Group field theory with non-commutative metric variables

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We introduce a dual formulation of group field theories, making them a type of non-commutative field theories. In this formulation, the variables of the field are Lie algebra variables with a clear interpretation in terms of simplicial geometry. For Ooguri-type models, the Feynman amplitudes are simplicial path integrals for BF theories. This formulation suggests ways to impose the simplicity constraints involved in BF formulations of 4d gravity directly at the level of the group field theory action. We illustrate this by giving a new GFT definition of the Barrett-Crane model.

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Introduction

Group field theories [1] (GFTs) are growing up as a promising formalism for quantum gravity, combining elements from several other approaches [2]. They extend to higher dimensions the idea of matrix models for 2d gravity [3] of define the quantum dynamics of geometry (and topology) as a sum over simplicial complexes weighted by quantum amplitudes. They build up on the achievements of loop quantum gravity [4] and spin foam models [5]. Loop quantum gravity describes the states of quantum space in terms of spin networks; spin foam models define their dynamics in a covariant language. GFTs subsume this dynamics, as every spin foam model can be interpreted as the Feynman amplitude of some GFT [6].

Several results in spin foam models, and the historic roots in matrix models, suggest a close relation between GFTs and simplicial gravity path integrals, as used in other discrete approaches [7]. The very construction of spin foam amplitudes is usually based on the geometric quantization of simplicial geometry [5, 8], and there are close relations between simplicial and LQG canonical data [9]. In 3d, where BF theory coincides with 1st order gravity, the Ponzano-Regge model for a simplicial manifold without boundary can be derived from a path integral for simplicial BF theory [5]. In 4d, where gravity can be understood as a constrained BF theory [10], most work aimed at identifying classical and quantum constraints turning discrete B variables into tetrad variables [5, 8]. The resulting single-simplex amplitude behaves in a semi-classical limit as the cosine of the Regge action for simplicial gravity [11]. Furthermore, the partition function of recent 4d models has been either derived from a path integral perspective [12], or recast in the form of a simplicial path integral, although with a non-standard action [13].

However much more remains to be understood in this direction. GFTs seem the most convenient setting to do so. They provide the most complete definition of spin foam models, and provide a 2nd quantization of fundamental building blocks of space represented dually as spin networks or simplices, suggesting a similar duality between spin foam models and simplicial path integrals in the corresponding Feynman amplitudes. For BF models, they realize nicely the duality between the spin foam representation of the dynamics and the representation as a lattice path integral in connection variables. This is the covariant counterpart of the two representations of LQG states (and of the GFT field) in terms of cylindrical functions or spin networks. All the above also motivated previous work attempting to encode simplicial geometry in a GFT formulation with explicit B variables, and whose Feynman amplitudes had the form of simplicial gravity path integrals [14].

In parallel, interesting connections between spin foam/GFT models and non-commutative geometry have been discovered. The spin foam amplitudes for 3d gravity coupled to point particles can be recast in the form of Feynman amplitudes for an effective matter non-commutative field theory [15] (and derived from an extended GFT action [16]). The same non-commutative field theory describes the effective dynamics of perturbations around classical solutions of the pure gravity GFT [17], and a similar result holds in the 4-dimensional Lorentzian setting [18], as well as in other GFT models [19] (the results of [19] also show the importance of encoding correctly the non-commutativity of B variables when including them in a GFT formalism). On top of their interest for quantum gravity phenomenology [21], these results suggest that non-commutative structures lie hidden at the very foundations of the GFT formalism.

In this paper we realize in a natural way the above suggestions. We introduce a representation of GFTs as non-local, non-commutative quantum field theories on Lie algebras, which we relate to the B variables of simplicial BF theory. We prove that the GFT Feynman amplitudes for arbitrary diagram, thus arbitrary simplicial complex, are simplicial BF path integrals. So we realize an explicit duality between spin foam models and simplicial gravity path integrals, stemming from a duality of representations for the GFT field, obtained by Peter-Weyl decomposition and by non-commutative Fourier transform [15, 22]. This new representation clarifies the encoding of simplicial geometry in the GFT action. Moreover, the imposition of simplicity constraints can be performed in a geometrically transparent manner directly at the GFT level. We show this by giving a new GFT definition of the Barrett-Crane model. The details of our construction and results will be left to a following publication.

