Impact of the density of line service stations on overall performance in Bi-modal public transport settings

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Abstract

Human mobility is mostly dominated by the use of private cars, leading to disproportionate carbon emissions, resource consumption, traffic jams, and pollution. Public transport, with buses, trains, etc., can mitigate these issues via its higher pooling potential. However, often times, public transport is considered less convenient and is therefore avoided. Here, we study a bi-modal public transport system consisting of a rail bound line service and a fleet of on-demand shuttles providing connections to the line service stops, aiming at fast transit at low energy and resource consumption. By means of agent-based simulations and analytical theory, we demonstrate that bi-modal transit indeed has the potential to significantly reduce energy consumption of human mobility at reasonable service quality. We further investigate the influence of the stop density along the rails upon the performance of the bi-modal system. We find that within a range of realistic technical parameters, additional stops tend to impede train speed without significantly enhancing the overall performance of bi-modal transit in terms of service quality and energy consumption. Hence, it can be beneficial to reduce the number of stops within an existing railway system and to implement bi-modal transit as a complement.

1. Introduction

Transportation plays a vital role in our society, enabling people to move from one place to another. However, the traditional motorized individual vehicle (MIV, i.e., the private car) used for passenger transportation is highly inefficient, requiring the movement of a ton of material to transport just one person (MacKenzie et al., 2014; Tachet et al., 2017). This wastefulness, causing air pollution (Cattazzo et al., 2013; European Environment Agency, 2020a) and other environmental impact (European Environment Agency, 2020b; Joireman et al., 2004) as well as traffic congestion (Arnott and Small, 1994; Barth and Boriboomsomsin, 2009; Chin, 1996; Kožlak, Aleksandra and Wach, Dagmar, 2018), highlights the need for more sustainable and efficient transportation solutions.

In stark contrast to MIV, line services can carry hundreds of passengers at a time (e.g., light rails). This makes them an ideal candidate for sustainable public transportation (PT) (Ferbrache and Knowles, 2017; Kato and Kaneko Y. & Soyama, 2014; Pietrzak and Pietrzak, 2019). However, owing to its seemingly higher convenience (Kent, 2013), the MIV dominates the global mobility market (Eurostat, 2022; Fiorello et al., 2016), compared to line services, with their downsides of fixed routes, fixed schedules, and a limited set of fixed stops for accessing and leaving the vehicles.

Demand responsive ride-pooling (DRRP) services, on the other hand, offer flexible routes and schedules by deploying shuttles which pick up and drop off users at the desired locations. Combining individual user requests into an appropriate set of routes of the shuttles (Alonso-Mora et al., 2017), they provide door-to-door transport, similar to the MIV, but at higher occupancy. In such systems,
Fig. 1. Bi-modal transport network on a square grid. (a) Idealized line service network. Green nodes at the intersection of railway lines represent the train stations. (b) A network as in (a), but with additional (intermediate) stations. (c) A snapshot of a simulation where passengers use MIVs (red dots) as the only mode of transportation. (d) A snapshot of bi-modal simulations on a network with one intermediate station ($\theta = 1$). Demand is the same as in (c). Red dots represent shuttles (DRRP), gray rectangles represent trains, green diamonds represent train stations. The number of required shuttles in a bi-modal system (d) is much lower than the number of MIVS required in (c). See Section 3.4 for a quantitative analysis.

however, users necessarily experience some undesirable detour with respect to the direct route due to the necessity to pick up and drop off other passengers (Herminghaus, 2019; Lobel and Martin, 2020). This trade-off between pooling and detouring severely limits the achievable pooling efficiency (Zwick et al., 2021), and therefore the achievable reduction in traffic and emissions.

A promising solution is bi-modal transport, in which DRRP is combined with line services. Line services, with their fixed routes and schedules, facilitate high vehicle occupancy, while DRRP shuttles provide on-demand transport to and from the line service stops. Additionally, shuttles can serve short distance requests door-to-door, where usage of line services is inefficient. This integration of line services and DRRP has the potential to achieve high pooling efficiency while maintaining user convenience.

In a previous, mean-field based approach, Sharma et al. (2023) identified user convenience and energy consumption as the relevant conflicting objectives for optimization of bi-modal transit. They found that with a diligent choice of system parameters, energy consumption may be reduced to about 20% relative to MIV, and traffic volume to less than 10% relative to MIV. Their findings suggested that bi-modal public transport systems have the potential to outperform customary public transportation as well as MIV in several respects. This appears promising, because most agglomerations have a passenger transport line service system already in place, and DRRP shuttles can be deployed on the streets in between. It therefore suggests itself to study bi-modal transport in a geometry akin to what one finds in real settings, in order to come close to advising authorities and policy makers.

