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Narrowing Uncertainty of Projections of the Global Economy-Climate System Dynamics via Mutually Compatible Integration within Multi-Model Ensembles

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Abstract

Any model used to derive projections of future climate or assess its impact constitutes a particular simplification of reality. To date, no model building process can guarantee full “objectivity” in the choice of model assumptions and parameterization. In this connection, researchers have introduced a number of stylized integrated assessment models, which attempt to represent the full time-dynamic non-linear causal loop between accumulated emissions, economy and climate, yet in a aggregated, simplified fashion to enable extensive uncertainty analysis with respect to both structural and parametric uncertainty.

In this work, we put forward a simplified system dynamics integrated assessment model which simulates the global economic growth, corresponding emissions, global warming and caused by its secondary effects economic losses. While generally our model follows the same logic as DICE and other models of this kind, it pays more attention to the mechanism of the emission reduction. Mitigation is assumed to be done through the allocation of a certain fraction of the total output into enhancing carbon and energy efficiency. The model enables exploring effects of mitigation scenarios defined via carbon tax. We explore the structural sensitivity by examining five alternative climate sensitivity functions and use the "mutual compatibility integration" approach to synthesize the information from the five alternative model versions.

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1 Introduction

Adoption of the Global Sustainability Goals by the 193 countries of the UN General Assembly in 2015 explicitly acknowledged the countries' commitment to include mitigation and adaptation to major global environmental changes, notably climate change, into their national policies. Science is challenged to provide information relevant to societal decision-making, and when models are expected to provide input, appropriate interpretation of their results becomes very important (Stainforth et al., 2007). Decision-makers would like science to provide exact and unique numbers on future climate conditions, but this demand is contrasted by uncertainties inherent in any future climate projection (Weigel et al., 2010).

Any model used to derive projections of future climate or assess its impact constitutes a particular simplification of reality. To date, no model building process can guarantee full "objectivity" in the choice of model assumptions and parameterization; instead, this choice is typically done by researchers based on their expert knowledge and experience (Knutti, 2010). Any uncertainty due to choices made in the model design, i.e. going beyond uncertainty in parameter values, is usually referred to as structural uncertainty (Tebaldi and Knutti, 2007).

Thus, multiple models have been put forward by individual scientific groups to represent and analyze parts of the complex dynamic climate-society system. Since there is a little opportunity to verify future climate forecasts on the timescale of decades, the skill or performance of models is often defined by comparing simulated patterns of present-day climate or its impact to observations. Such performance metrics are useful but not unique, and often it is unclear how they relate to future projections. Defining a set of criteria for a model to be 'credible', or agreeing on a quality metric is therefore difficult (IPCC, 2010).

Using collective information for decision-making is common sense in both everyday life and professional business (Hagedorn, 2005; Knutti, 2010). In particular, the greater the complexity of the involved processes, the more helpful the input to decision-making might be (Branzei et al., 2000). On the other hand, overload of possibly contradictory information can lead to suboptimal decisions. In general decision-making theory, it is under debate whether more information leads to more success or whether 'simplicity rules the world'. For example, in short- and medium-range weather forecasting it has been demonstrated, in the early 1960s, that combining different forecasts from individual forecasters can be beneficial (Hagedorn, 2005; therein one can see also citations of original works).

Estimates of impacts from anthropogenic climate change rely on projections from climate models. Uncertainties in those have often been a limiting factor (Knutti and Sedlacek, 2013). For time horizons of several decades or longer, the dominant sources of uncertainty at regional or larger spatial scales are model uncertainty and scenario uncertainty (Tebaldi et al., 2004; Weigel et al., 2010). Weigel et al. (2010) indicate that in practice, the scenario uncertainty is addressed by explicitly conditioning climate projections on a range of well-defined scenarios (concentration, e.g., RCP, scenarios - Moss et al., 2010; van Vuuren et al., 2011).

Some processes in the climate system are not fully understood or are impossible to resolve due to computational constraints (IPCC, 2010), which leads to the model uncertainty (also known as response uncertainty). That is, in response to the same radiative forcing, different models simulate somewhat different changes in climate (Hawkins and Sutton, 2009). The consequences of model uncertainty are receiving particular attention (Stainforth et al., 2007). A pragmatic and well-accepted approach to addressing model uncertainty is given by the concept of multi-model combination (Tebaldi and Knutti, 2007, Weigel et al., 2010).

An important issue for decision makers is to what degree the uncertainty can be narrowed down through investments in science – because the costs of adaptation are expected to be very large, the clear implication is that reducing uncertainty in climate predictions is potentially of enormous economic value (Hawkins and Sutton, 2009).

Using multiple models as a tool to potentially narrow uncertainty was pioneered by climate science. Improving forecasts and projections by combining models rests on the assumption that if models are independent, their errors might at least partly cancel, resulting in a multi-model average that is more skillful than its constitutive terms (Tebaldi and Knutti, 2007). Progress in science may sometimes broaden rather than narrow uncertainty (Hawkins and Sutton, 2009); the heterogeneity in the new generation of climate models and an increasing emphasis on estimates of uncertainty in the projections raise questions about how best to evaluate and combine model results in order to improve the reliability of projections (IPCC, 2010).

Using probabilistic approach to represent uncertainties becomes common. Tebaldi et al. (2004) point that determining probabilities of future global temperature change has flourished as a research topic in recent years. Exploration of uncertainty can be done by sampling of uncertain initial model states, parameter values or structural differences (IPCC, 2010).

A systematic approach was proposed to explore the uncertainty of a single climate model to model parameterization, a so-called perturbed physics ensemble (PPE) (Murphy et al., 2004; Stainforth et al., 2005; Tebaldi and Knutti, 2007). In each experiment, model parameters are set to a range of values derived from multiple prior distributions estimated by experts. For example, in climateprediction.net study (Sanderson et al., 2008), 15 parameters of a single model, HadSM3, were perturbed. There is certainly tremendous value in exploring parametric uncertainties by the PPE approach, and its success might be partly related to the simplicity of generating those ensembles. Apart from the enormous computational capacity required, this exploration of the parameter space is rather straightforward (Tebaldi and Knutti, 2007). Clearly, the PPE approach is limited in its ability to capture the full range of uncertainties in the models' representation of the true climate system, as there are many ways to design a parametrization (Tebaldi and Knutti, 2007). Multi-model combination is a pragmatic approach to estimating model uncertainties and to making climate projections more reliable (Weigel et al., 2010). Stainforth et al. (2007) consider climate ensembles exploring model uncertainty as potentially providing a lower bound on the maximum range of uncertainty. A variety of applications, not only limited to the weather and climate prediction problems (e.g., Palmer et al., 2005), have demonstrated that combining models generally increases the skill, reliability and consistency of model forecasts. Examples include model forecasts in the sectors of public health (e.g., malaria; Thomson et al., 2006) and agriculture (e.g., crop yield; Cantelaube and Terres, 2005), where the combined information of several models is reported to be superior to a single-model forecast Tebaldi and Knutti (2007).

Tebaldi and Knutti (2007) sketch a history of analyzing model uncertainty. Namely, in 1990, the Atmospheric Model Intercomparison Project (AMIP; Gates, 1992) developed a standard experimental protocol for atmospheric general circulation models (GCMs). For the first time, a systematic framework in support of model diagnosis, validation and intercomparison was put forward and since then the international community of climate modelling has participated and benefited from it widely. The natural follow-up to AMIP was CMIP, the Coupled Model Intercomparison Project (Meehl et al., 2000), whereby the output from coupled atmosphere–ocean general circulation models (AOGCMs) became the object of study. Output from the control runs and 1% (annual) increase CO_2 experiments represent the most scientifically straightforward response of the climate system to an unambiguous change in external forcing (Meehl et al., 2007). This ever increasing availability of model experiments under common scenarios, whose output is standardized and to which access is facilitated, has naturally inspired the analysis of multi-model ensembles since the beginning of 2000. Subsequently there have been several next phases of CMIP (IPCC, 2010). Sillmann et al. (2013a, 2013b) provide a first overview of the performance of state-of-the-art global climate models participating in CMIP5 in simulating climate extremes indices.

