

Broad search for gravitational waves from subsolar-mass binaries through LIGO and Virgo’s third observing run

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We present a search for gravitational waves from the coalescence of binaries which contain at least one subsolar mass component using data from the LIGO and Virgo observatories through the completion of their third observing run. The observation of a merger with a component below $1 M_{\odot}$ would be a clear sign of either new physics or the existence of a primordial black hole population; these black holes could also contribute to the dark matter distribution. Our search targets binaries where the primary has mass M_1 between 0.1 - $100 M_{\odot}$ and the secondary has mass M_2 from 0.1 - $1 M_{\odot}$ for $M_1 < 20 M_{\odot}$ and 0.01 - $1 M_{\odot}$ for $M_1 \geq 20 M_{\odot}$. Sources with $M_1 < 7 M_{\odot}$, $M_2 > 0.5 M_{\odot}$ are also allowed to have orbital eccentricity up to $e_{10} \sim 0.3$. This search region covers from comparable to extreme mass ratio sources up to $10^4 : 1$. We find no statistically convincing candidates and so place new upper limits on the rate of mergers; our analysis sets the first limits for most subsolar sources with $7 M_{\odot} < M_1 < 20 M_{\odot}$ and tightens limits by $\sim 8 \times$ ($1.6 \times$) where $M_1 > 20 M_{\odot}$ ($M_1 < 7 M_{\odot}$). Using these limits, we constrain the dark matter fraction to below 0.3 (0.7)% for 1 (0.5) M_{\odot} black holes assuming a monochromatic mass function. Due to the high merger rate of primordial black holes beyond the individual source horizon distance, we also use the lack of an observed stochastic background as a complementary probe to limit the dark matter fraction. We find that although the limits are in general weaker than those from the direct search they become comparable at $0.1 M_{\odot}$.

I. INTRODUCTION

Gravitational waves are now regularly observed by the ground-based observatories Advanced LIGO [1] and advanced Virgo [2]. Their accomplishments include more than 90 observed binary black hole (BBH) mergers [3–5] and a handful of binary neutron star [6, 7] and neutron star – black hole mergers [8]. These observations provide a wealth of knowledge for understanding the population of stellar-mass black holes [9], which may have arisen through standard stellar evolution [10]. Field evolution [11–14] and dynamical formation channels [15–20] for compact binaries have been proposed to describe this process. Several observations though have challenged current understanding of stellar formation; these include GW190521 [21, 22], the observation of a merger that includes a black hole that may be in the “upper mass gap” caused by pair-instability supernovae [23–25]. Observations also confirm the existence of compact objects with secondary mass $1 - 3 M_{\odot}$ (e.g. GW190814) [9, 26]. It has been proposed that such events may be composed of primordial black holes (PBHs) [27–31]. Studies have also shown that the current population of observed binary black holes is compatible with and may include contributions from a population of merging PBHs [32–34].

PBHs are hypothesized to form by direct collapse of overdensity in the very early Universe [35, 36], and have implications for several astrophysical and cosmological scenarios [37]; these include seeding the first galaxies and the formation of supermassive black holes [38, 39], explaining the recent excess power detected by pulsar tim-

ing arrays [40–42], and most intriguingly, as a candidate for dark matter [36]. A variety of astrophysical observations have put constraints on PBH abundance (for a review see e.g. [43]). So far, dark matter has evaded all direct searches based on experiments on the earth, including the hunt for weakly interacting massive particles (WIMPs) [44] and axion dark matter [45]. The interest in macroscopic dark matter candidates such as PBHs has been revived in light of the discovery of black hole mergers by LIGO and Virgo [46–48].

Among the gravitational-wave catalog, GW190814 is a source of particular interest due to its mass ratio ($q \sim 10$) and low spin [26]. Its spin is consistent with a merger of primordial origin [28, 49], however more mundane explanations are also possible [50]. Like GW190814, GW190425 [51], GW191219, GW200105, GW20015 [8], and GW200210 all contain secondary components with mass less than $3 M_{\odot}$ [9], which is lighter than the lower limit observed from X-ray binaries [52–54]. If there exists a distribution of binaries composed of PBHs, the population could be convincingly demonstrated by the observation of a similar source with a subsolar-mass black hole secondary. The detection of a subsolar-mass black hole would confidently establish the existence of PBHs [43, 55, 56] or other exotic physics able to produce subsolar-mass black holes [57–59] due to their inability to form through standard stellar evolution [60, 61].

In this letter, we conduct a search to answer if LIGO and Virgo have observed any mergers that include a subsolar-mass secondary using the most recent data from the three completed observation runs (O1-O3) [62, 63]. Previous work has directly searched for comparable mass subsolar mass sources up to the first half of the third observation run (O3a) [64–69], and high-mass-ratio sources

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through the second observation run (O2) [70]. We perform a broad analysis designed to be sensitive to sources where the primary mass M_1 is $0.1-100 M_\odot$ while the secondary can range from $0.01 - 1 M_\odot$ for $M_1 > 20 M_\odot$ and $0.1 - 1 M_\odot$ for $M_1 < 20 M_\odot$. Fig. 1 shows the boundaries of this search and how it compares to previous analyses; our search encompasses the mass ranges of prior searches and for the first time includes the full range where the primary mass is $7 - 20 M_\odot$. The most significant candidate in our search had a false alarm rate of ~ 1 per 1.8 years. Due to the time searched, we consider this a null observation. With the inclusion of the most recent data, our limits on the rate of mergers are $\sim 8\times$ more stringent than our prior high-mass-ratio search [70] and $1.6\times$ the comparable mass search [64, 65] which only included up to O2 and O3a, respectively. We find the merger rate of $0.5-0.5 (1-1) M_\odot$ binaries is $< 4400(700) \text{ Gpc}^{-3}\text{yr}^{-1}$ and for $1 - 20 M_\odot$ sources is $< 65 \text{ Gpc}^{-3}\text{yr}^{-1}$.

Our non-detection constrains astrophysical population models which predict the binary PBH merger rate. For a fiducial monochromatic mass population, we find that the dark matter mass fraction of PBHs is $\leq 6\%(0.3\%)$ for component mass $0.1(1) M_\odot$. For a two-point mass function where the primary mass is fixed to be $37 M_\odot$ (approximately the average value for all detections), the mass fraction is $\leq 3\%(0.03\%)$ for secondary mass $0.01(1) M_\odot$. Because the stochastic gravitational-wave background [71–73] provides information about the population of unresolved, high-redshift sources and the PBH merger rate increases with redshift beyond the detection horizon for individual sources, we also use the non-detection of a stochastic background in O3 data [74] to infer additional constraints.

II. SUBSOLAR MASS SEARCH RESULTS

Our search is conducted using the compact-binary analysis included in the open-source PyCBC toolkit [77]. The analysis identifies candidates [78–80], checks for consistency between the data and astrophysical sources [81–83], and assesses each candidate’s statistical significance. Our analysis targets the parameters space shown in Fig. 1 and is configured similarly to our previously conducted analyses for comparable mass sources in Refs. [64, 65] and high-mass-ratio binaries in Ref. [70].

