COMPLEX Final Scientific Report, Volume 2
Non-linearities and System-Flips

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With contributions from the COMPLEX Consortium
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10. A hierarchy of Out-of-equilibrium Actor-based System-dynamic Nonlinear Economic Models

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The process of modelling the dynamics of the coupled climate-socioeconomic system within the actor-based system dynamics approach implies developing system-dynamic models in the form of a model hierarchy, beginning with simple models that are successively made more complex as the simpler models are understood. The dimensionality of system-dynamic models accordingly increases when moving through the hierarchy from simpler to more complex models, until the level of complexity is reached where modelling results can no longer be firmly supported by real-world macroeconomic data.[8]

In the present paper we illustrate the basic concepts underlying the process of developing the model hierarchies within the actor-based system dynamics approach and gradually increasing their dimensionality by discussing a strongly simplified model hierarchy. The model economy consists of only two aggregate actors (an aggregate producer/firm and an aggregate consumer/worker/household). We start with a simplified two-dimensional (2D) dynamic system, and then

[8] Hasselmann, K. (2009): Simulating human behavior in macroeconomic models applied to climate change. European Climate Forum, Working Paper 3/2009, ISBN: 978-3-941663-03-9. URL: http://www.globalclimateforum.org/fileadmin/ecf-documents/publications/ecf-working-papers/hasselmann__simultating-human-behavior-in-macroeconomic-models-applied-to-climate-change.pdf.

explore its various alternative three-dimensional (3D) extensions. As we will see, depending on the model dimensionality and on the assumptions made about the control strategies of key actors determining their decision-making, the models can produce very different dynamic regimes, including stable exponential growth or instabilities, the latter in the form of nonlinear oscillations or economic collapse, etc.

All models developed in the present paper deliberately go beyond the market clearing assumption – one of the cornerstones of the mainstream general equilibrium paradigm in economic modelling. In the models presented below, market clearing is not assumed, supply is generally not equal to demand, and there exists a stock of unsold goods.[9] Therefore, the models presented below describe the substantially out-of-equilibrium economic dynamics.

As simplest dynamic model economy we consider a **2D model** (the state variables are the physical capital and the stock of unsold goods) governed by the interaction of two aggregate actors (an aggregate producer and an aggregate consumer). We assume that the share of production of consumer goods in the output is equal to ρ_G (the remainder of the output goes as investment in physical capital), while the share of consumption of consumer goods in the output is

[9] Other system dynamics models going beyond the market clearing paradigm reported in the literature include e.g. the Non-Equilibrium Dynamic Model (NEDyM) – Hallegatte, S., Ghil, M., Dumas, P., Hourcade, J.-C. (2008): Business cycles, bifurcations and chaos in a neo-classical model with investment dynamics. *Journal of Economic Behavior & Organization* **67**, 57-77.

equal to ρ_C. In the basic 2D model setup, and for illustrative and reference purposes only, we make an (unrealistic) assumption that ρ_G and ρ_C are both constant. The balanced case $\rho_G = \rho_C$ is the standard mainstream textbook case. The case $\rho_G > \rho_C$ implies the over-production regime with an exponentially growing stock of unsold goods; the opposite case $\rho_G < \rho_C$ implies the over-consumption regime (that is, of course, unsustainable in the long term) where the initial stock of unsold goods (if any) rapidly declines to zero.

Various alternative extensions of this basic 2D model are possible. We first extend the 2D model to a **3D model** from the **supply** side, no longer assuming that ρ_G is constant, and implying instead that ρ_G is a new dynamic variable governed by the producer's control strategy. From now on, the goal of the producer is to adapt production to the consumer's demand by adjusting ρ_G (the assumption $\rho_C = \text{const}$ is still retained).

However, this general producer's goal can be formalized as a control strategy in a plethora of alternative ways. In particular, we consider two alternative producer's control strategies: the **'stocks'** control strategy and the **'flows'** control strategy.

In case of the 'stocks' control strategy, the producer strives to adapt the level of production in such a way that the physical capital stock would be proportional to the unsold goods stock. Quite counterintuitively, this dynamic system manifests the oscillatory regime for realistic values of model parameters (Fig. 10.1). We note in passing that, from a mathematical perspective, this 3D model can be reduced to a closed second-order nonlinear ODE for ρ_g equivalent to a nonlinear damped oscillator equation.