According to Tebaldi and Knutti (2007), Raisanen (1997) is probably the first one to explicitly advocate the need of quantitative model comparison and the importance of inter-model agreement in assigning confidence to the forecasts of different models. But it was only Raisanen and Palmer (2001) who first proposed a probabilistic view of climate change projections on the basis of multi-model experiments. On this basis, they evaluated probabilities of threshold events such as 'the warming at the time of doubled CO_2 will be greater than 1C° by computing the fraction of models that simulated such an event.

In its simplest form, a multi-model ensemble forecast is produced by merging individual forecasts with equal weights (Hagedorn et al., 2005). However, more complex methods of optimally combining single-model outputs have been described (Krishnamurti et al., 1999; Pavan and Doblas-Reyes, 2000; Rajagopalan et al., 2002). Even an equally weighted average of several coupled climate models is usually found to agree better with observations than any single model (Tebaldi and Knutti, 2007; Lambert and Boer, 2001). In case of seasonal forecast models, the multi-model approach improves both deterministic and probabilistic performances of seasonal predictions compared to single-model forecasts (Hagedorn et al., 2005).

There are obviously different ways to derive model weights. In many cases, Bayesian methods (e.g., Robertson et al., 2004; Tebaldi and Knutti, 2007) where weights are determined by using the historical relationship between forecasts and observations perform better than simple averages. Intuitively, it makes perfect sense to trust, and thus weigh, the better models more. The difficulty, however, is in quantifying model skill and deriving model weights accordingly. Referring to Min and Hense (2006), Tebaldi and Knutti (2007) highlight that for a given metric and for present day climate, weighted averages of models compare better to observations than simple averages. However, they point that it seems to be rather unlikely, that the weights for future projections should be the same as those derived for present-day climate. The problem of constructing a weighted average for climate projection, where no verification is available, is discussed in Tebaldi and Knutti (2007).

Weigel et al. (2010) review proposed metrics serving as a basis for model weights, including the magnitude of observed systematic model biases during the control period (Giorgi and Mearns, 2002; Giorgi and Mearns, 2003; Tebaldi et al., 2005), observed trends (Greene et al., 2006; Hawkins and Sutton, 2009; Boe et al., 2009), or composites of a larger number of model performance diagnostics (Murphy et al., 2004), and promote an approach to weighting models from the angle of the expected error of the final outcome. They present results that confirm that equally weighted multi-models on average outperform the single models, and that projection errors can in principle be further reduced by optimum weighting. However, this not only requires accurate knowledge of the single model skill, but the relative contributions of the joint model error and unpredictable noise also need to be known to avoid biased weights. If weights are applied that do not appropriately represent the true underlying uncertainties, weighted multi-models perform on average worse than equally weighted ones, which is a scenario that is not unlikely, given that at present there is no consensus on how skill-based weights can be obtained. These results indicate that for many applications equal weighting may be the safer and more transparent way to combine models.

In addition, multi-model ensemble members may not represent estimates of the climate system behavior (trajectory) entirely independent of one another, for example, this is likely true of members that simply represent different versions of the same model or use the same initial conditions (IPCC, 2010).

In light of these complexities, IPCC (2010) formulated their Recommendations for Model Selection, Averaging and Weighting, including

- There should be no minimum performance criteria for entry into the multi-model database;
- Testing methods in perfect model settings (i.e., one model is treated as observations with complete coverage and no observational uncertainty) is encouraged;
- Arguments for providing code are full transparency of the analysis and that discrepancies and errors may be easier to identify;
- Options for information from multi-model simulations could include medians, percentile ranges of model outputs, scatter plots of temperature, precipitation and other variables, regions of high/low confidence, changes in variability and extremes, stability of teleconnections, decadal time-slices, and weighted and unweighted representations of any of the above.

Eventually, climate predictions are sued to assess its impacts onto different sectors of the economy at different locations (e.g., Hayhoe et al., 2006). Water-MIP, ISI-MIP, Ag-MIP etc., extend the model inter-comparison to other dimensions relevant to integrated assessment. For example, ISI MIP project focuses on providing cross-sectorial global impact assessments, Water-MIP focuses on inter-comparison of the land surface hydrology models and global hydrology models. To name a few other studies – Kriegler et al. (2014) carried out inter-comparisons of energy-economy models and IAMs in their Energy Modeling Forum studies; using multi-model ensembles, Semenov and Stratonovitch (2010) evaluated the impact of climate change on the probability of heat stress during flowering of wheat, which can result in significant yield losses.

In all above works, attempts have been made to work with detailed models, which take into account as many aspects of the processes as the current knowledge and computing power allow. This is achieved by modeling only a part of a system – normally in models discussed above the feedback between the climate change and economy, and therefore, emissions, is missing. On the other hand, scientists put forward a number of stylized integrated assessment models, which attempt to represent the full time-dynamic non-linear causal loop between accumulated emissions, economy and climate, yet in a aggregated, simplified fashion. The advantage of stylized IAMs is that they enable extensive uncertainty analysis with respect to both structural and parametric uncertainty – contrasting with large models which have reached a level of complexity prohibiting a large ensemble of perturbed initial condition simulations with each model with current computational resources. Stylized IAMs can be used to construct mitigation scenarios, which can then be tested by more detailed climate models (e.g., Johns et al., 2011).

Of these, Nordhaus's DICE model family (Nordhaus and Boyer, 2000; Nordhaus, 2008; Nordhaus, 2013) has been perhaps the most influential. Here, in presence of climate change, the world output is reduced directly by the climate damages and indirectly by diverting a part of the available funds for abatement instead investing them in growth. The climate damages are parameterized by a climate damage function gradually increasing with the increase in the global mean surface air temperature. Abatement measures aim at reducing GHG emissions with the costs of emissions reductions being a function of the emissions-reduction rate. The global warming is caused by GHG emissions, of which only the industrial CO_2 emissions are endogenous, while all other GHG emissions (including CO_2 emissions arising from land-use changes) are exogenous. The climate module includes the carbon cycle represented by a three-reservoir model: the atmosphere, the upper oceans and the biosphere, and the deep oceans. Radiative forcing/climate sensitivity link the GHG concentrations and the dynamics of the mean temperature.

There are available in literature a vast number of alternative models of different complexity and focus. For example, Greiner (2005) uses a very similar modeling framework to DICE to study the effects of cooperation between nations in combating climate change.

Non-Equilibrium Dynamic Model (NEDyM; Hallegatte and Ghil, 2008) describes the impacts of random shocks on the economy caused by natural disasters, including climate-related disasters, partially destroying the stock of productive capital and causing short-term economics disequilibria. Via appropriate stylized economy-climate models augmented with a stochastic damage function Rovenskaya (2010) and Kryazhimskiy et al. (2008) analyze how the anticipation of climate-driven natural disasters changes the abatement and investment decisions.

In this work, we build on the climate module of a stylized climate-emissions model presented in Kellie-Smith and Cox (2011). We put forward a simplified system dynamics integrated assessment model which simulates the global economic growth, corresponding emissions, global warming and caused by its secondary effects economic losses. While generally our model follows the same logic as DICE and other models of this kind, it pays more attention to the mechanism of the emission reduction. Mitigation is assumed to be done through the allocation of a certain fraction of the total output into enhancing carbon and energy efficiency. Such mechanism is currently recognized as a constructive solution to the society's unwillingness to deprive themselves from consumption today to avoid negative effects of climate change in the future. The model enables exploring effects of mitigation scenarios defined via carbon tax. We explore the structural sensitivity by examining five alternative climate sensitivity functions and use the "mutual compatibility integration" approach to synthesize the information from the five alternative model versions.

2 Model

2.1 Equations

In order to build long-term projections of the coupled economy-climate system trajectory, we employ a stylized integrated assessment model (IAM) with stochastic climate sensitivity and a nonlinear climate damage function taking out a part of the global output. The model we present here is a simplified and modified version of the previously developed Structural Dynamic Economic Model (SDEM) initially presented by Barth (2003) in an inter-temporal optimization setting, also extensively studied by Kovalevsky and Hasselmann (2014) and Kovalevsky (2014) in a system dynamics setting.

In this paper we further simplify the SDEM model from Kovalevsky and Hasselmann (2014) by ignoring the role of human capital and labor market. We will refer to this version of the SDEM model as SDEM-AK, as it relies on the AK production function (see below).

