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Universal Kaluza–Klein reductions of type IIB to $N = 4$ supergravity in five dimensions

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Abstract

We construct explicit consistent Kaluza–Klein reductions of type IIB supergravity on $HK_4 \times S^1$, where HK_4 is an arbitrary four-dimensional hyper-Kähler manifold, and on SE_5, an arbitrary five-dimensional Sasaki–Einstein manifold. In the former case we obtain the bosonic action of $D = 5$ $N = 4$ (ungauged) supergravity coupled to two vector multiplets. For the SE_5 case we extend a known reduction, which leads to minimal $D = 5$ $N = 2$ gauged supergravity, to also include a multiplet of massive fields, containing the breathing mode of the SE_5. We show that the resulting $D = 5$ action is also consistent with $N = 4$ gauged supergravity coupled to two vector multiplets. This theory has a supersymmetric AdS_5 vacuum, which uplifts to the class of supersymmetric $AdS_5 \times SE_5$ solutions, that spontaneously breaks $N = 4$ to $N = 2$, and also a non-supersymmetric AdS_5 vacuum which uplifts to a class of solutions first found by Romans.

1 Introduction

Consistent Kaluza-Klein (KK) reductions provide a powerful tool to construct exact solutions of $D = 10$ and $D = 11$ supergravity. For example, it has been shown, at the level of the bosonic fields, that there is a consistent KK reduction of type IIB supergravity on an arbitrary five-dimensional Sasaki-Einstein space, SE_5, to minimal $N = 2$ gauged supergravity in $D = 5$ [1]. By definition, this means that any solution of the $D = 5$ gauged supergravity can be uplifted on an arbitrary SE_5 space to obtain an infinite class of exact solutions of type IIB supergravity, one for each choice of SE_5. In particular, the supersymmetric AdS_5 vacuum solution uplifts to the class of supersymmetric $AdS_5 \times SE_5$ solutions which are dual to $N = 1$ SCFTs in $d = 4$. There is a similar consistent KK reduction of $D = 11$ supergravity on an arbitrary seven dimensional Sasaki-Einstein space, SE_7, to minimal $N = 2$ gauged supergravity in $D = 4$ [2]. In this case the supersymmetric AdS_4 vacuum solution of this theory uplifts to the class of supersymmetric $AdS_4 \times SE_7$ solutions dual to $N = 2$ SCFTs in $d = 3$.

These two examples form part of a more general story. For any supersymmetric solution of $D = 10$ or $D = 11$ supergravity consisting of a warped product of an AdS_{d+1} space with an internal manifold M and fluxes preserving the symmetries of AdS_{d+1}, it is expected [2] that there is always a consistent KK reduction on M to a $D = d + 1$ gauged supergravity theory where one only keeps the fields of the supermultiplet containing the metric. In the d dimensional SCFT dual to the supergravity solution, these fields are dual to the superconformal current multiplet. In the above SE examples, the bosonic field content of the $D = 5, 4$ minimal supergravities consist of a metric and a single gauge-field which are indeed dual to the energy-momentum tensor and the abelian R-symmetry current of the superconformal current multiplet in the dual SCFTs. Other examples include the $N = 8$ $SO(8)$ gauged supergravity arising from the KK reduction of $D = 11$ supergravity on S^7 and the $N = 8$ $SO(6)$ gauged supergravity arising from type IIB supergravity on S^5, where the consistency has been partially demonstrated in [3] and [4, 5, 6, 7] , respectively, and also examples discussed in [8, 9]. It is worth noting that almost all work on consistent KK reductions works at the level of the bosonic fields, with the expectation that the fermions will come along for the ride. A notable exception is [10, 11] where a complete reduction of $D = 11$ supergravity on S^4 to maximal $D = 7$ $SO(5)$ gauged supergravity was carried out. Also, in some cases [1, 8, 2] the fermions have been taken into consideration to the extent that one can show that any bosonic solution of the lower dimensional gauged supergravity that preserves supersymmetry will uplift to a bosonic solution of $D = 10$ or $D = 11$ supergravity that also preserves supersymmetry.

It has recently been shown that the consistent KK reduction of $D = 11$ supergravity on an arbitrary SE_7 space to minimal $D = 4$ $N = 2$ gauged supergravity that we mentioned above can be generalised [12]. At the level of bosonic fields it can be shown that in addition to the massless graviton supermultiplet one can also include the massive supermultiplet that contains the breathing mode. The resulting consistent KK reduction gives a $D = 4$ $N = 2$ gauged supergravity coupled to a vector multiplet and a tensor multiplet. This matter content and the supersymmetry can be understood in the following way. First recall that one can consistently KK reduce type IIA supergravity on an arbitrary Calabi-Yau three-fold to obtain a universal $D = 4$ $N = 2$ (ungauged) supergravity coupled to universal tensor multiplet and a universal hypermultiplet. The details of this reduction utilise the fact that the Calabi-Yau three-fold has an $SU(3)$ structure, specified by the Kähler form and the $(3, 0)$ form, both of which are closed. Returning to the reduction of $D = 11$ supergravity on the SE_7 we next recall that it also has a globally defined $SU(3)$ structure which implies that, locally, the SE_7 space can be viewed as a $U(1)$ fibration over a six-dimensional Kähler-Einstein space KE_6. Thus, after first reducing on the $U(1)$, the reduction has the structure of a type IIA reduction on an $SU(3)$ manifold [13], the KE_6 space. Thus one expects the same field content and the same off-shell supersymmetry as in the universal sector of the reduction of type IIA on the CY_3 space, but the twisting of the $U(1)$ fibration, the fact that the $(3, 0)$ form on the KE_6 is not closed and the presence of the background four-form flux lead to a gauging of the $D = 4$ $N = 2$ supergravity theory. See [14] for a related reduction of $D = 11$ supergravity in a non-supersymmetric context.

In this paper we will show that there is an analogous generalisation of the KK reduction of type IIB supergravity on SE_5. We will show that the consistent KK reduction to minimal $D = 5$ $N = 2$ gauged supergravity of [1] can also be extended to include the massive supermultiplet containing the breathing mode. We will show that the reduction is consistent with $D = 5$ $N = 4$ gauged supergravity coupled to two vector multiplets with a gauging as described in [15][16]. To understand this matter content, and the origin of the increased supersymmetry, we now view, locally, the SE_5 space as a $U(1)$ fibration over a four-dimensional Kähler-Einstein space, KE_4. The previous discussion suggests that we should expect a gauged supergravity with the same field content and supersymmetry as that arising in the universal sector of the reduction of type IIB supergravity on $HK_4 \times S^1$, where HK_4 is an arbitrary four-dimensional hyper-Kähler space (not necessarily compact). In fact this reduction has not yet been analysed[1], so in this paper we will show that there is a consistent KK reduction of type IIB on $HK_4 \times S^1$ to a universal sector that is consistent with $N = 4$ (ungauged) supergravity coupled to two vector multiplets. For the reduction on SE_5 the twisting of the $U(1)$ fibration, the fact that the $(2, 0)$ form on the KE_4 is not closed and the presence of the background five-form flux lead to a gauging of the $D = 5$ $N = 4$ supergravity theory. We show that the gauging is given by a $H_3 \times U(1) \subset SO(5, 2)$ subgroup of the duality symmetry group $SO(1, 1) \times SO(5, 2)$ of the ungauged theory, where H_3 is the three-dimensional Heisenberg group.

The $D = 5$ $N = 4$ gauged supergravity that we obtain from the reduction on an SE_5 space admits a supersymmetric AdS_5 vacuum which uplifts to the supersymmetric $AdS_5 \times SE_5$ solutions of type IIB. An interesting feature is that this AdS_5 vacuum spontaneously partially[2] breaks the $N = 4$ supersymmetry down to $N = 2$. There is also another AdS_5 vacuum that doesn't preserve any supersymmetry which uplifts to the type IIB solutions found by Romans [19] generalising those found in $D = 11$ by Pope and Warner [20][21]. Without supersymmetry, the stability of these type IIB solutions should be investigated; for the special case that the SE_5 space is the round S^5 it is expected that they are not stable [22].

The plan of the rest of the paper is as follows. The reduction of type IIB supergravity on $HK_4 \times S^1$ is analysed in section 2 and the reduction on SE_5 is analysed in section 3. We have included some details of our calculations, which are rather long, in an appendix. Section 4 concludes with some brief final comments concerning how our results might be generalised for the special case of S^5. We also briefly comment on the consistent KK reduction of $D = 11$ supergravity on tri-Sasaki manifolds and argue that it will lead to an $N = 4$ gauged supergravity in $D = 4$ with an AdS_4 vacuum that spontaneously breaks $N = 4$ to $N = 3$.

Note Added

When we were writing this work up we became aware of [23] with which there is considerable overlap.

[1]For related work, see [17].

[2]Note that a general analysis of such partial supersymmetry breaking has recently been carried out for general $N = 2$ $D = 4$ gauged supergravities in [18] .

2 Type IIB reduced on $HK_4 \times S^1$

Our starting point is the class of $\mathbb{R}^{1,4} \times HK_4 \times S^1$ solutions of type IIB supergravity. Recall that the bosonic fields of type IIB supergravity [24][25] consist of the metric, the dilaton Φ and the NS three-form field strength $H_{(3)}$, and the RR form field-strengths $F_{(1)} = dC_{(0)}$, $F_{(3)}$, and $F_{(5)}$. The equations of motion and Bianchi identities are given in appendix A. The $\mathbb{R}^{1,4} \times HK_4 \times S^1$ solution is given by

$$ds^2_{10} \;=\; ds^2(\mathbb{R}^{1,4}) + ds^2(HK_4) + \eta \otimes \eta \tag{2.1}$$

with $F_{(5)} = F_{(3)} = H_{(3)} = 0$ and has constant dilaton and constant axion ($F_{(1)} = 0$). The hyper-Kähler space HK_4 has a Kähler two-form J and a $(2,0)$ form Ω that satisfy algebraic conditions that are given in appendix B. They are both closed as is the one-form η on the S^1 factor:

$$dJ = d\Omega = d\eta = 0 \tag{2.2}$$

This solution generically preserves $N = 4$ supersymmetry. Note that if HK_4 is compact then it is either K_3 or T^4 and in the latter case all $N = 8$ supersymmetry is preserved.