Sharma et al. (2023) adopted a simplified ‘cartesian’ network as sketched in Fig. 1a, where they assumed line service stations to exist at each crossing. In most cities, however, a line has more stops than crossings with other lines, as sketched in Fig. 1b. The
impact of these intermediate stops on the overall performance of the system is hitherto not known. On the one hand, intermediate stops increase the user’s proximity to transit stations, resulting in shorter shuttle trips to and from train stations. On the other hand, additional stations slow down the trains, potentially impacting the service quality for longer trips.

In the present paper, we investigate the impact of intermediate stops on bimodal-transit via agent-based simulations. By studying realistic sets of parameters, we aim to provide valuable insights into how the addition of intermediate stops affects factors such as travel time, passenger waiting time, and vehicle occupancy. In addition, we validate the findings of our previous study (Sharma et al., 2023), which were based on a mean-field approach.

We find that 1) intermediate stops reduce average shuttle occupancy, detours, and waiting times. The maximum size of this effect increases with demand. 2) Intermediate stops do not improve overall performance because reduction in quality due to slower trains dominates over slight improvements in energy consumption and traffic volume. 3) With or without intermediate stops, average shuttle occupancy, detours, and waiting times vary significantly with the fraction of bi-modal trips, and increase with demand, in contrast with the assumption of constant waiting times and detours in Sharma et al. (2023). 4) None the less, our agent-based simulations validate the performance potential of bi-modal transit as suggested in Sharma et al. (2023), however, they provide a refined quantitative picture, especially regarding achievable service quality.

In the following, we first introduce the geometry and parameters of the system under consideration (Section 2.1). Subsequently, we describe the objectives of optimization and the parameters of operation (Section 2.2). We then present results from our simulations and compare them to theoretical modeling in Section 3. In the results part we discuss performance of DRRP (shuttles) within bi-modal transit (Section 3.1), energy consumption and service quality of bi-modal transit as a whole (Section 3.2), Pareto-optimal solutions (Section 3.3), and potential for traffic reduction (Section 3.4). We end with a discussion of our findings (Section 4).

2. Methods

2.1. Definition of the system

For simulating a system of bi-modal transport, with on-demand shuttles and trains operating on lines, we deploy the open-source, multi-agent transport simulation framework, MATSim (Horni et al., 2016). In contrast to the mean-field study by Sharma et al. (2023), where observables like waiting time, vehicle occupancy, or mean detour were estimated by heuristics, in this agent-based framework, transportation requests are served explicitly by individual vehicles (the agents), such that the aforementioned quantities emerge solely based on the user environment and the parameters of operation of the bi-modal transit system. Our methodological contribution lies in the implementation of a bi-modal transit system within an agent-based simulation framework.

To assess the validity of the assumptions, as well as findings on potential reductions in energy consumption and traffic volume in Sharma et al. (2023), we choose the same parameters in our simulations as in Sharma et al. (2023), where applicable. In addition, we introduce intermediate stations between line crossings (see Fig. 1 b) to study the impact of the density of line service stations on the overall performance in bi-modal public transport settings. We refer to Supporting Information for algorithmic details of this framework. In the following we present the characteristic parameters of this model.

*User environment.* In simulations, we consider a uniformly populated planar region of side length $L = 20 \text{ km}$, i.e., an area of $A = 400 \text{ km}^2$, and a total number of transit requests $N$, uniformly randomly spread over a time $T = 1 \text{ h}$. Introducing population density $E$ and average request frequency per inhabitant $v$, the number of transit requests per time $N/T = vE/A$. Users are assumed to place transit requests in an uncorrelated fashion, each consisting of a desired pick-up ($P$) and drop-off ($D$) location with a requested distance $d = PD$ following a distribution $p(d)$. As average requested distance we choose $D = 5 \text{ km}$ in simulations. We define $D$ as the intrinsic length scale of our system. Shuttles and MIV are assumed to have a characteristic road vehicle velocity, $v_0$, which we choose to be $30 \text{ km/h}$ in simulations. We can thus obtain the intrinsic time scale $t_0 = D/v_0 = 10 \text{ min}$. This is the average time a travel request would need to be completed by MIV. We denote (non-dimensional) variables measured in these units ($D, t_0$) with the  symbol. The demand of transport within the system can be characterized by the dimensionless parameter $\Lambda = \frac{AD}{v_0} = vE$, which measures the number of requests for transport in an area of $D^2$ during time $D/v_0$ (Sharma et al., 2023). Typical values range from $\Lambda = 10^2$ to $10^4$ for rural up to dense urban transportation scenarios (Sharma et al., 2023).

*Bi-modal transit system.* We distinguish two types of train stations. Train stations at railway intersections are called junction stations. They are separated by the lattice constant $\xi$. Additional train stations are called intermediate stations. The distance between two adjacent train stations along a train line is $\xi$. The number of intermediate stations between two junctions is then $\Theta = \frac{\xi}{\xi} - 1$. All train stations (intermediate and junction) serve as the connection points between DRRP and line service (see Fig. 1b). We restrict our analysis to values of up to three ($\Theta \in \{0, 1, 3\}$) intermediate stations, motivated by agglomerations like Berlin and New York, where we find about one intermediate stop on average, and only rarely more than two (see Supporting Information for details). We choose a grid of length $\xi = 2 \text{ km}$ (Sharma et al., 2023), since we observed previously that the performance of the system is optimal for $\xi/D = 0.4$ across an extensive range of user demand (Sharma et al., 2023). This value is also close to what is found, e.g., for Berlin (Sharma et al., 2023). Therefore, we fix $\xi/D = 0.4$ throughout this study.