The SDEM-AK model consists of two dynamically interconnected blocks:

- Global economy – with the key state variables being the physical capital stock K (translated into the global output Y), carbon efficiency f_c, and energy efficiency f_e
- Global climate – with the key state variables being GHG atmospheric concentrations C and the global temperature T, and the diagnostic variable being GHG emissions E.

Following the established tradition in the literature on stylized IAMs (see, e.g., Kellie-Smith and Cox, 2011), we further use global CO_2 emissions, which are the largest contributor to anthropogenic climate change, as a proxy for all GHG emissions driving the global warming.

The dynamics of the coupled economy-climate system is controlled by the carbon tax rate $\tau \geq 0$, which determines the amount of tax to be paid for a unit of emitted GHG, the share of carbon tax revenue $\sigma \in [0,1]$ allocated into improving the carbon efficiency and the saving rate $s \in [0,1]$.

The global output is assumed to evolve according to the AK model (see, e.g., Acemoglu, 2009) augmented with a temperature-based climate damage function $d(T)$ and the flow of carbon tax revenues Tax as follows:

$$Y = AK, \tag{1}$$

$$\dot{K} = s\big(1 - d(T)\big)Y - Tax - \delta K. \tag{2}$$

Dynamics (1)-(2) is independent on the population growth. We intend to use our model to produce scenarios for year 2100. According to the IIASA population projections (Lutz et al., 2014), the world population that reached 7.3 billion as of mid-2015 will peak at about 9.4 billion in 2070 and will then drop to 9.0 billion in 2100. So, as a first approximation, in our model we neglect the contribution of the population/ labor force to growth[1].

The carbon tax revenues are proportional to emissions

$$Tax = \tau E. \tag{3}$$

In this paper, we consider the (already calibrated) damage function suggested by Weitzman (2012) of the form

$$1 - d(T) = \frac{1}{1 + \left(\frac{T}{20.46}\right)^2 + \left(\frac{T}{6.081}\right)^{6.754}}. \tag{4}$$

It is assumed that the collected carbon tax revenue amounted to Tax is redirected into the economy in the form of purpose-oriented “green” R&D investment aimed at increasing both energy and carbon efficiency. Here $\sigma \in [0,1]$ defines the distribution of investment between the two sectors, μ_c and μ_e are corresponding investment efficiencies. Additionally, it is assumed that even in the absence of special investment both efficiencies autonomously improve over time with rates λ_c and λ_e (autonomous energy efficiency improvement (AEEI) is discussed in more details by Azar and

[1] AK production function is widely used in stylized models in the economics of climate change (e.g., Greiner, 2005) and more generally in the environmental economics (e.g., the Rebelo model (Rebelo, 1991) and its followers (Michel and Rotillon, 1995; Withagen, 1995); see also a review paper by (Withagen and Vellinga, 2001).

Dowlatabadi (1999). Thus the carbon and energy efficiency evolve according to (Weber et al., 2005)

$$\dot{f}_c = \mu_c \sigma Tax + \lambda_c f_c, \tag{5}$$

$$\dot{f}_e = \mu_e (1-\sigma) Tax + \lambda_e f_e. \tag{6}$$

GHG emissions are assumed to be proportional to the global output and inverse proportional to the carbon and energy efficiency

$$E = \frac{Y}{f_c f_e}. \tag{7}$$

The CO_2 atmospheric concentration grows due to the anthropogenic carbon emissions E, but is reduced by the carbon sink term with the characteristic time scale (relaxation time) τ_C (C_{PI} is the preindustrial CO_2 concentration) as follows

$$\dot{C} = \gamma E - \frac{C - C_{PI}}{\tau_C}. \tag{8}$$

Growing atmospheric CO_2 concentration gives rise to the global temperature dynamics with the radiative forcing assumed to be logarithmic in CO_2 concentration and the characteristic time scale τ_T set by the thermal capacity of the oceans (Kellie-Smith and Cox, 2011)

$$\dot{T} = \frac{1}{\tau_T}\left(\frac{\Delta T^*}{\ln 2}\ln\frac{C}{C_{PI}} - T\right) \tag{9}$$

for more details on the climate module). Here ΔT^* is the climate sensitivity that is the equilibrium global mean surface temperature increase response to doubling atmospheric CO_2 concentration.

All variables and parameters of equations (1)-(9) with the sources of their values are listed in Table 1. We start all simulations in the year 2010 and set the initial conditions from the historical data available for this year.

Table 1. Description of variables and parameters in the SDEM-AK model.

Notation	Name	Units	Values
		Variables	
Y	World output (at market prices)	constant 2010 trln USD/year	Eq.(1)
K	Global physical capital stock (at market prices)	constant 2010 trln USD	Eq.(2) with initial condition $K(0) = \frac{Y(0)}{A}$ =164.0 trln USD, where $Y(0) =$ 65.6 trln USD (World Bank, 2016)
C	Global atmospheric CO_2 concentration	ppmv	Eq.(8) with initial condition $C(0) =$ 388.58 ppmv (Dlugokencky and Tans, 2016)
E	Global CO_2 emissions	$GtCO_2$/year	Eq.(7)
T	Global mean surface air temperature increase above the preindustrial level	°C	Eq.(9) with initial condition $T(0)$ =0.85°C (IPCC AR5 SYR SPM, 2014)
f_c	Carbon efficiency	-	Eq.(5) with initial condition $f_c(0)$ =1.2 (Weber et al., 2005)
f_e	Energy efficiency	constant 2010 trln USD/$GtCO_2$	Eq.(6) with initial condition $f_e(0) =$ 1.75 trln USD/$GtCO_2$ (Weber et al., 2005; and units conversion)
$d(T)$	Climate damage function	-	Eq.(4) (Weitzman, 2012)
Tax	Carbon tax revenues	constant 2010 trln USD/year	Eq.(3)
		Control parameters	
s	Saving rate	-	SSP3 scenario: 0.156 SSP5 scenario: 0.201 (Authors' own calculations; $s = \frac{pcGDP\ growth\ rate+\delta+n}{A}$ where $pcGDP\ growth\ rate$ = 0.01 (SSP3) and 0.028 (SSP5) (Leimbach et al., 2015); $n = 0.0025$ is the average population growth rate, calculated based on the IIASA's population projections (Lutz et al.,

			2014))
σ	Share of carbon tax revenue allocated into improving the carbon efficiency vis-à-vis energy efficiency	-	0.2 (Calibrated to minimize the mean temperature increase in 2100 - see Appendix)
τ	Carbon tax rate	constant 2010 USD/tCO_2	BAU scenario: 0 USD/tCO_2 Mitigation scenario: 0 USD/tCO_2 before the year 2025; 30 USD/tCO_2 starting from year 2025 (Broadly corresponds to an 'optimal' level derived by Nordhaus (2008))
		Parameters	
A	Technology coefficient	1/year	0.4 1/year (Econometric estimates by Weber et al. (2005) and Barth (2003))
δ	Depreciation rate	1/year	0.05 (Edenhofer et al. 2005)
γ	Emissions to concentrations units conversion factor	ppmv/GtCO_2	0.12 ppmv/ GtCO_2 (Kellie-Smith and Cox, 2011; and units conversion)
μ_c	Efficiency coefficient for investments in f_c	1/Constant 2010 trln USD	0.03 1/trln USD (Weber et al., 2005; and units conversion)
μ_e	Efficiency coefficient for investments in f_e	1/GtCO_2	0.13 1/GtCO_2 (Weber et al., 2005; and units conversion)
λ_c	Rate of exogenous improvement of carbon efficiency	1/year	0 1/year (IPCC AR5 WGIII, Technical Summary Fig. TS.7d presents almost flat historical trajectory (1970-2010) of carbon intensity of energy (in our notations f_c). Since $Tax = 0$ from Eq.(5) we put here $\lambda_c = \frac{\dot{f}_c}{f_c} = 0$)
λ_e	Rate of exogenous improvement of energy efficiency (AEEI)	1/year	0.008 1/year (IPCC AR5 WGIII, Technical Summary Fig. TS.7c reports that the historical trend (1970-2010) of the energy intensity of GDP (in our notations $\frac{E}{Y}$) declines at rate 0.8%. Assuming a constant f_c and given that $Tax = 0$ from Eqs.(6) and (7) we obtain that energy efficiency increases at the same rate 0.8% equal to λ_e)
C_{PI}	Pre-industrial level	ppmv	280 ppmv (Kellie-Smith and Cox,

	of CO2		2011)
τ_C	A characteristic carbon time scale	year	50 years (Kellie-Smith and Cox, 2011)
τ_T	A characteristic climate time scale	year	50 years (Kellie-Smith and Cox, 2011)
ΔT^*	Climate sensitivity	°C	baseline value: 3°C (IPCC, 2014) Sensitivity analysis: Five parametric families of probabilistic distributions (Dietz, 2011; Weitzman, 2012; IPCC, 2014) – see Table 3

In the absence of climate damage and carbon tax, in accordance with the standard AK model, the global output grows exponentially at rate $sA - \delta$. The per capita GDP growth rate, therefore, can be expressed as $sA - \delta - n$, where n is the average population growth rate. In this paper, we examine two economic growth scenarios, reproducing SSP3 and SSP5 with the annual per capita GDP growth rates 1% and 2.8% respectively (Leimbach et al., 2015).