2.1 The consistent Kaluza-Klein reduction on $HK_4 \times S^1$

Our KK ansatz for the metric of type IIB supergravity is given by

$$ds^2_{10} = e^{-\frac{2}{3}(4U+V)} ds^2_{(E)} + e^{2U} ds^2(HK_4) + e^{2V} (\eta + A_1) \otimes (\eta + A_1) \tag{2.3}$$

where $ds^2_{(E)}$ is an arbitrary metric on an external five-dimensional space-time (it will turn out to be in the Einstein frame and hence the subscript E), U and V are scalar fields and A_1 is a one-form defined on the external five-dimensional space. Following [12], the ansatz for the form field strengths is constructed using the two-forms J, Ω

and η, and is given by

$$
\begin{aligned}
F_{(5)} =& e^{-\frac{4}{3}(U+V)} * K_2 \wedge J + K_1 \wedge J \wedge J \\
&+ \left[-2e^{-8U} * K_1 + K_2 \wedge J\right] \wedge (\eta + A_1) \\
&+ \left[e^{-\frac{4}{3}(U+V)} * L_2 \wedge \Omega + L_2 \wedge \Omega \wedge (\eta + A_1) + c.c.\right] \\
F_{(3)} =& G_3 + G_2 \wedge (\eta + A_1) + G_1 \wedge J + (N_1 \wedge \Omega + c.c.) \\
H_{(3)} =& H_3 + H_2 \wedge (\eta + A_1) + H_1 \wedge J + (M_1 \wedge \Omega + c.c.) \\
C_{(0)} =& a \\
\Phi =& \phi
\end{aligned}
\tag{2.4}
$$

Here, $*$ is the Hodge dual corresponding to the five-dimensional metric $ds^2_{(E)}$ in (2.3), with volume form $\mathrm{vol}_5^{(E)}$; a, ϕ, are real scalars, G_3, H_3, G_2, H_2, K_2, K_1 real forms, and L_2, M_1, N_1, complex forms, all of them defined on the external five-dimensional spacetime. Note that we have ensured that the five-form $F_{(5)}$ is self-dual with respect to the metric (2.3). Also note that we can add the terms $(G_0 J + N_0 \Omega) \wedge (\eta + A_1)$ to $F_{(3)}$ and $(H_0 J + M_0 \Omega) \wedge (\eta + A_1)$ to $H_{(3)}$, where G_0,H_0 are real scalars and N_0, M_0 are complex scalars. However, an analysis of the type IIB supergravity equations imply that they can be set to zero. Similarly we have also set to zero a possible factor e^Z, where Z is a scalar, that would multiply $\mathrm{vol}_5^{(E)}$ and $J \wedge J \wedge (\eta + A_1)$ terms in $F_{(5)}$.

We now substitute into the equations of motion and Bianchi identities of type IIB supergravity that are given in appendix A. The calculations are rather involved, so we will simply summarise the main results here, referring to appendix B for some details. We find that the physical degrees of freedom are 7 real scalars U, V, ϕ, a, b, c, h; 2 complex scalars ξ, χ; 4 real one-form potentials A_1, B_1, C_1, E_1; 1 complex one-form potential D_1 and 2 real two-form potentials B_2, C_2 with

$$
\begin{aligned}
H_3 &= dB_2 - B_1 \wedge F_2 \\
H_2 &= dB_1 \\
H_1 &= db \\
M_1 &= d\xi
\end{aligned}
\tag{2.5}
$$

where $F_2 \equiv dA_1$,

$$
\begin{aligned}
G_3 &= dC_2 - C_1 \wedge F_2 - a dB_2 + a B_1 \wedge F_2 \\
G_2 &= dC_1 - a dB_1 \\
G_1 &= dc - a db \\
N_1 &= d\chi - a d\xi
\end{aligned} \tag{2.6}
$$

and

$$
\begin{aligned}
K_2 &= dE_1 - c dB_1 + b dC_1 \\
L_2 &= dD_1 - \chi dB_1 + \xi dC_1 \\
K_1 &= dh + \tfrac{1}{2}(b dc - c db) + \xi^* d\chi + \xi d\chi^* - \chi d\xi^* - \chi^* d\xi
\end{aligned} \tag{2.7}
$$

We also find that the equations of motion for all of the fields can be obtained by varying a $D = 5$ action with Lagrangian given by

$$
\mathcal{L}^{(E)} = \mathcal{L}^{(E)}_{\rm kin} + \mathcal{L}_{\rm top} \tag{2.8}
$$

where the kinetic term is given by

$$
\begin{aligned}
\mathcal{L}^{(E)}_{\rm kin} = & R^{(E)} \, {\rm vol}^{(E)}_5 - \tfrac{28}{3} dU \wedge *dU - \tfrac{8}{3} dU \wedge *dV - \tfrac{4}{3} dV \wedge *dV - \tfrac{1}{2} e^{2\phi} da \wedge *da \\
& - \tfrac{1}{2} d\phi \wedge *d\phi - 4 e^{-4U-\phi} M_1 \wedge *M_1^* - 4 e^{-4U+\phi} N_1 \wedge *N_1^* - 2 e^{-8U} K_1 \wedge *K_1 \\
& - e^{-4U-\phi} H_1 \wedge *H_1 - e^{-4U+\phi} G_1 \wedge *G_1 - \tfrac{1}{2} e^{\frac{8}{3}(U+V)} F_2 \wedge *F_2 \\
& - e^{-\frac{4}{3}(U+V)} K_2 \wedge *K_2 - 4 e^{-\frac{4}{3}(U+V)} L_2 \wedge *L_2^* - \tfrac{1}{2} e^{\frac{4}{3}(2U-V)-\phi} H_2 \wedge *H_2 \\
& - \tfrac{1}{2} e^{\frac{4}{3}(2U-V)+\phi} G_2 \wedge *G_2 - \tfrac{1}{2} e^{\frac{4}{3}(4U+V)-\phi} H_3 \wedge *H_3 - \tfrac{1}{2} e^{\frac{4}{3}(4U+V)+\phi} G_3 \wedge *G_3
\end{aligned} \tag{2.9}
$$

and the topological term is given by

$$
\begin{aligned}
\mathcal{L}_{\rm top} = & A_1 \wedge \Big[- K_2 \wedge K_2 - 4 L_2 \wedge L_2^* + 2 K_1 \wedge \big(C_1 \wedge dB_1 - B_1 \wedge dC_1 \big) \\
& \qquad - 2 K_2 \wedge (B_1 \wedge dc - C_1 \wedge db) - \big[4 L_2^* \wedge \big(B_1 \wedge d\chi - C_1 \wedge d\xi \big) + c.c. \big] \Big] \\
& - 2 dC_2 \wedge X_2 + 2 dB_2 \wedge Y_2
\end{aligned} \tag{2.10}
$$

where

$$
\begin{aligned}
X_2 &= \big(h + \tfrac{1}{2} bc + \xi^* \chi + \xi \chi^* \big) dB_1 - \big(\tfrac{1}{2} b^2 + 2|\xi|^2 \big) dC_1 - b dE_1 - 2\xi^* dD_1 - 2\xi dD_1^* \\
Y_2 &= \big(h - \tfrac{1}{2} bc - \xi^* \chi - \xi \chi^* \big) dC_1 + \big(\tfrac{1}{2} c^2 + 2|\chi|^2 \big) dB_1 - c dE_1 - 2\chi^* dD_1 - 2\chi dD_1^*
\end{aligned} \tag{2.11}
$$

To summarise, by explicit construction, we have shown that any solution of the equations of motion arising from this $D = 5$ action can be uplifted on an arbitrary $HK_4 \times S^1$ space via (2.3) and (2.4) to obtain exact solutions of type IIB supergravity. In other words we have identified the consistent KK reduction of type IIB supergravity on $HK_4 \times S^1$ to a universal $D = 5$ theory. In the next subsection we will argue that this $D = 5$ theory is consistent with the bosonic sector of $N = 4$ supergravity coupled to two vector multiplets.

2.2 $N = 4$ ungauged supergravity

We first recall some aspects of $N = 4$ ungauged supergravity coupled to n vector multiplets [26] (see also [15][16]). The global symmetry group of the theory is given by $SO(1,1) \times SO(5,n)$. The bosonic content includes a metric and $6 + n$ vector fields, $\mathcal{B}^0, \mathcal{B}^M$ with $M = 1, \ldots 5 + n$ transforming in the $(-1, \mathbf{1})$ and $(+1/2, \mathbf{5} + \mathbf{n})$ representation, where the first entry indicates the $SO(1,1)$ charge. In addition there are $1 + 5n$ scalar fields which parametrise the coset $SO(1,1) \times SO(5,n)/SO(5) \times SO(n)$. The scalar corresponding to the $SO(1,1)$ factor is described by a real scalar field Σ which is a singlet under $SO(5,n)$ and carries $SO(1,1)$ charge $-1/2$. The remaining $5n$ scalars are described by a coset representative $\mathcal{V}$ of $SO(5,n)/SO(5) \times SO(n)$ with zero $SO(1,1)$ charge. The bosonic action of $D = 5$ $N = 4$ supergravity can be written

$$\begin{aligned}\mathcal{L}^{N=4} = &R\text{vol}_5^{(E)} - \Sigma^2 M_{MN}\mathcal{H}^M \wedge *\mathcal{H}^N - \Sigma^{-4}\mathcal{H}^0 \wedge *\mathcal{H}^0 \\ &- 3\Sigma^{-2}d\Sigma \wedge *d\Sigma + \tfrac{1}{8}\text{tr}(dM \wedge *dM) \\ &+ \sqrt{2}\eta_{MN}\mathcal{B}^0 \wedge \mathcal{H}^M \wedge \mathcal{H}^N\end{aligned} \tag{2.12}$$

where $M \equiv \mathcal{V}^T\mathcal{V}$, $\mathcal{H}^0 \equiv d\mathcal{B}^0$, $\mathcal{H}^M \equiv d\mathcal{B}^M$ and η_{MN} is the invariant tensor of $SO(5,n)$.

On quite general grounds we can argue that our consistent truncation on $HK_4 \times S^1$ should be consistent with $N = 4$ supersymmetry. Firstly, we recall that the uplifted vacuum solution preserves $N = 4$ supersymmetry of the type IIB supergravity. Furthermore, the $SU(2)$ structure of the $HK_4 \times S^1$ factor, specified by the forms J, Ω, η that we used in our KK reduction, can be constructed from the type IIB Killing spinors. Now our KK reduction ansatz that we considered was the most general universal ansatz using just J, Ω, η. Thus, given that we have shown that our ansatz was a consistent KK reduction we expect it to be the bosonic part of an $N = 4$ theory in $D = 5$. An immediate check of this logic is provided by counting the degrees of freedom. In addition to the metric, our $D = 5$ theory has eleven real scalar degrees of freedom, six real vectors and two real two-forms. Since we can dualise the two-forms to vectors we have the bosonic matter content of $N = 4$ supergravity coupled to $n = 2$ vector multiplets.

