Space-based Gravitational Wave Observatories Will Be Able to Use Eccentricity to Unveil Stellar-mass Binary Black Hole Formation

Han Wang(王晗)[1], Ian Harry[2], Alexander Nitz[3,4] and Yi-Ming Hu(胡一鸣)[1,*]

[1] *MOE Key Laboratory of TianQin Mission, TianQin Research Center for Gravitational Physics & School of Physics and Astronomy, Frontiers Science Center for TianQin, Gravitational Wave Research Center of CNSA, Sun Yat-sen University (Zhuhai Campus), Zhuhai 519082, China*

[2] *University of Portsmouth, Portsmouth, PO1 3FX, United Kingdom*

[3] *Department of Physics, Syracuse University, Syracuse NY 13244, USA*

[4] *Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), D-30167 Hannover, Germany*

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The measurement of eccentricity would provide strong constraints on the formation channel of stellar-mass binary black holes. However, current ground-based gravitational wave detectors will, in most cases, not be able to measure eccentricity due to orbital circularization. Space-based observatories, in contrast, can determine binary eccentricity at 0.01Hz to $e_{0.01} \gtrsim \mathcal{O}(10^{-4})$. Directly observing stellar-mass binary black holes with space-based observatories remains a challenging problem. However, observing such systems with ground-based detectors allows the possibility to identify the same signal in archival data from space-based observatories in the years previous. Since ground-based detectors provide little constraints on eccentricity, including eccentricity in the archival search will increase the required number of filter waveforms for the archival search by 5 orders of magnitudes [from $\sim \mathcal{O}(10^3)$ to $\sim \mathcal{O}(10^8)$], and will correspondingly need $\sim 8 \times 10^5$ core hours (and $\sim 10^5$ GB of memory), even for a mild upper limit on eccentricity of 0.1. In this work, we have constructed the first template bank for an archival search of space-based gravitational wave detectors, including eccentricity. We have demonstrated that even though the inclusion of eccentricity brings extra computational burden, an archival search including eccentricity will be feasible in the time frame of planned space-based observatories, and will provide strong constraints on the eccentricities of stellar-mass binary black holes.

I. INTRODUCTION

Stellar-mass black holes (sBBHs) detected before 2015 were mainly observed through X-ray binaries [1, 2], with measured masses $\lesssim 20M_\odot$ [3]. The first gravitational wave (GW) signal GW150914 observed by LIGO and Virgo has been identified as the coalescence of stellar-mass binary black hole (sBBH) with component masses $36^{+5}_{-4}M_\odot$ and $29^{+4}_{-4}M_\odot$ [4]. The observed masses posed a significant challenge to our understanding of the formation mechanism of sBBHs [5]. To date, nearly 100 sBBH mergers have been reported, many of them as heavy as GW150914 [6, 7]. With the accumulation of GW observations, numerous models have been proposed to explain the formation of these sBBHs [8]. The eccentricity of a sBBH system is a key probe in unveiling the system's formation mechanism. However, among all GW detections, none was claimed to have measurable eccentricity (eccentricity at 10 Hz $e_{10} \gtrsim 0.1$) [9–11] until GW190521, which some argue could be eccentric [12–14]. The sensitive frequency band of current ground-based detectors makes them only capable of observing sBBHs seconds before coalescence. Advanced LIGO/Virgo can measure the eccentricity for binaries with $e_{10} \gtrsim 0.05$ [15], but most sBBHs cannot retain eccentricity that high because of orbital circularization due to gravitational wave emission before entering the ground-based frequency band [16]. Therefore, it is challenging for ground-based detectors to distinguish and identify the formation channels of sBBHs [17].

Space-based GW observatories, like TianQin [18] and Laser Interferometer Space Antenna (LISA) [19], offer a promising solution to this question. They have longer baselines than their ground-based counterparts and therefore are sensitive in a lower frequency band and could observe sBBHs for years. This makes space-based observatories capable of precise mass measurements and unveiling the evolution of eccentricity and spin of sBBH sources [20–25]. For example, eccentricity evolves as $e \sim e_{\rm i}(f/f_{\rm i})^{-19/18}$ at leading order [26]. If the GW of a binary system evolves to the ground-based detector frequency band at $f \gtrsim 1$Hz with eccentricity equal to 10^{-3}, the system has a significantly larger eccentricity, $e_{\rm i} \sim 0.1$, at a frequency $f_{\rm i} \sim 0.01$Hz, which is a typical sensitive frequency for space-based observatories.

