Letter

Loop corrections versus marginal deformations in celestial holography

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Four-dimensional all-loop amplitudes in quantum electrodynamics and gravity exhibit universal IR singularities with a factorization structure. This structure is governed by tree amplitudes and a universal IR-divergent factor representing the exchange of soft particles between external lines. This Letter offers a precise dual interpretation of these universal IR-divergent factors within celestial holography. Considering the tree amplitude as the foundation of the celestial conformal field theory (CCFT), these universal factors correspond to marginal deformations with a *double-current* construction in the CCFT. Remarkably, a novel geometric representation of these deformations through topological gauging provides an exact description of transitions within bulk vacuum moduli spaces which extends the celestial holography beyond the perturbative level. Our findings establish a concrete dictionary for celestial holography and offer a holographic lens to understand loop corrections in scattering amplitudes.

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Introduction. The holographic principle [1,2], initially formulated within the context of AdS/CFT [3], has been a cornerstone in the quest to understand quantum gravity, proposing that the information within a space can be encoded on its boundary. Thriving in anti-de Sitter spaces, its extension to flat spacetime has both theoretical and applied necessity urgently [4–15]. In flat spacetime, scattering amplitudes are of fundamental importance in perturbative quantum field theory (QFT). They are the main concerns of the recently constructed celestial holography [16–22], which formally establishes the holographic principle for flat spacetime.

Crucially, Mellin transform establishes a concrete link for celestial holography between 4D massless scattering amplitudes in perturbative QFT and the correlators in celestial conformal field theory (CCFT). While the precise properties of the CCFT are yet to be fully defined, which leads to the recent inquiries from various angles, including fundamental conformal field theory (CFT) characteristics [23–29], symmetries [30–35], loop corrections [36–41], double-copy relations [42,43], supersymmetric extensions [44–46], Carrollian perspectives [47–51], and diverse deformations [52–60], underscoring the transformative potential of celestial holography.

Notably, current investigations have been predominantly bottom-up by exploring the transformation from amplitude to correlators of CCFT and by examining properties from the dual perspective [61–63], leaving a top-down quantitative verification of celestial holography, namely exactly matching of independent computations beyond symmetry perspective on both sides, an open challenge.[1] This extremely limits the implications of celestial holography for a broader understanding of scattering amplitudes. Moreover, scattering amplitude is defined by nature perturbatively. Hence, the nonperturbative feature is definitely beyond the current scope of celestial holography.

This Letter contributes to the growing body of research in celestial holography by addressing both critical challenges:

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[1]A top-down realization [64–66] of holography for asymptotically flat spacetimes has been recently constructed in the context of twisted holography.

the quantitative verification of this intriguing framework and its nonperturbative extension. Specifically, we focus on the universal IR singularities exhibited by four-dimensional all-loop amplitudes in quantum electrodynamics (QED) and gravity. The bulk theory's all-loop corrected amplitudes exhibit both UV and IR divergences. The IR divergences follow a universal factorization structure [67] resulting from the exchange of soft virtual gauge bosons and gravitons. The leading IR singularity of the loop amplitude is determined by the tree amplitude and a universal factor.

Considering the tree amplitude as seed CCFT, we systematically establish marginal deformation for the seed CCFT via double-current construction. An exact agreement has been uncovered between the universal part (the leading IR singularity) of the loop QED [26,67] and gravity [68–70] amplitudes and marginal deformations of the corresponding seed CCFT correlators. Our findings offer a *concrete dictionary* of celestial holography and extend the original avenue of celestial holography. A concrete holographic interpretation of loop corrections to the amplitudes is revealed. This will definitely open a new avenue for the understanding of the structure of loop amplitudes. Then, we propose a geometric representation of the marginal deformation, which remarkably equates marginal deformations in the CCFT with bulk vacua's moduli spaces. The celestial holography is thus verified at the nonperturbative level.