In this paper, we assume mitigation of the GHG emissions via carbon tax. For the *mitigation scenario* to be analyzed, starting from the year 2025 we set the carbon tax rate τ to 30 USD per ton of CO_2. This broadly corresponds (after averaging over time) to an "optimal" level derived by Nordhaus using the DICE model as described in Nordhaus (2008). We will compare the mitigation scenario with the *business-as-usual scenario* (BAU) in which the carbon tax rate τ is set to zero.

The SDEM-AK model roughly mimics the two-pronged RCP/SSP framework, which serves as a basis in the IPCC analysis of feedbacks between climate change and socioeconomic factors, like world population growth, economic development, and technological progress (IPCC AR5, 2014).

Indeed, we calibrate the economic module of SDEM-AK directly based on the assumptions of SSP3 and SSP5 scenarios (Leimbach et al., 2015). Because of the feedback effects of the climate change damages, the actual output per capita average growth rates within the period of simulations (2010-2100) under SSP3 and SSP5 scenarios (business-as-usual case) and the baseline value of the climate sensitivity 3°C are 0.0095 per annum (vs. 0.01 per annum in Leimbach et al. (2015)) for SSP3, and 0.0269 per annum (vs. 0.028 per annum in Leimbach et al. (2015)) for SSP5. The SDEM-AK generates global CO_2 concentration and mean temperature increase endogenously based on the level of global production and energy and carbon efficiency; the values in 2100 under SSP3 and SSP5 and the baseline value of the climate sensitivity 3°C roughly correspond to RCP4.5 and RCP8.5 scenarios (RCPs; IPCC AR5, 2014) (see Table 2).

Table 2. CO_2 concentrations and mean surface temperature increases in the SDEM-AK model and in four RCPs, RCP2.6, RCP4.5, RCP6, and RCP8.5 (named after a possible range of radiative forcing values in the year 2100 relative to pre-industrial values (+2.6, +4.5, +6.0, and +8.5 W/m2, respectively) (IPCC AR5 WG1, Summary for Policymakers, Table SPM.2, 2014).

	CO_2 concentrations in 2100, ppmv	**Mean surface temperature increase and likely range, °C**
	SDEM-AK model	
SSP3, business-as-usual scenario	497.7	1.9
SSP5, business-as-usual scenario	924.8	3.1
	Representative Concentration Pathways	
RCP2.6	420.9	1.0 (0.3 to 1.7)
RCP4.5	538.4	1.8 (1.1 to 2.6)
RCP6	669.7	2.2 (1.4 to 3.1)
RCP8.5	935.9	3.7 (2.6 to 4.8)

Thus, the SDEM-AK model incorporates major dynamic feedbacks between the economic and climate blocks; it can be used for the sensitivity analysis and for testing a broad range of different policies, including the mitigation scenario, which is in the special focus of this paper.

2.2 Uncertain climate sensitivity

Climate sensitivity may be the most prominent example of an important parameter to which models are sensitive (Tebaldi and Knutti, 2007). The range of climate sensitivities derived from the existing GCMs is concentrated approximately between 2.0 and 4.5°C. Most of the results indicate a substantial probability that climate sensitivity might be higher than 4.5°C, maybe up to 6°C or more (Andronova and Schlesinger, 2001; Forest et al., 2002; Forest et al., 2006; Knutti et al., 2002; Murphy et al., 2004; Frame et al., 2005; Piani et al., 2005; Stainforth et al., 2005; Hegerl et al., 2006). The significant expected sensitivity of IAMs with respect to climate sensitivity motivates the research question of this paper set to explore the structural sensitivity of the considered stylized economy-climate model to a probabilistic distribution describing the climate sensitivity.

In this paper, we adopt five alternative probabilistic distribution functions, which have been suggested in literature (to our knowledge, no other one has been suggested) to describe uncertain climate sensitivity. Following the Stern review (2007) and Stainforth et al. (2005), Dietz (2011) suggests triangular and log-logistic probability distributions respectively. Weitzman (2012) suggests a normal, log-normal and Pareto probability distribution functions.

Despite both Dietz (2011) and Weitzman (2012) provide also parameters for these functions, we recalibrate all five alternative PDFs based on most recent information from the IPCC Fifth Assessment report (IPCC, 2014). Namely, in the Summary for Policymakers, we find that "equilibrium climate sensitivity is likely in the range 1.5°C to 4.5°C (high confidence), extremely unlikely less than 1°C (high confidence), and very unlikely greater than 6°C (medium confidence). The lower temperature limit of the assessed likely range is thus less than the 2°C in the AR4, but the upper limit is the same. This assessment reflects improved understanding, the extended temperature record in the atmosphere and ocean, and new estimates of radiative forcing." We interpret the above description as the following conditions

$$\begin{gathered} P(\Delta T^* \in [1.5; 4.5]) \in [0.66; 1] \cap \\ P(\Delta T^* \leq 1) \in [0; 0.05] \cap \\ P(\Delta T^* \geq 6) \in [0; 0.1]. \end{gathered} \tag{10}$$

For each of the five parametric families of climate sensitivity functions, we define the set of its parameters, such that the distribution satisfies (10). Table 3 summarizes the input distributions of climate sensitivity

Table 3. Parametric families of probabilistic distributions describing uncertain climate sensitivity.

Normal distribution (Weitzman, 2012)	
Cdf	$F(x) = \frac{1}{2}\left[1 + erf\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$
Parameters	μ – mean, $\sigma^2 > 0$ – variance
Constraints	$0 < \mu < 5$, $0 < \sigma < 3$, IPCC constraints from (10)
Log-normal distribution (Weitzman, 2012)	
Cdf	$F(x) = \frac{1}{2}\left[1 + erf\left(\frac{\ln(x)-\mu}{\sigma\sqrt{2}}\right)\right]$
Parameters	μ – mean, $\sigma^2 > 0$ – variance
Constraints	$0 < \mu < 2$, $0 < \sigma < 1$,

	IPCC constraints from (10)
Pareto distribution (Weitzman, 2012)	
Cdf	$F(x) = 1 - \left(\frac{x_m}{x}\right)^{\alpha}, \;\; x \geq x_m$
Parameters	x_m – scale, $\alpha > 0$ – shape
Constraints	$0 < x_m < 5$, $0 < \alpha < 6$, IPCC constraints from (10)
Triangular distribution (Dietz, 2011; Stern, 2007)	
Cdf	$F(x) = \begin{cases} 0, \;\; x \leq a, \\ \frac{(x-a)^2}{(b-a)(c-a)}, \;\; a < x \leq c, \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)}, \;\; c < x < b, \\ 1, \;\; b \leq x. \end{cases}$
Parameters	a – lower limit, b – upper limit, c – mode
Constraints	$-2 < a < 6$, $0 < b < 10$, $-1 < c < 7$, IPCC constraints from (10)
Shifted log-logistic distribution (Dietz, 2011; Stainforth et al., 2005)	
Cdf	$F(x) = \frac{1}{1 + \left(\frac{x-\gamma}{\alpha}\right)^{-\beta}}, \;\; x > \gamma$
Parameters	$\alpha > 0$ – scale, $\beta > 0$ – shape, γ – location
Constraints	$1 < \alpha < 2$, $1 < \beta < 5$, $1 < \gamma < 3$, IPCC constraints from (10)

For each functional family, each particular admissible parameter set is considered to be equally probable; on this basis, we define a meta-distribution of climate sensitivity for each climate sensitivity function under consideration.

