numerical computation that indeed efficiently captures nonperturbative effects. More broadly, instead of CFT, any solvable QFT can be used as the starting point for the expansion.

One of the central properties of quantum physics out of equilibrium is the *horizon effect* introduced by Cardy and Calabrese [57–59]. A quantum system is initially prepared in a short range correlated nonequilibrium state, $\langle O(x)O(y)\rangle \propto e^{-|x-y|/\xi}$ with a local observable O, the

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correlation length ξ, and let to evolve dynamically for $t > 0$—a protocol commonly termed a *quantum quench*. The horizon bound states that the connected correlations following the quench spread within the horizon: $|\langle O(t,x)O(t,y)\rangle| < \kappa e^{-\max\{(|x-y|-2ct)/\xi_h,0\}}$ for some constant κ, where ξ_h is called the *horizon thickness* and c is the maximal velocity of the theory—speed of light in QFT and the Lieb-Robinson velocity in discrete systems [60]. The intuition is that correlations spread by pairs of entangled particles created in initially correlated region $|x-y| \lesssim \xi$ and traveling to opposite directions. This bound has been rigorously proven in CFT [57,58,61] and demonstrated, analytically and numerically, in a large set of interacting systems [61–97] as well as observed in experiments [98–100]. It has therefore been believed to be a universal property of quantum physics.

In a recent publication together with Sotiriadis and Takács [51], we have demonstrated that the horizon bound can be violated in QFT with nontrivial topological properties. We have proved this in the case of the *sine-Gordon* (SG) field theory, a prototypical example of strongly correlated QFT:

$$\mathcal{L}_{\mathrm{SG}} = \frac{1}{2}(\partial_\mu\Phi)(\partial^\mu\Phi) + \frac{\mu^2}{\beta^2}\cos(\beta\Phi). \tag{1}$$

Starting from short range correlated states, SG dynamics within a short time generates infinite range correlations oscillating in time and clearly violating the horizon bound. The mechanism is the following: Quenches in the SG model create cluster violating four-body correlations between solitons (S) and antisolitons (A), the topological excitations of the theory, written schematically:

$$\lim_{|x-y|\to\infty}\langle A(x)S(x+a)A(y)S(y+b)\rangle \neq \langle A(x)S(x+a)\rangle\langle A(y)S(y+b)\rangle. \tag{2}$$

The dynamics of the model then converts these solitonic correlations into two-point correlations of local bosonic fields $\langle\Phi(t,x)\Phi(t,y)\rangle$, $\langle\Pi(t,x)\Pi(t,y)\rangle$ and $\langle\partial_x\Phi(t,x)\partial_y\Phi(t,y)\rangle$. There is no violation of relativistic causality involved because the cluster violating correlations (2) are created by a quench, a global simultaneous event and not by the unitary dynamics of the model which is strictly causal. The horizon violation effect is graphically represented in Fig. 1. The mechanism of the effect suggests that the horizon violation should be found in any QFTs with nontrivial field topologies, an important class of them being gauge field theories. The results presented in this manuscript represent the first steps towards establishing that.

As a consequence of the Lieb-Robinson bound [60,101,102] and the Araki theorem [103,104], the horizon violation is expected not to be present in short-range interacting discrete systems with finite local Hilbert space dimension and is likely a genuinely field theoretical phenomenon. Therefore discretizing a model and simulating using DMRG or TNS [35,36,105–112] is not an option so methods working directly in the continuum are needed and THM seem to be the best class of methods for the task.