4D amplitudes as 2D correlators. The isomorphism between the 4D Lorentz group and the 2D global conformal group suggests the possibility of restructuring 4D quantum field theory using 2D CFT data. To manifest this connection, the conformal basis was introduced [6,16,17,71]. In the case of masslessness, the transition from momentum space to the conformal basis is achieved through a Mellin transform. Specifically, we parametrize null momenta and polarization vectors in terms of energy ω and angular parameters $\vec{z}=(z,\bar{z})$ as follows[2]:

$$q^\mu(\omega,\vec{z})=\frac{\tilde{\epsilon}\omega}{2}(1+z\bar{z},z+\bar{z},-i(z-\bar{z}),1-z\bar{z}),$$
$$\varepsilon_\mu^+(q)=\frac{1}{\sqrt{2}}(-\bar{z},1,-i,-\bar{z}),\qquad \varepsilon_\mu^-(q)=\bar{\varepsilon}_\mu^+(q),\quad (1)$$

where $\tilde{\epsilon}=1,-1$ for outgoing and incoming particles, respectively, and $\vec{z}$ indicates the location on the celestial plane. The n-point amplitude of massless particles is identified as a correlator as follows:

$$\mathcal{A}_n=\langle \text{out}|\mathcal{S}|\text{in}\rangle=\langle \mathcal{O}_1\cdots\mathcal{O}_n\rangle, \quad (2)$$

where $\mathcal{O}_k$ is the annihilation or creation operator of kth particle, which can be recast in terms of operators smeared along null infinity in the extrapolate dictionary [77–79]. Mellin transforming the correlator in Eq. (2), we have [18]

$$\langle X_n\rangle=\prod_{k=1}^n\left(\int_0^\infty d\omega_k\omega_k^{\Delta_k-1}\right)\mathcal{A}_n, \quad (3)$$

where $X_n=\prod_{k=1}^n\mathcal{O}_{\Delta_k,s_k}$.

Perturbatively, the amplitude in momentum space can be expressed as a series in powers of the bulk theory's coupling constant g: $\mathcal{A}_n=\sum_{\ell=0}^\infty\mathcal{A}_n^{(\ell)}$, where

$$\mathcal{A}_n^{(\ell)}=\frac{g^\ell}{\ell!}\left[\frac{(B_n)^\ell}{\epsilon^\ell}\mathcal{A}_n^{(0)}+\mathcal{N}^{(\ell)}\right]. \quad (4)$$

Here, ℓ denotes the loop order, and ϵ acts as the dimensional regularization parameter governing IR singularities. The right-hand side of Eq. (4) consists of two parts. The first one, primarily universal, arises from Abelian Feynman diagrams [26,68]. The second term encompasses nonuniversal infrared contributions, which are power suppressed in the dimensional regularization.

For scalar QED, $g_e=-\frac{e^2}{8\pi^3}$, and the leading IR factor [67,80] from Eq. (4) is

$$B_n^e=\pi\sum_{i,j}Q_iQ_j\log\frac{-2q_i\cdot q_j}{\mu^2}$$
$$=\pi\sum_{i,j}Q_iQ_j\log|z_{ij}|^2, \quad (5)$$

where Q_k represents the electric charge of the kth particle, $z_{ij}=z_i-z_j$, and μ is an arbitrary energy scale. The terms involving the energy scale can be eliminated by utilizing the conservation law $\sum_k Q_k=0$, with the parametrization provided in the second equality. Implementing Mellin transform to the full amplitude in Eq. (4) in QED, denoted as $\mathcal{A}_n^e$, one obtains[3]

$$\langle X_n\rangle_e^{(\ell)}=\frac{g_e^\ell}{\ell!}\left[\frac{(\sigma_n^e)^\ell}{\epsilon^\ell}\langle X_n\rangle_e^{(0)}+N_e^{(\ell)}\right], \quad (6)$$

where $\langle X_n\rangle_e^{(0)}$ is the correlator of CCFT that is the Mellin transform of the tree QED amplitude, $\sigma_n^e=B_n^e$, and $N_e^{(\ell)}$

[2]Here, we focus on 4D scattering in Minkowski spacetime. Massless particles enter and exit at past and future null infinity $\mathcal{I}^\mp$, parametrized by advanced $v=t+r$ and retarded $u=t-r$ time coordinates. In what follows, we will use stereographic coordinates $(z,\bar{z})$ to parametrize the celestial plane which is conformally mapped from the celestial sphere. See, e.g., Ref. [72] for the properties of the celestial plane and Refs. [73–76] for the transformations between the celestial plane and sphere.