To detect sources, our matched-filter based analysis requires a model of the expected gravitational-wave signal. We model the signal using the TaylorF2 waveform template derived from the post-Newtonian expansion for non-eccentric sources [84–87] where $M_1 < 7 M_\odot$ and EOBNRv2 [88], which includes a model of the merger and ringdown phase of the signal for higher masses, elsewhere. We use TaylorF2e [89–91] to model eccentric sources up to $e_{10} \sim 0.3$ where $0.5 M_\odot < M_1 < 7 M_\odot, M_2 > 0.5 M_\odot$ and e_{10} is the binary eccentricity at a reference gravitational-wave frequency of 10 Hz. Our template models only include the dominant mode

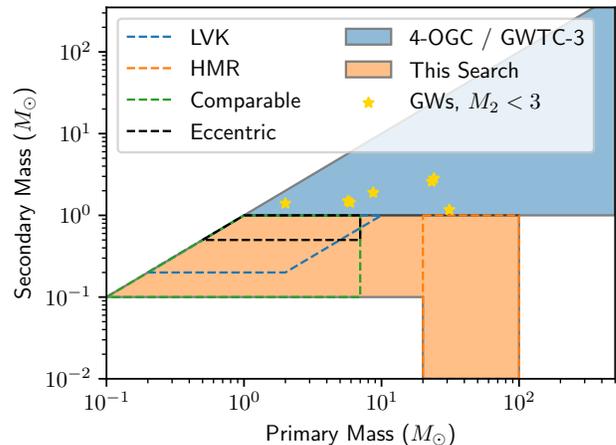


FIG. 1. The primary and secondary (redshifted) masses of the sources searched by our analysis (orange), 4-OGC/GWTC-3 (blue) [75, 76], and the subsolar mass LVK search (blue dotted) [66]. The boundaries where our search also includes sources with eccentricity up to 0.3 (black dashed), along with the boundaries of our prior comparable mass (green dashed) [64, 65] and high-mass-ratio (orange dashed) searches [70] are shown. For comparison, we show the masses of reported gravitational-wave sources closest to our search region (yellow stars); if any of these is primordial in origin, the population may extend into our search region.

of the gravitational-wave signal. A discrete set of template waveforms is selected using a stochastic placement algorithm [92] to ensure the signal-to-noise ratio (SNR) loss due to template bank density is 3 – 5% on average. To control for computational cost, the search analyzes data from a minimum of 20 Hz, but this frequency is raised independently for each template to ensure its duration is no more than 512s where $M_1 < 7 M_\odot$ and 60s everywhere else.

The public LIGO and Virgo dataset now contains data from all three observing runs through 2021 which amounts to 1.2 years of multi-detector time [62, 63]. Our analysis finds no convincing gravitational-wave detections; the most significant candidate has a false alarm rate of 1 per 1.8 years which is consistent with a null observation given the observation time. Using a null detection, we can set an upper limit at 90% confidence on the rate of mergers (shown in Fig. 2) as a function of the binary parameters using

$$R_{90}(m_{1,2}) = \frac{2.3}{VT(m_{1,2})}, \quad (1)$$

where VT is the measured volume-time of the search [93]. We measure the VT of our analysis as a function of the component masses of a binary by empirically measuring the response of our analysis to simulated signal populations. Sources are assumed to have isotropic orientation and sky location in addition to a uniform

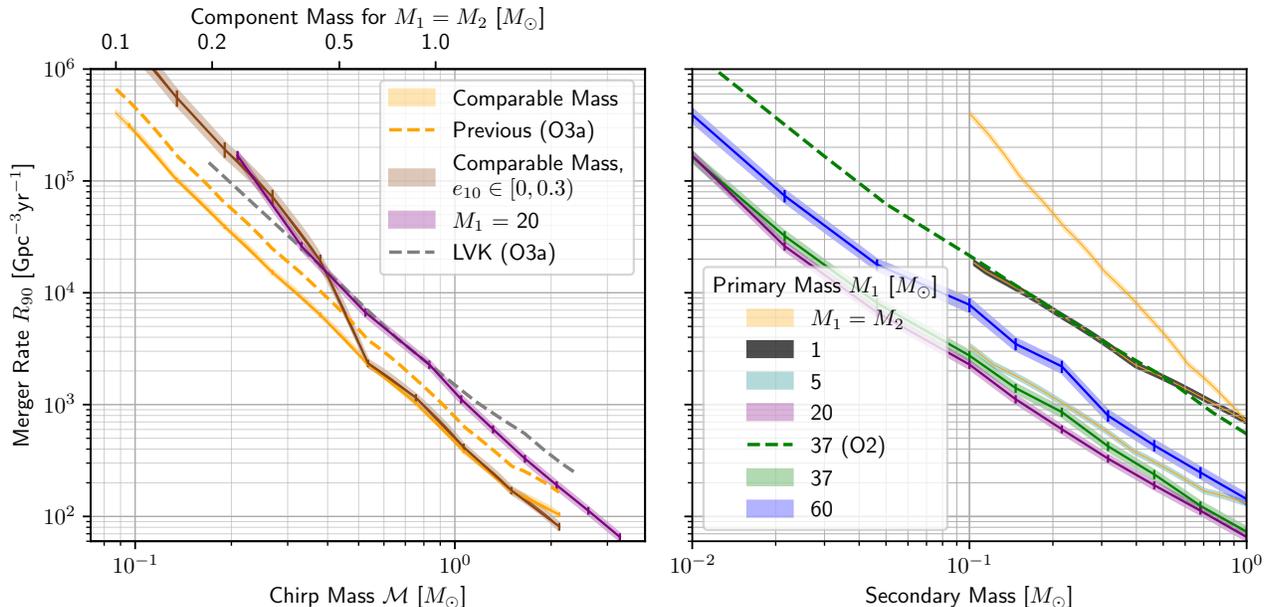


FIG. 2. The 90% upper limit on the rate of mergers from the null detection in our direct search for compact-binary mergers. Left: The upper limit as a function of chirp mass. For sources with comparable mass components, the search sensitivity can be approximated as only a function of the chirp mass. The most recent results from our prior analysis (orange dashed) [65] and the LVK (gray dashed) [66] are shown for reference. Our limits are up to 3x more constraining than the most recent LVK results, and have improved by $\sim 60\%$ over our prior limits. For comparison, a 20:1 mass ratio system is also shown; high-mass-ratio sources will have reduced sensitivity in comparison to comparable mass at the same chirp mass due to the effect of higher-order gravitational-wave modes and the merger moving into the sensitive frequency band of the observatories. Right: The upper limit as function of the secondary mass for a selection of primary masses. Our new limits for $M_1 = 37 M_{\odot}$ (green solid) have improved by 8x over the prior state-of-the art (green dashed) [70]. The one sigma Monte-Carlo statistical uncertainty is shown with shading.

distribution in volume. For comparable mass sources we use TaylorF2(e) as a source model. For $M_1 > 7$ where mass ratio can be up to 10^4 , we use the EOB-NRv2HM model which includes sub-dominant modes of the gravitational-wave signals [88, 94]; this allows us to account for the effect of neglecting sub-dominant modes in our search. In addition, we assume that PBHs will have negligible spin which is consistent with the predicted spin distribution [95–99]. For large total mass binaries, $M_1 + M_2 > \sim 35 - 80 M_{\odot}$, however, we note that non-negligible spin may be induced depending on the level of accretion [100].