Non-commutative representation of 3d GFT

We first consider Boulatov's group field formulation of 3d Riemannian gravity [20]. The variables are fields $\varphi_{123} := \varphi(g_1, g_2, g_3)$ on $\mathrm{SO}(3)^3$ satisfying the invariance:

$$\varphi_{123} = P\varphi_{123} := \int \mathrm{d}h \, \varphi(hg_1, hg_2, hg_3) \qquad (1)$$

The dynamics is governed by the action:

$$S = \frac{1}{2} \int [\mathrm{d}g]^3 \, \varphi_{123}^2 - \frac{\lambda}{4!} \int [\mathrm{d}g]^6 \, \varphi_{123}\varphi_{345}\varphi_{526}\varphi_{641}$$

The Feynman graphs generated by this theory are 2-complexes dual to 3d triangulations: the combinatorics of the field arguments in the interaction vertex is that of a tetrahedron, while the kinetic term dictates the gluing rule for tetrahedra along triangles.

By Peter-Weyl expansion of φ into irreducible SO(3) representations $j_i \in \mathbb{N}$, the field is pictured as 3-valent spin network vertex; it is interpreted as a quantized triangle, the three field arguments are associated to its edges. The following will substantiate further this geometrical picture. In this representation, the interaction term can be written in terms of 6j-symbols, and the amplitude of a Feynman graph gives the Ponzano-Regge model.

We now introduce an alternative formulation of the model, obtained by means of a 'group Fourier transform' [15, 22] mapping functions on a group to (non-commutative) functions on its Lie algebra $\mathfrak{g}$. This transform stems from the definition of plane waves $\mathrm{e}_g(x) = e^{i\vec{p}_g \cdot \vec{x}}$ as functions on $\mathfrak{g} \sim \mathbb{R}^n$, depending on a choice of coordinates $\vec{p}_g$ on the group manifold. In the sequel we will identify functions of SO(3) with functions of SU(2) invariant under $g \to -g$.

We choose the coordinates $\vec{p}_g = \mathrm{Tr}|g|\vec{\tau}$, where $|g| := \mathrm{sign}(\mathrm{Tr}g)g$, $\vec{\tau}$ are i times the Pauli matrices and 'Tr' is the trace in the fundamental representation. For $x = \vec{x} \cdot \vec{\tau}$ and $g = e^{\theta\vec{n}\cdot\vec{\tau}}$, we thus have

$$\mathrm{e}_g(x) = e^{i\mathrm{Tr}x|g|} = e^{-2i\sin\theta\vec{n}\cdot\vec{x}}$$

The Fourier transform of functions $f(g)$ on SU(2) is

$$\widehat{f}(x) = \int \mathrm{d}g f(g) \, \mathrm{e}_g(x)$$

where $\mathrm{d}g$ is the normalized Haar measure.

The image of the Fourier transform inherits an algebra structure from the convolution product on the group, given by the $\star$-product defined on plane waves as

$$\mathrm{e}_{g_1} \star \mathrm{e}_{g_2} = \mathrm{e}_{g_1 g_2} \; .$$

There is a relation [22] between the $\star$-product and a differential operator acting on ordinary functions on $\mathbb{R}^3$, encoded in the relation $\int f \star g = \int_{\mathbb{R}^3} f \sqrt{1+\Delta}\, g$, with Δ the Laplacian on $\mathbb{R}^3$. On functions of SO(3), the Fourier is invertible:

$$f(g) = \frac{1}{\pi} \int \mathrm{d}^3x \, (\widehat{f} \star \mathrm{e}_{g^{\text{-}1}})(x)$$

With a bit more work the above construction is extended to an invertible SU(2) Fourier transform [22].

Fourier transform and $\star$-product extend to functions of several variables like the Boulatov field as

$$\widehat{\varphi}_{123} := \hat{\varphi}(x_1, x_2, x_3) = \int [\mathrm{d}g]^3 \, \varphi_{123} \, \mathrm{e}_{g_1}(x_1)\mathrm{e}_{g_2}(x_2)\mathrm{e}_{g_3}(x_3)$$

The first feature of the dual formulation is that the constraint (1) acts on dual fields as a 'closure constraint' for the variables x_j. Indeed, a simple calculation gives

$$\widehat{P\varphi} = \widehat{C} \star \widehat{\varphi}, \quad \widehat{C}(x_1, x_2, x_3) = \delta_0(x_1 + x_2 + x_3)$$

where δ_0 is the element $x = 0$ of the family of functions:

$$\delta_x(y) := \int \mathrm{d}g \, \mathrm{e}_{g^{\text{-}1}}(x)\mathrm{e}_g(y) \qquad (2)$$