[3]The parametrization of the factor in the exponential here is different than the one in Ref. [26] because we keep the collinear-divergent terms; see, e.g., the comments in footnote 11 of that reference. The aim of the present work is to reveal a concrete correspondence between 4D amplitudes and 2D correlators. Subtractions in 4D and 2D are different in general, and we will not consider the collinear divergences in this paper.

corresponds to Mellin transformed nonuniversal contributions $\mathcal{N}_e^{(\ell)}$.

For gravity, the coupling constant is set to $g_G = \frac{G_N}{16\pi^2}$. The leading universal IR factor from Eq. (4) is [67–70]

$$B_n^G = 8\pi \sum_{i,j} q_i \cdot q_j \log \frac{-2q_i \cdot q_j}{\mu^2} = -4\pi \sum_{i,j} |z_{ij}|^2 \omega_i \omega_j \log |z_{ij}|^2. \tag{7}$$

Implementing the Mellin transform to the full graviton amplitude $\mathcal{A}_n^G$ yields

$$\langle X_n \rangle_G^{(\ell)} = \frac{g_G^\ell}{\ell!} \left[\frac{(\sigma_n^G)^\ell}{\epsilon^\ell} \langle X_n \rangle_G^{(0)} + N_G^{(\ell)} \right], \qquad \sigma_n^G = \pi \sum_{i,j} |z_{ij}|^2 \mathcal{G}_i \mathcal{G}_j \log |z_{ij}|^2, \tag{8}$$

where

$$\mathcal{G}_k \mathcal{O}_{\Delta_k, s_k} = \mathcal{O}_{\Delta_k + 1, s_k} \tag{9}$$

and $N_G^{(\ell)}$ corresponds to Mellin transformed $\mathcal{N}_G^{(\ell)}$ for gravity amplitude.

Seed theories and currents. We select for our seed CCFT correlator the Mellin transform of the tree-level scalar QED amplitude $\langle X_n \rangle_e^{(0)}$ and gravity amplitude $\langle X_n \rangle_G^{(0)}$, respectively. The seed CCFTs admit symmetry currents originated from soft theorems. In the conformal basis, they are represented by conformally soft operator $\mathcal{O}_{\Delta,s}$ with an integer conformal dimension Δ. The conformally soft operators of relevance for constructing deformations in the present work are with dimension 1. The Ward identity of such soft operators $\mathcal{O}_{1,s}$ are divergent, which can be regulated by slightly shifting the conformal dimension similar to dimensional regularization as $\Delta = 1 + \epsilon$ [81],

$$\langle \mathcal{O}_{1,s} X_n \rangle^{(0)} = \frac{W^{(0)\pm}}{\epsilon} \langle X_n \rangle^{(0)} + O(\epsilon^0), \tag{10}$$

where the leading factor $W^{(0)\pm}$ is universal with nonuniversal subleading corrections from the regularization.

In scalar QED, the seed CCFT features $U(1)$ symmetry [82–84] is derived from $\mathcal{O}^e_{1,\pm 1}$ with the leading universal factors

$$W_e^{(0)+} = \sum_{k=1}^n \frac{Q_k}{z - z_k}, \qquad W_e^{(0)-} = \sum_{k=1}^n \frac{Q_k}{\bar{z} - \bar{z}_k}, \tag{11}$$

which corresponds to the leading soft photon theorem originally obtained in Refs. [67,85–87]. The soft current in the context of 2D CCFT [83,84] is $(J, \bar{J}) = (\mathcal{O}^e_{1,+1}, \mathcal{O}^e_{1,-1})$ with the Ward identity in Eq. (10), where $\mathcal{O}^e_{1,-1}$ is the shadow [84,88] of $\mathcal{O}^e_{1,+1}$, generating another copy of $U(1)$ symmetry. Therefore, the symmetries generated by $\mathcal{O}^e_{1,\pm 1}$ can be split into a $U(1) \times U(1)$ symmetry, where $J_A^1 = (J, 0)$ and $J_A^2 = (0, \bar{J})$. The indices 1 and 2 distinguish the currents from different copies of $U(1)$. It is worth noting that the operator product expansion (OPE) of JJ and $J\bar{J}$ vanishes at the leading order of the regularization [25,30]. This observation can also be understood more intuitively: the leading soft photon theorem originates from large gauge transformations, and the symmetry algebra is Abelian with no central extension [89,90]. From the boundary perspective, these currents arise from the fluctuations of the gauge field A_A^i residing on the celestial sphere, given by $J_i^A = \frac{\delta S_e}{\delta A_A^i}$, with $i = 1, 2$. Here, S_e represents the CCFT action incorporating $U(1) \times U(1)$ symmetry.