Figure 1 shows eccentricity distributions predicted by different evolution models. sBBHs formed in isolation are likely to have $e_{0.01} \lesssim 10^{-3}$ [27, 28]. sBBHs dynamically formed in globular clusters and subsequently ejected into the field have similar distributions, with $e_{0.01} \lesssim 10^{-2}$ [28, 29]. However, sBBHs that evolve inside clusters can retain a high eccentricity with $e_{0.01} \gtrsim 10^{-2}$ [30], and eccentricities can reach extreme values ($e_{0.01} \sim 1$) for systems involved in various triplets [31–33] or in active galactic nuclei (AGN) disks [34]. Space-based observatories have the capability to detect eccen-

*Electronic address: huyiming@mail.sysu.edu.cn

tricities $e_{0.01} \gtrsim 10^{-3}$ [20, 23]. Therefore, sBBH detections with space-based observatories, alongside observations with ground-based facilities, offer a unique opportunity to identify the formation channel of sBBHs.

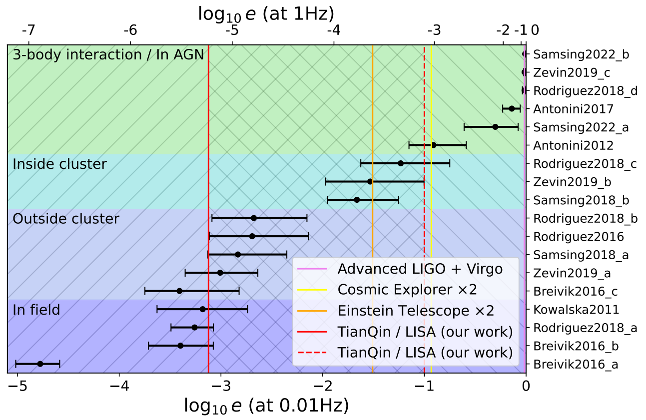

FIG. 1: Predicted eccentricity distributions from different evolution models.The black dots and error bars represent the median values and 50% credible intervals, respectively. The vertical solid (dashed) lines indicate the minimum (maximum) detectable eccentricities of different GW observatories.

Considering eccentricity for the sBBHs can bring additional benefits. The inclusion of eccentricity can break parameter degeneracy [35], improve the precision of measuring source distance and sky localization [36, 37], and make future tests of general relativity more reliable [38, 39].

Matched-filtering methods have been widely used in ground-based GW detection [40]. These searches require a suitable set of waveform filters, or "template bank". Applying this method to TianQin or LISA will be challenging because of the number of waveform templates required. An example search for compact binary mergers in LIGO/Virgo data requires $\lesssim 4 \times 10^5$ templates [41]. In contrast Moore et al. [42] predicts that a bank of order 10^{30} templates would be needed to cover the whole sBBH parameter space for LISA, far exceeding a reasonable computational cost.

It has been proposed that a search of archival data from space-based observatories, triggered by detection with ground-based facilities, can achieve the multiband detection of sBBHs [25, 43–45]. Next-generation ground-based detectors, like Einstein Telescope (ET) [46] and Cosmic Explorer (CE) [47] will be able to detect GW events with signal-to-noise ratios (SNRs) $\mathcal{O}(10^{2-3})$ and will therefore place tight constraints on the source parameters, for example measuring the chirp mass to one part in 10^6 [48]. Therefore, the parameter space of an archival search of TianQin/LISA data can be greatly reduced and the required template bank size reduced to the level of 10^4 templates [45].

However, the impact of the eccentricity on archival searches has not been explored. In this paper, for the first time, we implement a matched-filtering bank generation process for an archival search in space-based observatories incorporating eccentricity, triggered by an observation using next-generation ground-based detectors. Using GW150914 and GW190521 as examples, we find that even though the inclusion of eccentricity would enlarge the template bank by a factor of $\sim \mathcal{O}(10^5)$, the task is still tangible. This work provides a practical solution to the realistic multiband GW observation scenario.