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1. We are using the inverse-gamma distribution as it has been observed in NYC, for example (Herminghaus, 2019).
2. Introduction of intrinsic length and time scales, $D$ and $t_0$, as units reduces the number of parameters by two.
3. $D = 5 \text{ km}$ is at the lower end of typical values for cities/regions and $v_0 = 30 \text{ km/h}$ is at the upper end (see table in Sharma et al. (2023)), leading to a comparatively short estimate for the average driving time by car.
The trains operate along orthogonal lines from source to end, the first and the last station of the transit network line in the direction of travel (see Fig. 1). The trains arrive at stations with a frequency \( \mu = 1/10 \text{min}^{-1} \) (as in metropolitan regions like Berlin (S-Bahn Berlin, 2023)) and travel at an average speed \( v_{\text{train}} \). We assume that trains attain a maximum speed of \( 3 \cdot v_0 \) and the time taken to reach this speed starting from rest is assumed to be \( 0.05 \cdot t_0 \). The trains stop at each connecting station for \( 0.05 \cdot t_0 \). These values are inspired from real data (see Supporting Information for details) and are equal to the ones in Sharma et al. (2023). Note that due to these factors, the effective train velocity, \( v_{\text{train}} \), depends on the inter-station distance \( \epsilon \). Trains require energy \( v_{\text{train}} \) per unit distance of travel.

The transit system is further characterized by a number of shuttles, \( S \), in the plane. For the sake of conciseness and simplicity, we assume that the number of shuttles \( S \) is just sufficient to serve all user requests emanating in the system over time \( T \). Shuttles require energy \( v_{\text{shuttle}} \) per unit distance of travel. User requests served by DRRP/shuttles are subject to the constraint that the maximum accepted detour (travel distance / direct distance) is \( \delta_m = 3 \), the maximum waiting time is \( t_{w,\text{max}} = 5 \text{ min} \) and the maximum travel time is \( a \cdot t_{\text{direct}} + \gamma \), where \( a = 3, \gamma = 10 \text{ min} \) are simulation parameters and \( t_{\text{direct}} \) is the direct travel time\(^4\). Requests are assigned to feasible shuttles so as to minimize the total distance driven by the shuttles.

We let trains and DRRP operate for a time \( 2T = 2 \text{ h} \) in order to ensure that all user requests are served during simulation time.

2.2. Parameters and objectives of operation

**Choosing the type of transport service.** A single user in the model system may either be transported by uni-modal service, i.e., by shuttle (DRRP) only, or by bi-modal service, i.e., be brought from \( P = (x_p, y_p) \) to the nearest train station by means of a shuttle, followed by a train journey, which is again followed by a shuttle journey to \( D = (x_d, y_d) \). It is the task of the dispatcher system to decide, for each individual request \( (P, D) \), whether the desired door-to-door service should be completed by uni-modal transportation (shuttles only) or bi-modal transportation (shuttle-train(s)-shuttle). Sharma et al. (2023) showed that the requested travel distance, \( d = |PD| \), irrespective of the direction of travel, may serve as a reasonable discriminating parameter. Therefore, in order to choose the mode of transportation for an individual user request, we adhere to the previous policy (Sharma et al., 2023) of assigning user requests with travel distance \( d > d_{\text{cut-off}} \) to bi-modal transportation (shuttle-train(s)-shuttle). Shorter trips, i.e., user requests with travel distance \( d \leq d_{\text{cut-off}} \), are assigned to uni-modal transportation (shuttle only). The cut-off distance, \( d_{\text{cut-off}} \), is the control parameter we will use to optimize the performance of the system. It is in one-to-one correspondence to the fraction of bi-modal transportation \( F(d_{\text{cut-off}}) = \int_{d_{\text{cut-off}}}^\infty p(x) \, dx \), with \( p(\cdot) \) the probability density of requested distances.

**Service quality.** We define the service quality as the ratio between the average travel times by MIV and bi-modal transit, respectively,

\[
Q = \frac{1}{1-F} \cdot t_{\text{uni}} + F \cdot t_{\text{bi}},
\]

where \( F = F(d_{\text{cut-off}}) \) is the fraction of requests served by bi-modal transportation.

To compute the average travel time by MIV for a simulated scenario, we perform independent simulations where the MIV is the only allowed mode of transportation. If \( t_{\text{MIV},i} \) represents the travel time for user \( i \), then \( t_{\text{uni}} \) can be obtained by averaging over all users in the system, i.e., \( t_{\text{uni}} = 1/N \sum_i t_{\text{MIV},i} \). Similarly, we compute the denominator in Eq. (1), from simulations by averaging over all users in the system.

