Letter

Transport in Stark many-body localized systems

Guy Zisling,[1] Dante M. Kennes,[2,3] and Yevgeny Bar Lev[1]

[1]*Department of Physics, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel*

[2]*Institute for Theory of Statistical Physics, RWTH Aachen University, and JARA Fundamentals of Future Information Technology, 52056 Aachen, Germany*

[3]*Max Planck Institute for the Structure and Dynamics of Matter, Center for Free Electron Laser Science, Luruper Chaussee 149, 22761 Hamburg, Germany*

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Using numerically exact methods we study transport in an interacting spin chain which for sufficiently strong spatially constant electric field is expected to experience Stark many-body localization. We show that starting from a generic initial state, a spin excitation remains localized only up to a finite delocalization time, which depends exponentially on the size of the system and the strength of the electric field. This suggests that bona fide Stark many-body localization occurs only in the thermodynamic limit. We also demonstrate that the transient localization in a finite system and for electric fields stronger than the interaction strength can be well approximated by a Magnus expansion up to times which grow with the electric field strength.

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Introduction. Statistical mechanics assumes that isolated, interacting systems with many degrees of freedom approach the state of thermal equilibrium. More than a decade ago, it was argued that in the presence of a sufficiently strong disorder, this assumption can be defied, using a mechanism known as many-body localization (MBL) [1–7]. If such systems are isolated from the environment they will never thermalize. Perfect isolation from the environment is challenging in conventional condensed matter systems due to inevitable presence of phonons [8,9]; however, evidence of MBL was obtained in numerous experiments in cold atoms in both one-dimensional [10–12] and two-dimensional systems [13]. While coupling to an external environment or a noise source is detrimental to MBL [14,15], it was shown to be stable to periodic driving at sufficiently high frequencies, a phenomenon known as Floquet MBL [16–19]. Theoretical arguments in favor of MBL require the localization of *all* the single-particle states [1,20]. For quenched disorder this requirement is naturally satisfied in one- and two-dimensional systems due to Anderson localization [21]. Various attempts to relax this requirement were performed by considering models where some of the single-particle states are delocalized [22–29] as also translationally invariant models where all of the states are delocalized in the absence of interactions [28,30–38]. However, the observed localization is far from being convincing and typically suffers from severe finite-size effects [36]. Moreover, while some of these models show robust localization for special initial states, most initial states are apparently delocalized [28,30].

Anderson localization is not the only mechanism which can be used to localize the single-particle states. Single-particle states can be localized by a periodic-in-time, spatially uniform electric field at certain drive frequencies [39,40], and also by a static uniform electric field and any field strength [41]. The former is known as dynamic localization, and the latter as Wannier-Stark localization. While it was shown that dynamic localization is not stable to the addition of interactions [42], Wannier-Stark localization was argued to be stable to interactions for sufficiently strong electric fields [43,44], a phenomenon dubbed Stark MBL. Via a gauge change, constant electric field can be replaced by a time-dependent vector potential (see Fig. 1). Therefore the Stark problem is equivalent to a periodically driven translationally invariant interacting model; see Eq. (3). The mechanism behind Stark MBL is currently under debate, since many of the arguments of Refs. [1,20] cannot be readily applied due to proliferation of resonances, which are known to induce asymptotic delocalization in certain cases [45]. It was proposed that Stark MBL follows from an approximate "shattering" of the Hilbert space due to an almost conservation of the dipole moment [44,46,47]. This argument is however applicable only for an infinite electric field, γ, where jumps between sites are prohibited due to energy conservation (see Fig. 1), and cannot be easily generalized for finite and modest electric fields where the Stark-MBL transition ostensibly occurs [43,44,48].

The dynamics in both localized and delocalized phases was studied theoretically [43,44,49–51] and experimentally [52–54] starting from special initial states. Two-dimensional systems are delocalized and show subdiffusive transport [49,52]. For one-dimensional systems and sufficiently strong electric fields both charge density wave (CDW) [43,44,53,54] and domain-wall initial states [51] do not appear to melt completely. In fact in Ref. [51] it was argued that the system is localized in the thermodynamic limit, for any nonzero electric field, though Ref. [55] suggested that this is a special property of domain-wall initial states.