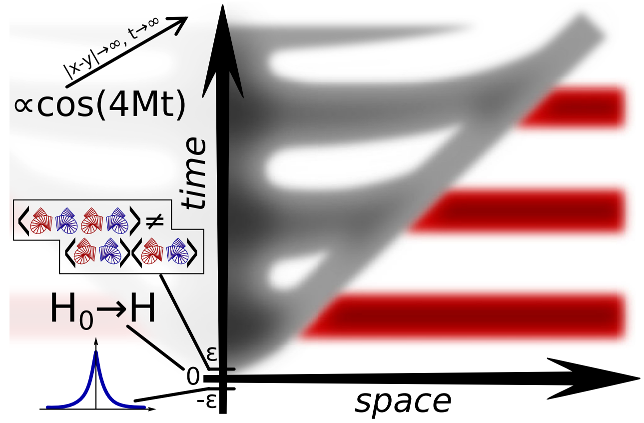

FIG. 1. Dynamical horizon violation as found in the sine-Gordon model [51]. The system is prepared in the ground state of a gaped Hamiltonian H_0 with short range correlations $\propto e^{-|x-y|/\xi}$. At time $t = 0$ the Hamiltonian is quenched to H. This generates cluster violating four-point correlations of solitons and antisolitons, Eq. (2) (here symbolically pictured using classical solitons), which are not observable at $t = 0$ but result in oscillating out-of-horizon correlations of local observables $\langle O(-x)O(x)\rangle$ at later times. The horizon is depicted here with gray color and the horizon violating correlations with red. Asymptotically, the latter oscillate with a frequency 4 times the soliton mass M, respectively twice the breather masses in the attractive regime.

II. THE SCHWINGER MODEL

We focus here on the simplest example of a gauge field theory, the $1+1$D *quantum electrodynamics* (QED), i.e., the (massive) *Schwinger model*:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu\partial_\mu - e\gamma^\mu A_\mu - m)\Psi,$$

with $\Psi = (\Psi_-, \Psi_+)^T$ the Dirac fermion, m the electron mass and e the electric charge. As a consequence of invariance under large gauge transformations, the model has infinitely degenerate vacuum states, the θ vacua for a parameter $\theta \in [0, 2\pi)$ that enters the bosonized form of the Hamiltonian and plays the physical role of the constant background electric field [113,114]. The Schwinger model thus has two physical parameters, the ratio m/e and θ.

The massless $m = 0$ version of the model was solved exactly by Schwinger [115] and has a gap of $e/\sqrt{\pi}$ corresponding to a meson, a bound state of a fermion and an antifermion. The full massive $m > 0$ version of the model is not integrable and has a rich phase diagram where the number of mesons depends on the values of the parameters m/e and θ [105,106,113,114,116–128].

The Schwinger model displays confinement and has been extensively studied for pair creation and string breaking [108–113,129–137].

Finally, it is known that due to the vacuum degeneracy, the massless version of the Schwinger model exhibits cluster violation of correlators of chiral fermion densities $\rho_\pm(x) = N[\bar{\psi}(x)\frac{1\pm\gamma^5}{2}\psi(x)]$ [138–140],

$$\langle \rho_-(x_1)\cdots\rho_-(x_n)\rho_+(y_1)\cdots\rho_+(y_n)\rangle, \quad (3)$$

closely related to the correlators from Eq. (2). This makes the model a good candidate for the horizon violation. The cluster violation is also intimately related to confinement of gauge theories [141–144].

Here we study general quenches of the massive Schwinger model and focus on the spreading of the current-current correlators:

$$C_\mu(t,x,y) = \langle J^\mu(t,x)J^\mu(t,y)\rangle. \quad (4)$$

We prepare the system in the ground state of the model with the prequench values of the parameters $m_0/e_0, \theta_0$ and at time $t=0$ switch the parameters to their postquench values m/e, θ.

III. THE METHOD

We implement a THM for the Schwinger model in finite volume L with antiperiodic boundary conditions (Neveu-Schwarz sector). We eliminate the gauge redundancy of degrees of freedom alongside the bosonization of the model [145].