2.3 Simulations

For each of the five parametric families of climate sensitivity functions, we performed Monte-Carlo simulations from a probability distribution of the climate sensitivity with 500 randomly chosen parameter tuples from the domain of admissible values. In each of 500 realizations of the probability distribution functions, we derived 500 realizations of the climate sensitivity and run the model dynamics. Thus, for each output variable we have 500 samples with 500 points in each sample. Each point can either fall into a variable value domain or take the *no answer* value. The latter case corresponds to the climate sensitivity parameter being admissible, but taking a non-positive value in the simulation run. We represent each output variable sample for each input family of climate sensitivity functions as an empirical probability distribution.

Continuous output variables

We put a uniform grid S_i ($i = 1, \dots, 5$) on an output variable axis. The step size of the grid for the *i*-th family of climate sensitivity functions is defined as a Freedman-Diaconis step size (Freedman and Diaconis, 1981) taken over all points in 500 samples:

$$h_i = 2\frac{IQR(X_i)}{n_i^{1/3}}, \tag{11}$$

where X_i is a union of all sample points with exclusion of points with *no answer* value, $IQR(X_i)$ is an interquartile range of the sample X_i and n_i is the number of points in X_i.

Next, we define a probability mass function $P_i^0(s_{ij})$ on the grid S_i for the *i*-th input family of climate sensitivity functions as a conditional probability on the event that simulations returned some output.

That is,

$$P_i^0(s_{ij}) \coloneqq \Pr_i(s_{ij}|E) = \frac{\Pr_i(s_{ij})}{\Pr_i(s_{i1}) + \dots + \Pr_i(s_{ip_i})} = \frac{f_i(s_{ij})}{f_i(s_{i1}) + \dots + f_i(s_{ip_i})} , \tag{12}$$

where s_{ij} is the *j*-th grid cell, p_i is the total number of cells in the grid S_i and $f_i(s_{ij})$ denotes a frequency of sample points, which fall into the *j*-th grid cell. E is an event that a simulation value falls into the segment $[0; 1)$. Here, each particular admissible parameter tuple is considered to be equally probable.

Thus, we obtain empirical probability distributions for each of the five parametric families of climate sensitivity functions. After that, we project P_i^0 on S_i to a probability distribution P_i on S, where S is the grid with the minimum step size among all five grids. In general, the left and right border cells of the grid S are determined from the minimum and maximum values over all points in the dataset, respectively.

If a cell s_k of the grid S falls entirely into some cell s_{ij} of the grid S_i, then

$$P_i(s_k) := \frac{h}{h_i} P_i^0(s_{ij}),$$

otherwise, a cell s_k intersects with two cells s_{ij} and $s_{i(j+1)}$ of the grid S_i and

$$P_i(s_k) := \frac{l_{ij}}{h_i} P_i^0(s_{ij}) + \frac{l_{i(j+1)}}{h_i} P_i^0(s_{i(j+1)}),$$

where h is a minimum step size, l_{ij} is a length of the fraction in s_k that falls into s_{ij} and $l_{i(j+1)}$ is a length of the fraction in s_k that falls into $s_{i(j+1)}$.

The distributions $P_1, \dots, P_5$ on S are considered as the input distributions for the integration.

Discrete output variables

The sample space Ω_i of a discrete variable for the *i*-th input family of climate sensitivity functions ($i = 1, \dots, 5$) is a union of k_i elementary events $\omega_{i1}, \dots, \omega_{ik_i} \in \mathbb{N}^+$ (e.g., $\omega_{ij} = 2050$) and the event X^0 that a variable takes a *non-defined* value.

We put a uniform grid S_i with fixed step size $h_i \in \mathbb{N}^+$ (e.g., h_i = 5 years) on an output variable axis such that:

$$s_{ij} = [v_{i1} + (j-1)h_i;\ v_{i1} + jh_i), \qquad j = 1, \dots, p_i - 1,$$

$$s_{ip_i} = [v_{i1} + (p_i - 1)h_i;\ v_2], \qquad j = p_i,$$

where s_{ij} is the *j*-th grid cell, p_i is the total number of cells in the grid S_i (i.e., $S_i = \bigcup_{j=1}^{p_i} s_{ij}$) and $v_{i1} = \left\lfloor \frac{\min(\omega_{i1}, \dots, w_{ik_i})}{h_i} \right\rfloor \cdot h_i$. Here, $v_2 \geq \max(\omega_{i1}, \dots, w_{ik_i})$ is a threshold value (e.g., 2100).

Next, we define a probability mass function $P_i^0\ (X = X_i^j)$ on the sample space $\Omega_i' = S_i \cup X^0$ for the *i*-th input family of climate sensitivity functions as a conditional probability on the event that simulations returned some output. That is,

$$P_i^0(\omega_{ij}') \coloneqq \Pr_i(\omega_{ij}' | E) = \frac{\Pr_i(\omega_{ij}')}{\Pr_i(s_{i1}) + \dots + \Pr_i(s_{ip_i}) + \Pr_i(X^0)} = \frac{f_i(\omega_{ij}')}{f_i(s_{i1}) + \dots + f_i(s_{ip_i}) + f_i(X^0)}\ , \qquad (13)$$

where $\omega_{ij}' \in \Omega'$ and $f_i(\cdot)$ denotes a frequency function. E is an event that a simulation value falls into Ω_i. Here, each particular admissible parameter tuple is considered to be equally probable.

Thus, we obtain empirical probability distributions for each of the five parametric families of climate sensitivity functions. After that, we project P_i^0 on Ω' to a probability distribution P_i on Ω, where Ω is the union of X^0 and a consecutive sequence of unique ω_{ij} ($i = 1, \dots, 5$ and $j = 1, \dots, k_i$). That is, if $\omega_k \in \Omega$ equals ω'_{ij} (i.e., it equals either some s_{ij} or X^0), then

$$P_i(\omega_k) \coloneqq P_i^0\big(\omega'_{ij}\big),$$

otherwise, $P_i(\omega_k) \coloneqq 0$.

The distributions $P_1, \dots, P_5$ on Ω are considered as the input distributions for the integration.

2.4 Output

Simulations in the SDEM-AK model are run for four scenarios. The scenario matrix is given in Table 4.

Table 4. Matrix of simulation scenarios.

	SSP3 scenario	**SSP5 scenario**
Business-as-usual scenario	economic growth scenario without introducing carbon tax or mitigation, with the annual per capita GDP growth rate 1%	economic growth scenario without introducing carbon tax or mitigation, with the annual per capita GDP growth rate 2.8%
Mitigation scenario	economic growth scenario with global carbon tax, with the annual per capita GDP growth rate 1%	economic growth scenario with global carbon tax, with the annual per capita GDP growth rate 2.8%

In this paper, we are focusing on comparing and reconciling stochastic output of the global economy, emissions, CO_2 concentration and temperature increase in the SDEM-AK model in the year 2100 according to the five alternative model versions. Additionally, we perform cost benefit analysis by comparing gains from mitigation to its costs. Table 5 describes output variables for the end year of simulations t_1 ($t_1 = 2100$).

Table 5. Output variables of the SDEM-AK model.

Notation	Units	Name
Continuous domain $[\mathbf{0}, +\infty)$		
$\left(1 - d(T(t_1))\right) \cdot Y(t_1)$	constant 2010 trln USD2010/year	Output of the global economy corrected for climate damages
$d(T(t_1)) \cdot Y(t_1)$	constant 2010 trln USD2010/year	Global economic losses from climate change
$E(t_1)$	$GtCO_2$/year	Global carbon emissions
$C(t_1)$	ppmv	Atmospheric CO_2 concentration
$T(t_1)$	°C	Global mean surface air temperature increase above the pre-industrial level
Discrete domain		
t^{*}[1]	year	The first year when the gains from mitigation surpass its costs

3 Integration

3.1 Methodology

Inserting five alternative climate sensitivity distributions into the SDEM-AK model, we obtain five alternative stochastic models of the coupled global climate-economy system. We treat five distributions of a model output variable as priors and employ the Bayesian framework to combine the priors into the joint probability distribution based on the two principal assumptions:

A1. There is no ground for giving a preference to any prior distribution;
A2. The posterior event is the one when stochastic variables in all models have the same realization.