In this Letter, we consider the nonequilibrium dynamics in a one-dimensional Stark-MBL system starting from a *generic*

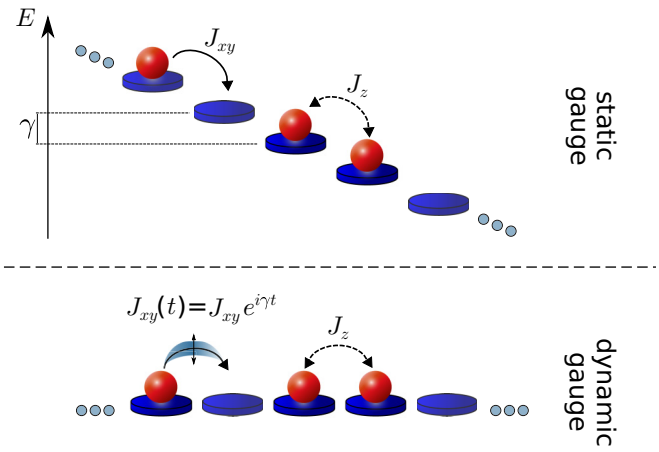

FIG. 1. A schematic representation of the Stark localization problem in two gauges. The upper panel shows the static gauge, where particles are subject to a tilted potential. The bottom panel shows a dynamic gauge, where the scalar potential is written as a "vector potential," which produces time-dependent hopping.

initial state, which corresponds to an average over *all* possible initial states. We demonstrate that in both presumably delocalized and localized regions, a local spin excitation remains localized for increasingly long times when the system size is increased, suggesting that transport might be completely suppressed only in the thermodynamic limit.

Model. The interacting Stark model is described by the following Hamiltonian,

$$\hat{H} = \sum_{j=1}^{L-1} \frac{J_{xy}}{2}(\hat{S}_j^+ \hat{S}_{j+1}^- + \text{H.c.}) + J_z \hat{S}_j^z \hat{S}_{j+1}^z + \sum_{j=1}^{L} W_j \hat{S}_j^z, \quad (1)$$

where L is the length of the spin chain, "H.c." denotes the Hermitian conjugate, $\hat{S}_j^{\pm}$, $\hat{S}_j^z$ are spin-1/2 operators, J_{xy} is the strength of the flip-flop term, J_z is the strength of the Ising term, and $W_j = (\gamma j + \alpha j^2/L^2)$ is a spatially varying potential, where γ corresponds to an electric field; α/L^2 is the magnitude of a shallow parabolic trap that we add in order to break some of the symmetries of the system, following Ref. [43]. The system conserves the total magnetization, $\hat{M} = \sum_j \hat{S}_j^z$, and in the thermodynamic limit is translationally invariant (for $\alpha = 0$). Through this work we use open boundary conditions and set $J_{xy} = 2$, $J_z = 1$, and $\alpha = 0.5$, verifying that our results do not change qualitatively for other α's and J_z's, as also boundary conditions (see [48]). Via the Jordan-Wigner transformation [56], the model is equivalent to a system of spinless interacting fermions moving in a uniform electric field; however for the clarity of the presentation we proceed using the spin formalism.

A number of works show an apparent ergodicity breaking for $\gamma \gtrsim 1.5$ [43,44] (see also [48]). In this Letter, using two numerically exact methods we study spin transport in this model.

Methods. To assess spin transport in the system for various electric fields, γ, we calculate the infinite-temperature spin-spin correlation function,

$$G_n(t) = \frac{1}{\mathcal{N}} \text{Tr}\left[\hat{S}_n^z(t)\hat{S}_{L/2}^z\right], \quad (2)$$

where $\mathcal{N}$ is the Hilbert space dimension, and $\hat{S}_n^z(t)$ is the Heisenberg evolution of $\hat{S}_n^z$. This correlation function describes the spatial spreading of an initially local spin excitation on top of an infinite temperature state. The squared width of the excitation is given by $x^2(t) = \sum_n n^2[G_n(t) - G_n(0)]$ and is analogous to the mean-squared displacement (MSD). For diffusive transport, $x^2 \sim 2Dt$, with D coinciding with the diffusion coefficient calculated from the corresponding Kubo formula [57–60].