II. METHODOLOGY

To detect GWs by matched filtering, we use `EccentricFD` [26, 49], a nonspinning inspiral-only frequency-domain waveform approximant with eccentricity at the initial frequency $e_{\rm i}$ valid up to 0.4, for constructing the template bank. `EccentricFD` includes post-Newtonian (PN) corrections up to 3.5PN order and has been included into `LALSuite` [50]. The eccentricity in `EccentricFD` is expanded to $\mathcal{O}(e^8)$ and then further expanded in $e_{\rm i}$ up to $\mathcal{O}(e_{\rm i}^8)$. The parameter set follows $\lambda^\mu = (\mathcal{M}, \eta, D_L, t_c, \phi_c, \iota, \lambda, \beta, \psi, e_{\rm i})$, where $\mathcal{M} \equiv (m_1 m_2)^{3/5}(m_1+m_2)^{-1/5}$ and $\eta \equiv (m_1 m_2)\,(m_1+m_2)^{-2}$ given by the component masses m_1 and $m_2 (m_1 > m_2)$ are the chirp mass and symmetric mass ratio, D_L is the luminosity distance, t_c and ϕ_c are the coalescence time and phase, ι is the inclination angle, (λ, β) are ecliptic longitude and ecliptic latitude, ψ is the polarization angle and $e_{\rm i}$ is the eccentricity at the initial frequency $f_{\rm i}$ in the quadrupolar GW mode. For space-based observatories, $f_{\rm i}$ is determined by the evolution time T from the beginning of observation to the merger. In this work, we assume a fully continuous five-year observation for both TianQin and LISA, and the merger happens at the end of the five-year period. For $M_{\rm tot} \lesssim 10^5 M_\odot$ and $T \gtrsim 1$yr, the correction for $f_{\rm i}$ from the eccentricity can be neglected(see Ref. [26], Appendix E), so we will use the noneccentric frequency-time relation at leading PN order in the following calculation: $f_{\rm i} = (5/256)^{3/8} \pi^{-1} \mathcal{M}^{-5/8} T^{-3/8}$.

The size of the parameter space that would need to be searched in an archival search depends on the parameter estimation precision of the next-generation ground-based detectors. One can use the Fisher information matrix (FIM) Γ_{ij} to estimate the statistical uncertainties in measuring parameters. $\Gamma_{ij} = \left(\frac{\partial h}{\partial \lambda^i} \middle| \frac{\partial h}{\partial \lambda^j}\right)$, where $(h|g) \equiv 4\Re \int_0^{+\infty} \frac{\tilde{g}^*(f)\tilde{h}(f)}{S_n(f)} {\rm d}f$, $S_n(f)$ is the one-sided detector noise power spectral density, $\tilde{h}(f) = \tilde{h}(f, \lambda^\mu)$ is the Fourier transform of the waveform $h(t)$, and λ^μ is the parameter set. The overall FIM of a detector network is the summation of the FIM of each detector. Under the Gaussian stationary assumption, the covariance matrix can be approximated by $\Sigma = \Gamma^{-1}$, and the marginalized parameter uncertainties can be estimated as $\sigma_{\lambda^i} = \sqrt{\Sigma_{ii}}$.

Here we consider a ground-based detector network including ET and two CEs, with their sites randomly chosen. Since GW emission will cause a binary orbit to circularize over time [16], we assume that events are noneccentric in the ground-based observation window. Higher-order modes, however, will be important for ET or CE, especially given the large SNR that events visible to LISA and TianQin will have. We therefore use the noneccentric `IMRPhenomHM` [51] waveform to estimate the precision with which next-generation ground detectors can measure source parameters. We choose a low-frequency cutoff of $f_{\rm low} = 1$Hz for both CE and ET during the calculation. This is motivated by the result that one would acquire 20% of the whole SNR between 1 and 10Hz with ET [52]. Our estimation is consistent with previous studies [23, 43, 45], which show that for a GW event that retains no eccentricity when entering the ground-based observation window, the only two parameters that space-based observatories can measure more precisely are the chirp mass $\mathcal{M}$ and initial eccentricity $e_{\rm i}$. Therefore we assume that all the parameters except for chirp mass and eccentricity are known exactly when performing an archival search, and the chirp mass range is determined by the uncertainty from the network of the ET and two CEs, i.e., $\mathcal{M} \in [\mathcal{M}_0 - 10\sigma_{\mathcal{M}}, \mathcal{M}_0 + 10\sigma_{\mathcal{M}}]$. In the future, we should directly use the posterior from Bayesian inference in ground-based detectors, but for this study, the uncertainty range generated by the FIM is a reasonable and conservative estimate.