When a soft graviton is emitted in a scattering process, the amplitude satisfies the Weinberg's soft theorem [67]. Correspondingly, one can introduce conformal soft operators $\mathcal{O}^G_{1,\pm 2}$ with dimension 1 with the insertion in 2D correlators as

$$W_G^{(0)+} = \sum_{k=1}^n \frac{\bar{z} - \bar{z}_k}{z - z_k} \mathcal{G}_k, \qquad W_G^{(0)-} = \sum_{k=1}^n \frac{z - z_k}{\bar{z} - \bar{z}_k} \mathcal{G}_k, \tag{12}$$

where $\mathcal{G}_k$ is defined in Eq. (9). Then, we define the soft current as $P = \mathcal{O}^G_{1,+2}$ and its shadow [84], $\bar{P} = \mathcal{O}^G_{1,-2}$, which are components of a traceless current P_{AB}. It is important to note that P_{AB} is Abelian in the leading order because of the vanishing of the PP and $P\bar{P}$ OPEs [25,30]. However, it is not a Kac-Moody current, as discussed in Ref. [83]. The scattering of soft gravitons from bulk to boundary induces fluctuations in the boundary metric γ_{AB}, associated with different soft graviton modes, each dual to distinct currents. These fluctuations are represented in the basis $E^a{}_A$, enabling the decomposition of the 2D transverse metric as $\gamma_{AB} = g_{ab} E^a{}_A E^b{}_B$, where a and b indices use g_{ab} for raising and lowering. We denote $E^a{}_A$ as the basis for the soft graviton mode. $P^a{}_A$ is defined as $P^a{}_A = P^B{}_A E^a{}_B = \frac{\delta S_G}{\delta E_a{}^A}$, where S_G represents the action of the CCFT with symmetries generated by P_{AB}, related to bulk supertranslations.

Deformations. We propose that a standard 2D CCFT double-trace deformation captures the universal IR structure of the all-loop corrected amplitude in Eq. (4). This deformation, constructed by two commutative currents, can be defined as [91]

$$\frac{\partial S^{[\lambda]}}{\partial \lambda} = \int d^2x \mathcal{O}^{[\lambda]}_{j\bar{j}}, \qquad \mathcal{O}^{[\lambda]}_{j\bar{j}} = \frac{1}{2} \epsilon_{\bar{a}\bar{b}} \epsilon^{AB} j_A^{\bar{a}[\lambda]} j_B^{\bar{b}[\lambda]}. \tag{13}$$

Here, $S^{[\lambda]}$ serves as the action deformed from the seed CCFT[4]; λ is a coupling constant; $\bar{a}$ is an abstract index, which can be chosen as i or a in this Letter; ϵ_{AB}, ϵ^{ab} are Levi-Civita tensors; and ϵ_{ij} is Levi-Civita symbol.

In scalar QED, we utilize the soft current J_A^i to induce a dimensionless marginal deformation within the CCFT. The $J\bar{J}$ deformed action by specifying the abstract indices as $\bar{a}\,\bar{b} = ij$ and the current as $j_A^{\bar{a}} = J_A^i$ within the generic form Eq. (13).[5] Applying the Ward identities in Eqs. (10) and (11), one obtains the ℓth order correction of the deformed correlator

$$\langle X_n\rangle^{(\ell)}_{[\lambda_e]} = \frac{1}{\ell!}\left(-\frac{\lambda_e}{\epsilon}\right)^{\ell}\left[\left(-\frac{1}{\epsilon}\sum_{i,j=1}^{n} Q_i Q_j I^{1\bar{1}}_{i\bar{j}}\right)^{\ell}\langle X_n\rangle^{(0)}_{e} + N^{(\ell)}_{\lambda_e}\right], \tag{14}$$

where $N^{(\ell)}_{\lambda_e}$ is the nonuniversal part which is from the flow effect of the current J_A^i and is power suppressed in the expansion of ϵ. The integral $I^{r\bar{s}}_{i\bar{j}}$ is defined as