III. OBSERVATIONAL CONSTRAINTS ON PRIMORDIAL BLACK HOLE DARK MATTER CONTRIBUTION

Limits on the observed merger rate can constrain the dark matter mass fraction of PBHs given a population model for the binary merger rate. We use the model in Refs. [47, 101–103], which assumes a Poisson spatial distribution for PBHs when they initially form from large overdensity collapse in the early Universe. A nearby pair of black holes form a binary after decoupling from the

cosmic expansion and then inspiral due to gravitational radiation. Given a general mass distribution $P(m)$, the binary merger rate in units of $\text{Gpc}^{-3}\text{yr}^{-1}$ is

$$R(\tilde{f}_{\text{PBH}}, m_{1/2}, t) = 3 \times 10^6 \tilde{f}_{\text{PBH}}^2 (0.7 \tilde{f}_{\text{PBH}}^2 + \sigma_{\text{eq}}^2)^{-\frac{21}{4}} \times (m_1 m_2)^{\frac{3}{37}} (m_1 + m_2)^{\frac{36}{37}} \min\left(\frac{P(m_1)}{m_1}, \frac{P(m_2)}{m_2}\right) \times \left(\frac{P(m_1)}{m_1} + \frac{P(m_2)}{m_2}\right) \left(\frac{t}{t_0}\right)^{-\frac{34}{37}}, \quad (2)$$

where $m_{1/2}$ are in unit of M_{\odot} , t is the cosmic time and t_0 is the age of the Universe, both in units of years, $\sigma_{\text{eq}} = 0.005$ characterizes the variance of density inhomogeneity from dark matter at the mass density equality era. The effective mass fraction is defined to $\tilde{f}_{\text{PBH}}^{53/37} = S f_{\text{PBH}}^{53/37}$ where f_{PBH} is the true mass fraction and the suppression term S accounts for binary disruption after formation. Refs. [104–106] showed the disruption by nearby PBH clusters can reduce the merger rate by orders of magnitude if PBHs account for nearly all dark matter but become negligible when $f_{\text{PBH}} \lesssim \mathcal{O}(0.1\%)$.

In this letter we focus on a common fiducial mass distribution following [64, 65, 70] where the component masses are fixed to chosen values. Limits for arbitrary extended

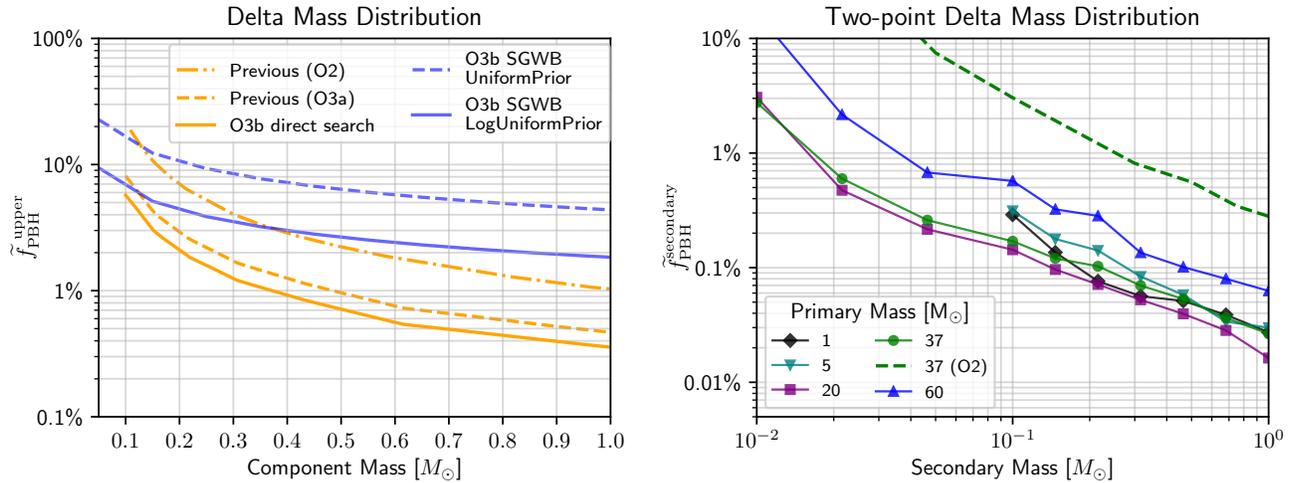


FIG. 3. Left: 90% upper limits on the fraction of dark matter accounted for by PBHs assuming a delta function mass distribution. The orange lines are constraints from our direct searches with data through O2, O3a, and O3b. The blue solid and dashed lines are constraints from the stochastic background. Right: 90% upper limits on the abundance of PBHs for the secondary black hole assuming a two-point delta function for mass. To allow consistent comparison with previous work, we follow Ref. [64] to fix the abundance of the primary mass to be $\tilde{f}_{\text{PBH}}^{\text{primary}} = 3 \times 10^{-3}$ which is obtained by fitting the observed black hole merger population [34]. The solid lines are constraints for different representative primary mass. The dash line is the prior constraint based on data only through O2.

distributions can also be derived from our limits (see e.g. [64, 107, 108]).

A. Constraints from our direct search

We first use our direct search limits to constrain the dark matter mass fraction from PBHs. Results are shown in Fig. 3. In the left panel the PBH mass function is assumed to be a delta distribution. For 1 and 0.1 M_{\odot} reference masses, the fraction of dark matter composed of PBHs can not exceed 0.3% and 6%, respectively. The right panel of Fig. 3 shows constraints where we choose the mass function to be a two-point delta distribution. We fix $\tilde{f}_{\text{PBH}} = 0.3\%$ for the primary mass and constrain the \tilde{f}_{PBH} of the secondary PBH. This choice is motivated by Ref. [34] which fitted a PBH mass function and abundance with the current black hole merger observations [3] assuming they are primordial in origin. We consider five representative primary component masses 1, 5, 20, 37 (approximately the average mass of all gravitational wave events) and 60 M_{\odot} , and secondary mass range in $[0.01, 1] M_{\odot}$. The previous constraint for $M_1 = 37 M_{\odot}$, which included data only up to O2, are also shown for comparison. The O3b constraints improved over our previous by one order of magnitude due to upgrades of the advanced detectors for O3 and the longer duration of observation [3, 109]

B. Constraints from the stochastic background

In addition to individually resolvable sources, the incoherent superposition of all binary PBH coalescences produces a stochastic background of gravitational waves [110, 111]. While the astrophysical binary black hole merger rate approximately follows the star formation rate peaking at redshift ~ 2 [112], the PBH merger rate gets higher when redshift increases [47, 101]. The search for an isotropic stochastic gravitational-wave background by the LVK did not find any significant excess energy in their O3 analysis [74]. We investigate the implication of this non-detection of a stochastic background for the PBH abundance.

The stochastic background is characterized by

$$\Omega_{\text{GW}} = \frac{\nu}{\rho_c} \frac{d\rho}{d\nu} \quad (3)$$

where $d\rho$ is the gravitational-wave energy density in a frequency bin $[\nu, \nu + d\nu]$, normalized by critical energy density for a flat Universe ρ_c . Given the merger rate model for PBH binaries from Eq. 2, the stochastic background can be computed as

$$\Omega_{\text{GW}} = \frac{\nu}{\rho_c} \int \frac{R(z; \tilde{f}_{\text{PBH}}, m_{1/2})}{(1+z)H(z)} \frac{dE_s}{d\nu_s} dz \quad (4)$$

where $H(z)$ is the Hubble parameter at redshift z , dE_s/df_s is the energy spectrum from a single inspiraling source evaluated in the source frame. We use the waveform template IMRPhenomD [113, 114] to compute

dE_s/df_s and integrate Eq.4 up to $z = 20$; the contributions from higher redshifts can be neglected.