We compute $G_n(t)$ using two complementary numerically exact methods. In the first method we work at a zero-magnetization sector, with the Hilbert space dimension $\mathcal{N} = \binom{L}{L/2}$, and utilize dynamical typicality to reduce the trace in Eq. (2) to a unitary propagation of a random initial state taken from the Haar distribution [59,61]. We then average over a small number of such samples. Our initial condition therefore corresponds to a generic initial state with volume-law entanglement. We would like to stress that while the generation of such a highly entangled pure state is probably close to impossible experimentally, we could equally well take a random product state, which *can* be realized experimentally. Such a state would produce an equivalent result, though it would require more averaging over the initial states to sample the correlation function in Eq. (2).

The unitary evolution is performed using a Krylov subspace method [62]. Given the exponential scaling of the Hilbert space dimension we are able access system sizes of $L \lesssim 24$, which correspond to $\mathcal{N} \leqslant 2\,704\,156$, though we can propagate the system for quite long times. As a complimentary method, which provides us access to large system sizes, we use the time-dependent density matrix renormalization group (tDMRG) [63]. In this method the wave function is represented as a matrix product state (MPS), built of matrices with a maximal dimension χ, called the bond dimension. The bond dimension sets the maximum entanglement that the MPS can accommodate. If the bond dimension is set to be smaller than $\chi < d^{L/2}$, where d is the local Hilbert space dimension, the error in the MPS representation of the wave function is bounded by the truncation weight. In our simulations, we set the truncation weight to 10^{-7} allowing the bond dimension to grow dynamically during the propagation. Since for ergodic systems the entanglement is typically increasing linearly with time, the computational effort increases exponentially. We checked for convergences of our results by decreasing the truncation weight to 10^{-8}. In tDMRG we use *all* magnetization sectors, and obtain $G_n(t)$ by the Heisenberg evolution of $\hat{S}_n^z(t)$, using the computational method detailed in Ref. [64]. Due to the equivalence between the ensembles of fixed and varying magnetization, in the thermodynamic limit, both Krylov-based and tDMRG results are expected to agree up to some finite time when the finite-size effects become important.

Results. We calculate the MSD for a number of electric fields, $\gamma = 0.75$–3, and various system sizes $L = 14$–24 using the Krylov subspace method, and for sizes $L = 20$–100 using tDMRG. The results are presented in Fig. 2. For times $t \leqslant t^\star(\gamma, L)$ an initial growth of the MSD is followed by a localization plateau. This plateau is visible for $\gamma \geqslant 1$, and becomes even more pronounced for larger system sizes.

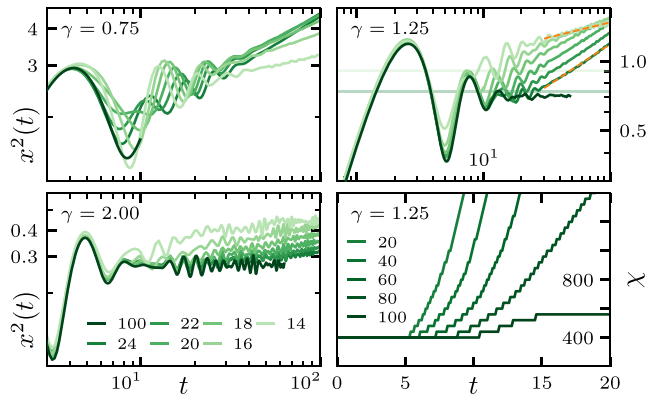

FIG. 2. Mean-squared displacement (MSD) as a function of time (top panels and bottom left) for $L \in [14, 24]$ (Krylov-based method) and $L = 100$ (tDMRG). The orange dotted line corresponds to power-law fits, while the horizontal lines indicate the plateau of the MSD calculated by taking the mean of the MSD between the 2nd and the 3rd peaks. The color of the plateau lines matches the coloring of the corresponding system size. Bottom right: Bond dimension χ as a function of time obtained using tDMRG for $L \in [20, 100]$ and with a fixed discarded weight 10^{-7}. All plots were obtained for $J_{xy} = 2$, $J_z = 1$, and $\gamma = 1.25$.