Assumption A2 is motivated by the fact that all the prior stochastic estimates are supposed to represent the same deterministic element. This implies, in particular, that if

[1] $t^* = \min_{\bar{t}} \left\{ \bar{t} \colon \sum_{t=t_\tau}^{\bar{t}} \left(d\left(T^{BaU}(t)\right) * Y^{BaU}(t) - d\left(T^{mit.}(t)\right) * Y^{mit.}(t) \right) > \sum_{t=t_\tau}^{\bar{t}} Tax^{mit.}(t) \right\}$, $t_\tau = 2025$ – year of carbon tax introduction.

a certain elementary event (outcome) is not possible in at least one model, it should be excluded from the set of admissible posterior elementary events, i.e., we consider only mutually compatible outcomes of models from a multi-model ensemble. Kryazhimskiy (2015) operationalized the use of the Bayesian framework under assumptions A1 and A2 given the independence of priors.

For simplicity, we operate with discrete distributions. First consider two priors. Let Z be the set of all possible elementary events (outcomes) in both distributions; $z \in Z$ be an elementary event from Z. Therefore, the posterior probability distribution $\pi(z)$ $(z \in Z)$ describing mutually compatible priors $\pi_1(z)$ and $\pi_2(z)$ is defined as follows

$$\pi(z) = \frac{\pi_1(z)\pi_2(z)}{\sum_{z' \in Z} \pi_1(z')\pi_2(z')} \ . \tag{14}$$

The numerator in (14) is the probability of z to be realized in both models simultaneously and the denominator is the probability of the entire posterior event (see A2). More than two priors can be treated in the same way iteratively. The theoretical foundations of this approached are discussed in Kryazhimskiy (2013) and Kryazhimskiy (2016). An example of application to reconciling alternative models of the net primary production of carbon by forests is presented in Kryazhimskiy et al. (2015).

3.2 Performance Metrics

We look for robustness with respect to the assumption regarding how the global warming impacts the economy. The latter process is described in a stylized fashion, by the combination of the climate sensitivity with the damage function that takes away a part of the global output depending on the level of global warming. We are interested in estimating the most probable realization of a model output variable in 2100, as well as in estimating the resulting uncertainty.

Continuous output variables

We summarize original and integrated distributions by their mean and standard deviation.

The mean values are compared relative to average and spread of the mean in the five original distributions. That is, we compute a relative mean of the probability distribution as follows

$$\mu^* = \frac{\mu - \bar{\mu}}{\bar{\sigma}}, \tag{15}$$

where μ is a mean of the probability distribution, $\bar{\mu}$ is an average mean over all original distributions $P_1, \dots, P_5$ and $\bar{\sigma}$ is a population standard deviation of the sample, which includes means of all original distributions $P_1, \dots, P_5$.

Discrete output variables

We summarize original and integrated distributions by their conditional mean and standard deviation with respect to an event $\overline{X^0}$ that an output variable takes some integer value (i.e., $\overline{X^0} = \Omega \setminus X^0$), as follows

$$\mu^* = E\left(X\middle|\overline{X^0}\right) = \sum_{j=1,..,k} P(s_j)\frac{s_j^1 + s_j^2}{2}, \tag{16}$$

$$\sigma^* = \sqrt{E\left(X^2\middle|\overline{X^0}\right) - E\left(X\middle|\overline{X^0}\right)^2} = \sum_{j=1,..,k} P(s_j)\left(\frac{s_j^1 + s_j^2}{2}\right)^2 - (\mu^*)^2, \tag{17}$$

where s_j^1 and s_j^2 are the left and right border points of the bin s_j $(j = 1, \dots, k)$ respectively, and k is a number of bins in Ω.

Additionally, we compute probability of the non-defined value $P(X^0)$ in original distributions $P_1, \dots, P_5$ and in the integrated distribution.

4 Results

Table 6. Statistics of the *global economic output* in the original models and in the ensemble integrated product. The SDEM-AK versions differ in the probability distribution of the climate sensitivity.

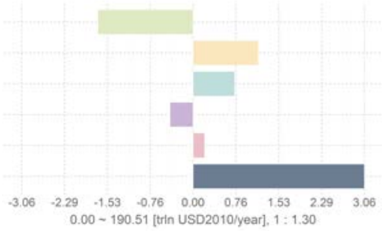
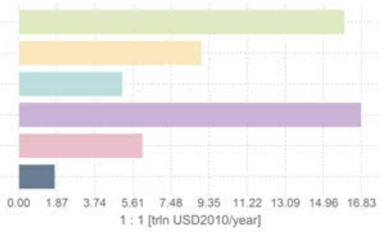
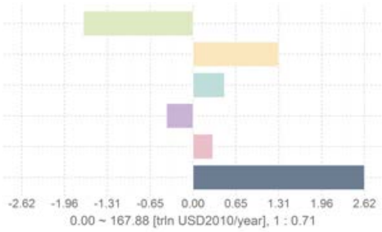
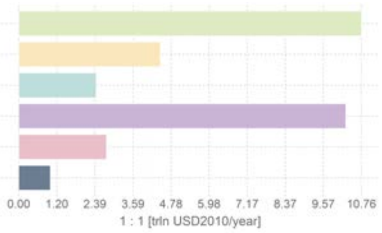
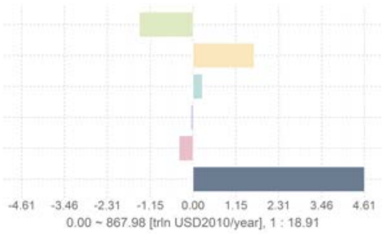
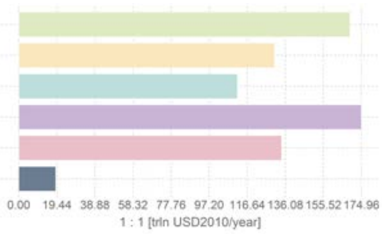
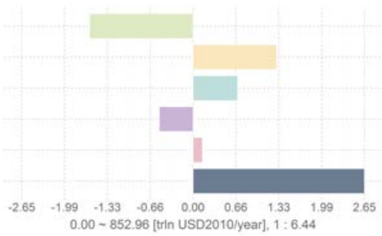
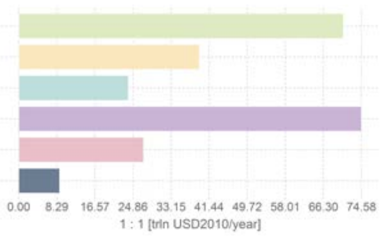
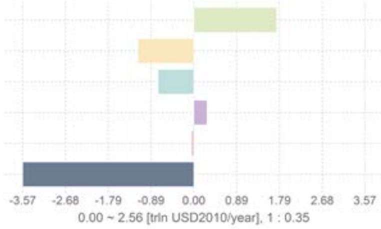
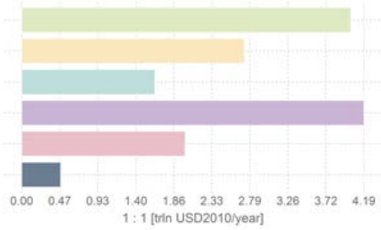
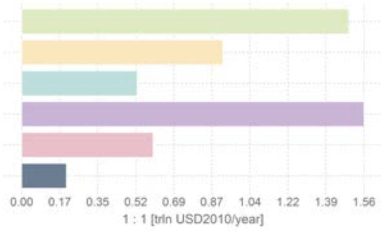
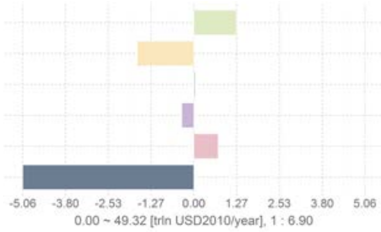
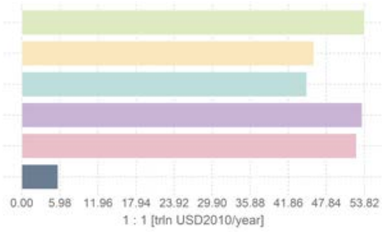
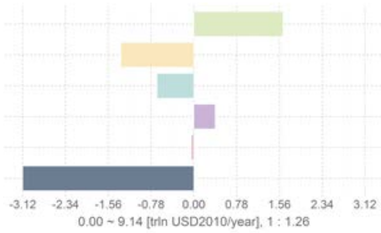
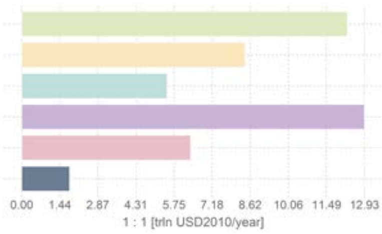
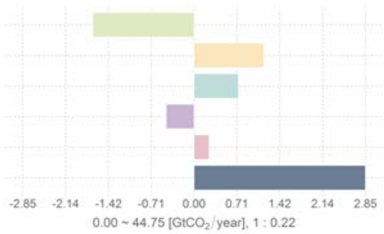
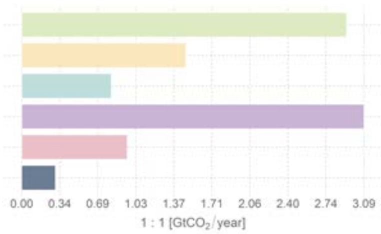
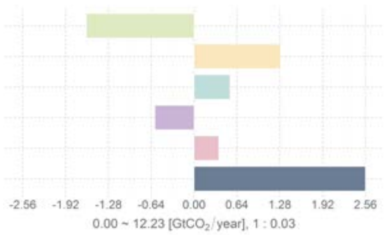
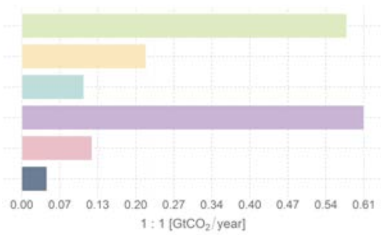
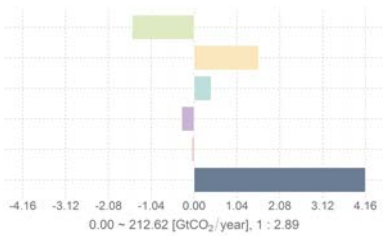
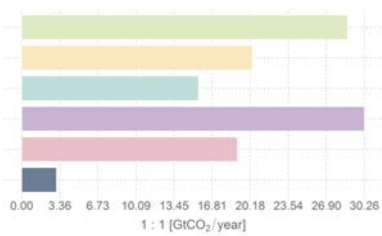
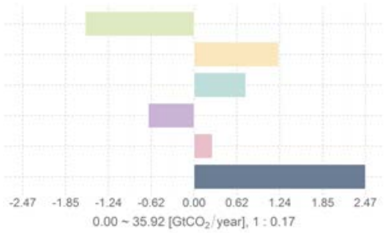
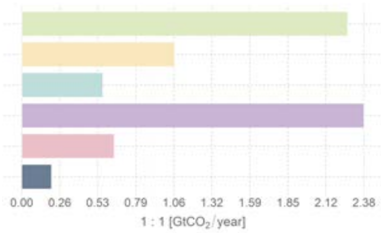
	Mean (relative)	**Standard deviation**
	SSP3	
Business-as-usual scenario		
Mitigation scenario		
	SSP5	
Business-as-usual scenario		
Mitigation scenario		