In order to make the supersymmetry structure of our theory manifest, we now dualise the two-forms B_2 and C_2 into two vectors C_1' and B_1' by defining $H_3' = dB_2$ and $G_3' = dC_2$, and adding the term

$$\mathcal{L}' = C_1' \wedge dH_3' + B_1' \wedge dG_3' \tag{2.13}$$

to the Lagrangian $\mathcal{L}^{(E)}$ in (2.8). Integrating out H_3' and G_3', we find that H_3 and G_3 are now given by

$$\begin{aligned} H_3 &= -e^{-\frac{4}{3}(4U+V)+\phi} * G_2' \\ G_3 &= -e^{-\frac{4}{3}(4U+V)-\phi} * H_2' \end{aligned} \tag{2.14}$$

where we have defined

$$\begin{aligned} H_2' &= dB' - 2X_2 \\ G_2' &= dC_1' + 2Y_2 + aH_2' - 2aX_2 \end{aligned} \tag{2.15}$$

and X_2, Y_2 are given in (2.11). Substituting H_3', G_3' back into $\mathcal{L}^{(E)} + \mathcal{L}'$ we obtain a dual Lagrangian $\mathcal{L}^{\text{dual}}$ which contains eight vector fields. With a little further effort we can show that the topological Lagrangian simplifies considerably and in particular all dependence on the scalar fields drops out. Before writing this action we first introduce new scalar fields given by

$$\Sigma = e^{-\frac{2}{3}(U+V)}, \quad \varphi_1 = \tfrac{1}{\sqrt{2}}(\phi - 4U), \quad \varphi_2 = -\tfrac{1}{\sqrt{2}}(\phi + 4U)\,. \tag{2.16}$$

The dual Lagrangian can then be written as

$$\mathcal{L}^{\text{dual}} = R^{(E)}\text{vol}_5^{(E)} + \mathcal{L}_{\text{scalars}} + \mathcal{L}_{\text{vectors}} + \mathcal{L}_{\text{top}}^{\text{dual}} \tag{2.17}$$

where the scalar kinetic terms are given by

$$\begin{aligned} \mathcal{L}_{\text{scalars}} = &-3\Sigma^{-2} d\Sigma \wedge * d\Sigma - \tfrac{1}{2} d\varphi_1 \wedge * d\varphi_1 - \tfrac{1}{2} d\varphi_2 \wedge * d\varphi_2 \\ &- \tfrac{1}{2} e^{\sqrt{2}(\varphi_1 - \varphi_2)} da \wedge * da - 2e^{\sqrt{2}(\varphi_1 + \varphi_2)} K_1 \wedge * K_1 \\ &- e^{\sqrt{2}\varphi_1} G_1 \wedge * G_1 - 4e^{\sqrt{2}\varphi_1} N_1 \wedge * N_1^* \\ &- e^{\sqrt{2}\varphi_2} H_1 \wedge * H_1 - 4e^{\sqrt{2}\varphi_2} M_1 \wedge * M_1^*\,, \end{aligned} \tag{2.18}$$

the kinetic terms for the vectors are given by

$$\mathcal{L}_{\rm vectors} = -\tfrac{1}{2}\Sigma^{-4}F_2\wedge *F_2 - \Sigma^2\Big[K_2\wedge *K_2 + 4L_2\wedge *L_2^* + \tfrac{1}{2}e^{\sqrt{2}\varphi_2}H_2'\wedge *H_2' + \tfrac{1}{2}e^{\sqrt{2}\varphi_1}G_2'\wedge *G_2' + \tfrac{1}{2}e^{-\sqrt{2}\varphi_1}H_2\wedge *H_2 + \tfrac{1}{2}e^{-\sqrt{2}\varphi_2}G_2\wedge *G_2\Big] \qquad (2.19)$$

and the topological term is

$$\mathcal{L}_{\rm top}^{\rm dual} = -A_1\wedge\Big[dE_1\wedge dE_1 + 4dD_1\wedge dD_1^* - dB_1\wedge dC_1' - dC_1\wedge dB_1'\Big] \qquad (2.20)$$

We can now identify with the degrees of freedom of $N=4$ supergravity. For the scalars we see that Σ corresponds to the $\mathbb{R}\sim SO(1,1)$ factor in the scalar manifold. The remaining dilatons, φ_1, φ_2, and the axions a, b, c, h, ξ, χ, therefore parametrise the homogeneous space $SO(5,2)/(SO(5)\times SO(2))$. To make this manifest we find it convenient to resort to the solvable Lie algebra approach [27, 28]. According to this method, a parametrisation of the supergravity scalar manifold G/H can be obtained via the exponentiation of a suitable solvable subalgebra of the Lie algebra of G, including as many Cartan generators as dilatons, and as many positive root generators as axions that are contained in G/H. For $SO(5,2)/(SO(5)\times SO(2))$, the relevant ten-dimensional subalgebra of $so(7)$ is accordingly spanned by two Cartan generators H^1, H^2, and eight positive root generators T^i, $i=1,\ldots,8$, which, in the fundamental of $so(7)$, can be taken to be[3] [17]

$$\begin{aligned}
&\mathsf{H}^1=\sqrt{2}(E_{22}-E_{77}), && \mathsf{T}^1=E_{67}-E_{21}, && \mathsf{T}^4=E_{23}+E_{37}, && \mathsf{T}^7=E_{24}+E_{47},\\
&\mathsf{H}^2=\sqrt{2}(E_{11}-E_{66}), && \mathsf{T}^2=E_{17}-E_{26}, && \mathsf{T}^5=E_{14}+E_{46}, && \mathsf{T}^8=E_{25}+E_{57},\\
& && \mathsf{T}^3=E_{13}+E_{36}, && \mathsf{T}^6=E_{15}+E_{56},
\end{aligned} \qquad (2.21)$$

where E_{ij} denotes the 7×7 matrix with 1 in the i-th row and j-th column and 0 elsewhere.

We find the coset representative of $SO(5,2)/(SO(5)\times SO(2))$ to be given by

$$\mathcal{V} = e^{\frac{1}{2}(\varphi_1\mathsf{H}^1+\varphi_2\mathsf{H}^2)}e^{a\mathsf{T}^1}e^{(2h-bc-4\xi^*\chi-4\xi\chi^*)\mathsf{T}^2}e^{b\sqrt{2}\mathsf{T}^3}e^{c\sqrt{2}\mathsf{T}^4}e^{2\sqrt{2}{\rm Re}(\xi)\mathsf{T}^5}e^{2\sqrt{2}{\rm Im}(\xi)\mathsf{T}^6} \times e^{2\sqrt{2}{\rm Re}(\chi)\mathsf{T}^7}e^{2\sqrt{2}{\rm Im}(\chi)\mathsf{T}^8}\,. \qquad (2.22)$$

Note that in this basis we have $\mathcal{V}^T\eta\mathcal{V}=\eta$ with $\eta=E_{33}+E_{44}+E_{55}-E_{16}-E_{61}-$

[3]These generators close into the (solvable) Lie algebra with commutators specified in (3.12) of [17]. With $N=3$ there, we identify the positive root generators as $\mathsf{T}^1={E_2}^3$, $\mathsf{T}^2=V^{23}$, $\mathsf{T}^3=U_1^2$, $\mathsf{T}^4=U_1^3$, $\mathsf{T}^5=U_2^2$, $\mathsf{T}^6=U_3^2$, $\mathsf{T}^7=U_2^3$, $\mathsf{T}^8=U_3^3$. Our explicit realisation (2.21) of these generators follows from (3.31) of [17].

$E_{27} - E_{72}$. The Maurer-Cartan form $d\mathcal{V}\mathcal{V}^{-1}$ takes values in the solvable Lie algebra,

$$\begin{aligned} d\mathcal{V}\mathcal{V}^{-1} = &\tfrac{1}{2}d\varphi_1 \mathsf{H}^1 + \tfrac{1}{2}d\varphi_2 \mathsf{H}^2 + e^{\frac{\sqrt{2}}{2}(\varphi_1-\varphi_2)} da \mathsf{T}^1 + 2e^{\frac{\sqrt{2}}{2}(\varphi_1+\varphi_2)} K_1 \mathsf{T}^2 + \sqrt{2} e^{\frac{\sqrt{2}}{2}\varphi_2} H_1 \mathsf{T}^3 \\ &+\sqrt{2} e^{\frac{\sqrt{2}}{2}\varphi_1} G_1 \mathsf{T}^4 + 2\sqrt{2} e^{\frac{\sqrt{2}}{2}\varphi_2} \mathrm{Re}(M_1) \mathsf{T}^5 + 2\sqrt{2} e^{\frac{\sqrt{2}}{2}\varphi_2} \mathrm{Im}(M_1) \mathsf{T}^6 \\ &+2\sqrt{2} e^{\frac{\sqrt{2}}{2}\varphi_1} \mathrm{Re}(N_1) \mathsf{T}^7 + 2\sqrt{2} e^{\frac{\sqrt{2}}{2}\varphi_1} \mathrm{Im}(N_1) \mathsf{T}^8 \ , \end{aligned} \tag{2.23}$$

with coefficients along the generators (2.21) corresponding to the axion one-form field strengths defined in (2.5)–(2.7). Note that the transgression terms in these one-forms arise as a consequence of the non-trivial commutation relations among the positive root generators. Finally, we can construct the quadratic form

$$M = \mathcal{V}^T \mathcal{V} \ , \tag{2.24}$$

to bring the scalar kinetic terms (2.18) to the form

$$\mathcal{L}_{\text{scalars}} = -3\Sigma^{-2} d\Sigma \wedge *d\Sigma + \tfrac{1}{8}\mathrm{tr}(dM^{-1} \wedge *dM) \ . \tag{2.25}$$

exactly as in (2.12).

For the vectors we identify

$$\begin{aligned} \mathcal{B}_1^0 &= -\tfrac{1}{\sqrt{2}} A_1 \\ \mathcal{B}_1^M &= \{\tfrac{1}{\sqrt{2}} C_1',\ \tfrac{1}{\sqrt{2}} B_1',\ E_1,\ 2\mathrm{Re}(D_1),\ 2\mathrm{Im}(D_1),\ \tfrac{1}{\sqrt{2}} B_1,\ \tfrac{1}{\sqrt{2}} C_1\} \ , \end{aligned} \tag{2.26}$$

In particular, for our Chern-Simons term we then have

$$\mathcal{L}_{\text{top}}^{\text{dual}} = \sqrt{2}\eta_{MN} \mathcal{B}^0 \wedge \mathcal{H}_2^M \wedge \mathcal{H}_2^N \ . \tag{2.27}$$

and we have verified that the kinetic terms for the vectors given in (2.19) can be written as

$$\mathcal{L}_{\text{vectors}} = -\Sigma^2 M_{MN} \mathcal{H}^M \wedge *\mathcal{H}^N - \Sigma^{-4} \mathcal{H}^0 \wedge *\mathcal{H}^0 \tag{2.28}$$

as in (2.12). This completes our demonstration that we indeed have the bosonic action of $N = 4$ supergravity coupled to two vector multiplets.

3 Type IIB reduced on SE_5

We now turn our attention to reductions on SE_5 spaces. We begin by recalling the class of $AdS_5 \times SE_5$ solutions of type IIB supergravity given by

$$\begin{aligned} ds^2_{10} &= ds^2(AdS_5) + ds^2(SE_5) \\ F_{(5)} &= 4\mathrm{vol}(SE_5) + 4\mathrm{vol}(AdS_5) \end{aligned} \tag{3.1}$$

with $F_{(3)} = H_{(3)} = 0$ and constant dilaton and constant axion ($F_{(1)} = 0$). Generically these solutions preserve $N = 2$ supersymmetry (i.e. dual to $N = 1$ SCFTs in $d = 4$).