We construct a template bank using `sbank` [53–55], a `PYTHON` package for generating stochastic template banks for compact binaries. When generating template banks stochastically we need to determine how much those two waveforms overlap with each other. The fitting factor (FF) is used to define the maximum "similarity" between a given waveform and the best matching template in a bank: [56]

$$\mathrm{FF}\left(\lambda^{\mu}\right) \equiv \max_{\lambda^{\mu'}} \frac{\left(h(\lambda^{\mu})\middle|h(\lambda^{\mu'})\right)}{\sqrt{\left(h(\lambda^{\mu})|h(\lambda^{\mu})\right)\left(h(\lambda^{\mu'})|h(\lambda^{\mu'})\right)}}. \quad (1)$$

Here $\lambda^{\mu'}$ denotes the parameter set for a template in the bank, and λ^{μ} is the parameter set for the test waveform. For a template bank to be complete (or "valid"), any GW signal in its parameter range should have FF $\geq M$, where M is the minimal match. Here we set $M = 0.97$, which is a commonly used value [6, 7, 41].

Ground-based detectors observe the GW signal over a period of only seconds before coalescence, so that the Doppler frequency modulation from the movement at Earth's orbit can be ignored. However, the long observation time and the orbital motion of space-based observatories make the response time dependent, and one must consider these time-dependent response terms during bank generation. Additionally, unlike ground-based detectors that have fixed arm lengths during operation, the relative spacecraft motion results in unequal arm lengths. The method of time delay interferometry (TDI) has been proposed for canceling out the laser phase noise from different arms. It constructs particular combinations to make virtual equal arm interferometers. This is further complicated when considering eccentric waveforms. Here we use the frequency-domain TDI response [57, 58] and combine it with `EccentricFD` which contains a set of eccentric harmonics. We follow the arm length and noise budget in Luo et al. [18] for TianQin, and $L = 2.5 \times 10^9$m with noise budget from Babak et al. [59] for LISA. We consider the response in the A channel as an example during all the calculations in this work.

Since different eccentric harmonics have different correspondences with the Fourier frequency, we should provide a frequency cutoff during the calculation to avoid the waveform generation exceeding the valid range for a specific GW detector: $\tilde{h}_{\rm det} = \sum_j \tilde{h}_j \times \Theta\left(j \cdot f_{\rm high} - 2f\right)\Theta\left(2f - j \cdot f_{\rm low}\right)$, where $\Theta(x)$ is the Heaviside step function and j denotes the jth eccentric harmonic [26]. For TianQin or LISA, we have $f_{\rm low} = \max\left[10^{-4}\mathrm{Hz}, f_0\right], f_{\rm high} = \min\left[f_{\rm ISCO}, 1\mathrm{Hz}\right]$, where $f_{\rm ISCO} = (6^{3/2}\pi(m_1 + m_2))^{-1}$ is the quadrupolar frequency at innermost-stable circular orbit (ISCO).

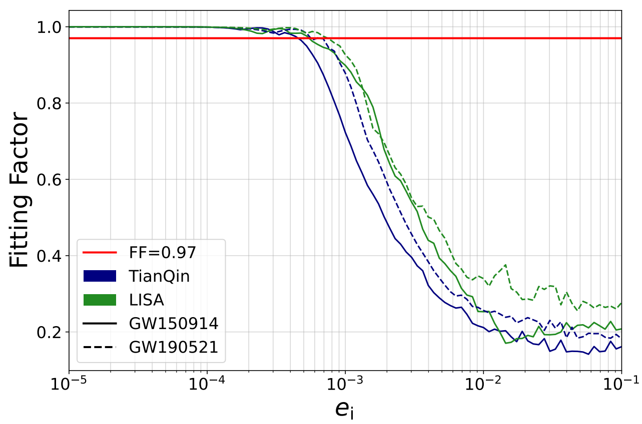

FIG. 2: The fitting factor between a noneccentric template bank and a signal with different eccentricities. The blue(green) lines denote the banks of TianQin(LISA), the solid(dashed) lines correspond to the banks of a GW150914-like(GW190521-like) scenario.

III. STOCHASTIC TEMPLATE BANK GENERATION

If a signal has small eccentricity, it could be that a circular waveform would be sufficient to recover it. The question is, how small is small enough? We therefore use a noneccentric bank (i.e. banks of $\mathcal{M}$ in Table I) and match it with an eccentric signal. In Fig 2, we plot the fitting factor between the injected eccentric waveform and the template bank. As expected, the mismatch increases as eccentricity gets larger and we find that the eccentricity is distinguishable for TianQin/LISA when $e_{\rm i} \gtrsim 5 \times 10^{-4}$. Many models, for example, dynamical interactions mechanisms [21, 27, 30], predict larger initial eccentricity at ~ 0.01Hz. We also investigate the bias between the injected and recovered chirp mass when neglecting eccentricity, which increases from $\lesssim 10^{-6}M_\odot$ at $e_\mathrm{i}=0$ to $\gtrsim 10^{-3}M_\odot$ at $e_\mathrm{i}=0.1$. Such systematic bias could be even larger in the full parameter space. It is therefore necessary for searches to take eccentricity into account.