These play the role of Dirac distributions in the non-commutative setting, in the sense that[1]

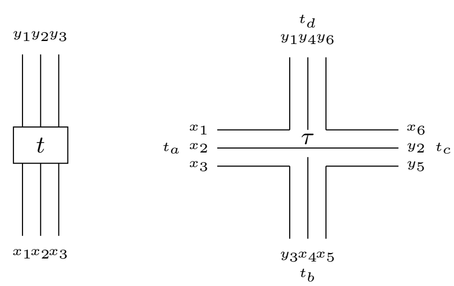
$$\int \mathrm{d}^3y \, (\delta_x \star f)(y) = \int \mathrm{d}^3y \, (f \star \delta_x)(y) = f(x) \qquad (3)$$

We may thus interpret the variables of the Boulatov dual field as the edges vectors of a triangle in $\mathbb{R}^3$, and the dual fields themselves as (non-commutative) triangles.

Since the $\star$-product is dual to group convolution, the combinatorial structure of the action in terms of the dual field matches the one in terms of the original field. We may thus show that

$$S = \frac{1}{2} \int [\mathrm{d}x]^3 \, \widehat{\varphi}_{123} \star \widehat{\varphi}_{123} - \frac{\lambda}{4!} \int [\mathrm{d}x]^6 \, \widehat{\varphi}_{123} \star \widehat{\varphi}_{345} \star \widehat{\varphi}_{526} \star \widehat{\varphi}_{641}$$

where it is understood that $\star$-products relate repeated indices as $\phi_i \star \phi_i := (\phi \star \phi_{\text{-}})(x_i)$, with $\phi_{\text{-}}(x) = \phi(\text{-}x)$. The structure of this action is best visualized in terms of diagrams. Thus, kinetic and interaction terms identify a propagator and a vertex



(4)

given by:

$$\int \mathrm{d}h_t \prod_{i=1}^{3} (\delta_{\text{-}x_i} \star \mathrm{e}_{h_t})(y_i), \qquad \int \prod_t \mathrm{d}h_{t\tau} \prod_{i=1}^{6} (\delta_{\text{-}x_i} \star \mathrm{e}_{h_{tt'}})(y_i) \qquad (5)$$

[1] Seen as a function of $\mathbb{R}^3$, δ_0 is the regular function, peaked on $x = 0$: $\delta_0(x) \propto J_1(|x|)/|x|$, with J_1 the 1st Bessel function [22].

with $h_{tt'} := h_{t\tau} h_{\tau t'}$. We have used '$t$' for triangle and '$\tau$' for tetrahedron. The group variables h_t and $h_{t\tau}$ arise from (1), and should be interpreted as parallel transports through the triangle t for the former, and from the center of the tetrahedron τ to triangle t for the latter (see [14]).

The integrands in (5) factorize into a product of functions associated to strands (one for each field argument), with a clear geometrical meaning: the pair of variables (x_i, y_i) associated to the same edge i corresponds to the edges vectors seen from the frames associated to the two triangles t, t' sharing it. The vertex functions state that the two variables are identified, up to parallel transport $h_{tt'}$, and up to a sign labeling the two opposite edge orientations inherited by the triangles t, t'. The propagator encodes a similar gluing condition, allowing for the possibility of a further mismatch between the reference frames associated to the same triangle in different tetrahedra.

Feynman amplitudes as simplicial path integrals

We now sketch the calculation of Feynman amplitudes. In building up a closed graph, propagator and vertex strands are joined to one another using the $\star$-product, keeping track of the ordering of functions associated to the various building blocks of the graph. Each loop of strands bound a face of the 2-complex, which is dual to an edge of the triangulation.

Under the integration over the group variables $h_t, h_{t\tau}$, the amplitude factorizes into a product of *face* amplitudes. Let then f_e be a face of the 2-complex, dual an edge e in the triangulation, and consider the loop of strands that bound it. The choice of an orientation and a reference vertex defines an ordered sequence $\{\tau_j\}_{0 \le j \le N}$ of vertices on the loop. It corresponds to an ordered set of tetrahedra around e. Using (3), each vertex τ_j, after contraction with the propagator t_j joining τ_j and τ_{j+1}, contributes with $(\delta_{x_j} \star \mathrm{e}_{h_{jj+1}})(x_{j+1})$ to the face amplitude, where $h_{jj+1} = h_{\tau_j t_j} h_{t_j} h_{t_j \tau_{j+1}}$ parallel transports j to $j+1$.