Now any SE_5 space has a globally defined one-form η, that is dual to the Reeb Killing vector, and so locally we can always write the metric on the SE_5 space as

$$ds^2(SE_5) = ds^2(KE_4) + \eta \otimes \eta$$

where $ds^2(KE_4)$ is a local Kähler-Einstein metric with positive curvature, normalised so that the Ricci tensor is six times the metric. On SE_5 there is also a globally defined two form J and a $(2,0)$ form Ω that locally define the Kähler and complex structures on $ds^2(KE_4)$ respectively. Note that they satisfy the *same* algebraic conditions as those associated with the $HK_4 \times S^1$ solution (2.1) and are given in appendix B. By contrast, however, they are no longer closed and instead satisfy

$$\begin{aligned} d\eta &= 2J \\ d\Omega &= 3i\eta \wedge \Omega \end{aligned} \tag{3.2}$$

The fact that $HK_4 \times S^1$ and SE_5 have globally defined $SU(2)$ structures specified by the forms J, Ω, η implies that the universal KK reduction on these spaces are very similar, as we shall see. For the SE_5 case the conditions (3.2) as well as the background five-form flux appearing in (3.1) will lead to a gauging of the $N = 4$ supergravity coupled to two vector multiplets that we saw for the $HK_4 \times S^1$ case in the last section.

3.1 The consistent Kaluza-Klein reduction on SE_5

Our KK ansatz for the metric of type IIB supergravity is given by

$$ds^2_{10} = e^{-\frac{2}{3}(4U+V)} ds^2_{(E)} + e^{2U} ds^2(KE_4) + e^{2V}(\eta + A_1) \otimes (\eta + A_1) \tag{3.3}$$

where again $ds^2_{(E)}$ is an arbitrary metric on an external five-dimensional space-time, U and V are scalar fields and A_1 is a one-form defined on the external five-dimensional space. The ansatz for the form field strengths is given by

$$
\begin{aligned}
F_{(5)} =& 4e^{-\frac{8}{3}(4U+V)}\mathrm{vol}_5^{(E)} + e^{-\frac{4}{3}(U+V)} * K_2 \wedge J + K_1 \wedge J \wedge J \\
&+ \left[2e^Z J \wedge J - 2e^{-8U} * K_1 + K_2 \wedge J\right] \wedge (\eta + A_1) \\
&+ \left[e^{-\frac{4}{3}(U+V)} * L_2 \wedge \Omega + L_2 \wedge \Omega \wedge (\eta + A_1) + c.c.\right] \\
F_{(3)} =& G_3 + G_2 \wedge (\eta + A_1) + G_1 \wedge J + [N_1 \wedge \Omega + N_0 \Omega \wedge (\eta + A_1) + c.c.] \\
H_{(3)} =& H_3 + H_2 \wedge (\eta + A_1) + H_1 \wedge J + [M_1 \wedge \Omega + M_0 \Omega \wedge (\eta + A_1) + c.c.] \\
C_{(0)} =& a \\
\Phi =& \phi
\end{aligned}
\tag{3.4}
$$

Here, $\mathrm{vol}_5^{(E)}$ and $*$ are the volume form and Hodge dual corresponding to the five-dimensional metric $ds^2_{(E)}$ in (3.3), Z, a, ϕ, are real scalars, M_0, N_0 complex scalars, G_3, H_3, G_2, H_2, K_2, K_1 real forms, and L_2, M_1, N_1, complex forms, all of them defined on the external five-dimensional spacetime. We have also ensured the self duality of the five-form $F_{(5)}$ with respect to the metric (3.3). Note that we can also add the terms $G_0 J \wedge (\eta + A_1)$ to $F_{(3)}$ and $H_0 J \wedge (\eta + A_1)$ to $H_{(3)}$, where G_0 and H_0 are real scalars, but the type IIB equations imply that $G_0 = H_0 = 0$. This is as in the $HK_4 \times S^1$ case, but, by contrast, note that we now must include the scalar fields M_0, N_0 and Z.

We now substitute into the equations of motion and Bianchi identities of type IIB supergravity that we have given in appendix A. The calculations are rather involved, so we will simply summarise the main results here, referring to appendix B for some details. We find that the physical degrees of freedom are 7 real scalars U, V, ϕ, a, b, c, h and 2 complex scalars ξ, χ; 4 one-form potentials A_1, B_1, C_1, E_1; 2 two-form potentials B_2, C_2 plus the complex two-form L_2. This is exactly the same field content that arose in the reduction on $HK_4 \times S^1$. In particular, the extra scalars M_0, N_0 and Z that we introduced in (3.4) are not independent degrees of freedom as we shall see in detail below. Furthermore, the field L_2 should also be regarded as a potential, satisfying a first-order, self-duality-type equation of motion (see the second equation in (B.5)).

In more detail we find that

$$\begin{aligned}
H_3 &= dB_2 + \tfrac{1}{2}(db - 2B_1) \wedge F_2 \\
H_2 &= dB_1 \\
H_1 &= db - 2B_1 \\
M_0 &= 3i\xi \\
M_1 &= D\xi
\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}
G_3 &= dC_2 - a dB_2 + \tfrac{1}{2}(dc - 2C_1 - a db + 2aB_1) \wedge F_2 \\
G_2 &= dC_1 - a dB_1 \\
G_1 &= dc - 2C_1 - a db + 2aB_1 \\
N_0 &= 3i(\chi - a\xi) \\
N_1 &= D\chi - a D\xi
\end{aligned} \tag{3.6}$$

where $F_2 = dA_1$, $D\xi \equiv d\xi - 3iA_1\chi$ and $D\chi \equiv d\chi - 3iA_1\chi$. This gauging can be traced back to the fact that these scalar fields enter the KK ansatz (3.4) with Ω and that $d\Omega = 3i\eta \wedge \Omega$. In addition, after fixing an integration constant, we find that the scalar field Z is fixed by the scalars χ, ξ:

$$e^Z = 1 + 3i(\xi^* \chi - \xi \chi^*) \tag{3.7}$$

and that

$$\begin{aligned}
K_2 &= dE_1 + \tfrac{1}{2}(db - 2B_1) \wedge (dc - 2C_1) \\
K_1 &= dh - 2E_1 - 2A_1 + \xi^* D\chi + \xi D\chi^* - \chi D\xi^* - \chi^* D\xi
\end{aligned} \tag{3.8}$$

We also find that the equations of motion for all of the fields can be obtained by varying a $D = 5$ action with Lagrangian given by

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{top}} \tag{3.9}$$

where the kinetic and scalar potential terms are given by

$$\begin{aligned}
\mathcal{L}^{(E)}_{\text{kin}} = R^{(E)}\ \text{vol}^{(E)}_5 &- \tfrac{28}{3} dU \wedge *dU - \tfrac{8}{3} dU \wedge *dV - \tfrac{4}{3} dV \wedge *dV - \tfrac{1}{2} e^{2\phi} da \wedge *da \\
&- \tfrac{1}{2} d\phi \wedge *d\phi - 4e^{-4U-\phi} M_1 \wedge *M_1^* - 4e^{-4U+\phi} N_1 \wedge *N_1^* - 2e^{-8U} K_1 \wedge *K_1 \\
&- e^{-4U-\phi} H_1 \wedge *H_1 - e^{-4U+\phi} G_1 \wedge *G_1 - \tfrac{1}{2} e^{\frac{8}{3}(U+V)} F_2 \wedge *F_2 \\
&- e^{-\frac{4}{3}(U+V)} K_2 \wedge *K_2 - 4e^{-\frac{4}{3}(U+V)} L_2 \wedge *L_2^* - \tfrac{1}{2} e^{\frac{4}{3}(2U-V)-\phi} H_2 \wedge *H_2 \\
&- \tfrac{1}{2} e^{\frac{4}{3}(2U-V)+\phi} G_2 \wedge *G_2 - \tfrac{1}{2} e^{\frac{4}{3}(4U+V)-\phi} H_3 \wedge *H_3 - \tfrac{1}{2} e^{\frac{4}{3}(4U+V)+\phi} G_3 \wedge *G_3
\end{aligned} \tag{3.10}$$

and

$$\mathcal{L}^{(E)}_{\rm pot} = \Big[24e^{-\frac{2}{3}(7U+V)} - 4e^{\frac{4}{3}(-5U+V)} - 8e^{-\frac{8}{3}(4U+V)}\left[1+\tfrac{i}{3}(M_0^*N_0 - M_0N_0^*)\right]^2$$
$$-4e^{-\frac{4}{3}(5U+2V)-\phi}|M_0|^2 - 4e^{-\frac{4}{3}(5U+2V)+\phi}|N_0|^2\Big]{\rm vol}^{(E)}_5 \ , \qquad (3.11)$$

while the topological terms are given by the intimidating expression

$$\begin{aligned}
\mathcal{L}_{\rm top} = & -A_1\wedge K_2\wedge K_2 - (dh-2E_1-2A_1)\wedge[dB_2\wedge(dc-2C_1)+(db-2B_1)\wedge dC_2]\\
&+A_1\wedge(dh-2E_1)\wedge[(db-2B_1)\wedge dC_1 - dB_1\wedge(dc-2C_1)]\\
&+2A_1\wedge(dE_1)\wedge(db-2B_1)\wedge(dc-2C_1)\\
&+A_1\wedge(db-2B_1)\wedge(dc-2C_1)\wedge F_2\\
&+\tfrac{i}{3}A_1\wedge(M_0^*N_1 - M_0N_1^* - N_0M_1^* + N_0^*M_1)\wedge(H_1\wedge G_2 + G_1\wedge H_2)\\
&+\tfrac{2i}{3}A_1\wedge(N_0N_1^* - N_0^*N_1)\wedge H_1\wedge H_2 + \tfrac{2i}{3}A_1\wedge(M_0M_1^* - M_0^*M_1)\wedge G_1\wedge G_2\\
&+\Big[\tfrac{4i}{3}\big(-\tfrac{1}{2}L_2^*\wedge DL_2 + N_0L_2^*\wedge H_3 - M_0L_2^*\wedge G_3 - L_2^*\wedge N_1\wedge H_2 + L_2^*\wedge M_1\wedge G_2\big)+c.c.\Big]\\
&-4C_2\wedge dB_2 - \tfrac{4i}{3}C_2\wedge G_2\wedge(M_0M_1^* - M_0^*M_1)\\
&-\tfrac{2i}{3}C_2\wedge H_2\wedge(M_0^*N_1 - M_0N_1^* - N_0M_1^* + N_0^*M_1)\\
&-\tfrac{2i}{3}B_2\wedge(G_2 - aH_2)\wedge(M_0^*N_1 - M_0N_1^* - N_0M_1^* + N_0^*M_1)\\
&-\tfrac{4i}{3}B_2\wedge\left[H_2\wedge(N_0N_1^* - N_0^*N_1) - aG_2\wedge(M_0M_1^* - M_0^*M_1)\right]\\
&+\tfrac{4}{9}A_1\wedge(M_0G_2 - N_0H_2)\wedge(M_0^*G_2 - N_0^*H_2)\\
&+\left[\tfrac{2}{9}(M_1\wedge G_2 - N_1\wedge H_2)\wedge(M_0^*G_2 - N_0^*H_2) + c.c.\right] \ . \qquad (3.12)
\end{aligned}$$

3.2 $N = 4$ gauged supergravity

Our KK reduction of type IIB supergravity was based on the most general ansatz using the $SU(2)$ structure (J, Ω, η) on the SE_5 space. For reasons similar to that discussed in section 3.2 for the universal KK reduction on $HK_4 \times S^1$ we expect to obtain a supersymmetric theory. Indeed we have already noted that the field content of our KK reduction on SE_5 is identical to that of the universal KK reduction on $HK_4 \times S^1$ and hence to that of $N = 4$ supergravity coupled to two vector multiplets. A key difference with the $HK_4 \times S^1$ case is that the $SU(2)$ structure forms are no longer closed (3.2). This difference can be viewed as the addition of "geometric fluxes". Another difference is the extra five-form flux that we have in (3.4) compared with (2.4). On rather general grounds [13] it is expected that these flux contributions lead to a gauging of the $N = 4$ supergravity theory.