TABLE I: Template bank sizes for GW150914- and GW190521-like events with different parameter spaces.

	Parameter space	GW150914-like	GW190521-like
TianQin	$e_\mathrm{i}\in[0,0.1]$	117202	49943
	$\mathcal{M}\in\mathcal{M}_0\pm 10\sigma_\mathcal{M}$	3034	4250
LISA	$e_\mathrm{i}\in[0,0.1]$	100403	44867
	$\mathcal{M}\in\mathcal{M}_0\pm 10\sigma_\mathcal{M}$	2070	3088

In Table I we show the size of the stochastic template banks, with different parameter spaces for both GW150914- and GW190521-like sources. We first assume that all the parameters (including chirp mass) are known exactly except for eccentricity, and thus generate a one-dimensional bank with $e_\mathrm{i}\in[0,0.1]$. The bank size is as large as $\mathcal{O}(10^5)$ when only searching over eccentricity, and requires $\lesssim 80$ core hours (and $\lesssim 100$GB of memory). Therefore, for TianQin/LISA, we consider $e_\mathrm{i}=5\times10^{-4}$ ($e_\mathrm{i}=0.1$) as the smallest distinguishable eccentricity (the upper limit by the current computational cost), which corresponds to the red solid (dashed) line in Fig. 1. Figure 3 shows the distribution of the eccentricity, which follows an e^2 cumulative distribution. It agrees with the theoretical estimate in previous studies [20], subject to Poisson fluctuation as indicated by the shaded region. We then generate a one-dimensional bank covering only a range of chirp mass. Since the range is small, $\mathcal{M}$ appears to be uniformly distributed.

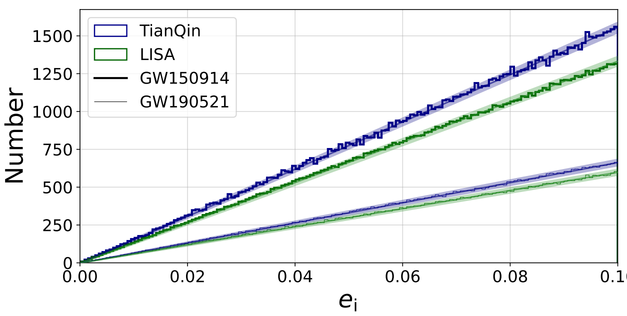

FIG. 3: The distribution of the eccentricity in the archival search template bank. The shaded regions represent the 1σ Poisson fluctuation.

As our current result fits well with a theoretical distribution for both eccentricity and chirp mass, we can give a reasonable estimation for the full 2D parameter space of the archival search. By assuming the 2D bank follows the same relationship as the 1D eccentric bank as the eccentricity range increases, the full 2D archival search banks are expected to have $N_T\sim\mathcal{O}(10^8)$ templates, if we consider the maximal valid range for `EccentricFD`, i.e. $e_\mathrm{i}\in[0,0.4]$, N_T will be up to $\mathcal{O}(10^9)$.

To evaluate if we have overestimated the magnitude of 2D bank size due to any degeneracy between the eccentricity and the chirp mass [60–62], we generate a 2D bank for both detectors and for both sources. Restricted by the huge computational burden, we choose to verify the estimate through a bank within a smaller eccentricity range of $e_\mathrm{i}\in[0,0.001]$. All 2D banks have $N_T\sim\mathcal{O}(10^4)$, which is smaller but of the same order as the direct multiplication of bank sizes that are calculated separately in their parameter spaces. Such results do not change our magnitude estimation of the full 2D archival search bank size. This indicates the challenge of computational cost: an example 2D bank with $e_\mathrm{i}\in[0,0.001]$ includes 13372 templates, and would need ~ 80hr for one core (and 18 GB of memory to cache waveforms) to generate. By slicing the full parameter space along eccentricity and generating the 2D bank in parallel, a bank with $N_T\sim\mathcal{O}(10^8)$ needs $\sim 8\times10^5$ core hours (and $\sim 10^5$GB of memory).