$$I^{r\bar{s}}_{i\bar{j}} = \frac{i}{2}\int \frac{d^2z}{(z-z_i)^r(\bar{z}-\bar{z}_j)^s}, \tag{15}$$

which can be computed using the Stokes formula, as detailed in Ref. [56], and their evaluations are provided in the Supplementary Material [92]. Finally,

$$\langle X_n\rangle^{(\ell)}_{[\lambda_e]} = \frac{1}{\ell!}\left(-\frac{\lambda_e}{\epsilon}\right)^{\ell}\left[\frac{(\sigma^e_n)^{\ell}}{\epsilon^{\ell}}\langle X_n\rangle^{(0)}_{e} + N^{(\ell)}_{\lambda_e}\right], \tag{16}$$

where charge conservation lets us eliminate any scale for the celestial plane introduced in the evaluation of Eq. (15). Here, σ^e_n is precisely the one introduced in Eq. (6). Hence, by identifying the regularization parameters in Eqs. (4) and (10) and setting the deformation coupling constant λ_e as $-\epsilon g_e$, an exact equivalence emerges between the universal IR divergent factor of the all-loop amplitude and the $J\bar{J}$ deformed correlator.

A parallel correspondence holds true for gravity. Utilizing the operator P_{AB}, we formulate a 2D marginal deformation, addressing the 4D loop correction of amplitudes exhibiting an infrared leading divergence. The action governing the double-trace $P\bar{P}$ deformation specifies the abstract indices as $\bar{a}\,\bar{b} = ab$ and designates the current as $j = P$ in Eq. (13), with λ_G representing a dimensionless coupling constant. The ℓth-order correction of the $P\bar{P}$ deformed correlator can be obtained by configuring $j\bar{j} = P\bar{P}$. Employing the Ward identities in Eqs. (10) and (12) and evaluating the integrals $I^{r\bar{s}}_{i\bar{j}}$ (detailed in the Supplemental Material [92]) yields the following result:

$$X_{n\,[\lambda_G]}^{(\ell)} = \frac{1}{\ell!}\left(-\frac{\lambda_G}{\epsilon}\right)^{\ell}\left[\frac{(\sigma^G_n)^{\ell}}{\epsilon^{\ell}}\langle X_n\rangle^{(0)}_{G} + N^{(\ell)}_{\lambda_G}\right], \tag{17}$$

where σ^G_n is the same as that introduced in Eq. (8). By identifying λ_G as $-\epsilon g_G$,[6] the ℓ-loop corrected 4D gravity amplitudes, particularly for the leading term of IR divergence, can be regarded as correlation functions in a seed CCFT with the ℓth marginal deformation $P\bar{P}$. For consistency, we reformulate soft theorems [81,99–101] within the deformed CCFT. It is shown that the leading soft theorem does not flow under the deformation. This consolidates the fact that the leading IR singularity in Eq. (4) is universal. Our rigorous confirmation also shows that the deformed Ward identity of the stress tensor, arising as a shadow transformation of the deformed subleading soft theorem, precisely agrees with the amplitude-based derivation [102]. Explicit computations are available in the Supplementary Material [92].

Geometric realization of moduli spaces of bulk vacua. In standard holographic scenarios, marginal deformations in the boundary conformal field theory correspond to continuous moduli spaces of bulk gravitational vacua. Recent works [53,57] in celestial holography have extended this idea to transitions in bulk vacua at the perturbative level, characterized by introducing coherent states of zero-energy soft particles. The marginal deformation discussed in the previous sections aligns with such transitions, representing changes in electromagnetic or gravitational field vacua. To explore these transitions in more detail from a nonperturbative perspective, we employ a geometric realization of the marginal deformation, as proposed by Refs. [103–106].

In celestial holography, the nature of the symmetry breaking and relation of the degenerate vacua are very different from the standard case of spontaneous breaking of a global symmetry [107]. The parameter that generates the spontaneously broken symmetry is an arbitrary local function on the celestial sphere. The vacuum state is changed by soft particle creation, which is classically

[4]It is important to note that calculating the universal part of deformed correlators only requires the data of correlators in the seed CCFT, the precise form of the seed CCFT action is not needed.