A general description of $N = 4$ gauged supergravity coupled to vector multiplets is presented in [16], which uses the embedding tensor formalism of [29] (for a review see [30]). The gauging is described by promoting a subgroup $G_0 \subset G$ of the global non-abelian duality symmetry group of the ungauged theory, G (in our case $SO(1,1)\times SO(5,2)$), to a local symmetry. This requires that the ordinary derivatives get replaced by covariant derivatives via

$$d \to d - g\mathcal{B}_1^{\mathcal{M}} X_{\mathcal{M}} \equiv d - g\mathcal{B}_1^{\mathcal{M}} \Theta_{\mathcal{M}}{}^{\alpha} t_{\alpha} \tag{3.13}$$

where g is the gauge coupling constant, $\mathcal{B}_1^{\mathcal{M}} \equiv (\mathcal{B}_1^0, \mathcal{B}_1^M)$ are the vector gauge fields, t_α are the generators of G and the explicit embedding of G_0 in G is given by an embedding tensor $\Theta_{\mathcal{M}}{}^{\alpha}$. In addition to the vector gauge fields it is necessary to also include two-form gauge fields $B_2^{\mathcal{M}}$ as off-shell degrees of freedom. As usual, the gauging leads to a scalar potential that is determined by the embedding tensor.

We now return to our $D = 5$ theory (3.9)-(3.12) obtained from KK reduction on SE_5. By again introducing

$$\Sigma = e^{-\frac{2}{3}(U+V)}\,, \quad \varphi_1 = \tfrac{\sqrt{2}}{2}(\phi - 4U)\,, \quad \varphi_2 = -\tfrac{\sqrt{2}}{2}(\phi + 4U)\,. \tag{3.14}$$

we find that the Lagrangian can be written as

$$\mathcal{L}^{(E)} = R^{(E)}\mathrm{vol}_5^{(E)} + \mathcal{L}_{\mathrm{scalars}} + \mathcal{L}_{\mathrm{vectors/2\text{-}forms}} + \mathcal{L}_{\mathrm{pot}}^{(E)} + \mathcal{L}_{\mathrm{top}} \tag{3.15}$$

where the scalar kinetic terms are given by

$$\begin{aligned}\mathcal{L}_{\mathrm{scalars}} = & -3\Sigma^{-2}d\Sigma\wedge *d\Sigma - \tfrac{1}{2}d\varphi_1\wedge *d\varphi_1 - \tfrac{1}{2}d\varphi_2\wedge *d\varphi_2\\ & -\tfrac{1}{2}e^{\sqrt{2}(\varphi_1-\varphi_2)}da\wedge *da - 2e^{\sqrt{2}(\varphi_1+\varphi_2)}K_1\wedge *K_1\\ & -e^{\sqrt{2}\varphi_1}G_1\wedge *G_1 - 4e^{\sqrt{2}\varphi_1}N_1\wedge *N_1^*\\ & -e^{\sqrt{2}\varphi_2}H_1\wedge *H_1 - 4e^{\sqrt{2}\varphi_2}M_1\wedge *M_1^*\,.\end{aligned} \tag{3.16}$$

the kinetic terms for the vectors and two-forms are given by

$$\begin{aligned}\mathcal{L}_{\mathrm{vectors/2\text{-}forms}} = & -\tfrac{1}{2}\Sigma^{-4}F_2\wedge *F_2 - \Sigma^2\Big[K_2\wedge *K_2 + 4L_2\wedge *L_2^* + \tfrac{1}{2}e^{\sqrt{2}\varphi_2}H_3\wedge *H_3\\ & +\tfrac{1}{2}e^{\sqrt{2}\varphi_1}G_3\wedge *G_3 + \tfrac{1}{2}e^{-\sqrt{2}\varphi_1}H_2\wedge *H_2 + \tfrac{1}{2}e^{-\sqrt{2}\varphi_2}G_2\wedge *G_2\Big]\end{aligned} \tag{3.17}$$

and the potential (3.11) can be rewritten as

$$\begin{aligned}\mathcal{L}_{\mathrm{pot}}^{(E)} = \Big[& 24\Sigma e^{\frac{\sqrt{2}}{2}(\varphi_1+\varphi_2)} - 4\Sigma^{-2}e^{\sqrt{2}(\varphi_1+\varphi_2)}\\ & -4\Sigma^4\big[2e^{\sqrt{2}(\varphi_1+\varphi_2)}\big(1+3i(\xi^*\chi-\xi\chi^*)\big)^2 + 9e^{\sqrt{2}\varphi_1}|\chi - a\xi|^2 + 9e^{\sqrt{2}\varphi_2}|\xi|^2\big]\Big]\mathrm{vol}_5^{(E)}\,,\end{aligned} \tag{3.18}$$

We observe that the general structure of all these terms is consistent with the general form of the corresponding terms in the $N = 4$ gauged supergravity action given in [16]. Furthermore, noting that the covariant derivative acting on Σ in (3.16) is simply the ordinary covariant derivative, and comparing with (3.13) we immediately conclude that the gauging does not lie within the $SO(1,1)$ factor but just within the $SO(5,2)$ factor (and hence is in the class considered by [15]).

By analysing the way in which the vector fields are entering the scalar derivatives in (3.5), (3.6) and (3.8) and comparing with (3.13) it is straightforward to deduce the precise gauged subgroup of $SO(5,2)$. We find that

$$\begin{aligned}\mathcal{B}^0 X_0 + \mathcal{B}^M X_M &= -\sqrt{2}\mathcal{B}_1^0(3\mathsf{R} + 2\mathsf{S}^2) + 2\mathcal{B}_1^3\ \mathsf{S}^2 + 2\mathcal{B}_1^6\ \mathsf{S}^3 + 2\mathcal{B}_1^7\ \mathsf{S}^4 \\ &= A_1(3\mathsf{R} + 2\mathsf{S}^2) + 2E_1\ \mathsf{S}^2 + \sqrt{2}B_1\ \mathsf{S}^3 + \sqrt{2}C_1\ \mathsf{S}^4\ , \end{aligned} \tag{3.19}$$

where $\mathsf{S}^i \equiv [\mathsf{T}^i]^T$, $i = 1, \ldots, 8$, and $\mathsf{R} \equiv E_{45} - E_{54}$ are generators of $SO(5,2)$ supplementing those in (2.21). The only non-vanishing commutators among the generators of the gauge algebra is $[\mathsf{S}^3, \mathsf{S}^4] = -\mathsf{S}^2$ and hence we see that our $D = 5$ theory corresponds to a gauging of an $H_3 \times U(1)$ subgroup of $SO(5,2)$, where H_3 is the three-dimensional Heisenberg group.

It would be satisfying to see that the rest of our Lagrangian is in accord with [15][16], especially our Chern-Simons terms (3.12), but we leave that to future work.

3.3 AdS_5 vacua

By analysing the scalar potential (3.11) appearing in our $D = 5$ $N = 4$ gauge supergravity theory we find that there are both supersymmetric and non-supersymmetric AdS_5 vacua. We first discuss the former and then the latter.

3.3.1 The supersymmetric AdS_5 vacuum

Setting $U = V = \xi = \chi = 0$ and allowing for arbitrary constant axion a and dilaton ϕ we obtain an AdS_5 vacuum solution with unit radius (and all other fields trivial). This solution uplifts to the class of supersymmetric $AdS_5 \times SE_5$ solutions of type IIB given in (3.1). As a ten-dimensional solution this preserves eight supercharges and hence as a five dimensional solution it spontaneously partially breaks the $N = 4$ supersymmetry to $N = 2$.

We can determine the masses of the different fields in this vacuum. For the scalars

ϕ, a, U, V, ξ, χ, we employ

$$\begin{aligned} U &= \tfrac{1}{2}u + \tfrac{3}{4}v \\ V &= -2u + \tfrac{3}{4}v \ . \end{aligned} \tag{3.20}$$

and

$$\begin{aligned} \xi &= \tilde{\xi} + i\tilde{\chi} \\ \chi &= i\tilde{\xi} + \tilde{\chi} \ . \end{aligned} \tag{3.21}$$

to obtain the masses:

$$m_\phi^2 = 0 \ , \ \ m_a^2 = 0 \ , \ \ m_u^2 = 12 \ , \ \ m_v^2 = 32 \ , \ \ m_{\tilde{\xi}}^2 = -3 \ , \ \ m_{\tilde{\chi}}^2 = 21 \ , \tag{3.22}$$

Observe that v is a breathing mode which controls the overall volume of the SE_5 space in (3.3), while u is a volume preserving squashing mode.

Turning now to the contributions from the vectors A_1, E_1, B_1, C_1 and the scalars h, b, c, we find that the transformation

$$\begin{aligned} A_1 &= \tilde{A}_1 + 2\tilde{E}_1 \\ E_1 &= -\tilde{A}_1 + \tilde{E}_1 \end{aligned} \tag{3.23}$$

leads to the following terms in the Lagrangian

$$\begin{aligned} \mathcal{L}_2 = &-\tfrac{3}{2}d\tilde{A}_1 \wedge *d\tilde{A}_1 - 3d\tilde{E}_1 \wedge *d\tilde{E}_1 - 2(dh - 6\tilde{E}_1) \wedge *(dh - 6\tilde{E}_1) \\ &-\tfrac{1}{2}dB_1 \wedge *dB_1 - \tfrac{1}{2}dC_1 \wedge *dC_1 - (db - 2B_1) \wedge *(db - 2B_1) \\ &-(dc - 2C_1) \wedge *(dc - 2C_1) \ . \end{aligned} \tag{3.24}$$

We now see that $\tilde{A}_1$, $\tilde{E}_1$, B_1 and C_1 are massive vectors with masses given by

$$m_{\tilde{A}_1}^2 = 0 \ , \qquad m_{\tilde{E}_1}^2 = 24 \ , \qquad m_{B_1}^2 = m_{C_1}^2 = 8 \ , \tag{3.25}$$

and that the scalars h, b and c are just the associated Stückelberg fields.