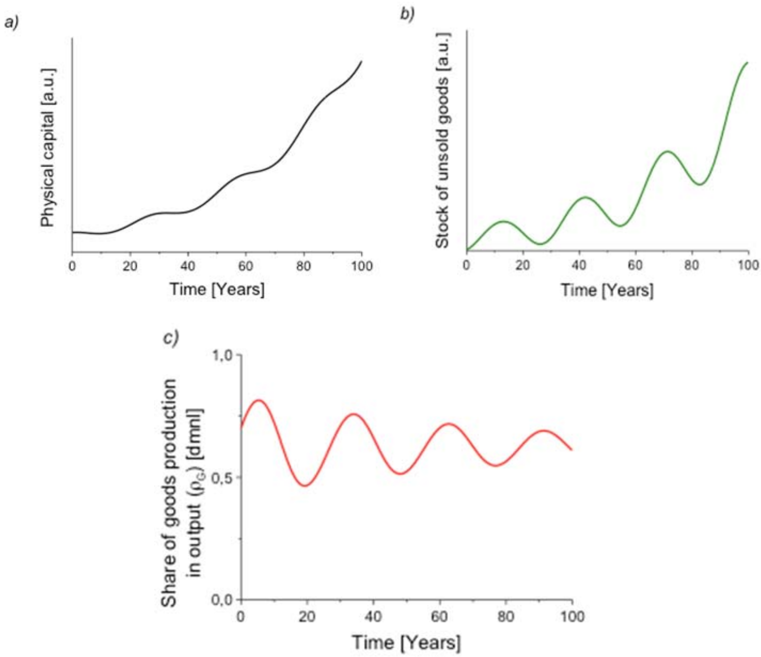

Figure 10.1. *Oscillatory dynamics of a 3D model with the supply-side 'stocks' control strategy: a) physical capital [arbitrary units]; b) stock of unsold goods [arbitrary units]; c) the share of production of consumer goods* in the output (ρ_G) [dimensionless]

Alternatively, in case of the 'flows' control strategy the goal of the producer is to balance the input and output flows of goods, rather than to maintain a constant ratio of the stock of goods to capital. Under the 'flows' control strategy, the model dynamics strongly differs from the 'stocks' case: after the transitional regime, the capital growth becomes (asymptotically) exponential, while the stock of unsold goods and the share of production of consumer goods ρ_G both converge in the long term to their stationary values (Fig. 10.2).

Another option to upgrade the basic 2D model to a 3D model is to improve it from the **demand** side.

Assuming again, like in the basic model, ρ_G = const, and introducing the consumption as the third state variable governed by a dynamic equation much in a spirit of the wage rate dynamics equation used in many members of MADIAMS model family,[10] we finally come to a 3D linear model, where all three state variables (physical capital, stock of unsold goods, and consumption) exert (asymptotically) exponential growth after the transitional regime.

Alternatively, introducing the price dynamics into the model in the form of the (slightly modified) textbook Walrasian price adjustment law[11] tending to equalize supply and demand, and assuming that the share of consumption $\rho_C(t)$ is now price-dependent, we ultimately come to the dynamic behaviour in many respects similar to (although not identical with) that of the model with the supply-side 'flows' control strategy outlined above (after the transitional regime, the capital growth becomes (asymptotically) exponential, while

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[10] Ibid.

[11] cf. Intriligator, M.D. (1971): *Mathematical Optimization and Economic Theory*, Prentice-Hall, Englewood Cliffs, New Jersey.

the stock of unsold goods and the share of consumption ρ_C both converge in the long term to their stationary values).

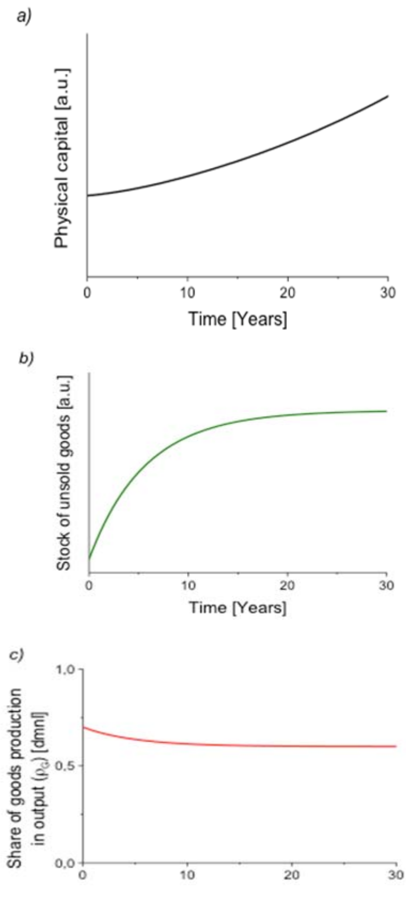

Figure 10.2. *Dynamics of a 3D model with the supply-side 'flows' control strategy: a) physical capital [arbitrary units]; b) stock of unsold goods [arbitrary units]; c) the share of production of consumer goods in the output (ρ_G) [dimensionless]*