In a mean-field approach, Sharma et al. (2023) derived an analytical expression for \( Q \) as

\[
Q^{-1} = (1 - F) \cdot \left( \frac{\bar{t}_{\text{w}} + \delta \langle d \rangle_{d \leq d_{\text{cut-off}}} + F \cdot \left( 2\bar{t}_{\text{w}} + 2\beta \bar{\epsilon} \delta + \frac{1}{\beta} + \frac{4}{\pi} \frac{\langle d \rangle_{d \leq d_{\text{cut-off}}}}{v_{\text{train}}} \right) }{t_{\text{bi}}} \right)
\]

The \( \bar{\cdot} \) indicates quantities non-dimensionalized by division with the respective unit \( (D, t_{\text{w}}, \text{and} \ v_0) \) as units for length, time, and velocity, respectively). \( \langle d \rangle_{d \leq d_{\text{cut-off}}} \) is the mean of requested distances larger than \( d_{\text{cut-off}} \) and \( \delta \) is the average detour incurred by a user during the DRRP trip, \( \bar{t}_{\text{w}} \) is the average waiting time for shuttles and \( \beta \) the train frequency. \( \beta = \frac{1}{2} (\sqrt{2} + \log(1 + \sqrt{2})) \approx 0.383 \) is a geometrical constant, and \( 4\pi^{-1} \langle d \rangle_{d \leq d_{\text{cut-off}}} \) is the average distance traveled on trains.

**Energy consumption.** To assess the overall energy consumption by the bi-modal transportation system, we define the dimensionless energy consumption \( \mathcal{E} \) as the ratio of total energy consumed by bi-modal transportation and a fleet of MIV, mathematically, \( \mathcal{E} \) can be written as

\[
\mathcal{E} \equiv \frac{\Delta_{\text{shuttle}} \cdot v_{\text{shuttle}} + \Delta_{\text{train}} \cdot v_{\text{train}}}{\Delta_{\text{MIV}} \cdot v_{\text{MIV}}}.
\]

where \( \Delta \) denotes the (mode-specific) total distance traveled in a unit cell of area \( \epsilon^2 \) per unit time. \( e \) is the vehicle-specific energy consumption per unit distance. Note that \( \mathcal{E} \) is already normalized with respect to the MIV energy consumption (denominator), as this is the door-to-door transportation system we intend to compare with. For \( \mathcal{E} > 1 \) \((<1)\), energy requirement for bi-modal transportation is larger (smaller) than for MIV serving the same requests.

\(^4\) For the mean-field theory, Sharma et al. (2023) require averages, they assume \( \langle \delta \rangle = \delta_m/2 \) and \( \langle t_{\text{w}} \rangle = t_{w,\text{max}}/2 \).
Both, uni-modal (shuttle only) and bi-modal trips, contribute to the total distance driven by shuttles per unit time due to requests from a unit cell of area \( \ell^2 \), hence

\[
\Delta_{\text{shuttle}} = \frac{\nu E \ell^2}{\eta} \left( (d)_{d<\ell d} (1-F) + \frac{2\beta \ell F}{\eta} \right),
\]

where \( \eta \) is the DRRP pooling efficiency, which is the ratio of direct distance requested by the users and the distance actually driven by the shuttles (for MIV, \( \eta = 1 \)). \( \text{Mühle (2023)} \) has observed that \( \eta \) scales with demand \( \Lambda \) roughly in an algebraic manner, \( \eta(\Lambda) \approx \Lambda^\gamma \), with \( \gamma \approx 0.12 \). In a bi-modal system, however, none of the demand \( \Lambda \) is directed towards trains. Therefore, we need to compute an adjusted demand, \( \Lambda_{\text{shuttle}} = (\mathcal{E} \nu_{\text{shuttle}} D_{\text{shuttle}}^3)/\nu_0 \) considering shuttle trips only. \( \nu_{\text{shuttle}} \) is the effective request frequency for shuttle trips and \( D_{\text{shuttle}} \) is the average distance of a shuttle trip. One can show that (\text{Sharma et al., 2023})

\[
\Lambda_{\text{shuttle}} = \Lambda (1 + F)^{-2}((1-F)(d)_{d<\ell d} + 2\beta \ell F)^3.
\]

We compute the theoretical pooling efficiency, \( \eta \), according to the power law mentioned above

\[
\eta \equiv \Lambda_{\text{shuttle}}^{0.12}.
\]

The distance travelled per unit cell by line service, \( \Delta_{\text{train}} \), remains constant throughout our study because trains operate at a constant frequency \( \mu \), mathematically,

\[
\Delta_{\text{MIV}} = 4 \cdot \mu \cdot \ell^2.
\]

\( \Delta_{\text{MIV}} \) is the total distance requested by users per unit time,

\[
\Delta_{\text{MIV}} = \nu \ell^2 D.
\]