We assume a delta mass distribution for PBH binaries and have verified its stochastic background spectrum follows a power law with slope index 2/3 within the LIGO/Virgo’s most sensitive frequency band ($\sim 20 - 100$ Hz). Therefore, we apply the LVK O3 upper limit for a power law spectrum with index 2/3 that $\Omega_{\text{GW}}(\nu = 25\text{Hz}) < 3.4 \times 10^{-9}$ for a log-uniform prior on Ω_{GW} and $\Omega_{\text{GW}}(\nu = 25\text{Hz}) < 1.2 \times 10^{-8}$ for a uniform prior [74] to constrain the PBH mass fraction. The result is shown in the left panel of Fig. 3 accompanied by the results from our direct search. As shown, the constraint is generally weaker than the direct search by one order of magnitude, but becomes comparable for $0.1 M_{\odot}$ where the individual source search is less sensitive. The stochastic background from PBH mergers at high redshift complements the direct search which only probes the local Universe.

IV. CONCLUSIONS

Subsolar mass black holes would establish the existence of a population of PBHs [43, 55, 56] or hint at new physics outside of the standard model of stellar evolution [57–59]. We have performed a broad search for compact-binary coalescences where at least one component is less massive than one solar mass; binary sources span from comparable mass to extreme mass ratios up to 10^4 . We use our null detection to set new upper limits on the rate of mergers and the implied fraction of dark matter composed of PBHs. The top candidates from our analysis along with the configuration

files necessary to reproduce the search are available at <https://github.com/gwastro/subsolar-O3-search>.

We have investigated the constraints on PBH abundance from the non-detection of the direct search for individual sources and a search for a stochastic background from the superposition of unresolved sources [74]; these probe gravitational-wave sources from the local and high redshift Universe, respectively. The observation of binary black hole mergers at high redshift is a key way to distinguish primordial from stellar origin because the former can merge before star formation [115–117].

Assuming a non-detection, we expect the next observing run (O4), with expected data available at the end of 2024, will be able to improve constraints by another factor of $2-4\times$ [118]. The future direct detection of subsolar mass black hole binaries or an excess in the merger rate at high redshift would give decisive evidence for PBHs.

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- [1] J. Aasi *et al.* (LIGO Scientific Collaboration), “Advanced LIGO,” *Class. Quantum Grav.* **32**, 074001 (2015), [arXiv:1411.4547 \[gr-qc\]](https://arxiv.org/abs/1411.4547).
 - [2] F. Acernese *et al.* (VIRGO), “Advanced Virgo: a second-generation interferometric gravitational wave detector,” *Class. Quantum Grav.* **32**, 024001 (2015), [arXiv:1408.3978 \[gr-qc\]](https://arxiv.org/abs/1408.3978).
 - [3] R. Abbott *et al.* (LIGO Scientific, VIRGO, KAGRA), “GWTC-3: Compact Binary Coalescences Observed by LIGO and Virgo During the Second Part of the Third Observing Run,” (2021), [arXiv:2111.03606 \[gr-qc\]](https://arxiv.org/abs/2111.03606).
 - [4] Alexander H. Nitz, Sumit Kumar, Yi-Fan Wang, Shilpa Kastha, Shichao Wu, Marlin Schäfer, Rahul Dhurkunde, and Collin D. Capano, “4-OGC: Catalog of gravitational waves from compact-binary mergers,” (2021), [arXiv:2112.06878 \[astro-ph.HE\]](https://arxiv.org/abs/2112.06878).
 - [5] Seth Olsen, Tejaswi Venumadhav, Jonathan Mushkin, Javier Roulet, Barak Zackay, and Matias Zaldarriaga, “New binary black hole mergers in the LIGO–Virgo O3a data,” (2022), [arXiv:2201.02252 \[astro-ph.HE\]](https://arxiv.org/abs/2201.02252).
 - [6] B. P. Abbott *et al.* (LIGO Scientific Collaboration, Virgo Collaboration), “GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral,” *Phys. Rev. Lett.* **119**, 161101 (2017), [arXiv:1710.05832 \[gr-qc\]](https://arxiv.org/abs/1710.05832).
 - [7] B. P. Abbott *et al.* (LIGO Scientific, Virgo), “GW190425: Observation of a Compact Binary Coalescence with Total Mass $\sim 3.4M_{\odot}$,” *Astrophys. J. Lett.* **892**, L3 (2020), [arXiv:2001.01761 \[astro-ph.HE\]](https://arxiv.org/abs/2001.01761).
 - [8] R. Abbott *et al.* (LIGO Scientific, KAGRA, VIRGO), “Observation of Gravitational Waves from Two Neutron Star–Black Hole Coalescences,” *Astrophys. J. Lett.* **915**, L5 (2021), [arXiv:2106.15163 \[astro-ph.HE\]](https://arxiv.org/abs/2106.15163).
 - [9] R. Abbott *et al.* (LIGO Scientific, VIRGO, KAGRA), “The population of merging compact binaries inferred using gravitational waves through GWTC-3,” (2021), [arXiv:2111.03634 \[astro-ph.HE\]](https://arxiv.org/abs/2111.03634).
 - [10] Ilya Mandel and Floor S. Broekgaarden, “Rates of Compact Object Coalescences,” (2021), [arXiv:2107.14239 \[astro-ph.HE\]](https://arxiv.org/abs/2107.14239).
 - [11] Vassiliki Kalogera, K. Belczynski, C. Kim, Richard W. O’Shaughnessy, and B. Willems, “Formation of Double Compact Objects,” *Phys. Rept.* **442**, 75–108 (2007), [arXiv:astro-ph/0612144](https://arxiv.org/abs/astro-ph/0612144).