Choosing the Weyl (time) gauge, $A_t = 0$, and defining $A \equiv A_x$, the Hamiltonian of the model is $H = \int_0^L dx(\frac{1}{2}\dot{A}^2 - \bar{\Psi}[\gamma^1(i\partial_x - eA) - m]\Psi)$. Expanding the fermion currents $J_\sigma(x) = \Psi_\sigma^\dagger(x)\Psi_\sigma(x) = \frac{1}{L}[Q_\sigma - \sigma\sum_{n>0}\sqrt{n}(b_{\sigma,n}e^{-\sigma in\frac{2\pi}{L}x} + b_{\sigma,n}^\dagger e^{\sigma in\frac{2\pi}{L}x})]$, with the chirality $\sigma = \pm$, its modes obey bosonic canonical commutation relations. Further defining the N_σ vacua as $|0;N_-\rangle \equiv \prod_{n=N_-}^\infty c_{-,n}^\dagger|0\rangle$, $|0;N_+\rangle \equiv \prod_{n=-\infty}^{N_+-1} c_{+,n}^\dagger|0\rangle$, with $c_{\sigma,n}$ the fermion mode operators, the Hilbert space spanned by bosonic modes $b_{\sigma,n}^\dagger$ on top of $|0;N_-\rangle \otimes |0;N_+\rangle$ is equivalent to the Hilbert space spanned by $c_{\sigma,n}^\dagger$ acting on top of $|0\rangle$. This is the foundation for the bosonization of the model. Because of the invariance under large gauge transformations, the true vacua of the system are the infinitely degenerate θ vacua $|\theta\rangle = \sum_{N\in\mathbb{Z}} e^{-iN\theta}|0;N\rangle$ for $\theta \in [0,2\pi)$. Gauge invariance further implies that the only mode of the EM potential A that is not fixed by the Gauss law is the zero mode $\alpha = \frac{1}{L}\int_0^L dx A(x)$ along with its dual $i\partial_\alpha = \int_0^L dx\dot{A}(x)$.

By setting $B_0 = \sqrt{\frac{1}{2ML}}(-\sqrt{\pi}\{Q_+ - Q_-\} + \frac{\partial}{\partial\alpha})$, the part of the Hamiltonian involving the zero modes transforms into a harmonic oscillator with the mass $M = \frac{e}{\sqrt{\pi}}$. Complemented with a Bogoliubov transform of the non-zero momentum modes into massive bosonic modes, $B_{\sigma,n} = \frac{1}{2}(\frac{\sqrt{E_n}}{\sqrt{k_n}} + \frac{\sqrt{k_n}}{\sqrt{E_n}})b_{\sigma,n} - \frac{1}{2}(\frac{\sqrt{E_n}}{\sqrt{k_n}} - \frac{\sqrt{k_n}}{\sqrt{E_n}})b_{-\sigma,n}^\dagger$, with $k_n = \frac{2\pi n}{L}$ and $E_n = \sqrt{M^2 + k_n^2}$, the massless part of the Hamiltonian is transformed into the Hamiltonian of a massive free boson with the mass M. The mass term of the Hamiltonian is written in the bosonic form using the bosonization relation $\Psi_\sigma(x) = F_\sigma\frac{1}{\sqrt{L}}:e^{-\sigma i(\sqrt{4\pi}\Phi_\sigma(x) - \frac{\pi}{L}x)}:$ with $\partial_x\Phi_\sigma(x) = \sqrt{\pi}J_\sigma(x) + \frac{\sigma e}{2\sqrt{\pi}}A(x)$ the chiral boson field and F_σ the Klein factor. Then using $F_\sigma^\dagger F_{-\sigma}|\theta\rangle = e^{\sigma i\theta}|\theta\rangle$, the Schwinger model Hamiltonian takes the bosonized form,

$$\begin{aligned} H &= H_M + U, \\ H_M &= M(B_0^\dagger B_0) + \sum_{n>0} E_n(B_{+,n}^\dagger B_{+,n} + B_{-,n}^\dagger B_{-,n}), \\ U &= -\frac{mM}{2\pi}e^\gamma\int_0^L dx\,:\cos(\sqrt{4\pi}\Phi(x) + \theta):_M \end{aligned} \quad (5)$$

with $\Phi(x) = \Phi_- + \Phi_+$ with Bogoliubov transformed modes, $:\bullet:_M$ denotes normal ordering with respect to the mass M and γ is the Euler-Mascheroni constant.

The form of the Hamiltonian (5) offers a natural THM splitting into the massive free part and the cosine potential. To implement the numerical method, the cosine potential and the observables, are represented as matrices in the Hilbert space of the free part—the Fock space generated by applying the $B_{\sigma,n}^\dagger$ modes on the θ vacuum. Finally, an energy cutoff $\langle\Psi|H_M|\Psi\rangle \leq E_{\mathrm{cut}}$ is imposed on the states $|\Psi\rangle$ of the THM Hilbert space. Momentum conservation implied by translation invariance and the decoupling of the B_0 mode from the rest of the modes are used to further reduce the dimension of the Hilbert space by diagonalizing each sector separately. We use the Hilbert spaces with up to 20 000 states per sector. The full details of the method can be found in the Supplemental Material [146].