Table 7. Statistics of the *global economic losses from climate change* in the original models and in the ensemble integrated product. The SDEM-AK versions differ in the probability distribution of the climate sensitivity.

	Mean (relative)	**Standard deviation**
	SSP3	
Business-as-usual scenario		
Mitigation scenario		
	SSP5	
Business-as-usual scenario		
Mitigation scenario		

1: Log-logistic (shifted) 3: Normal 5: Triangular
2: Log-normal 4: Pareto Product

Table 8. Statistics of the *global carbon emissions* in the original models and in the ensemble integrated product. The SDEM-AK versions differ in the probability distribution of the climate sensitivity.

	Mean (relative)	Standard deviation
	SSP3	
Business-as-usual scenario		
Mitigation scenario		
	SSP5	
Business-as-usual scenario		
Mitigation scenario		

1: Log-logistic (shifted) 3: Normal 5: Triangular
2: Log-normal 4: Pareto Product

Table 9. Statistics of the *atmospheric CO_2 concentration* in the original models and in the ensemble integrated product. The SDEM-AK versions differ in the probability distribution of the climate sensitivity.

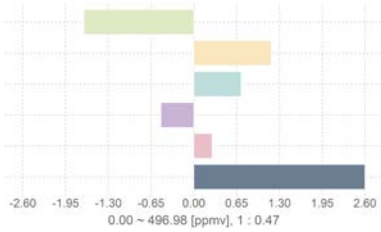
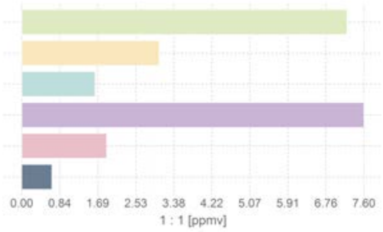
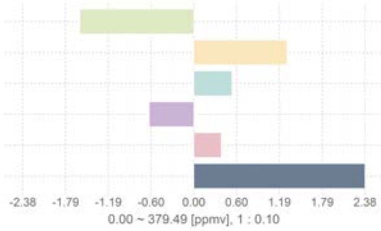
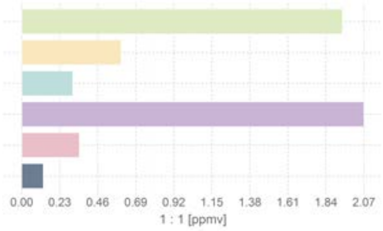
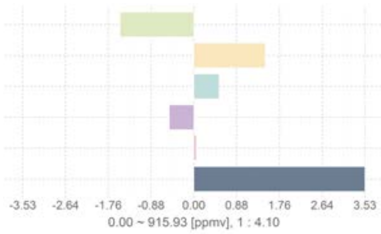
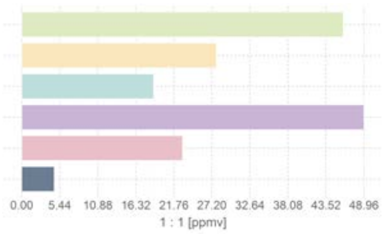
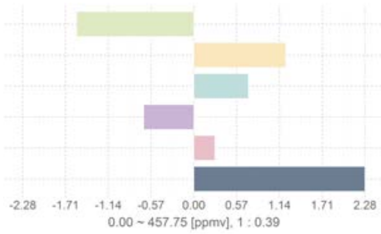
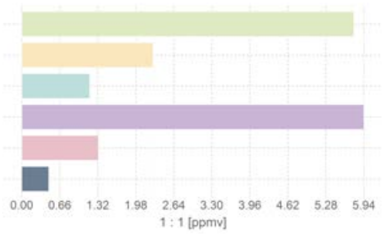
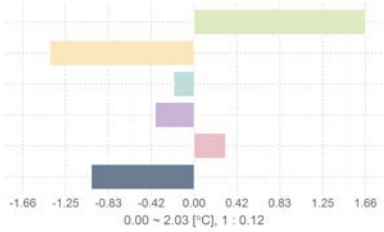
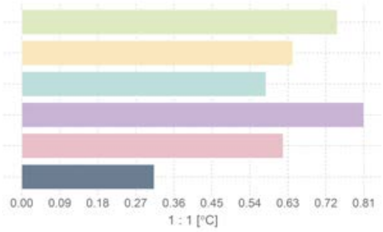
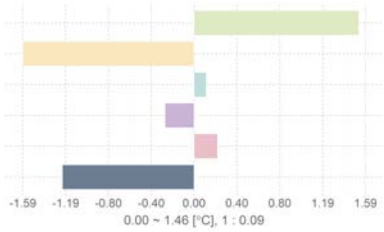
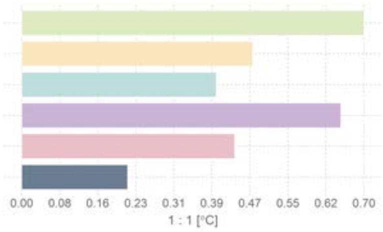
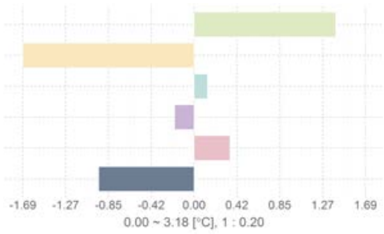
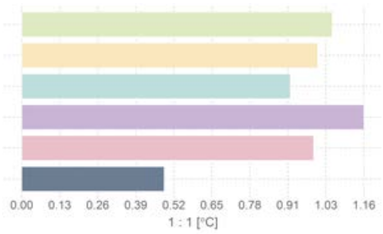
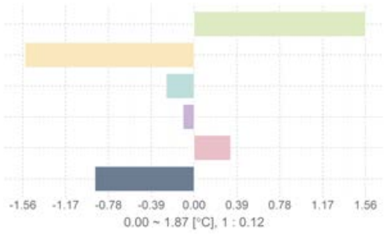
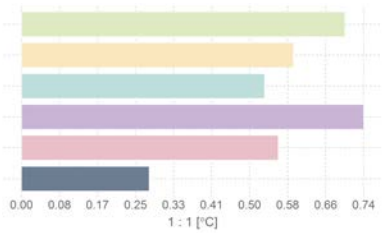
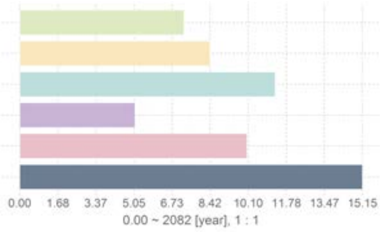
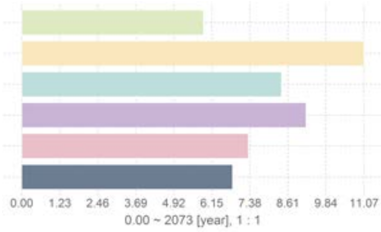
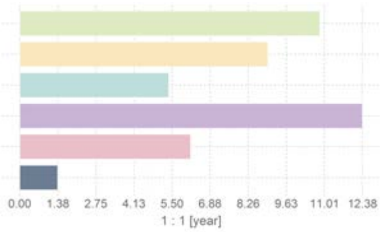
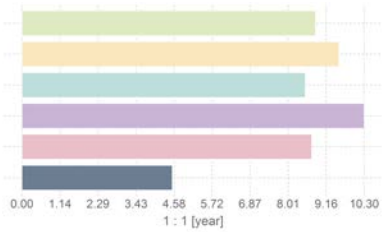
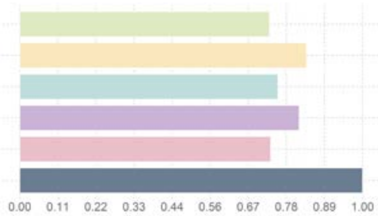
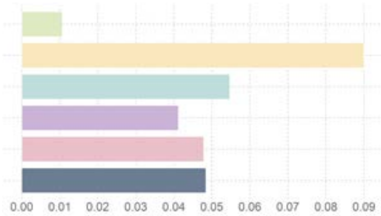
<table>
<tr><th></th><th>Mean (relative)</th><th>Standard deviation</th></tr>
<tr><td></td><td colspan="2">**SSP3**</td></tr>
<tr><td>**Business-as-usual scenario**</td><td></td><td></td></tr>
<tr><td>**Mitigation scenario**</td><td></td><td></td></tr>
<tr><td></td><td colspan="2">**SSP5**</td></tr>
<tr><td>**Business-as-usual scenario**</td><td></td><td></td></tr>
<tr><td>**Mitigation scenario**</td><td></td><td></td></tr>
</table>

Table 10. Statistics of the *global mean surface air temperature increase* in the original models and in the ensemble integrated product. The SDEM-AK versions differ in the probability distribution of the climate sensitivity.

<table>
<tr><th></th><th>Mean (relative)</th><th>Standard deviation</th></tr>
<tr><td></td><td colspan="2">SSP3</td></tr>
<tr><td>Business-as-usual scenario</td><td>
</td><td>
</td></tr>
<tr><td>Mitigation scenario</td><td>
</td><td>
</td></tr>