- [12] Michal Dominik, Emanuele Berti, Richard O’Shaughnessy, Ilya Mandel, Krzysztof Belczynski, Christopher Fryer, Daniel E. Holz, Tomasz Bulik, and Francesco Pannarale, “Double Compact Objects III: Gravitational Wave Detection Rates,” *Astrophys. J.* **806**, 263 (2015), [arXiv:1405.7016 \[astro-ph.HE\]](#).
- [13] Krzysztof Belczynski, Daniel E. Holz, Tomasz Bulik, and Richard O’Shaughnessy, “The first gravitational-wave source from the isolated evolution of two 40-100 Msun stars,” *Nature* **534**, 512 (2016), [arXiv:1602.04531 \[astro-ph.HE\]](#).
- [14] Nicola Giacobbo and Michela Mapelli, “The progenitors of compact-object binaries: impact of metallicity, common envelope and natal kicks,” *Mon. Not. Roy. Astron. Soc.* **480**, 2011–2030 (2018), [arXiv:1806.00001 \[astro-ph.HE\]](#).
- [15] Fabio Antonini, Sourav Chatterjee, Carl L. Rodriguez, Meagan Morscher, Bharath Pattabiraman, Vicky Kalogera, and Frederic A. Rasio, “Black hole mergers and blue stragglers from hierarchical triples formed in globular clusters,” *Astrophys. J.* **816**, 65 (2016), [arXiv:1509.05080 \[astro-ph.GA\]](#).
- [16] Carl L. Rodriguez, Meagan Morscher, Bharath Pattabiraman, Sourav Chatterjee, Carl-Johan Haster, and Frederic A. Rasio, “Binary Black Hole Mergers from Globular Clusters: Implications for Advanced LIGO,” *Phys. Rev. Lett.* **115**, 051101 (2015), [Erratum: *Phys.Rev.Lett.* 116, 029901 (2016)], [arXiv:1505.00792 \[astro-ph.HE\]](#).
- [17] Carl L. Rodriguez, Sourav Chatterjee, and Frederic A. Rasio, “Binary Black Hole Mergers from Globular Clusters: Masses, Merger Rates, and the Impact of Stellar Evolution,” *Phys. Rev. D* **93**, 084029 (2016), [arXiv:1602.02444 \[astro-ph.HE\]](#).
- [18] Dawoo Park, Chunglee Kim, Hyung Mok Lee, Yeong-Bok Bae, and Krzysztof Belczynski, “Black Hole Binaries Dynamically Formed in Globular Clusters,” *Mon. Not. Roy. Astron. Soc.* **469**, 4665–4674 (2017), [arXiv:1703.01568 \[astro-ph.HE\]](#).
- [19] Giacomo Fragione and Bence Kocsis, “Black hole mergers from an evolving population of globular clusters,” *Phys. Rev. Lett.* **121**, 161103 (2018), [arXiv:1806.02351 \[astro-ph.GA\]](#).
- [20] Davide Gerosa and Maya Fishbach, “Hierarchical mergers of stellar-mass black holes and their gravitational-wave signatures,” *Nature Astron.* **5**, 8 (2021), [arXiv:2105.03439 \[astro-ph.HE\]](#).
- [21] R. Abbott *et al.* (LIGO Scientific, Virgo), “Properties and Astrophysical Implications of the 150 M_{\odot} Binary Black Hole Merger GW190521,” *Astrophys. J. Lett.* **900**, L13 (2020), [arXiv:2009.01190 \[astro-ph.HE\]](#).
- [22] R. Abbott *et al.* (LIGO Scientific, Virgo), “GW190521: A Binary Black Hole Merger with a Total Mass of $150M_{\odot}$,” *Phys. Rev. Lett.* **125**, 101102 (2020), [arXiv:2009.01075 \[gr-qc\]](#).
- [23] S. E. Woosley, “Pulsational Pair-Instability Supernovae,” *Astrophys. J.* **836**, 244 (2017), [arXiv:1608.08939 \[astro-ph.HE\]](#).
- [24] Pablo Marchant, Mathieu Renzo, Robert Farmer, Kaliroe M. W. Pappas, Ronald E. Taam, Selma E. de Mink, and Vassiliki Kalogera, “Pulsational pair-instability supernovae in very close binaries,” *Astrophys. J.* **882**, 36 (2019).
- [25] Simon Stevenson, Matthew Sampson, Jade Powell, Alejandro Vigna-Gómez, Coenraad J. Neijssel, Dorottya Szécsi, and Ilya Mandel, “The impact of pair-instability mass loss on the binary black hole mass distribution,” (2019), [10.3847/1538-4357/ab3981](#), [arXiv:1904.02821 \[astro-ph.HE\]](#).
- [26] R. Abbott *et al.* (LIGO Scientific, Virgo), “GW190814: Gravitational Waves from the Coalescence of a 23 Solar Mass Black Hole with a 2.6 Solar Mass Compact Object,” *Astrophys. J. Lett.* **896**, L44 (2020), [arXiv:2006.12611 \[astro-ph.HE\]](#).
- [27] V. De Luca, V. Desjacques, G. Franciolini, P. Pani, and A. Riotto, “GW190521 Mass Gap Event and the Primordial Black Hole Scenario,” *Phys. Rev. Lett.* **126**, 051101 (2021), [arXiv:2009.01728 \[astro-ph.CO\]](#).
- [28] Sebastien Clesse and Juan Garcia-Bellido, “GW190425, GW190521 and GW190814: Three candidate mergers of primordial black holes from the QCD epoch,” (2020), [arXiv:2007.06481 \[astro-ph.CO\]](#).
- [29] Kyriakos Vattis, Isabelle S. Goldstein, and Savvas M. Koushiappas, “Could the 2.6 M_{\odot} object in GW190814 be a primordial black hole?” *Phys. Rev. D* **102**, 061301 (2020), [arXiv:2006.15675 \[astro-ph.HE\]](#).
- [30] Karsten Jedamzik, “Consistency of Primordial Black Hole Dark Matter with LIGO/Virgo Merger Rates,” *Phys. Rev. Lett.* **126**, 051302 (2021), [arXiv:2007.03565 \[astro-ph.CO\]](#).
- [31] Sai Wang and Zhi-Chao Zhao, “GW200105 and GW200115 are compatible with a scenario of primordial black hole binary coalescences,” *Eur. Phys. J. C* **82**, 9 (2022), [arXiv:2107.00450 \[astro-ph.CO\]](#).
- [32] Gabriele Franciolini, Vishal Baibhav, Valerio De Luca, Ken K. Y. Ng, Kaze W. K. Wong, Emanuele Berti, Paolo Pani, Antonio Riotto, and Salvatore Vitale, “Quantifying the evidence for primordial black holes in LIGO/Virgo gravitational-wave data,” (2021), [arXiv:2105.03349 \[gr-qc\]](#).
- [33] V. De Luca, G. Franciolini, P. Pani, and A. Riotto, “Bayesian Evidence for Both Astrophysical and Primordial Black Holes: Mapping the GWTC-2 Catalog to Third-Generation Detectors,” *JCAP* **05**, 003 (2021), [arXiv:2102.03809 \[astro-ph.CO\]](#).
- [34] Zu-Cheng Chen, Chen Yuan, and Qing-Guo Huang, “Confronting the primordial black hole scenario with the gravitational-wave events detected by LIGO-Virgo,” (2021), [arXiv:2108.11740 \[astro-ph.CO\]](#).
- [35] Ya. B. Zel’dovich and I. D. Novikov, “The Hypothesis of Cores Retarded during Expansion and the Hot Cosmological Model,” *Astronomicheskii Zhurnal* **43**, 758 (1966).
- [36] Stephen Hawking, “Gravitationally Collapsed Objects of Very Low Mass,” *Monthly Notices of the Royal Astronomical Society* **152**, 75–78 (1971), <https://academic.oup.com/mnras/article-pdf/152/1/75/9360899/mnras152-0075.pdf>.
- [37] Sebastien Clesse and Juan Garcia-Bellido, “Seven Hints for Primordial Black Hole Dark Matter,” *Phys. Dark Univ.* **22**, 137–146 (2018), [arXiv:1711.10458 \[astro-ph.CO\]](#).
- [38] Norbert Duechting, “Supermassive black holes from primordial black hole seeds,” *Phys. Rev. D* **70**, 064015 (2004), [arXiv:astro-ph/0406260](#).
- [39] Maxim Yu. Khlopov, “Primordial Black Holes,” *Res. Astron. Astrophys.* **10**, 495–528 (2010), [arXiv:0801.0116](#)