IV. RESULTS

Our THM implementation of the Schwinger model recovers the results from the literature for the meson masses and gives a region of highly dense states above them, referred to as the continuum in the $L \to \infty$ limit (Fig. 2). This serves as a sanity check of the method. We are able to get the masses of the vector meson precisely, while our THM method seems to be slightly less precise for the scalar meson mass. We have been able to simulate large system sizes $L \gg \frac{1}{M}$ where the finite size effects are exponentially suppressed.

The results shown in Fig. 3 indeed confirm that the Schwinger model exhibits the horizon violation effect—the correlation functions $C_x(t,x,y)$ are nonzero and oscillating for $|x-y| > 2t$. The effect is found in quenches in both e/m and θ as well as in quenches to and from the massless Schwinger model. The sign of the out-of-horizon

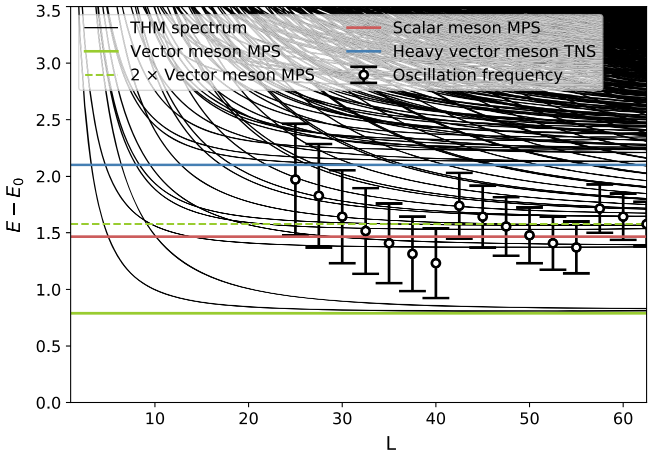

FIG. 2. The THM spectrum the Schwinger model at $m/e = 0.125$ in dependence of the system size L in the 0, 1 and 2 sectors of the total momentum. The spectral lines are compared with the $L \to \infty$ results of the matrix product states (MPS) computations [127] for the vector and the scalar particles and the TNS [128] for the heavy vector particle. On top of the spectrum, the dominant frequency of the oscillations of the out-of-horizon correlations are plotted.

correlations changes depending on whether the quenched parameter is increased or decreased. As is expected for periodic boundary conditions, the effect is present in the C_x and not present in the C_t channel.

To shed light on the origin of the effect, we study the clustering properties of correlators of chiral densities (3) (Fig. 3, lower right), more specifically, its component $\langle \psi^\dagger_\sigma(x)\psi_{-\sigma}(x)\psi^\dagger_{-\sigma}(y)\psi_\sigma(y)\rangle$. We find that the correlator violates clustering—when x and y are far apart, the correlator does not cluster into $\langle \psi^\dagger_\sigma(x)\psi_{-\sigma}(x)\rangle\langle \psi^\dagger_{-\sigma}(y)\psi_\sigma(y)\rangle$. In the case of the massless Schwinger model, this clustering violation is well known and can be computed analytically [138–140], in the case of the massive version of the model, this is to our knowledge a new result. Interestingly, in the massless case, the normal ordered version of the correlator does not exhibit the clustering violation while in the massive model, even the normal ordered correlator violates clustering. We expect that similarly as in the SG model [51], the nonlinear postquench dynamics rotates the initial clustering violation from such chiral correlators into the local nonchiral observables. We note that in the case of the ground states of the massive model, we observe numerically a tiny clustering violation also in the C_x correlators which is 2 orders of magnitude smaller than the cluster violation of $\langle \rho_\sigma \rho_{-\sigma}\rangle$. We expect, however, that this is not a physical fact but an artifact of the THM truncation. Such tiny artifacts are common in derivative fields but do not falsely produce the horizon violation effect, as was for example verified in the case of Klein-Gordon dynamics in the first version of [51,147]. As well as that, our THM simulation of the Schwinger model displays the horizon violation in the quenches starting from the massless model, where there are no such artifacts in the initial state. So we expect that the effect originates fully from the cluster violation of the chiral terms.