[5]If one considers the soft currents as classical fields, the deformation of the action is precisely the inverse shadow of the effective action of the soft photon modes in Ref. [84]. There the shadow transformation is introduced by the propagator in the effective action. However, in our construction, a shadow transformation is not involved. The situation is similar to the gravity case in the next section.

[6]In the main text, we have assumed that the length scale of the celestial plane is 1, using stereographic coordinates on the Riemann sphere $(z,\bar{z})$. One could also restore the length scale r_0 to regulate the collinear limits, in which case the identification should be $\lambda_G = -\frac{\epsilon G_N}{16\pi^2 r_0^2}$ for the identifications in Eq. (17).

measured by the memory effect. At the classical level, the degenerate vacua all have vanishing field strength tensor $F_{\mu\nu}$ or curvature tensor $R_{\mu\nu\alpha\beta}$ and are characterized by asymptotic symmetry transformation; see, e.g., detailed analysis in Chapter 2.11 of Ref. [107].

Suppose that the action $S[\phi, F]$ is associated with the complete seed CFT, where ϕ denotes a collection of arbitrary fundamental fields and F is the background gauge field. A double current deformation can be viewed as a form of topological gauging [103–106],

$$S^{[\lambda]} = S[\phi, F] - \frac{1}{2\lambda} S_M[F, Y],$$
$$S_M = \int \mathrm{d}^2x\, \epsilon^{AB} \epsilon_{\bar{a}\,\bar{b}} (F_A^{\bar{a}} - Y_A^{\bar{a}})(F_B^{\bar{b}} - Y_B^{\bar{b}}), \tag{18}$$

where $F_A^{\bar{a}}$ now is promoted into a dynamical gauge field and $Y_A^{\bar{a}}$ is the auxiliary field. Correspondingly, the double current deformed action results from integrating over $F_A^{\bar{a}}$ using a leading semiclassical approximation with the saddle point condition [106]. We set $Y_A^{\bar{a}}$ to represent the reference background gauge field, ensuring the proper recovery of the undeformed theory as λ approaches zero. The path integral transformation of Eq. (18) provides a comprehensive quantum definition for these deformations [106]. Different saddle points in the gauge field $F_A^{\bar{a}}$ lead to distinct action decay patterns, distinguishing theories with unique vacua. Notably, specific saddle point choices automatically induce $J\bar{J}$ and $P\bar{P}$ deformations

$$F_{*A}^{\bar{a}} = Y_A^{\bar{a}} + \lambda \epsilon^{\bar{a}\,\bar{b}} \epsilon_{AB} J_{\bar{b}}^{B[\lambda]}. \tag{19}$$

In a holographic understanding, the F_* can be regarded as the source field coming from the bulk, which generates fluctuation to the vacua of the bulk.

For scalar QED, $Y_A^{\bar{a}}$ is the background (undeformed) gauge field A_A^i, so the $J\bar{J}$ deformation will change the gauge field in the bulk from A_A^i to F_{*A}^i,

$$A^i \to F_*^i(\lambda_e) = A^i + \mathrm{d}\chi^i(\lambda_e), \tag{20}$$

where $\lambda_e \epsilon^{ij} \epsilon_{AB} J_j^{B[\lambda_e]} = \mathrm{d}\chi_A^i$ since $\mathrm{d}(\lambda_e \epsilon^{ij} \epsilon_{AB} J_j^{B[\lambda_e]}) = 0$ for conserved currents. This indicates that the $J\bar{J}$ deformation on the boundary is equivalent to implementing a large gauge transformation in bulk with dynamic terms, which leads to the transition between inequivalent vacua in bulk [90,107].