The face amplitude $A_{f_e}[h]$ is then the cyclic $\star$-product of all these contributions:

$$A_{f_e}[h] = \int \prod_{j=0}^{N} \mathrm{d}^3 x_j \, \vec{\bigstar}_{j=0}^{N+1} \, (\delta_{x_j} \star \mathrm{e}_{h_{jj+1}})(x_{j+1})$$

where $x_{N+1} := x_0$. This amplitude encodes the identification, up to parallel transport, of the metric variables associated to e in different tetrahedron frames. We may integrate over all metric variables x_j in $A_{f_e}[h]$, except for that of the reference frame. Introducing the holonomy $H_0 := h_{01} \cdots h_{N0}$ around the boundary of f_e, we obtain that

$$A_{f_e}[h] = \int \mathrm{d}^3 x_0 \, \mathrm{e}_{H_0}(x_0)$$

Finally the Feynman amplitude reads:

$$Z(\Gamma) = \int \prod_t \mathrm{d}h_t \prod_e \mathrm{d}^3 x_e \, e^{i \sum_e \mathrm{Tr}\, x_e H_e} \tag{6}$$

The variables h_t corresponds to the parallel transport between the two tetrahedra sharing t; H_e is the holonomy around the boundary of f_e, calculated from a chosen reference tetrahedron frame.

Equ.(6) is the usual expression for the simplicial path integral of first order 3d gravity (or 3d BF theory). The Lie algebra variables x_e, one per edge of the simplicial complex, play the role of discrete triad; the group elements h_t, one per triangle or link of the dual 2-complex, play the role of discrete connection, defining the discrete curvature H_e through holonomy around the faces dual to the edges of the simplicial complex [5].

Open GFT Feynman amplitudes have fixed boundary simplicial data. The one-vertex contribution to the 4-point functions, for example, is the function of twelve metric variables x_i, x'_i obtained by acting with a closure operator $\widehat{C}$ (propagator) on each external 3-stranded leg of the vertex diagram in (4), building up four triangles $t_a, \cdots t_c$. The amplitude is the $\star$-product of the identification functions $\delta_{-x'_i}(x_i)$ of the boundary metric with the BF action for a single simplex $e^{i \sum_i \mathrm{Tr} x_i h_i}$, $h_i = h_{t\tau} h_{\tau t'}$ being the parallel transport between the two triangles sharing i, integrated over the bulk connection $h_{t\tau}$. The integrand can be viewed as constraints on the gauge connection; in fact one can show that, in the limit of large scale boundary geometries, the constraints become those [14] characterizing a discrete Levi-Civita connection. For generic open graphs, the amplitudes are given by a path integral for the BF action augmented by the appropriate boundary terms (see also [23]). Note that the (exponential of the) BF action for a single simplex is already explicitly present in the interaction term of the GFT action. This can be useful to study the link with semi-classical/continuum gravity directly at the GFT level.

Polynomial gauge invariant GFT observables in this representation are labeled by spin-networks; they are expressed in terms of the Fourier dual of spin-network functionals on the group. These dual spin-network functionals show up as the basis states in a flux representation of loop quantum gravity [24].

These results show an exact duality between spin foam models and simplicial gravity path integrals, stemming from two equivalent representations of the GFT field: as a function of representation labels, following Peter-Weyl decomposition, and as a non-commutative function on Lie algebra variables, interpreted as metric variables.

Towards 4d gravity models

Going up dimensions, we consider the GFT model for SO(4) BF theory, defined in terms of a gauge invariant field $\varphi_{1234} = \int \mathrm{d}h \, \varphi(hg_1, hg_2, hg_3, hg_4)$ by the action:

$$S = \frac{1}{2} \int \varphi_{1234}^2 - \frac{\lambda}{5!} \int \varphi_{1234} \, \varphi_{4567} \, \varphi_{7389} \, \varphi_{962\,10} \, \varphi_{10\,851} .$$

The Feynman graphs are 2-complexes dual to 4d simplicial complexes: the combinatorics of the interaction term is that of a 4-simplex; the kinetic terms dictates the gluing rules for 4-simplices along tetrahedra. Using harmonic analysis on the group the vertex term is written in terms of SO(4) 15j symbols and the Feynman amplitudes gives the Ooguri state sum model.