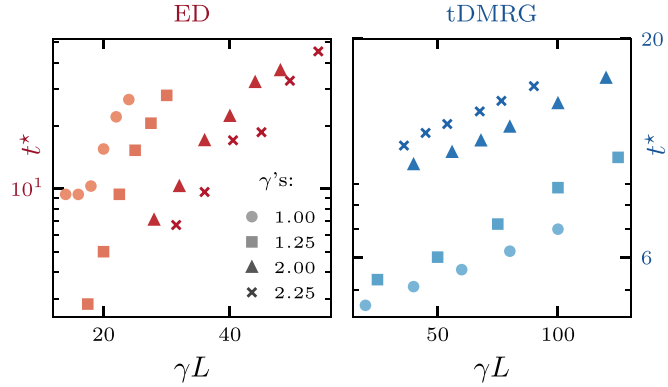

FIG. 3. Delocalization time $t^\star$ as a function of γL for $\gamma = 1, 1.25, 2, 2.25$, as extracted from Krylov-based method (left panel; $L \in [14, 24]$) and tDMRG (right panel; $L \in [20, 100]$). For all data points $J_z = 1$.

For all the studied γ's, including a regime where according to Refs. [43,44] (see also [48]) the system is expected to be strongly localized, the late-time dynamics of a *finite* system is always delocalized, which allows us to identify the time $t^\star(\gamma, L)$, as the delocalization time. Note that our data suggest that for $\gamma \geqslant 1$ the system becomes localized only in the thermodynamic limit. The observed, apparently subdiffusive growth of the MSD for $t > t^\star(\gamma, L)$, which is consistent with previous experimental [52] and theoretical works [49,55,65,66], is therefore a finite-size effect, and will not be considered further in this Letter (see however [48]). For $\gamma \leqslant 1$ our results are not conclusive, since the delocalization time, if it exists here, is very short, and the plateau in the MSD is not clearly visible. But, we do see that for $\gamma = 0.75$ the fast growth of the MSD is pushed to later times for larger system sizes, which hints that localization at the *thermodynamic limit* might occur for all $\gamma > 0$. A similar suggestion was recently raised in Ref. [51]. The localization-delocalization transition at a finite time, $t^\star(\gamma, L)$, can also be seen from the growth of the bond dimension in tDMRG to maintain a chosen accuracy of the results (discarded weight). For $t \leqslant t^\star(\gamma, L)$ a modest bond dimension is required, while for $t > t^\star(\gamma, L)$ to keep the same accuracy of the numerical evolution an increasingly larger bond dimension is required. We stress that the bond dimension is not a physical quantity and we only use it as an indicator of delocalization to obtain $t^\star(\gamma, L)$ [67].

To quantitatively study the dependence of $t^\star(\gamma, L)$ on γ and L, we extract it using two independent methods. For the Krylov-subspace method it is extracted from the intersection point between two straight lines on a log-log scale: the plateau of the MSD (see caption in Fig. 2) and the apparent subdiffusive growth (dashed orange lines in Fig. 2). For tDMRG we define $t^\star(\gamma, L)$ as the time when the bond dimension departs from its initial value (set to 400). While these definitions are of course arbitrary, using different definitions did not result in a qualitative change. In Fig. 3 we show the delocalization time, $t^\star(\gamma, L)$, plotted vs γL on a semilog scale for various tilts of the potential γ. We find that both Krylov-subspace and tDMRG methods suggest that the delocalization time increases exponentially both with γ and L, namely $t^\star \sim \exp[\gamma L]$, such that true localization is obtained only in the thermodynamic limit. Remarkably, the tDMRG simulation of this system becomes *easier*; namely, with the same computational resources for *larger* system sizes one can go to *longer* times. This indicates a change in the *bulk* dynamics, when the size of the system is increased, even though the Hamiltonian is local.