Analytically, Eq. (3) can then be written as

\[
\mathcal{E} = \eta^{-1} \left( (d)_{d<\ell d} (1-F) + \frac{2\beta \ell F}{\eta} \right) \cdot \frac{\nu_{\text{shuttle}}}{\nu_{\text{MIV}}} \cdot \frac{\ell_{\text{train}}}{\ell_{\text{shuttle}}} + \frac{4 \mu}{D}. \frac{\nu_{\text{train}}}{\nu_{\text{MIV}}}.
\]

In analogy to \text{Sharma et al. (2023)}, we consider electric light rails with a maximum seating-capacity \( k = 100 \) and \( e_{\text{train}} = 9.72 \, kN \) (\text{Knörr et al., 2016}) for the line service. For MIV we consider Diesel cars with \( e_{\text{MIV}} = 2.47 \, kN \) (\text{BMDV, 2022}). For the shuttles we choose Mercedes Sprinter (8.8 liters of Diesel per 100 km (\text{Mercedes-Benz, 2022})), resulting in \( e_{\text{shuttle}} = 3.28 \, kN \).

In order to compute the ratios above in Eq. (3) for a simulated scenario, we perform independent simulations for MIV and bi-modal transit with identical user requests. The denominator in Eq. (3) is obtained from MIV simulations by multiplying the total driven distance by \( e_{\text{MIV}} \). The numerator in Eq. (3) is obtained from bi-modal simulations by multiplying mode-specific total driven distance\(^5\) with the respective vehicle-specific energy consumption per unit distance.

3. Results

We organize our results as follows. First we present how DRRP performs within a bi-modal system in Section 3.1. Then we characterize the overall performance of bi-modal transit systems in terms of energy consumption and service quality in Sections 3.2 and 3.3. We conclude the results section with an analysis of potential reductions in traffic volume (Section 3.4).

3.1. DRRP performance

Occupancy. Fig. 2a shows the mean DRRP occupancy averaged over driving vehicles, namely \( b \), against the bi-modal fraction, \( F \), for various numbers of intermediate stations, \( \Theta \), and demands, \( \Lambda \). We observe that shuttles generally have a higher mean occupancy for large demands because of the greater possibility of pooling. Mean occupancy is observed to decrease with the involvement of trains (increasing \( F \)) because trips by shuttles are shortened, resulting in passengers spending less time in shuttles during their transit. We also observe a general trend of decreasing mean occupancy with more intermediate stations, because trips by shuttles for the users assigned to bi-modal transportation become 1) more dispersed and 2) shorter due to the higher number and proximity of train stations, therefore providing less opportunity for pooling.

Detours. From Fig. 2b, the general trend of increasing detour with demand is evident. This trend, together with the trend for mean occupancy, \( b \), in Fig. 2a, shows the well known trade-off between detour and pooling for DRRP, i.e., desirable pooling necessitates undesirable detours for passengers (\text{Herminghaus, 2019}). A trend of decreasing detours with increasing involvement of line services (increasing \( F \)) can be attributed to a ‘common stop effect’, that is, passengers are picked up or dropped off at the same train station, thereby reducing detours. We also observe a trend of decreasing detours with more intermediate stations which is due to the lower potential for pooling due to shorter trips, as well as a reduced ‘common stop effect’.

\(^5\) Note that we only consider the total distance that trains drive during the time interval \( T \).
Notice that the black dashed curve represents the assumed value of $\delta = 1.5$ for the theoretical analysis (Sharma et al., 2023). We observe in Section 3.3, that this assumption does not severely impact the agreement between the theoretical and simulated overall performance of the system.

**Pooling efficiency.** The ratio between mean occupancy and mean detour, shown in Fig. 2c, provides a reasonable estimate for pooling efficiency, $\eta$ (Mühle, 2023), which is defined as the ratio of requested direct distance by the users and driven distance by the shuttles (for MIV, $\eta = 1$). In Fig. 2c, we observe a general trend of increasing pooling efficiency, $\eta$, with demand, which suggests that deploying shuttles in a region with high demand is favourable. Involvement of line services decreases the DRRP pooling efficiency because user requests are diverted toward the lines, thus shortening the average distance a passenger travels on the shuttle during the entire journey. We observe that the effect of intermediate stations on $\eta$ is mostly insignificant, except for higher demand at large bi-modal fraction, $F$.

The dashed curves represent the theoretical prediction for pooling efficiency, determined by Eq. (6), which follows a trend similar to the simulations. We observe that the theoretical data underestimates the pooling efficiency when compared with the simulation data. This underestimation can be attributed to the 'common stop effect' mentioned earlier, which is not accounted for in theoretical predictions in Eq. (6).