- [astro-ph].
- [40] V. De Luca, G. Franciolini, and A. Riotto, “NANOGrav Data Hints at Primordial Black Holes as Dark Matter,” *Phys. Rev. Lett.* **126**, 041303 (2021), arXiv:2009.08268 [astro-ph.CO].
- [41] Kazunori Kohri and Takahiro Terada, “Solar-Mass Primordial Black Holes Explain NANOGrav Hint of Gravitational Waves,” *Phys. Lett. B* **813**, 136040 (2021), arXiv:2009.11853 [astro-ph.CO].
- [42] Ville Vaskonen and Hardi Veermäe, “Did NANOGrav see a signal from primordial black hole formation?” *Phys. Rev. Lett.* **126**, 051303 (2021), arXiv:2009.07832 [astro-ph.CO].
- [43] Bernard Carr, Kazunori Kohri, Yuuiti Sendouda, and Jun’ichi Yokoyama, “Constraints on primordial black holes,” *Rept. Prog. Phys.* **84**, 116902 (2021), arXiv:2002.12778 [astro-ph.CO].
- [44] Yue Meng *et al.* (PandaX-4T), “Dark Matter Search Results from the PandaX-4T Commissioning Run,” *Phys. Rev. Lett.* **127**, 261802 (2021), arXiv:2107.13438 [hep-ex].
- [45] C. Bartram *et al.* (ADMX), “Search for Invisible Axion Dark Matter in the 3.3–4.2 μeV Mass Range,” *Phys. Rev. Lett.* **127**, 261803 (2021), arXiv:2110.06096 [hep-ex].
- [46] Simeon Bird, Ilias Cholis, Julian B. Muñoz, Yacine Ali-Haïmoud, Marc Kamionkowski, Ely D. Kovetz, Alvise Raccanelli, and Adam G. Riess, “Did LIGO detect dark matter?” *Phys. Rev. Lett.* **116**, 201301 (2016), arXiv:1603.00464 [astro-ph.CO].
- [47] Misao Sasaki, Teruaki Suyama, Takahiro Tanaka, and Shuichiro Yokoyama, “Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914,” *Phys. Rev. Lett.* **117**, 061101 (2016), [Erratum: Phys.Rev.Lett. 121, 059901 (2018)], arXiv:1603.08338 [astro-ph.CO].
- [48] Sebastien Clesse and Juan García-Bellido, “The clustering of massive Primordial Black Holes as Dark Matter: measuring their mass distribution with Advanced LIGO,” *Phys. Dark Univ.* **15**, 142–147 (2017), arXiv:1603.05234 [astro-ph.CO].
- [49] Bernard Carr, Sebastien Clesse, Juan García-Bellido, and Florian Kühnel, “Cosmic conundra explained by thermal history and primordial black holes,” *Phys. Dark Univ.* **31**, 100755 (2021), arXiv:1906.08217 [astro-ph.CO].
- [50] Bin Liu and Dong Lai, “Hierarchical Black-Hole Mergers in Multiple Systems: Constrain the Formation of GW190412, GW190814 and GW190521-like events,” *Mon. Not. Roy. Astron. Soc.* **502**, 2049–2064 (2021), arXiv:2009.10068 [astro-ph.HE].
- [51] R. Abbott *et al.* (LIGO Scientific, Virgo), “GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run,” *Phys. Rev. X* **11**, 021053 (2021), arXiv:2010.14527 [gr-qc].
- [52] Charles D. Bailyn, Raj K. Jain, Paolo Coppi, and Jerome A. Orosz, “The Mass distribution of stellar black holes,” *Astrophys. J.* **499**, 367 (1998), arXiv:astro-ph/9708032.
- [53] Feryal Özel, Dimitrios Psaltis, Ramesh Narayan, and Jeffrey E. McClintock, “The Black Hole Mass Distribution in the Galaxy,” *Astrophys. J.* **725**, 1918–1927 (2010), arXiv:1006.2834 [astro-ph.GA].
- [54] Will M. Farr, Niharika Sravan, Andrew Cantrell, Laura Kreidberg, Charles D. Bailyn, Ilya Mandel, and Vicky Kalogera, “The Mass Distribution of Stellar-mass Black Holes,” *Astrophys. J.* **741**, 103 (2011), arXiv:1011.1459 [astro-ph.GA].
- [55] Misao Sasaki, Teruaki Suyama, Takahiro Tanaka, and Shuichiro Yokoyama, “Primordial black holes—perspectives in gravitational wave astronomy,” *Class. Quant. Grav.* **35**, 063001 (2018), arXiv:1801.05235 [astro-ph.CO].
- [56] Anne M. Green and Bradley J. Kavanagh, “Primordial Black Holes as a dark matter candidate,” *J. Phys. G* **48**, 043001 (2021), arXiv:2007.10722 [astro-ph.CO].
- [57] Sarah Shandera, Donghui Jeong, and Henry S. Grasshorn Gebhardt, “Gravitational Waves from Binary Mergers of Subsolar Mass Dark Black Holes,” *Phys. Rev. Lett.* **120**, 241102 (2018), arXiv:1802.08206 [astro-ph.CO].
- [58] Chris Kouvaris, Peter Tinyakov, and Michel H. G. Tytgat, “NonPrimordial Solar Mass Black Holes,” *Phys. Rev. Lett.* **121**, 221102 (2018), arXiv:1804.06740 [astro-ph.HE].
- [59] Basudeb Dasgupta, Ranjan Laha, and Anupam Ray, “Low Mass Black Holes from Dark Core Collapse,” *Phys. Rev. Lett.* **126**, 141105 (2021), arXiv:2009.01825 [astro-ph.HE].
- [60] F. X. Timmes, S. E. Woosley, and Thomas A. Weaver, “The Neutron Star and Black Hole Initial Mass Function,” *Astrophys. J.* **457**, 834 (1996), arXiv:astro-ph/9510136 [astro-ph].
- [61] Yudai Suwa, Takashi Yoshida, Masaru Shibata, Hideyuki Umeda, and Koh Takahashi, “On the minimum mass of neutron stars,” *Monthly Notices of the RAS* **481**, 3305–3312 (2018), arXiv:1808.02328 [astro-ph.HE].
- [62] Michele Vallisneri, Jonah Kanner, Roy Williams, Alan Weinstein, and Branson Stephens, “The LIGO Open Science Center,” *Proceedings, 10th International LISA Symposium: Gainesville, Florida, USA, May 18–23, 2014*, *J. Phys. Conf. Ser.* **610**, 012021 (2015), arXiv:1410.4839 [gr-qc].
- [63] Rich Abbott *et al.* (LIGO Scientific, Virgo), “Open data from the first and second observing runs of Advanced LIGO and Advanced Virgo,” (2019), arXiv:1912.11716 [gr-qc].
- [64] Alexander H. Nitz and Yi-Fan Wang, “Search for gravitational waves from the coalescence of sub-solar mass and eccentric compact binaries,” (2021), 10.3847/1538-4357/ac01d9, arXiv:2102.00868 [astro-ph.HE].
- [65] Alexander H. Nitz and Yi-Fan Wang, “Search for Gravitational Waves from the Coalescence of Subsolar-Mass Binaries in the First Half of Advanced LIGO and Virgo’s Third Observing Run,” *Phys. Rev. Lett.* **127**, 151101 (2021), arXiv:2106.08979 [astro-ph.HE].
- [66] R. Abbott *et al.* (LIGO Scientific, VIRGO, KAGRA), “Search for subsolar-mass binaries in the first half of Advanced LIGO and Virgo’s third observing run,” (2021), arXiv:2109.12197 [astro-ph.CO].
- [67] B. P. Abbott *et al.* (LIGO Scientific, Virgo), “Search for Subsolar Mass Ultracompact Binaries in Advanced LIGO’s Second Observing Run,” *Phys. Rev. Lett.* **123**, 161102 (2019), arXiv:1904.08976 [astro-ph.CO].