Next consider the two-forms B_2 and C_2, whose relevant contributions are given by

$$\mathcal{L}_3 = -\tfrac{1}{2}dB_2 \wedge *dB_2 - \tfrac{1}{2}dC_2 \wedge *dC_2 - 4C_2 \wedge dB_2 \ , \tag{3.26}$$

they combine to describe a massive two-form with mass

$$m_{C_2}^2 = 16 \ , \tag{3.27}$$

(see *e.g.* [31]). Finally, from the contribution

$$\mathcal{L}_4 = -\tfrac{2i}{3} L_2^* \wedge dL_2 + \tfrac{2i}{3} L_2 \wedge dL_2^* - 4 L_2 \wedge * L_2^* \tag{3.28}$$

we see that L_2 is a complex two-form satisfying a self-duality equation [32] has the same degrees of freedom as a massive real two-form[4] with

$$m_{L_2}^2 = 9 \tag{3.29}$$

To conclude, we quote here the scaling dimensions of the operators dual to the supergravity fields. Using the expressions

$$\Delta = 2 \pm \tfrac{1}{2}\sqrt{(4-2p)^2 + 4m^2}\ , \tag{3.30}$$

for four-dimensional operators dual to supergravity p-forms (subject to second-order equations of motion), and

$$\Delta = 2 + |m| \tag{3.31}$$

for the operator dual to a first-order two-form, we find

$$\Delta_\phi = 4\ ,\quad \Delta_a = 4\ ,\quad \Delta_u = 6\ ,\quad \Delta_v = 8\ ,\quad \Delta_{\tilde{\xi}} = 3\ ,\quad \Delta_{\tilde{\chi}} = 7\ , \tag{3.32}$$

for the operators dual to the scalars,

$$\Delta_{\tilde{A}_1} = 3\ , \qquad \Delta_{\tilde{E}_1} = 7\ , \qquad \Delta_{B_1} = \Delta_{C_1} = 5\ , \tag{3.33}$$

for the operators dual to the vectors, and

$$\Delta_{C_2} = 6\ , \qquad \Delta_{L_2} = 5 \tag{3.34}$$

for the operators dual to the two-forms.

These modes should form the bosonic fields of unitary irreducible representations of $SU(2,2|1)$. The KK modes we have kept are present for any SE_5 space and so, in particular, we can consider the special case $T^{1,1}$ for which the supermultiplet structure was analysed in detail in [33, 34]. We deduce that the metric and the vector $\tilde{A}_1$ form a massless graviton multiplet, the fields $\tilde{E}_1$, u, v and $\tilde{\chi}$ fill out a long vector multiplet, the fields ξ, a, ϕ form a hypermultiplet and finally, B_1, C_1, C_2 and L_2 form a semi-long massive gravitino multiplet.

It is also interesting to consider the special case of S^5. The $N = 8$ KK spectrum was computed in [35] and the modes were arranged in supermultiplets of $SU(2,2|4)$,

[4] To see this simply write L_2 as a real and an imaginary two form an observe that either one can be considered a Lagrange multiplier and eliminated.

in [36]. The various fields of our $D = 5$ theory can be identified with those presented in figures 1, 2 and 3 of [35]. Specifically, the metric, the scalars ϕ, a, $\tilde{\xi}$ and the vector $\tilde{A}_1$ belong to the supermultiplet with $p = 2$ (following the notation of [36]), namely, the $N = 8$ $SO(6)$ gauged supergravity multiplet. Similarly the vectors B_1, C_1, the two-forms C_2, L_2 belong to the supermultiplet with $p = 3$ and the scalars v, u, $\tilde{\chi}$, the vector $\tilde{E}_1$ belong to the supermultiplet with $p = 4$ (the breathing mode supermultiplet).

3.3.2 The Romans AdS_5 vacuum

The theory admits another AdS_5 vacuum solution where

$$e^{4U} = e^{-4V} = \frac{2}{3}, \quad \xi = \frac{1}{\sqrt{12}} e^{\phi/2} e^{i\theta}, \quad \chi - a\xi = ie^{-\phi}\xi \tag{3.35}$$

with arbitrary axion a and dilaton ϕ and θ is an arbitrary constant phase. The AdS_5 radius is $2\sqrt{2}/3$. This solution can be uplifted to a class of solutions that were first found by Romans in [19], generalising analogous solutions constructed in $D = 11$ supergravity in [20][21]. For the special case when the $SE_5 = S^5$ it is expected that this solution is unstable [22].

3.4 Further truncations

There are various additional truncations of the fields appearing in the ansatz (3.3), (3.4) that are consistent with the type IIB equations of motion. Let us discuss several of them and in particular make contact with some other works in the literature. In particular we will recover the truncations of [37] which helped to motivate the work of [12] and of this paper. Note that some cases that we discuss below can be combined. It is notationally convenient to label some of the forms as G_i, H_i, with $i = 1, 2, 3$ and M_a, N_a with $a = 0, 1$.

3.4.1 Self-dual five-form, dilaton and axion

It is consistent to truncate IIB supergravity itself to just the ten-dimensional metric, self-dual five-form $F_{(5)}$, dilaton Φ and axion $C_{(0)}$, by setting $H_{(3)} = F_{(3)} = 0$. It is also consistent to further truncate to just the ten-dimensional metric and self-dual five-form by further setting $\Phi = C_{(0)} = 0$. Accordingly, it is consistent to truncate all of the modes coming from $H_{(3)}$, $F_{(3)}$ by setting $H_i = G_i = M_a = N_a = 0$. It is also then consistent to further set $\phi = a = 0$.

3.4.2 NS sector

It is also consistent with the type IIB equations of motion to set all of the Ramond-Ramond fields to zero, $F_{(5)} = F_{(3)} = F_{(1)} = 0$. We can therefore set $e^Z = K_1 = K_2 = L_2 = G_i = N_a = a = 0$. Since this would be a universal reduction of type I supergravity on SE_5 incorporating the breathing mode, it seems quite plausible that the resulting theory should be the bosonic part of an $N = 2$ gauged supergravity theory. Indeed the truncated theory contains a metric, two vectors A_1, B_1, a two-form B_2 and four real scalars U, V, ϕ, b, and a complex scalar ξ which could comprise the bosonic part of a gravity multiplet, one vector multiplet, one tensor mutliplet and a single hypermultiplet. Note that this truncated $D = 5$ theory will no longer have an AdS_5 vacuum solution.

3.4.3 No R-charged fields

It is consistent to set all of the fields carrying non-zero R-charge to zero: $L_2 = M_a = N_a = 0$. Recall that these are the fields appearing with Ω in (3.4).

3.4.4 Minimal $N = 2$ gauged supergravity in $D = 5$

We can recover the KK reduction to minimal $D = 5$ $N = 2$ gauge supergravity of [1] (see also [38]) by setting $U = V = e^Z = K_1 = L_2 = G_i = H_i = M_a = N_a = 0$ and $K_2 = -F_2$. In fact our equations of motion reduce to (2.15), (2.16) of [1] with $F_2^{\text{here}} = (1/3)F_2^{\text{there}}$.

3.4.5 The truncations of [37]

Two consistent truncations of type IIB supergravity on SE_5 spaces were studied in [37] and both can be simply obtained from our results.

Firstly, if we set $e^Z = L_2 = K_1 = K_2 = H_3 = G_i = M_a = N_a = A_1 = a = 0$ we obtain the the truncation discussed in appendix D.1 of [37]. (We can identify $H_2 = F_2^{\text{there}}$ and $H_1 = -2A_1^{\text{there}}$.

Secondly, we can also set $e^Z = L_2 = G_i = H_i = M_a = N_a = a = \phi = 0$ to obtain the truncation discussed appendix D.2 of [37]. (We can identify $F_2 = \mathcal{F} = d\mathcal{A}$, $K_1 = 2\mathbf{A}$, $K_2 = -\mathbb{F} = -\mathcal{F} - d\mathbf{A}$ to find agreement after taking into account a different convention for the $D = 5$ orientation.)

3.4.6 Gravity and scalars

It is also consistent to set $A_1 = K_2 = K_1 = L_2 = H_i = G_i = e^Z = M_a = N_a = 0$ leaving only the metric and the scalars U, V, ϕ, a. It is consistent to then further set $\phi = a = 0$ and then $U = V$. The latter truncation was discussed in [39] [40] in the context of IIB reductions on S^5 (who also considered the addition of some other fields).

3.4.7 No $p = 3$ sector

At the end of section 3.3.1, for the special case when $SE_5 = S^5$, we argued that the modes we have kept arise from the $p = 2, 3$ and $p = 4$ sectors in the notation of [36]. Interestingly, for any SE_5, it is possible to set all of the fields corresponding to the $p = 3$ sector to zero, namely $H_i = G_i = L_2 = 0$, leaving only the $p = 2$ and $p = 4$ sectors. Along with the metric, this truncated theory contains five real scalars U, V, ϕ, a, h, two complex scalars ξ, χ and two one-forms A_1, E_1. It would be interesting to know if this theory is the bosonic part of an $N = 2$ gauged supergravity coupled to a vector multiplet and two hypermultiplets.

Having truncated out the $p = 3$ sector for general SE_5, it is consistent to further set $a = \phi = 0$ while setting one of the scalars to be proportional to the other: $\chi = i\xi$. (From (3.21) we see that this is tantamount to truncating the $p = 4$ charged scalar $\tilde{\chi}$ with mass $m^2_{\tilde{\chi}} = 21$). Along with the metric, this truncated theory contains three real scalars U, V, h, one complex scalar ξ and two one-forms A_1, E_1. This theory has a chance to be the bosonic part an $N = 2$ gauged supergravity coupled to a vector multiplet and a single hypermultiplet.

3.4.8 The truncation of [41]

For a general SE_5, having truncated out the $p = 3$ sector ($H_i = G_i = L_2 = 0$), the dilaton and axion ($a = \phi = 0$), and one of the complex scalars ($\chi = i\xi$), it is consistent to further set

$$e^{4U} = e^{-4V} = 1 - 4|\xi|^2 \tag{3.36}$$

while also truncating the massive vector $\tilde{E}_1$ defined in (3.23) by setting $K_2 = -F_2$ and $h = 0$. The resulting theory contains the metric, a massless vector A_1, and a charged scalar ξ with mass $m^2_\xi = -3$ in the supersymmetric AdS_5 vacuum. In fact we precisely recover the truncation first discussed in [41] in the context of holographic superconductivity (we should set $A^{\text{here}} = \frac{2}{3}A^{\text{there}}$, $L^{\text{there}} = 1$ and $\xi = \frac{1}{2}e^{i\theta}\tanh\frac{\eta}{2}$). Note that for the special case when $SE_5 = S^5$ the fields kept in this truncation all arise in the $p = 2$ sector and hence can be obtained as a truncation of $N = 8$ $D = 5$ $SO(6)$ gauged supergravity.