**Waiting time.** In Fig. 2d, we study the mean waiting time for trips with shuttles. Mean waiting time normalized with the average trip duration, $t_{\text{wp}}$, is plotted against bi-modal fraction $F$. The black dashed curve represents the assumed value for the theoretical study (see Supporting Information for details). We observe a trend of increasing waiting time for higher demands because shuttles become busier. Involvement of line services generally decreases the waiting time for trips with shuttles because of the 'common stop effect' and a lower share of distance travelled in shuttles. The latter holds, too, for more intermediate stations, thus explaining the lower waiting time for larger $\Theta$.

In summary, the main messages from Fig. 2 are: 1) Shuttles become more efficient with demand, while user experience suffers due to larger detours and waiting times. 2) Intermediate stations enhance the user experience due to shorter detours and waiting times. The effect on pooling efficiency is insignificant. 3) Trips with shuttles become more convenient for users with involvement of line services due to reductions in waiting time and detours.

### 3.2. Overall energy consumption and service quality of Bi-modal transit

We now analyze the overall objectives, i.e., energy consumption (Eq. 3) and service quality (Eq. 1) of the combined bi-modal system.

**Energy consumption.** In Fig. 3a, relative energy consumption, $\mathcal{E}'$, is plotted as a function of bi-modal fraction, $F$, for various numbers of intermediate stations, $\Theta$. We observe a general trend of decreasing energy consumption with involvement of line services. This is because $\Delta_{\text{shuttle}}$ decreases with involvement of line services (see Fig. 4a) while $\Delta_{\text{train}}$ is constant due to a constant service frequency $\mu$, etc.
Fig. 3. Effect of intermediate stations on the overall performance of bi-modal transit: The bimodal fraction $F$ has been varied in the range $[0,1]$ in simulations to obtain the data shown. Blue and red curves represent data for $\Lambda = \{123, 1201\}$, respectively. Triangles, circles and squares represent $\Theta = \{0, 1, 3\}$, respectively. (a) Energy consumption, $\varepsilon$, as a function of $F$. The dashed curves represent the theoretical data determined by Eq. (9). (b) Quality, $Q$, as a function of $F$. Black dashed curve represents the theoretical data determined by Eq. (2). Notice that the theoretical data for quality is assumed the same across all demands. (c) Pareto fronts of energy consumption, $\varepsilon$, vs. service quality, $Q$ determined from the data shown in (a), (b). Data not part of Pareto fronts is not shown. The dashed curves represent the theoretical data as in (a), (b). Black circle and triangle represent uni-modal transport (shuttles-only) data for $\Lambda = \{123, 1201\}$, respectively. (d) Pareto fronts as in (c), but normalized with respect to the performance, $(\varepsilon_0, \eta_0)$, of the uni-modal system (shuttles only).

Fig. 4. Impact of intermediate stations on traffic volume. Green, blue and red curves represent $\Lambda = \{13.7, 123, 1201\}$, respectively. Triangles, circles and squares represent $\Theta = \{0, 1, 3\}$, respectively. (a) Relative bi-modal traffic in simulations, $\Delta_{\text{shut\_trk}}$, as defined in Eq. (10), as a function of the bi-modal fraction, $F$. Black square, circle, and triangle represent uni-modal transport (shuttles-only) data for $\Lambda = \{13.7, 123, 1201\}$, respectively. Dashed curves represent the theoretical data, determined by Eq. (10). (b) Relative bi-modal traffic in simulations, $\Delta_{\text{shut\_trk}}$, determined along the Pareto fronts in Fig. 3, against corresponding service quality $Q$. Dashed curves and black symbols represent the theoretical and uni-modal data, respectively, as in (a).

thus decreasing the total energy consumption by the bi-modal system (see Eq. 3). We observe that the relative energy consumption is reduced with increasing demand, $\Lambda$, as is evident from the direct contribution in Eq. (9), as well as via the enhanced pooling efficiency, $\eta$ (see Fig. 2c). Fig. 3a reveals that the energy consumption can be lower than 25% of the energy consumption for MIV for demands in large cities like Berlin ($\Lambda \approx 5 \cdot 10^3$, Sharma et al. (2023)). Energy consumption is observed to decrease slightly with increasing number of intermediate stations, because the normalized distance driven by shuttles is reduced (see Fig. 4a). We find reasonable agreement between theoretical data (Eq. 9) and simulations.

Quality. In Fig. 3b, we plot the overall quality of the system against the bi-modal fraction, $F$. We observe that $Q$ is non-monotonic in $F$. This is due to the competing effects of decreasing waiting time for shuttles with more involvement of trains (see Fig. 2d), on the one hand, and additional waiting time for trains at the train stations for a larger user fraction ($F$), on the other hand.

We observe that the overall service quality, $Q$, decreases with demand, which is due to the trend we observed for waiting times for trips with shuttles in Fig. 2d.

The dashed black curve represents the theoretical predictions determined by Eq. (2). Notice that the theoretical prediction for quality does not depend on the demand because the train frequency $\mu$ is maintained at a constant value of 0.1 min$^{-1}$ across all demands.
(see Section 2.1). The difference between theory and simulation data primarily stems from the waiting time of shuttles, which we approximated as 0.25\( t_o \) in our theory, for all demands (see Supporting Information for motivation). However, we see in Fig. 2d that the waiting time varies with demand, bi-modal fraction and number of intermediate stations.