- [68] B. P. Abbott *et al.* (LIGO Scientific, Virgo), “Search for Substellar-Mass Ultracompact Binaries in Advanced LIGO’s First Observing Run,” *Phys. Rev. Lett.* **121**, 231103 (2018), [arXiv:1808.04771 \[astro-ph.CO\]](#).
- [69] Khun Sang Phukon, Gregory Baltus, Sarah Caudill, Sebastien Clesse, Antoine Depasse, Maxime Fays, Heather Fong, Shasvath J. Kapadia, Ryan Magee, and Andres Jorge Tanasijczuk, “The hunt for sub-solar primordial black holes in low mass ratio binaries is open,” (2021), [arXiv:2105.11449 \[astro-ph.CO\]](#).
- [70] Alexander Harvey Nitz and Yi-Fan Wang, “Search for Gravitational Waves from High-Mass-Ratio Compact-Binary Mergers of Stellar Mass and Substellar Mass Black Holes,” *Phys. Rev. Lett.* **126**, 021103 (2021), [arXiv:2007.03583 \[astro-ph.HE\]](#).
- [71] Bruce Allen, “The Stochastic gravity wave background: Sources and detection,” in *Les Houches School of Physics: Astrophysical Sources of Gravitational Radiation* (1996) pp. 373–417, [arXiv:gr-qc/9604033](#).
- [72] Bruce Allen and Joseph D. Romano, “Detecting a stochastic background of gravitational radiation: Signal processing strategies and sensitivities,” *Phys. Rev. D* **59**, 102001 (1999), [arXiv:gr-qc/9710117](#).
- [73] Joseph D. Romano and Neil J. Cornish, “Detection methods for stochastic gravitational-wave backgrounds: a unified treatment,” *Living Rev. Rel.* **20**, 2 (2017), [arXiv:1608.06889 \[gr-qc\]](#).
- [74] R. Abbott *et al.* (KAGRA, Virgo, LIGO Scientific), “Upper limits on the isotropic gravitational-wave background from Advanced LIGO and Advanced Virgo’s third observing run,” *Phys. Rev. D* **104**, 022004 (2021), [arXiv:2101.12130 \[gr-qc\]](#).
- [75] Alexander H. Nitz, Thomas Dent, Gareth S. Davies, Sumit Kumar, Collin D. Capano, Ian Harry, Simone Mozzon, Laura Nuttall, Andrew Lundgren, and Márton Tápai, “2-OGC: Open Gravitational-wave Catalog of binary mergers from analysis of public Advanced LIGO and Virgo data,” *Astrophys. J.* **891**, 123 (2019), [arXiv:1910.05331 \[astro-ph.HE\]](#).
- [76] B. P. Abbott *et al.* (LIGO Scientific, Virgo), “GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs,” *Phys. Rev. X* **9**, 031040 (2019), [arXiv:1811.12907 \[astro-ph.HE\]](#).
- [77] Alexander H. Nitz, Ian W. Harry, Joshua L. Willis, Christopher M. Biwer, Duncan A. Brown, Larne P. Pekowsky, T. Dal Canton, Andrew R. Williamson, Thomas Dent, Collin D. Capano, Thomas J. Massinger, Amber K. Lenon, Alex B. Nielsen, and Miriam Cabero, “PyCBC Software,” <https://github.com/gwastro/pycbc> (2018).
- [78] Samantha A. Usman *et al.*, “The PyCBC search for gravitational waves from compact binary coalescence,” *Class. Quant. Grav.* **33**, 215004 (2016), [arXiv:1508.02357 \[gr-qc\]](#).
- [79] Bruce Allen, Warren G. Anderson, Patrick R. Brady, Duncan A. Brown, and Jolien D. E. Creighton, “FIND-CHIRP: An Algorithm for detection of gravitational waves from inspiraling compact binaries,” *Phys. Rev. D* **85**, 122006 (2012), [arXiv:0509116 \[gr-qc\]](#).
- [80] Gareth S. Davies, Thomas Dent, Márton Tápai, Ian Harry, Connor McIsaac, and Alexander H. Nitz, “Extending the pycbc search for gravitational waves from compact binary mergers to a global network,” *Phys. Rev. D* **102**, 022004 (2020), [arXiv:2002.08291 \[astro-ph.HE\]](#).
- [81] Bruce Allen, “A χ^2 time-frequency discriminator for gravitational wave detection,” *Phys. Rev. D* **71**, 062001 (2005), [arXiv:gr-qc/0405045 \[gr-qc\]](#).
- [82] Alexander H. Nitz, Thomas Dent, Tito Dal Canton, Stephen Fairhurst, and Duncan A. Brown, “Detecting binary compact-object mergers with gravitational waves: Understanding and Improving the sensitivity of the PyCBC search,” *Astrophys. J.* **849**, 118 (2017), [arXiv:1705.01513 \[gr-qc\]](#).
- [83] Alexander Harvey Nitz, “Distinguishing short duration noise transients in LIGO data to improve the PyCBC search for gravitational waves from high mass binary black hole mergers,” *Class. Quant. Grav.* **35**, 035016 (2018), [arXiv:1709.08974 \[gr-qc\]](#).
- [84] B. S. Sathyaprakash and S. V. Dhurandhar, “Choice of filters for the detection of gravitational waves from coalescing binaries,” *Phys. Rev. D* **44**, 3819–3834 (1991).
- [85] Serge Droz, Daniel J. Knapp, Eric Poisson, and Benjamin J. Owen, “Gravitational waves from inspiraling compact binaries: Validity of the stationary-phase approximation to the fourier transform,” *Phys. Rev. D* **59**, 124016 (1999).
- [86] Luc Blanchet, “Gravitational radiation from post-Newtonian sources and inspiralling compact binaries,” *Living Rev. Rel.* **5**, 3 (2002), [arXiv:gr-qc/0202016](#).
- [87] Guillaume Faye, Sylvain Marsat, Luc Blanchet, and Bala R. Iyer, “The third and a half post-Newtonian gravitational wave quadrupole mode for quasi-circular inspiralling compact binaries,” *Class. Quant. Grav.* **29**, 175004 (2012), [arXiv:1204.1043 \[gr-qc\]](#).
- [88] Yi Pan, Alessandra Buonanno, Michael Boyle, Luisa T. Buchman, Lawrence E. Kidder, Harald P. Pfeiffer, and Mark A. Scheel, “Inspiraling-merger-ringdown multipolar waveforms of nonspinning black-hole binaries using the effective-one-body formalism,” *Phys. Rev. D* **84**, 124052 (2011), [arXiv:1106.1021 \[gr-qc\]](#).
- [89] Blake Moore and Nicolás Yunes, “A 3PN Fourier domain waveform for non-spinning binaries with moderate eccentricity,” *Classical and Quantum Gravity* **36**, 185003 (2019), [arXiv:1903.05203 \[gr-qc\]](#).
- [90] Blake Moore and Nicolás Yunes, “Data Analysis Implications of Moderately Eccentric Gravitational Waves,” *Class. Quant. Grav.* **37**, 225015 (2020), [arXiv:1910.01680 \[gr-qc\]](#).
- [91] Blake Moore, Travis Robson, Nicholas Loutrel, and Nicolás Yunes, “Towards a Fourier domain waveform for non-spinning binaries with arbitrary eccentricity,” *Classical and Quantum Gravity* **35**, 235006 (2018), [arXiv:1807.07163 \[gr-qc\]](#).
- [92] Ian W. Harry, Bruce Allen, and B.S. Sathyaprakash, “A Stochastic template placement algorithm for gravitational wave data analysis,” *Phys. Rev. D* **80**, 104014 (2009), [arXiv:0908.2090 \[gr-qc\]](#).
- [93] Rahul Biswas, Patrick R. Brady, Jolien D. E. Creighton, and Stephen Fairhurst, “The Loudest Event Statistic: General Formulation, Properties and Applications,” *Class. Quantum Grav.* **26**, 175009 (2009), [arXiv:arXiv:0710.0465 \[gr-qc\]](#).
- [94] “LSC Algorithm Library Suite,” <https://git.ligo.org/lscsoft/lalsuite>.
- [95] Takeshi Chiba and Shuichiro Yokoyama, “Spin Distribution of Primordial Black Holes,” *PTEP* **2017**, 083E01