Figure 2 shows how the dominant frequencies of the oscillations compare to the spectrum of the model. Due to simulation times limited to $t \leq L/4$, we are only able to see a few oscillations. Therefore, the frequencies have considerable error bars ($\Delta\omega \approx 2\pi/L$—half a frequency bin) and the values of the possible discrete frequencies move with L resulting in a chainsaw pattern. The error bars compare to both the scalar meson mass and twice the vector meson mass. Based on the mechanism of the effect in the SG

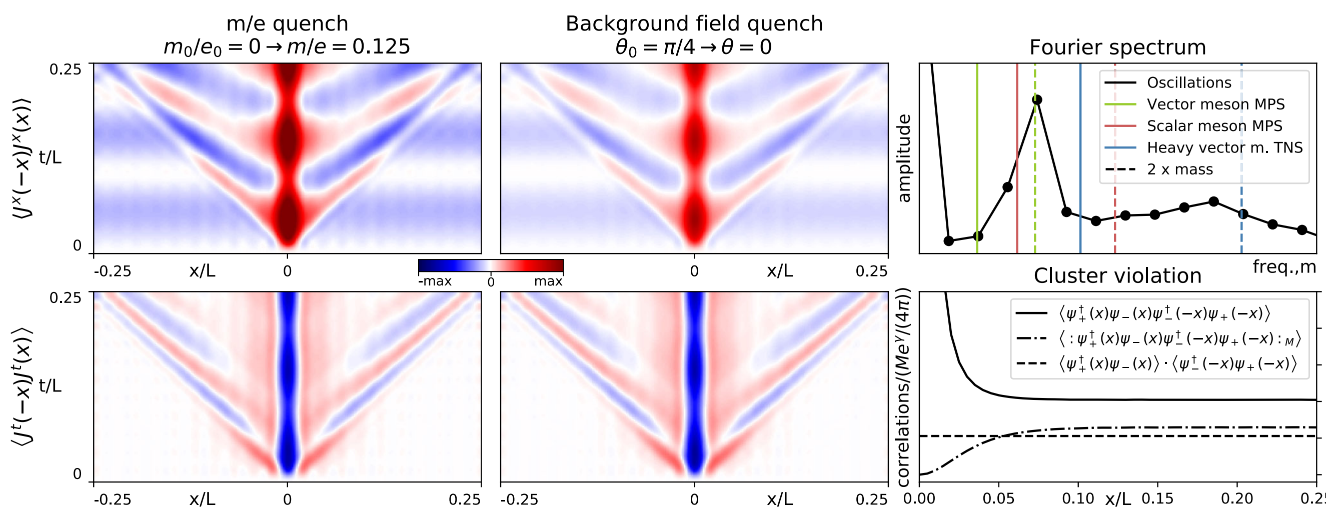

FIG. 3. Left: time dependent $\langle J^x(t,x)J^x(t,y)\rangle$ and $\langle J^t(t,x)J^t(t,y)\rangle$ correlations for different types of quenches in the Schwinger model (initial correlations subtracted): (1) Quench in m/e with $m_0 = 0$, $m = 0.125$, $\theta_0 = \theta = 0$; (2) Quench in θ with $\theta_0 = \frac{\pi}{4}$, $\theta = 0$, $m_0 = m = 0.125$. Both with $e_0 = e = 1$, $L = 40$. Upper right: frequency spectrum of the out-of-horizon component of the correlations (mass quenches to $m = 0.25$, $e = 1$, $L = 47.5$) compared to meson masses (full lines) and twice the values of meson masses (dashed lines). Lower right: cluster violation in the massive Schwinger model at $m = 0.125$, $\theta = 0$, $e = 1$, $L = 40$.

model [51], it is expected that the frequencies correspond to twice the mass of the lightest meson. This is supported by computations at higher values of m/e, where those masses can be better discriminated (Fourier spectrum in the upper right of Fig. 3). This suggests that the horizon violation is mediated through correlated vector meson pairs entangled by the quench. In some cases even subdominant peaks appear close to twice the masses of heavier mesons in the frequency spectra, suggesting that they could also be contributing to the effect.

V. DISCUSSION

We stress again that the observed phenomenon is in no contradiction with relativistic causality as guaranteed by the Lorentz invariance of the model the microcausality of the fields. Rather, the violation of horizon can be likely traced back to the cluster violation of chiral fermion fields as in the SG model [51].