In Fig. 3b, we observe that the overall service quality of the combined bi-modal system decreases with more intermediate stops, despite a trend of decreasing waiting times and detours for trips with shuttles (see Figs. 2b, d). Apparently, this trend is dominated by a slowing down of trains due to intermediate stations, which increases the average trip duration.

The general trend of reduction in consumption of energy with increasing demand and involvement of line services hints towards the merit of bi-modal transportation for high-demand scenarios. For service quality, we find that typical values are around one-half the service quality of MIV, i.e., about twice the travel time. This quality is customary for public transport systems (Liao et al., 2020; Salonen and Toivonen, 2013) and is generally well accepted by users. Note that our data for service quality represent a safe lower bound, as the (sometimes quite substantial) time required for parking spot search (Chaniotakis and Pel, 2015; Fulman and Benenson, 2021) is neglected in \( t_o \).

We observe that service quality attains a maximum for a bi-modal fraction, \( F \), where energy consumption is not minimal. Jointly optimizing such conflicting objectives can be done in the framework of Pareto optimization, which we will discuss next.

### 3.3. Pareto optimization

A tuple of parameter values, in our case \((E, Q)\), is called Pareto-optimal if none of the parameters (or objectives) can be further optimized without compromising on at least one of the others. The set of all such tuples of parameters is called the Pareto front (Debreu, 1959; Greenwald and Stiglitz, 1959; Magill and Quinzii, 2002). We now apply this concept to our results, keeping in mind that we aim at maximum service quality at minimum energy consumption. Hence, in diagrams spanned by \( Q \) as the abscissa and \( E \) as the ordinate, system operation as far as possible to the lower right is desirable.

In order to study the effect of density of line service stations on the overall performance of the bi-modal transit system, we have introduced \( \Theta = \{0, 1, 3\} \) as the number of intermediate stations. For \( \Theta = 0 \), transit stops are only at the intersection between two transit lines. For each value of \( \Theta \), we vary \( d_i \) to obtain the Pareto fronts.

In Fig. 3c, we show the Pareto fronts obtained for data in Figs. 3a, b. Note that for a better resolution of the curves, we only show the data for \( \Lambda = \{123, 1201\} \). We observe that intermediate stations reduce the energy consumption to as low as 20% of MIV for larger demand. However, the service quality is worsened due to reduced average speed of trains. Black triangle and circle represent the data for the uni-modal system (shuttles only), and dashed curves represent the theoretical estimates. We observe a fair agreement between previous theoretical estimates and simulation results for the overall performance of the bi-modal system. Discrepancies mainly result from the simplifying assumptions for mean waiting time, \( \bar{r}_w \) and detour, \( \delta \), as described in Section 3.1.

In Fig. 3d, the same Pareto fronts as in Fig. 3c are plotted normalized with respect to energy consumption and service quality of a uni-modal system, \((E, Q, 0, 0)\). We observe that the bi-modal system can provide a service quality superior to a uni-modal (shuttles only) system with a lower energy consumption. This observation holds for both demand values presented.

### 3.4. Traffic volume

Road traffic is a source of local noise and air pollution and occupies significant shares of urban space. Bi-modal transit aims at reduction of road traffic by utilizing line services for trips over larger distances. Fig. 1c and d provide qualitative evidence by comparing abundance of MIV and shuttles for the same request pattern.

To obtain a quantitative estimate, we define as bi-modal traffic volume, \( \Delta_{\text{shuttle}} \), the cumulative distance driven by shuttles, \( \Delta_{\text{shuttle}} \) (Eq. 4), normalized with respect to the equivalent of total MIV distance requested, \( \Delta_{\text{MIV}} \) (Eq. 8), or, equivalently, the relative number of driving shuttles as compared to MIV. For our theoretical estimates, we use the analytical expression,

\[
\Delta_{\text{shuttle}} \equiv \Delta_{\text{shuttle}} / \Delta_{\text{MIV}} = \eta^{-1} (1 + F) D_{\text{shuttle}},
\]

with pooling efficiency, \( \eta \), bi-modal fraction, \( F \), and average requested distance for trips by shuttles involved in bi-modal transit, \( D_{\text{shuttle}} \).

In Fig. 4a, we plot bi-modal traffic in simulations on a vertical logarithmic axis as a function of bi-modal fraction for various demands and number of intermediate stations. Dashed curves represent the theoretical data (Eq. 10) for an idealized square grid network without any intermediate stops (\( \Theta = 0 \)).