- (2017), [arXiv:1704.06573 \[gr-qc\]](#).
- [96] V. De Luca, V. Desjacques, G. Franciolini, A. Malhotra, and A. Riotto, “The initial spin probability distribution of primordial black holes,” *JCAP* **05**, 018 (2019), [arXiv:1903.01179 \[astro-ph.CO\]](#).
- [97] V. De Luca, G. Franciolini, P. Pani, and A. Riotto, “The Evolution of Primordial Black Holes and their Final Observable Spins,” *JCAP* **04**, 052 (2020), [arXiv:2003.02778 \[astro-ph.CO\]](#).
- [98] Mehrdad Mirbabayi, Andrei Gruzinov, and Jorge Noreña, “Spin of Primordial Black Holes,” *JCAP* **03**, 017 (2020), [arXiv:1901.05963 \[astro-ph.CO\]](#).
- [99] K. Postnov, A. Kuranov, and N. Mitichkin, “Spins of black holes in coalescing compact binaries,” *Phys. Usp.* **62**, 1153–1161 (2019), [arXiv:1907.04218 \[astro-ph.HE\]](#).
- [100] G. Franciolini and P. Pani, “Searching for mass-spin correlations in the population of gravitational-wave events: the GWTC-3 case study,” (2022), [arXiv:2201.13098 \[astro-ph.HE\]](#).
- [101] Takashi Nakamura, Misao Sasaki, Takahiro Tanaka, and Kip S. Thorne, “Gravitational waves from coalescing black hole MACHO binaries,” *Astrophys. J. Lett.* **487**, L139–L142 (1997), [arXiv:astro-ph/9708060](#).
- [102] Yacine Ali-Haïmoud, Ely D. Kovetz, and Marc Kamionkowski, “Merger rate of primordial black-hole binaries,” *Phys. Rev. D* **96**, 123523 (2017), [arXiv:1709.06576 \[astro-ph.CO\]](#).
- [103] Zu-Cheng Chen and Qing-Guo Huang, “Merger Rate Distribution of Primordial-Black-Hole Binaries,” *Astrophys. J.* **864**, 61 (2018), [arXiv:1801.10327 \[astro-ph.CO\]](#).
- [104] Martti Raidal, Christian Spethmann, Ville Vaskonen, and Hardi Veermäe, “Formation and Evolution of Primordial Black Hole Binaries in the Early Universe,” *JCAP* **02**, 018 (2019), [arXiv:1812.01930 \[astro-ph.CO\]](#).
- [105] Karsten Jedamzik, “Primordial Black Hole Dark Matter and the LIGO/Virgo observations,” *JCAP* **09**, 022 (2020), [arXiv:2006.11172 \[astro-ph.CO\]](#).
- [106] Gert Hütsi, Martti Raidal, Ville Vaskonen, and Hardi Veermäe, “Two populations of LIGO-Virgo black holes,” *JCAP* **03**, 068 (2021), [arXiv:2012.02786 \[astro-ph.CO\]](#).
- [107] Florian Kühnel and Katherine Freese, “Constraints on Primordial Black Holes with Extended Mass Functions,” *Phys. Rev. D* **95**, 083508 (2017), [arXiv:1701.07223 \[astro-ph.CO\]](#).
- [108] Bernard Carr, Martti Raidal, Tommi Tenkanen, Ville Vaskonen, and Hardi Veermäe, “Primordial black hole constraints for extended mass functions,” *Phys. Rev. D* **96**, 023514 (2017), [arXiv:1705.05567 \[astro-ph.CO\]](#).
- [109] Derek Davis *et al.* (LIGO), “LIGO detector characterization in the second and third observing runs,” *Class. Quant. Grav.* **38**, 135014 (2021), [arXiv:2101.11673 \[astro-ph.IM\]](#).
- [110] Vuk Mandic, Simeon Bird, and Ilias Cholis, “Stochastic Gravitational-Wave Background due to Primordial Binary Black Hole Mergers,” *Phys. Rev. Lett.* **117**, 201102 (2016), [arXiv:1608.06699 \[astro-ph.CO\]](#).
- [111] Sai Wang, Yi-Fan Wang, Qing-Guo Huang, and Tjonnie G. F. Li, “Constraints on the Primordial Black Hole Abundance from the First Advanced LIGO Observation Run Using the Stochastic Gravitational-Wave Background,” *Phys. Rev. Lett.* **120**, 191102 (2018), [arXiv:1610.08725 \[astro-ph.CO\]](#).
- [112] Pablo A. Rosado, “Gravitational wave background from binary systems,” *Phys. Rev. D* **84**, 084004 (2011), [arXiv:1106.5795 \[gr-qc\]](#).
- [113] Sascha Husa, Sebastian Khan, Mark Hannam, Michael Pürrer, Frank Ohme, Xisco Jiménez Forteza, and Alejandro Bohé, “Frequency-domain gravitational waves from nonprecessing black-hole binaries. I. New numerical waveforms and anatomy of the signal,” *Phys. Rev. D* **93**, 044006 (2016), [arXiv:1508.07250 \[gr-qc\]](#).
- [114] Sebastian Khan, Sascha Husa, Mark Hannam, Frank Ohme, Michael Pürrer, Xisco Jiménez Forteza, and Alejandro Bohé, “Frequency-domain gravitational waves from non-precessing black-hole binaries. II. A phenomenological model for the advanced detector era,” *Phys. Rev. D* **93**, 044007 (2016), [arXiv:1508.07253 \[gr-qc\]](#).
- [115] Savvas M. Koushiappas and Abraham Loeb, “Maximum redshift of gravitational wave merger events,” *Phys. Rev. Lett.* **119**, 221104 (2017), [arXiv:1708.07380 \[astro-ph.CO\]](#).
- [116] Zu-Cheng Chen and Qing-Guo Huang, “Distinguishing Primordial Black Holes from Astrophysical Black Holes by Einstein Telescope and Cosmic Explorer,” *JCAP* **08**, 039 (2020), [arXiv:1904.02396 \[astro-ph.CO\]](#).
- [117] Ken K. Y. Ng, Shiqi Chen, Boris Goncharov, Ulyana Dupletsa, Ssohrab Borhanian, Marica Branchesi, Jan Harms, Michele Maggiore, B. S. Sathyaprakash, and Salvatore Vitale, “On the single-event-based identification of primordial black hole mergers at cosmological distances,” (2021), [arXiv:2108.07276 \[astro-ph.CO\]](#).
- [118] B. P. Abbott *et al.* (LIGO Scientific Collaboration, Virgo Collaboration), “Prospects for Observing and Localizing Gravitational-Wave Transients with Advanced LIGO and Advanced Virgo,” *Living Rev. Relat.* **19**, 1 (2016), [arXiv:1304.0670 \[gr-qc\]](#).