So far we were considering the models with **stable** dynamics (at least within the domain of realistic values of model parameters). However, many alternative extensions of the basic 2D model are possible that will generate fundamentally **unstable** economic dynamics (that is unfortunately often observed in real-world economies).

Particularly, the 2D model can be extended to an unstable 3D recession model by introducing the employment level as the third state variable.

In many standard textbook models assuming full employment, a sudden stepwise decrease in consumption through some external factor would induce a decrease in the goods price, restoring again demand. In practice, however, the response of producers to a decrease in demand can also be to lower supply rather than to reduce prices. This is achieved by laying off workers and idling productive capital, leading to a further decrease in demand. This may result in a positive feedback loop producing a vicious cycle, culminating in a depression or (depending on further feedbacks) a business cycle.

Fig. 10.3 illustrates this unstable dynamic regime, assuming that at some point of time (model year 20 on Fig. 10.3) the employment level (initially equal to unity) is slightly reduced by some external shock. As is seen from Figure 3c, this would lead to a scenario of rapid catastrophic drop of the employment to zero level. In practice, the collapse will of course be arrested well before the employment level drops to zero by further feedback processes not considered in our simple model. For example, wage reductions induced by decreases in the employment level can lead to business cycles[12] or, depending on parameter settings, a slow recovery after a period of stagnation.

The actor-based system dynamic approach to economic modelling illustrated above by a simple 2D/3D model hierarchy – tractable to the extent that for many members of this model hierarchy it was even possible to obtain analytical solutions in closed form – was implemented (to a large extent – in the framework of EU FP7 COMPLEX project) in several more realistic members of the MADIAMS model family tailored to study the impacts of various global climate policies. These more complex and more realistic models of course require numeric simulations. Still, we conclude by mentioning that many dynamic features revealed (and often explained within analytical framework) for this simple 2D/3D model hierarchy are clearly visible in numeric solutions of 'larger' actor-based system dynamic models. Particularly, proper accounting for out-of-equilibrium dynamics (on which the simple modelling exercises outlined in the present paper were focused) indeed in many cases substantially affects the simulation results generated by 'larger' models.

[12] Ibid. 4.

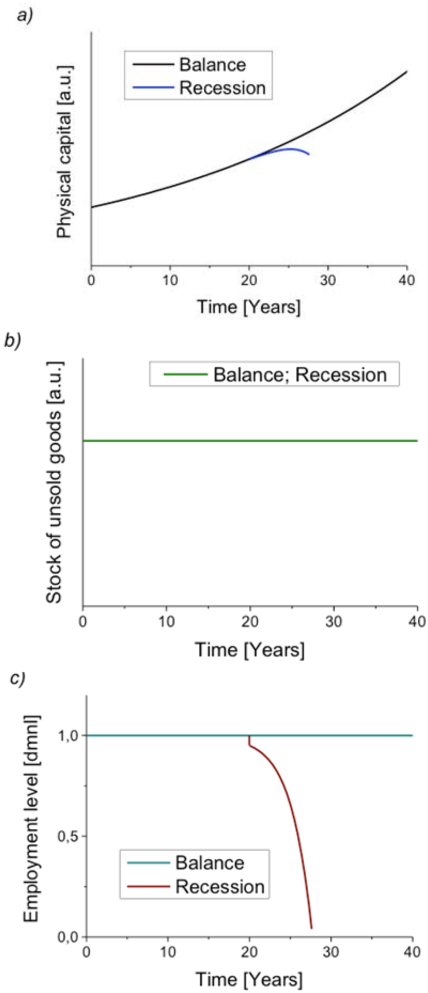

Figure 10.3. *Dynamics of a 3D recession model with an instability incurred at model year 20 (recession): a) physical capital [arbitrary units]; b) stock of unsold goods [arbitrary units]; c) employment level [dimensionless] (superimposed also the balanced solution from the basic 2D model – see text)*