<tr><td></td><td colspan="2">SSP5</td></tr>
<tr><td>Business-as-usual scenario</td><td>
</td><td>
</td></tr>
<tr><td>Mitigation scenario</td><td>
</td><td>
</td></tr>
</table>

1: Log-logistic (shifted) 3: Normal 5: Triangular
2: Log-normal 4: Pareto Product

Table 11. Statistics of the *year when the gains from mitigation surpass its costs*[1] in the SDEM-AK simulations in the original models and in their ensemble integrated products (bin size = 5 years). The SDEM-AK versions differ in the probability distribution of the climate sensitivity.

	SSP3	SSP5
Conditional mean		
Conditional standard deviation		
$\boldsymbol{P(t^* > 2100)}$		

[1] $t^* = \min_{\bar{t}} \left\{ \bar{t} \colon \sum_{t=t_\tau}^{\bar{t}} \left(d\left(T^{BaU}(t)\right) * Y^{BaU}(t) - d\left(T^{mit.}(t)\right) * Y^{mit.}(t) \right) > \sum_{t=t_\tau}^{\bar{t}} Tax^{mit.}(t) \right\}$, $t_\tau = 2025$ – year of carbon tax introduction.

Software

The estimates from the posterior integration method are obtained using the R package 'modelIntegration', which is available through the link http://www.iiasa.ac.at/web/home/research/researchPrograms/AdvancedSystemsAnalysis/modelIntegration-package.html at the International Institute for Applied Systems Analysis (IIASA).

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Appendix

Parameter σ in equations. (5)-(6) defines the share of the carbon tax revenue to be allocated into improving the carbon efficiency vis-a-vis energy efficiency. An optimal distribution of funds between these two allocations needs to be found in order to make these investments most efficient in terms of mitigation the global warming.

Therefore, in order to calibrate parameter σ, we investigate the sensitivity of the global temperature increase in 2100 in the SDEM-AK model under SSP3 and SSP5 scenarios and the baseline value of the climate sensitivity 3°C. We vary σ over its entire range of admissible values, from 0 to 1, with step 0.05, run the SDEM-AK model and display the output mean temperature increase in 2100 as a function of σ. Figures A.1 and A.2 present results of the SSP3 and SSP5 scenarios respectively. Maximal mean temperature reduction is achieved at $\sigma = 0.2$ in the SSP3 case and at $\sigma = 0.35$ in the SSP5 case (within the accuracy of the used grid). In both scenarios, the curve is rather flat around an interior minimum point, basically, in each case any value of σ between 0 and 0.4 gives a near-to-optimal value to the mean temperature increase. On this basis, we choose $\sigma = 0.2$ for the simulations.

Figure A.1. Global mean surface air temperature increase above the preindustrial level in the end year of simulations 2100 as a function of σ, case of SSP3 scenario.

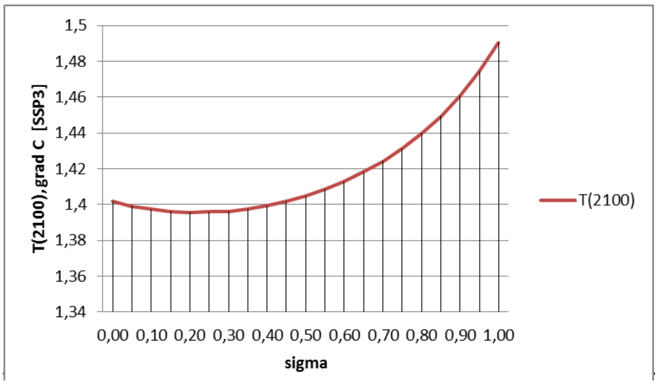

Figure A.2. Global mean surface air temperature increase above the preindustrial level in the end year of simulations 2100 as a function of σ, case of SSP5 scenario.