For gravity, $Y_A^{\bar{a}}$ is the background basis E_A^a of the undeformed metric, i.e., the Minkowskian bulk metric. It will be formally changed through the $P\bar{P}$ flow. In particular, the transverse metric will be switched from γ_{AB} to

$$\gamma_{AB}^{[\lambda_G]} = g_{ab} F_{*A}^a F_{*B}^b$$
$$= \gamma_{AB} + 2\lambda_G \hat{P}_{AB}^{[\lambda_G]} + \lambda_G^2 \hat{P}_{AC}^{[\lambda_G]} \hat{P}_{BD}^{[\lambda_G]} \gamma^{CD}, \tag{21}$$

where $\hat{P}_{AB}^{[\lambda_G]} = P_{AB}^{[\lambda_G]} - \gamma_{AB} P_C^{C[\lambda_G]}$. As a marginal deformation, the current $\hat{P}_{AB}^{[\lambda_G]}$ should obey the same properties as the seed theory from a nonperturbative perspective, which leads to the following decomposition that the current must satisfy:

$$\hat{P}_{AB}^{[\lambda_G]} = -2\partial_A \partial_B C(z, \bar{z}) + \gamma_{AB} \partial^D \partial_D C(z, \bar{z}). \tag{22}$$

Remarkably, the supertranslation shifts the transverse metric as [74,75]

$$\gamma_{AB}^{\mathrm{vac}} = \gamma_{AB} + \frac{1}{\rho} C_{AB}^{\mathrm{vac}} + \frac{1}{4\rho^2} C_{AC}^{\mathrm{vac}} C_{BD}^{\mathrm{vac}} \gamma^{CD}, \tag{23}$$

where the traceless vacuum shear tensor C_{AB}^{vac} satisfies the same type of decomposition as the current $\hat{P}_{AB}^{[\lambda_G]}$. After balancing the dimension, we can identify $\hat{P}_{AB}^{[\lambda_G]}$ as $\frac{C_{AB}^{\mathrm{vac}}}{2G_N \epsilon}$ in the bulk. Together with the identification $\lambda_G = \frac{\epsilon G_N}{\rho}$, one obtains

$$\gamma_{AB}^{\mathrm{vac}} = \gamma_{AB}^{[\lambda_G]}. \tag{24}$$

This verifies that the $P\bar{P}$ deformation on the boundary is equivalent to implementing a supertranslation in bulk, a dynamical coordinates transform. The vacuum of standard Minkowskian spacetime will flow to the inequivalent vacua [74,75,107].

Finally, it is worth noting that supertranslations and large gauge transformations in the bulk account for the exchange of virtual soft gravitons and gauge bosons, respectively [67,80,107]. This exchange generates the leading IR divergence in the amplitudes for scalar QED and gravity [84,88,108]. Therefore, it is reasonable to infer that the IR-divergent component of the amplitude relation in Eq. (4) corresponds to the deformed action on the boundary in Eq. (18) near the saddle point F_*, confirming the consistency of our framework with previous research, especially [53,57,109–111].

Concluding remarks. This paper establishes a precise correspondence between the leading-order universal infrared divergences in 4D massless scattering amplitudes within the bulk theory and marginal deformations of correlators in 2D CCFT. In the context of QED and gravity, we determine the soft current, J or P, by invoking the leading soft photon theorem [82–84,88] or soft graviton theorem [83,99,108], respectively. By deforming the seed CCFT with the composite operator $J\bar{J}$ or $P\bar{P}$, we uncover a universal part of the deformed correlators expressed in a factorial form. Notably, there is an exact agreement

between the correlator relations and the Mellin transform of the universal part of the amplitude relations in Eq. (4), underscoring the remarkable fact that the universal part of the loop amplitude can be precisely reproduced by the tree amplitude and a marginal deformation in the context of celestial holography. Lastly, we propose a geometric representation of the marginal deformation, linking it with the moduli spaces of bulk vacua, thereby offering a promising avenue to understand the interplay between boundary CCFT and bulk theories.

In summary, we offer quantitative verification for celestial holography at perturbative and nonperturbative levels, opening avenues for future exploration.

First, within our framework, soft theorems, known to extend beyond the leading order [85,87,112–114], allow systematic construction of marginal deformations using subleading soft currents. The Mellin transformation of resulting CCFT correlator relations is poised to unveil insightful holographic amplitude relations, potentially discovering novel results for loop amplitude. Despite amplitude relations being expressed in the low-energy expansion form in Eq. (4), they closely relate to the classical limit of loop amplitudes, crucial for recent amplitude-based gravitational wave investigations [115–130].

Second, utilizing the geometric representation to identify moduli spaces of bulk vacua corresponding to subleading soft theorems' marginal deformations is crucial. This uniquely determines the symmetry transformation of the bulk solution phase space, addressing distinct proposals in the literature [131–139].

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