This Type IIB truncation has a direct analogue in $D = 11$ supergravity reduced on SE_7 which was presented in [42] building on [12][43].

4 Final Comments

We conclude with some comments on type IIB reductions for the special case when $SE_5 = S^5$ and then on $D = 11$ reductions on seven-dimensional tri-Sasaki manifolds.

The spectrum of type IIB supergravity on S^5 was computed in [35] and the modes were arranged in supermultiplets of $SU(2,2|4)$ in [36]. We have already noted at the end of section 3.3.1 that the modes that we have kept in our consistent KK reduction belong to the supermultiplets with $p = 2, 3$ and 4 (following the notation of [36]). We have also seen in section 3.4.7 that is possible to truncate out the modes arising in the $p = 3$ sector consistently and possibly consistent with $N = 2$ supersymmetry.

In [12] it was conjectured that there might be a consistent truncation of type IIB on S^5 to the full massless graviton multiplet of the $p = 2$ sector combined with the full breathing mode multiplet of the $p = 4$ sector and consistent with $N = 8$ supersymmetry. The bosonic fields of the $p = 2$ multiplet consist of the metric, scalars in the $\mathbf{1_C}$, $\mathbf{10_C}$, $\mathbf{20}$, vectors in the $\mathbf{15}$ and a two-form in the $\mathbf{6_C}$ of the $SO(6)$ R-symmetry group, and correspond to the fields of maximal $SO(6)$ gauged supergravity. On the other hand the $p = 4$ multiplet has bosonic field content consisting of a massive graviton in the $\mathbf{20}$, scalars in the $\mathbf{105}$, $\mathbf{126_C}$, $\mathbf{20_C}$, $\mathbf{84}$, $\mathbf{10_C}$, $\mathbf{1}$, vectors in the $\mathbf{175}$, $\mathbf{64_C}$, $\mathbf{15}$ and two-forms in the $\mathbf{50_C}$, $\mathbf{45_C}$, $\mathbf{6_C}$. Note that the massive complex two-forms satisfy self-duality equations and hence have six real degrees of freedom [32] and also that the singlet scalar corresponds to the breathing mode.

In light of the results presented in this paper, where for the special case of $SE_5 = S^5$ we included modes in the $p = 3$ sector, we might expect that there is a truncation of type IIB on S^5 to an $N = 8$ theory that keeps the $p = 2, 4$ and also the $p = 3$ multiplet, whose bosonic content consists of a massive graviton in $\mathbf{6}$, scalars in the $\mathbf{50}$, $\mathbf{45_C}$, $\mathbf{6_C}$, vectors in the $\mathbf{64}$, $\mathbf{15_C}$ and two-forms in the $\mathbf{20_C}$, $\mathbf{10_C}$, $\mathbf{1_C}$. Going further one might conjecture that one could truncate the $p = 3$ sector of this conjectured theory to obtain the conjectured theory of [12] with the $p = 2$ and $p = 4$ sectors. The existence of both massive and massless gravitons combined with the $N = 8$ supersymmetry in these conjectured theories necessarily means that they would have to be very exotic.

Consistent KK reductions of $D = 11$ supergravity on SE_7 spaces, corresponding to $AdS_4 \times SE_7$ solutions preserving $N = 2$ supersymmetry, were presented in [12] and it was shown that the $D = 4$ reduced theory also preserves $N = 2$ supersymmetry. Similar reductions on manifolds with weak G_2 holonomy, M_7, corresponding to $AdS_4 \times M_7$ solutions preserving $N = 1$ supersymmetry, were also found and it was shown that the $D = 4$ reduced theory preserves $N = 1$ supersymmetry. It was conjectured in [12] that the analogous reduction on seven dimensional tri-Sasaki manifolds, T_7, corresponding to $AdS_4 \times T_7$ solutions preserving $N = 3$ supersymmetry, would give rise to a $D = 4$ reduced theory preserving $N = 3$ supersymmetry. However, in light of the results presented in this paper, we expect that this KK reduction will give rise to a gauged supergravity theory with $N = 4$ supersymmetry[5], with an AdS_4 vacuum solution that will spontaneously partially break the supersymmetry from $N = 4$ to $N = 3$. To see this, recall [45] that the tri-Sasaki space T_7 has a globally defined $SU(2)$ structure, specified by three two-forms, J^a, and three one-forms, η^a, satisfying $d\eta^a = 2(J^a - \epsilon^{abc}\eta^b \wedge \eta^c)$ (locally T^7 is an S^3 bundle over a four-dimensional quaternionic Kähler space). The supersymmetry and field content of the consistent KK reduction of $D = 11$ supergravity on T_7 will therefore be the same as the universal KK reduction of $D = 11$ on $HK_4 \times T^3$. Hence the consistent KK reduction on T_7 should lead to a $D = 4$ $N = 4$ gauged supergravity coupled to three vector multiplets. In particular, the scalars should parametrise the coset $SL(2)/SO(2) \times SO(6,3)/(SO(6) \times SO(3))$. As in the examples studied in [12], there should also be a skew-whiffed version of this $N = 4$ gauged supergravity theory where the basic AdS_4 vacuum will break all of the supersymmetry.

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[5]The possibility that the reduced theory will have $N = 4$ supersymmetry was first suggested in [44], using different arguments than we give. Note that a different coset for the scalar manifold was suggested than the one we argue for below.

A Type IIB supergravity conventions

The bosonic sector of IIB supergravity contains the RR forms $F_{(1)}$, $F_{(3)}$, $F_{(5)}$, the NS form $H_{(3)}$, the dilaton Φ and the metric. The forms satisfy the Bianchi identities

$$dF_{(5)} + F_{(3)} \wedge H_{(3)} = 0 \tag{A.1}$$

$$dF_{(3)} + F_{(1)} \wedge H_{(3)} = 0 \tag{A.2}$$

$$dF_{(1)} = 0 \tag{A.3}$$

$$dH_{(3)} = 0 \tag{A.4}$$

which can be solved by introducing potentials as $F_{(5)} = dC_{(4)} - C_{(2)} \wedge H_{(3)}$, $F_{(3)} = dC_{(2)} - C_{(0)} dB_{(2)}$, $F_{(1)} = dC_{(0)}$, $H_{(3)} = dB_{(2)}$.

The equations of motion read:

$$*F_{(5)} = F_{(5)} \tag{A.5}$$

$$d(e^{\Phi} * F_{(3)}) - F_{(5)} \wedge H_{(3)} = 0 \tag{A.6}$$

$$d(e^{2\Phi} * F_{(1)}) + e^{\Phi} H_{(3)} \wedge *F_{(3)} = 0 \tag{A.7}$$

$$d(e^{-\Phi} * H_{(3)}) - e^{\Phi} F_{(1)} \wedge *F_{(3)} - F_{(3)} \wedge F_{(5)} = 0 \tag{A.8}$$

$$d * d\Phi - e^{2\Phi} F_{(1)} \wedge *F_{(1)} + \tfrac{1}{2} e^{-\Phi} H_{(3)} \wedge *H_{(3)} - \tfrac{1}{2} e^{\Phi} F_{(3)} \wedge *F_{(3)} = 0 \tag{A.9}$$

$$\begin{aligned} R_{MN} = {} & \tfrac{1}{2} e^{2\Phi} \nabla_M C_{(0)} \nabla_N C_{(0)} + \tfrac{1}{2} \nabla_M \Phi \nabla_N \Phi + \tfrac{1}{96} F_{M P_1 P_2 P_3 P_4} F_N{}^{P_1 P_2 P_3 P_4} \\ & + \tfrac{1}{4} e^{-\Phi} \left(H_M{}^{P_1 P_2} H_{N P_1 P_2} - \tfrac{1}{12} g_{MN} H^{P_1 P_2 P_3} H_{P_1 P_2 P_3} \right) \\ & + \tfrac{1}{4} e^{\Phi} \left(F_M{}^{P_1 P_2} F_{N P_1 P_2} - \tfrac{1}{12} g_{MN} F^{P_1 P_2 P_3} F_{P_1 P_2 P_3} \right), \end{aligned} \tag{A.10}$$

B Details on the KK reduction

Here we shall provide some details of the KK reduction on SE_5. The calculations for the $HK_4 \times S^1$ case are very similar and we omit the details.

We first record some useful algebraic conditions satisfied by the globally defined forms (J, Ω, η) that specify the $SU(2)$ structure on the SE_5 space. We have $\Omega \wedge \Omega^* = 2J \wedge J$, $\mathrm{vol}(SE_5) = \frac{1}{2} J \wedge J \wedge \eta$, $*J = J \wedge \eta$, $*\Omega = \Omega \wedge \eta$. We also have $J_{ik} J^{jk} = \delta_i^j$, $\Omega_{ik} \Omega^{jk} = 0$, $\Omega_{ik} \Omega^{*jk} = 2\delta_i^j - 2i J_i{}^j$ and $J_{ik} \Omega^{jk} = -i \Omega_i{}^j$. In addition, $J_{[ik} J_{mn]} J^{[jk} J^{mn]} = J_{[ik} J_{mn]} J^{jk} J^{mn} = \frac{2}{3} \delta_j^i$.

The KK ansatz for the metric can be written as

$$ds^2_{10} = ds^2_5 + e^{2U} ds^2(KE_4) + e^{2V} (\eta + A_1) \otimes (\eta + A_1) \tag{B.1}$$

where here ds_5^2 is the line element of the external five-dimensional metric. At the end we will convert our results to the Einstein-frame metric $ds^2_{(E)}$ that we used in the main text. The ansatz for the form field-strengths can be written as

$$
\begin{aligned}
F_{(5)} =& 4e^{-4U-V+Z}\mathrm{vol}_5 + e^{-V} * K_2 \wedge J + K_1 \wedge J \wedge J \\
&+ \left[2e^Z J \wedge J - 2e^{-4U+V} * K_1 + K_2 \wedge J\right] \wedge (\eta + A_1) \\
&+ \left(e^{-V} * L_2 \wedge \Omega + L_2 \wedge \Omega \wedge (\eta + A_1) + c.c.\right) \\
F_{(3)} =& G_3 + G_2 \wedge (\eta + A_1) + G_1 \wedge J + [N_1 \wedge \Omega + N_0 \Omega \wedge (\eta + A_1) + c.c.] \\
H_{(3)} =& H_3 + H_2 \wedge (\eta + A_1) + H_1 \wedge J + [M_1 \wedge \Omega + M_0 \Omega \wedge (\eta + A_1) + c.c.] \\
C_{(0)} =& a \\
\Phi =& \phi
\end{aligned}
\tag{B.2}
$$

Here, vol_5 and $*$ are the volume form and Hodge dual corresponding to the five-dimensional metric ds_5^2 in (3.3). We use a mostly plus metric convention both in $D = 10$ and in $D = 5$ and the $D = 10$ volume form is give by $\epsilon_{10} = e^{4U+V}\mathrm{vol}_5 \wedge \mathrm{vol}(SE_5)$.