We observe a trend of decreasing bi-modal traffic with involvement of line services, i.e., with increasing \( F \). Also, \( \Delta_{\text{shuttle}} \) decreases with increasing demand, because shuttles become more efficient due to the possibility of larger pooling (see also Fig. 2c). We furthermore observe that, for low demand, bi-modal traffic decreases with more intermediate stations. This is despite an insignificant impact on pooling efficiency, \( \eta \) (see Fig. 2c). It is rather the reduced average requested distance for trips with shuttles, \( D_{\text{shuttle}} \), due to increased proximity of train stations, which accounts for the reduced traffic volume (see Eq. 10).

In Fig. 4b, we plot the relative bi-modal traffic volume, \( \Delta_{\text{shuttle}} \), for simulations on a vertical axis, determined along the Pareto fronts in Fig. 3c, against corresponding service quality \( Q \). The traffic volume for uni-modal (shuttle-only) scenarios is plotted with black symbols. We observe that the relative traffic volume for uni-modal scenarios decreases with demand due to increased pooling efficiency, \( \eta \) (see Fig. 2c). The relative traffic volume for the uni-modal case can be reduced to 80% of MIV for very low demand and down to 50% and 40% for low and medium demand, respectively.
Bi-modal public transportation allows for further reduction in relative traffic volume below the uni-modal case at a superior service quality. For an idealized square grid without any intermediate stations (represented by coloured triangles), bi-modal transportation reduces traffic volume down to about 30% of MIV for very low and low demand and even below 20% for medium demand. Intermediate stops allow for further reduction in traffic. This reduction, however, comes at the cost of reduced service quality. Due to computational limitations, we could only simulate scenarios with the demand of the order of \( \lambda = 10^4 \). However, the demand can be as high as \( 10^8 \) for very dense areas like New York. Earlier findings suggest that bi-modal public transportation can reduce traffic in such areas even below 10% of MIV (Sharma et al., 2023).

4. Discussion

Our investigation had two primary goals. First, to study the impact of the number of intermediate line service stops on the performance of bi-modal public transport systems. Second, to evaluate the analytical framework proposed by Sharma et al. (2023) through simulations. The parameters investigated in our study drew inspiration from real-world agglomerations. The grid constant \( \ell = D/\ell = 0.4 \) in our model is close to what is found for, e.g., New York and Berlin. Other technical parameters, including shuttle speed \( (\ell_{ssh}) \) and train speed \( (\ell_{train}) \) have been carefully chosen to reflect the observed real-life environments (see Section 2.1).

Below, we first discuss the comparison between agent-based simulations and the analytical study by Sharma et al. (2023). Simulations reveal that observables characterizing shuttle performance, namely detour and waiting time, vary with demand, bi-modal fraction and the number of intermediate stops, in contrast to the constant values assumed by the previous analytical study (Sharma et al., 2023). Furthermore, the previous analytical study approximated the pooling efficiency using an algebraic power-law (see Eq. 6), which underestimated the actual pooling efficiency due to a ‘common stop effect’ (see Section 3.1). The findings above highlight that shuttle waiting time, detours, and pooling efficiency are complex observables, which, in bi-modal transit, depend on line service operations, and thus must be modeled carefully to reflect the performance of bi-modal transit systems accurately.

Adding intermediate stations between line crossings had limited benefits. While there was a marginal reduction in energy consumption and traffic volume, the resulting slowdown of trains made them less suitable as a faster mode of transportation than MIVs and shuttles, i.e., lead to substantial reduction of service quality.

It should be noted that the network structure employed in our research, although inspired by real-world urban agglomerations, is still a simplified representation. Future studies should consider more complex and realistic network typologies to capture the nuances of different urban environments and evaluate the generality of our findings. Moreover, realistic demand patterns, like rush hours and spatial commuting patterns can be included.

In summary, our study confirms that bi-modal transit can provide door-to-door service with satisfactory service quality while consuming only a fraction of the energy required by an equivalent fleet of MIVs, and significantly reducing road traffic volume. These advantages hold for low-demand regions as well as medium-sized cities. Although our simulations were limited to a medium user demand of approximately \( \lambda = 10^4 \) due to computational constraints, we anticipate that bi-modal transit would outperform MIVs and uni-modal ride-pooling even more significantly under higher demand conditions, as suggested in previous research (Sharma et al., 2023). It is important to note that our analysis did not consider other MIV-specific drawbacks, such as parking time or traffic congestion during rush hours (Chaniotakis and Pel, 2015; Manville and Shoup, 2005; Mingardo et al., 2022), which would further enhance the relative performance of bi-modal transit.

The findings of this research paper have significant implications for the improvement of public transportation systems. One practical application of these findings is in the design and optimization of line service networks. Public transit authorities and city planners can consider reducing the number of intermediate stops within existing railway systems and complementing on-demand shuttle services to improve the overall performance of public transport. Furthermore, the research suggests that bi-modal transit is not only suitable for high-demand regions but also holds promise for medium-sized cities. As humanity must reduce its carbon footprint, insights from this study can guide the development of more sustainable and efficient public transportation solutions, benefiting both the environment and the quality of life for residents.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Supplementary material


References