Magnus expansion. In order to better understand the dependence of $t^\star(\gamma, L)$ on γ we apply a time-dependent unitary transformation $\hat{U}(t) \equiv e^{-i\gamma t \sum_j j\hat{S}^z_j}$ to Eq. (1), which corresponds to a gauge change, replacing the potential term in Eq. (1) by a time-dependent "vector potential." This yields the following time-dependent Hamiltonian,

$$\hat{H}(t) = \sum_{j=1}^{L-1} \left[\frac{J_{xy}}{2} \left(e^{-i\gamma t} \hat{S}^+_j \hat{S}^-_{j+1} + \text{H.c.} \right) + J_z \hat{S}^z_j \hat{S}^z_{j+1} \right], \quad (3)$$

where the electric field, γ, takes the role of a frequency. The static part of the Hamiltonian is trivially localized and has a spectrum composed of highly degenerate energy bands, which differ by a number of domain walls. It takes an energy of $J_z/4$ to annihilate or create a domain wall, and therefore the bands are equally spaced. The time-dependent hopping facilitates transport in the system by two possible processes: either by connecting the various bands, or by higher-order, virtual transitions from some state in a band to a different state in the same band. For $\gamma \gg J_z/4$ both processes are suppressed since multiple spin rearrangements are required to absorb the energy of the "photon" and the system is expected to be in a long-lived prethermal state described by the time-averaged Hamiltonian [which here coincides with the static part of $\hat{H}(t)$] up to times $t^\star \sim \exp[\gamma/J_z]$ [68–70]. A slightly different scaling was suggested in Ref. [47]. We have checked that for larger J_z, the apparent localization-delocalization transition shifted to larger γ's [48].

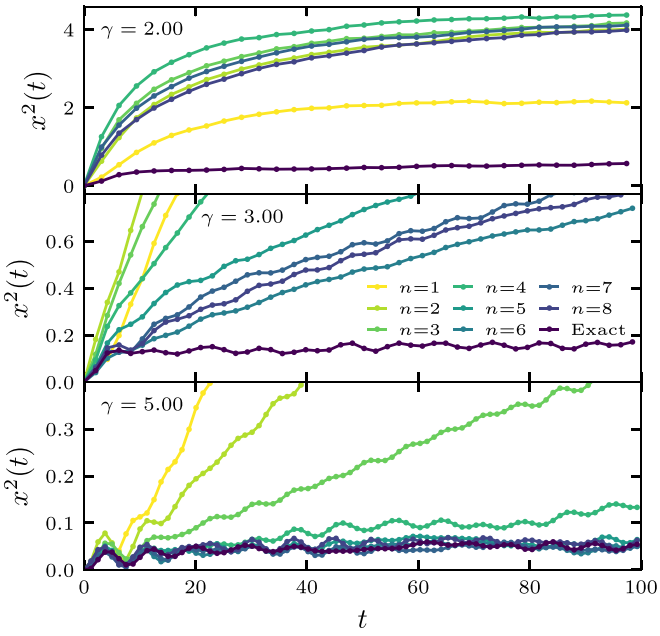

FIG. 4. Mean-squared displacement as a function of time for various electric fields. The darkest lines correspond to numerically exact results obtained by using Eq. (3) for propagation. The colored lines with increasing brightness correspond to evolution using effective Hamiltonians Eq. (4), obtained from a truncated Magnus expansion. For all panels, $J_{xy} = 2,\ J_z = 1,\ L = 14$.

The stroboscopic evolution of the system is determined by an effective Hamiltonian, which is defined from the one-period propagator,

$$\hat{U}(T) = e^{-i\hat{H}_{\text{eff}}T} = \mathcal{T}\exp\left[-i\int_0^T d\bar{t}\hat{H}(\bar{t})\right], \qquad (4)$$