Using the simplest representative, we have hereby demonstrated that the horizon violation occurs in gauge field theory. In the future, it would be interesting to explore higher gauge theories like SU(2) or SU(3) or study the Wess-Zumino-Witten models. It would be of crucial importance to answer whether the effect is present also in $D > 1 + 1$. There, gauge fields are dynamical, so the physics could be drastically different. The key to finding such generalizations could be a complete classification of the topological conditions for the horizon violation. This requires further analytical approaches which should also be found to get a better understanding of the effect in the Schwinger model.

The horizon violation presented in this work is a novel phenomenon in $(1 + 1)$D quantum electrodynamics. It is reasonable to expect that it could have interesting physical implications, in particular if it turns out that the effect is present also in higher dimensions. In condensed matter physics, phase transitions are an ubiquitous phenomenon and could serve as a trigger for horizon violation generating quenches. Here, already the $D = 1 + 1$ case could be an interesting candidate since at the present day there are numerous experiments available for probing $(1 + 1)$D physics [10]. An especially important class are ultra cold atoms in atom chips, where one dimensional QFTs are directly realised and correlation functions can be measured both in equilibrium states and nonequilibrium dynamics [148]. In cosmology, there several candidates for quenches like the end of inflation, the QCD and the electroweak transitions and topological symmetry breaking in grand unified theories [149–152]. Consider also the following example illustrated in fig. 4: a toy universe is created with an anisotropic initial condition—a nonzero background electric field. This is a possibility since the zero background field case is a special, fine-tuned, value. In $D = 1 + 1$ the background electric field is stable while in $D = 1 + 3$, it decays through the electric breakdown of

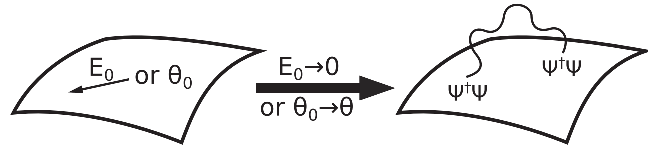

FIG. 4. Decay of the anisotropic initial condition or a θ term in a toy universe as a quench that generates long range correlations through the horizon violation effect. Long range correlations are the price that the toy universe has to pay for the initial anisotropy.

the vacuum [114]. The rapid decay of the background electric field would serve as a quench that causes a horizon violation effect in the QED degrees of freedom as we have seen here in the $\theta_0 \neq 0 \to \theta = 0$ quenches. This transforms the initial anisotropy of the toy universe into long range correlations. Similarly, in a higher gauge theory the effect could be triggered by a decay of the theta term which is linked in some models with the cosmological constant [153,154]. It would be interesting to explore the possible predictions for traces of this effect in the cosmic microwave background.

What concerns possible realizations of the horizon violation effect in discrete systems, lattices, the Lieb-Robinson bounds [60,101,102] guarantee that the quench dynamics of a locally interacting theory starting from a clustering initial state will preserve clustering at all times, the lattice analog of causality, and the Araki theorems [103,104] constrain the thermal states of locally interacting systems with finite dimensional local Hilbert spaces in $D = 1 + 1$ to cluster (in $D > 1 + 1$ an analog holds for a high enough temperature). Since in lattices, the tensor product of any complete algebras of local operators forms globally a complete algebra, it is not possible that clustering of a state is lost through a change of basis. This is prohibitive for the possibility of the horizon violation in the strict sense where the initial state clusters in terms of the prequench degrees of freedom but not in terms of the postquench degrees of freedom. However, if this condition is relaxed and nonclustering initial states are allowed, such states can be found, see for example [155]. One could further look for such states through construction of some global superselection rules analogous to the ones imposed by Klein factors (or zero modes) in the case of the SG and Schwinger models [51,140] or through nonlocal interactions as for example appearing in the light-cone discretizations of field theories [156]. It would be useful to find general conditions for cluster violations and obtain a further understanding of the relation between the continuum and the discrete through the perspective of the horizon violation effect.

Finally, it would be interesting to use THM to explore the confinement and string breaking phenomena in the Schwinger model and to use THM implementations [56] to study dynamics of higher gauge theories.

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