The SO(3) group Fourier transform naturally extends to a Fourier transform on $\mathrm{SO}(4) \simeq \mathrm{SU}(2) \times \mathrm{SU}(2)/\mathbb{Z}_2$, which is invertible on even functions $f(g) = f(-g)$. In the sequel we assume the further invariance of the Ooguri field under $g_i \to -g_i$ in each of the variables.

The dual Ooguri field is a function of four $\mathfrak{so}(4)$ Lie algebra elements, or bivectors, associated to the four triangles of each tetrahedron. Gauge invariance translates into a closure constraint for the bivectors, meaning that the four triangles close to form a tetrahedron. Kinetic and vertex terms again encode the identification, up to parallel transport, of the bivectors associated to the same triangle in different tetrahedral frames. The computation of Feynman amplitudes proceeds analogously to the 3d case. The result is again a simplicial path integral for BF theory analogue to (6), with integrals now over SO(4) group and Lie algebra elements.

The new representation of the Ooguri model provides a convenient starting point for imposing in a geometrically transparent manner the discrete simplicity constraint that turn BF theory into 4d simplicial gravity [8, 10]. Using the decomposition of $x \in \mathfrak{so}(4)$ into selfdual x^+ and anti-selfdual x^- $\mathfrak{su}(2)$-components, we impose that the four bivectors in each tetrahedron are orthogonal to the same normal vector $k \in \mathcal{S}^3 \sim \mathrm{SU}(2)$ to the tetrahedron, by means of the constraint projector

$$\widehat{S}_k(x_j^-, x_j^+) = \prod_{j=1}^{4} \delta_{-k x_j^- k^{-1}}(x_j^+)$$

where the δ functions are given by (2). It acts on the field as

$$(\widehat{S}_k \star \widehat{\varphi})(x) = \int_{\mathrm{SO}(4)} \mathrm{d}g \, S_k\varphi(g) \, \mathrm{e}_{g^-}(x^-) \mathrm{e}_{g^+}(x^+)$$

with

$$S_k\varphi(g) := \int_{\mathrm{SO}(3)} \mathrm{d}u \varphi(k^{-1} u k g^-, u g^+)$$

where we have decomposed the group elements g into selfdual g^+ and anti-selfdual g^- components. Hence $\widehat{S}_k$ acts dually as the projector onto fields on the homogeneous space $\mathcal{S}^3 \sim \mathrm{SO}(4)/\mathrm{SO}(3)_k$, where $\mathrm{SO}(3)_k$ is the stabilizer subgroup of the normal k. The case $k = 1$ corresponds to removing the diagonal SO(3) subgroup; it reproduces the standard Barrett-Crane projector [25].

By combining the simplicity projector $\widehat{S} := \widehat{S}_1$ with the closure projector, one may build up the field $\widehat{\Psi} := \widehat{S} \star \widehat{C} \star \widehat{\varphi}$, which reproduces the GFT field used in the standard GFT formulation of the Barrett-Crane model. More precisely, combining the interaction term:

$$\frac{\lambda}{5!} \int \widehat{\Psi}_{1234} \star \widehat{\Psi}_{4567} \star \widehat{\Psi}_{7389} \star \widehat{\Psi}_{962\,10} \star \widehat{\Psi}_{10\,851}$$

with the possible kinetic terms:

$$\frac{1}{2} \int \widehat{\Psi}_{1234}^{\star 2}\,, \ \frac{1}{2} \int (\widehat{C} \star \widehat{\varphi})_{1234}^{\star 2} \ \text{ or } \ \frac{1}{2} \int \widehat{\varphi}_{1234}^{\star 2}$$

gives the versions of the Barrett-Crane model derived in [27], [28] and [12] respectively. The origin of these different versions can be understood geometrically, thanks to the new GFT representation. In fact, given $h \in \mathrm{SO}(4)$, we may show that

$$(e_h \star \widehat{S}_k)(x) = (\widehat{S}_{h \triangleright k} \star e_h)(x)$$

with $h \triangleright k := h^+ k (h^-)^{-1}$. This expresses the fact that, after rotation by h, simple bivectors with respect to the normal k become simple with respect to the rotated normal $h \triangleright k$. Therefore closure and simplicity constraints do not commute. Moreover, whereas the model couples correctly the bivector variables x across simplices, the integration over holonomies effectively decorrelates the normal vectors k associated to the same tetrahedron in different 4-simplices. This implies a missing geometric condition on connection variables $h_{\tau\sigma}$. Work on a GFT model where simplicity constraints are imposed covariantly is currently in progress [26].