where $\mathcal{T}$ corresponds to time ordering, and $T = 2\pi/\gamma$ is the period. For $\gamma \gg J_z/4$ we can approximate $\hat{H}_{\text{eff}}$ by a Magnus expansion in γ^{-1} [71]. For γ smaller than the many-body bandwidth, this expansion is *not* guaranteed to converge, but it can approximate the dynamics of the system up to some optimal order [72]. We use a recursive formula described in Ref. [71] to obtain $\hat{H}_{\text{eff}}$ up to order $n = 10$ for $L = 14$. Figure 4 shows the stroboscopic evolution of the MSD computed numerically using $\hat{H}^{(n)}_{\text{eff}}$, which is $\hat{H}_{\text{eff}}$ truncated to an order n. We see that for $\gamma \leqslant 2$ the Magnus expansion fails to approximate the dynamics even for short times, while for $\gamma = 3, 5$, as the Magnus order n increases, the approximate solution approaches the exact solution for longer times (it is hard to reliably extract t_{Magnus} from our data to obtain the functional dependence on n, but see Ref. [73]). There is little to no dependence of t_{Magnus} on the system size (see [48]). Interestingly, the long-time dynamics of $\hat{H}^{(n)}_{\text{eff}}$ is diffusive with a diffusion coefficient which decreases with n [48], even for $\gamma = 5$, where the system is expected to be strongly localized [43,44].

Discussion. In this Letter, using two complementary numerically exact methods, we have examined the dynamics of a spin excitation starting from a generic initial condition in a spin chain which is expected to exhibit Stark MBL. For $\gamma > J_z$ we find strong evidence of a finite delocalization time, $t^\star(L,\gamma)$, which scales exponentially with both the size of the system and the electric field, namely $t^\star(L,\gamma) \sim \exp[\gamma L/J_z]$. For intermittent times $t < t^\star$ the spin excitation is localized, while for $t > t^\star$ it delocalizes in a manner consistent with subdiffusion [52]. This strongly suggests that for $\gamma \geqslant J_z$, Stark MBL strictly occurs only in the thermodynamic limit, $L \to \infty$, while any *finite* system is ultimately delocalized for sufficiently long times. For $\gamma \leqslant J_z$ and system sizes and times accessible to us, the localization regime is not apparent. Nevertheless, we do see that the dynamics is delayed with increasing system size, which can be consistent with a localization length larger than the system size $\xi(\gamma) \gg L$. It is therefore plausible to conjecture that Stark MBL in the thermodynamic limit occurs for all $\gamma > 0$, which is consistent with the conjecture in Ref. [51].

In the dynamic gauge, where the electric field is replaced by a periodically driven flip-flop term such that γ plays the role of the frequency, it is rigorously known that for $\gamma \gg J_z$ the heating time is exponential in γ/J_z [68–70]. We show that for sufficiently large electric fields, up to time t_{Magnus}, the dynamics is well approximated by a *static* effective Hamiltonian obtained from a Magnus expansion truncated up to order n. This time increases with both γ/J_z and n (cf. Ref. [73]). The first order of the expansion is given by $\hat{H}^{(1)}_{\text{eff}} = J_z\sum_{j=1}^{L-1}\hat{S}^z_j\hat{S}^z_{j+1} + O(J_z/\gamma)$. The spectrum of $\hat{H}^{(n)}_{\text{eff}}$ is composed of equally spaced bands, $J_z/4$ distance apart, with a bandwidth of $O(J_z/\gamma)$ [48]. Therefore, for $\hat{H}^{(n)}_{\text{eff}}$ the situation is similar to models of quasi-MBL, which show asymptotic delocalization [33,36,45]. Indeed all $\hat{H}^{(n>1)}_{\text{eff}}$ show diffusion at long times, with a diffusion constant decreasing with n [48]. We would like to stress that the delocalization of $\hat{H}^{(n)}_{\text{eff}}$ occurs *before* the delocalization in Eq. (1) and Eq. (3) at time $t^\star$, and therefore Magnus expansion does *not* capture the delocalization regime of Eq. (1) and Eq. (3). It does suggest that the localization mechanism of Stark MBL is probably different from Floquet MBL, where the effective Hamiltonian is expected to be nonergodic [16,18].

While the analysis we provided explains the transient localization regime, it does *not* explain why the delocalization time increases with the size of the system, suggesting that Stark MBL happens only in the thermodynamic limit. This conclusion remains qualitatively robust for both open and periodic boundary conditions, in the static and dynamic gauges, and with and without the parabolic potential in Eq. (1) [48]. A possible explanation could be that the measure of delocalized states is vanishing in the thermodynamic limit. This would also explain why localization appears to be robust for CDW and domain-wall initial states. We leave the exploration of this avenue to future studies.

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