To evaluate the performance of our template banks, we perform tests to quantify both the validity and redundancy. First we randomly generate 10,000 test waveforms with parameter values drawn from within the parameter space of the bank, and calculate the fitting factor for each waveform. If the bank is valid, all the test waveforms will have at least one template with which the match is larger than the minimal match threshold ($M=0.97$). In Fig. 4 we present the histogram of the fitting factor for the 10,000 injected waveforms. The red vertical line represents the threshold $M=0.97$, and we find that for almost all cases, the injected waveform has a FF larger than 0.97, only 0.44% of them fall lower than 0.97.

Then we move on to test the redundancy of the generated bank. We calculate the match between every template in the template bank. An ideal bank will have no redundancy, meaning the matches between all pairs of templates should be smaller than the minimal match threshold. In Fig 4, following the validity test, for each template we present the histogram of the fitting factor, which is calculated on a bank that excludes the template itself. We find that only 6.22% of all templates are redundant. This brings marginal extra computational cost.

IV. SUMMARY AND DISCUSSION

Numerous studies pointed out that the eccentricity of sBBHs will play a significant role in unveiling their origin. In this paper, we demonstrate that the archival search of the sBBHs from space-based observatories is highly sensitive to the eccentricity. Furthermore, for the first time, we successfully implement a GW template bank generation process that includes eccentricity.

We generate one-dimensional template banks for either initial eccentricity or for chirp mass. The upper limit of

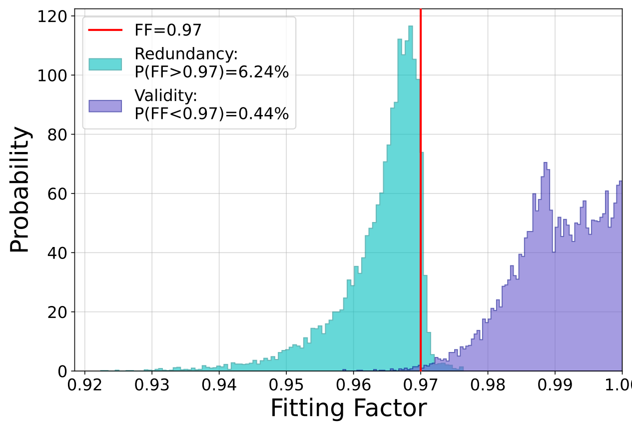

FIG. 4: Validity and redundancy test of the example 2D template bank. The histogram in purple (cyan) shows the result of the validity test (redundancy test). The vertical red line corresponds to the match criteria $M = 0.97$.

initial eccentricity at a system five years before merger is 0.1. The range of chirp mass is determined by the estimation with the ground-based network. By extrapolating the one-dimensional bank results, we conclude that a two-dimensional eccentric bank will comprise $N_T \sim \mathcal{O}(10^{8-9})$ templates, which is $\sim \mathcal{O}(10^5)$ larger compared to the zero eccentricity case, and will require $\sim \mathcal{O}(10^6)$ core hours [and $\sim \mathcal{O}(10^5)$GB of memory] for the pipeline to generate it [also $\sim \mathcal{O}(10^5)$ larger compared to noneccentric case]. This conclusion is verified by a small 2D bank, where the upper limit on the initial eccentricity is 0.001. Constructing and filtering a template bank of 10^{8-9} waveforms will therefore be a challenging task, but it is not outside the scope of the expected computational facilities in the late 2030s, and could be further improved with additional optimization of the relevant software techniques.

Our work provides a practical solution to the realistic multiband GW observation scenario, with which one can determine the formation mechanism of sBBHs with successful archival searches.

It should be noted that we use a nonspinning eccentric waveform model in the paper. It is already known that spin effects are largely negligible during the inspiral[63] phase. However, in our technique, this is not a concern at all because the spin would already be constrained by ground-based facilities. Our space-based archival search would then just search a range of chirp mass and eccentricity values, using the measured black hole spins. It is important to note though that for both ground- and space-based detectors, more precise waveform models will be needed in the future to avoid potential systematic errors [58, 64–67].

One caveat in the study is the duty cycle. We consider ET + dual CE for the ground-based detectors, whereas in reality the duty cycle cannot reach 100%; so the sky localization from realistic future networks might be worse than our calculation. Space-based observatories will also be limited by duty cycles[18, 68]. We leave the detailed calculation to future studies.

V. ACKNOWLEDGMENTS

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