A simplicial path integral formulation of the Barrett-Crane model, in, say, its version [12], is obtained by using the Feynman rules for the propagator and vertex:

$$\prod_{i=1}^{4} (\delta_{-x_i})(y_i), \qquad \int \prod_t \mathrm{d}h_{\tau\sigma} \prod_{i=1}^{10} (\delta_{-x_i} \star \widehat{S} \star \mathrm{e}_{h_{\tau\tau'}})(y_i)$$

The amplitude of a graph dual to a triangulation Δ takes the form of the $\star$-evaluation of a non-commutative observable in BF theory:

$$Z_{\mathrm{BC}}(\Delta) = \int \prod_{\tau\sigma} \mathrm{d}h_{\tau\sigma} \int \prod_t \mathrm{d}^6 x_t \, (\mathcal{O}_t \star e_{H_t})(x_t)$$

where the functions $\mathcal{O}_t(x_t)$ implement simplicity $\delta_{-h_{0j}^{--1} x_t^- h_{0j}^-}(h_{0j}^{+-1} x_t^+ h_{0j}^+)$ of the bivectors x_t in each of the 4-simplex frames $j = 0 \cdots N$ around t:

$$\mathcal{O}_t = \bigstar_{j=0}^{N} \, \delta_{-h_{0j}^{--1} \bullet^- h_{0j}^-}(h_{0j}^{+-1} \bullet^+ h_{0j}^+)$$

Conclusions and perspectives

We have introduced a new non-commutative representation of group field theories, based on the group Fourier transform, turning them into non-local and non-commutative field theories on Lie algebras. We have shown that the resulting Lie algebra variables correspond to the B variables of simplicial BF theory, in any dimension, and that the corresponding Feynman amplitudes for arbitrary simplicial complex have the form of simplicial BF path integral. This realizes an explicit GFT duality between spin foam models and simplicial gravity path integrals, and allows to understand clearly how simplicial geometry is encoded in the GFT formalism. We have also shown how the Barrett-Crane model for 4d gravity is obtained in the new representation, and pointed out the geometric insights this reformulation gives.

The new GFT representation, and the duality it realizes, can trigger much further progress. We mention here a few research directions, already ongoing, whose results will be reported elsewhere.

The interpretation of GFTs as 2nd quantized theories of spin networks suggests to apply the group Fourier transform to general cylindrical functions, namely to generic loop quantum gravity states [24]. This should allow to describe a *flux* representation of the theory, usually assumed to be intractable precisely because of the non-commutativity of flux operators.

The new representation should also help the identification of spacetime symmetries (e.g. diffeomorphisms [29]) which act on the B variables, at the level of the GFT action [30] (see also [31]). Understanding the role of diffeomorphisms can then guide the study of the continuum approximation of GFTs, and of their relation with general relativity.

Obviously, the goal is the construction of a satisfactory GFT model for quantum gravity in 4 dimensions. In the new GFT representation, guided by the manifest geometric meaning of the variables and the Feynman amplitudes, simplicity constraints on the B variables, with and without Immirzi parameter, can be imposed in a natural way. Work on this is in progress and can either lead to the definition of a new spin foam model for 4d quantum gravity, or to a complete and geometrically clear GFT formulation of the recently proposed ones. It can also, in one stroke, give a reformulation of these models as simplicial path integrals.

The new representation may help also in the study of GFT renormalization [32] and, by means of this, of their phase structure and continuum approximation [34]. It can be used for the introduction of scales (by re-expressing the star product in terms of differential operators) allowing for a multi-scale analysis, and for defining a new notion of (GFT) locality [33]. Moreover, it can be used to characterize the regime in which general GFTs reduce to the linearized approximation for which rigorous power counting theorems can be proven [35].

Finally, it should reinforce the links between the GFT formalism and non-commutative geometry, and allow to push further the approach to quantum gravity phenomenology [21] based on effective non-commutative matter field theories.

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