We now substitute the KK ansatz (B.1), (B.2) into the type IIB Bianchi equations and equations of motion given in (A.1)–(A.10). We first observe that the ansatz for the five-form has been constructed to be self dual and thus (A.5) is satisified.

Equation (A.4) gives:

$$
\begin{aligned}
&dH_3 + H_2 \wedge F_2 = 0 \\
&dH_2 = 0 \\
&dH_1 + 2H_2 = 0 \\
&DM_1 + M_0 F_2 = 0 \\
&DM_0 - 3iM_1 = 0
\end{aligned}
\tag{B.3}
$$

where $DM_1 \equiv dM_1 - 3iA_1 \wedge M_1$ and $DM_0 \equiv dM_0 - 3iA_1 M_0$.

Equation (A.2) gives

$$
\begin{aligned}
&dG_3 + G_2 \wedge F_2 + da \wedge H_3 = 0 \\
&dG_2 + da \wedge H_2 = 0 \\
&dG_1 + 2G_2 + da \wedge H_1 = 0 \\
&DN_1 + N_0 F_2 + da \wedge M_1 = 0 \\
&DN_0 - 3iN_1 + M_0 da = 0
\end{aligned}
\tag{B.4}
$$

where $DN_1 \equiv dN_1 - 3iA_1 \wedge N_1$ and $DN_0 \equiv dN_0 - 3iA_1 N_0$.

Equation (A.1) gives:

$$
\begin{aligned}
& dK_2 - H_1 \wedge G_2 + H_2 \wedge G_1 = 0 \\
& DL_2 - 3ie^{-V} * L_2 - H_3 N_0 + M_0 G_3 + H_2 \wedge N_1 - M_1 \wedge G_2 = 0 \\
& dK_1 + 2K_2 + 2e^Z F_2 - H_1 \wedge G_1 - 2M_1 \wedge N_1^* - 2M_1^* \wedge N_1 = 0 \\
& de^Z - M_1 N_0^* - M_1^* N_0 + M_0 N_1^* + M_0^* N_1 = 0 \\
& d(e^{-V} * K_2) - 4e^{-4U+V} * K_1 + K_2 \wedge F_2 - H_3 \wedge G_1 - H_1 \wedge G_3 = 0 \\
& D(e^{-V} * L_2) + L_2 \wedge F_2 - H_3 \wedge N_1 - M_1 \wedge G_3 = 0 \\
& d(e^{-4U+V} * K_1) + \tfrac{1}{2} H_3 \wedge G_2 - \tfrac{1}{2} H_2 \wedge G_3 = 0
\end{aligned}
\tag{B.5}
$$

where $DL_2 \equiv dL_2 - 3iA_1 \wedge L_2$

Equation (A.6) gives:

$$
\begin{aligned}
& d(e^{4U+V+\phi} * G_3) - 4e^Z H_3 + 2H_2 \wedge K_1 - 2H_1 \wedge K_2 - 4M_1 \wedge L_2^* - 4M_1^* \wedge L_2 \\
& \qquad +4e^{-V} M_0 * L_2^* + 4e^{-V} M_0^* * L_2 = 0 \\
& d(e^{4U-V+\phi} * G_2) - 4e^{V+\phi} * G_1 - e^{4U+V+\phi} * G_3 \wedge F_2 + 2H_3 \wedge K_1 + 2e^{-V} H_1 \wedge *K_2 \\
& \qquad +4e^{-V} M_1 \wedge *L_2^* + 4e^{-V} M_1^* \wedge *L_2 = 0 \\
& d(e^{V+\phi} * G_1) - H_3 \wedge K_2 + e^{-V} H_2 \wedge *K_2 + 2e^{-4U+V} H_1 \wedge *K_1 = 0 \\
& D(e^{V+\phi} * N_1) - H_3 \wedge L_2 + e^{-V} H_2 \wedge *L_2 + 2e^{-4U+V} M_1 \wedge *K_1 \\
& \qquad +e^{-V} \left(4e^{-4U+Z} M_0 + 3iN_0 e^{\phi}\right) \mathrm{vol}_5 = 0
\end{aligned}
\tag{B.6}
$$

Equation (A.8) gives:

$$
\begin{aligned}
&d(e^{4U+V-\phi} * H_3) + 4e^{Z} G_3 - 2G_2 \wedge K_1 + 2G_1 \wedge K_2 + 4N_1 \wedge L_2^* + 4N_1^* \wedge L_2 \\
&\quad -4e^{-V} N_0 * L_2^* - 4e^{-V} N_0^* * L_2 - e^{4U+V+\phi} da \wedge *G_3 = 0 \\
\\
&d(e^{4U-V-\phi} * H_2) - 4e^{V-\phi} * H_1 - e^{4U+V-\phi} * H_3 \wedge F_2 - 2G_3 \wedge K_1 - 2e^{-V} G_1 \wedge *K_2 \\
&\quad -4e^{-V} N_1 \wedge *L_2^* - 4e^{-V} N_1^* \wedge *L_2 - e^{4U-V+\phi} da \wedge *G_2 = 0 \\
\\
&d(e^{V-\phi} * H_1) + G_3 \wedge K_2 - e^{-V} G_2 \wedge *K_2 - 2e^{-4U+V} G_1 \wedge *K_1 - e^{V+\phi} da \wedge *G_1 = 0 \\
\\
&D(e^{V-\phi} * M_1) + G_3 \wedge L_2 - e^{-V} G_2 \wedge *L_2 - 2e^{-4U+V} N_1 \wedge *K_1 \\
&\quad -e^{-V} \left(4e^{-4U+Z} N_0 - 3iM_0 e^{-\phi}\right) \mathrm{vol}_5 - e^{V+\phi} da \wedge *N_1 = 0
\end{aligned}
\tag{B.7}
$$

Equation (A.7) gives:

$$
\begin{aligned}
&d(e^{4U+V+2\phi} * da) + e^{4U+V+\phi} H_3 \wedge *G_3 + e^{4U-V+\phi} H_2 \wedge *G_2 + 2e^{V+\phi} H_1 \wedge *G_1 \\
&\quad +4e^{V+\phi} M_1 \wedge *N_1^* + 4e^{V+\phi} M_1^* \wedge *N_1 + 4e^{-V+\phi} \left(M_0 N_0^* + M_0^* N_0\right) \mathrm{vol}_5 = 0
\end{aligned}
\tag{B.8}
$$

Equation (A.9) gives:

$$
\begin{aligned}
&d(e^{4U+V} * d\phi) - e^{4U+V+2\phi} da \wedge *da + \tfrac{1}{2} e^{4U+V-\phi} H_3 \wedge *H_3 - \tfrac{1}{2} e^{4U+V+\phi} G_3 \wedge *G_3 \\
&\quad +\tfrac{1}{2} e^{4U-V-\phi} H_2 \wedge *H_2 - \tfrac{1}{2} e^{4U-V+\phi} G_2 \wedge *G_2 + e^{V-\phi} H_1 \wedge *H_1 - e^{V+\phi} G_1 \wedge *G_1 \\
&\quad +4e^{V-\phi} M_1 \wedge *M_1^* - 4e^{V+\phi} N_1 \wedge *N_1^* + 4e^{-V} \left(e^{-\phi} |M_0|^2 - e^{\phi} |N_0|^2\right) \mathrm{vol}_5 = 0
\end{aligned}
\tag{B.9}
$$

Finally, we need to impose the Einstein equation (A.10). To calculate the Ricci tensor we use the orthonormal frame

$$
\begin{aligned}
\bar{e}^{\alpha} &= e^{\alpha}\,, \qquad \alpha = 0, \ldots, 4 \\
\bar{e}^{i} &= e^{U} e^{i}\,, \qquad i = 1, \ldots, 4 \\
\bar{e}^{5} &= e^{V} \hat{e}^{5} \equiv e^{V} (\eta + A_1).
\end{aligned}
\tag{B.10}
$$

We find the spin connection is given by

$$\begin{aligned}
\bar{\omega}^{\alpha\beta} &= \omega^{\alpha\beta} - \tfrac{1}{2}e^{2V}F^{\alpha\beta}\hat{e}^5 \\
\bar{\omega}^{\alpha i} &= -e^{U}\partial^{\alpha}U e^{i} \qquad \text{(B.11)}\\
\bar{\omega}^{\alpha 5} &= -e^{V}\partial^{\alpha}V\hat{e}^5 - \tfrac{1}{2}e^{V}F^{\alpha}{}_{\beta}e^{\beta} \\
\bar{\omega}^{ij} &= \omega^{ij} - e^{2V-2U}J^{ij}\hat{e}^5 \\
\bar{\omega}^{i5} &= -e^{V-U}J^{i}{}_{j}e^{j} \qquad \text{(B.12)}
\end{aligned}$$

and the Riemann tensor, $\bar{\Theta}^{AB} = d\bar{\omega}^{AB} + \bar{\omega}^{A}{}_{C}\wedge\bar{\omega}^{CB}$, by:

$$\begin{aligned}
\bar{\Theta}^{\alpha\beta} =& \Theta^{\alpha\beta} - \tfrac{1}{4}e^{2V}\left[F^{\alpha\beta}F_{\lambda\mu} + F^{\alpha}{}_{[\lambda}F^{\beta}{}_{\mu]}\right]\bar{e}^{\lambda\mu} - \left[\tfrac{1}{2}e^{-V}\nabla_{\lambda}(e^{2V}F^{\alpha\beta}) + e^{V}\left(F^{[\alpha}{}_{\lambda}\nabla^{\beta]}V\right)\right]\bar{e}^{\lambda 5} \\
\bar{\Theta}^{\alpha i} =& -\left[(\nabla_{\lambda}\nabla^{\alpha}U + \nabla_{\lambda}U\nabla^{\alpha}U)\delta^{i}_{j} + \tfrac{1}{2}e^{-2U+2V}J^{i}{}_{j}F^{\alpha}{}_{\lambda}\right]\bar{e}^{\lambda j} \\
&+ \left[e^{-2U+V}\nabla^{\alpha}(V-U)J^{i}{}_{j} - \tfrac{1}{2}e^{V}F^{\alpha\gamma}\nabla_{\gamma}U\delta^{i}_{j}\right]\bar{e}^{j5} \\
\bar{\Theta}^{\alpha 5} =& -\tfrac{1}{2}\left[\nabla_{\lambda}(e^{V}F^{\alpha}{}_{\mu}) + e^{V}\nabla^{\alpha}VF_{\lambda\mu}\right]\bar{e}^{\lambda\mu} - \left[\nabla_{\lambda}\nabla^{\alpha}V + \nabla_{\lambda}V\nabla^{\alpha}V + \tfrac{1}{4}e^{2V}F^{\alpha\gamma}F_{\gamma\lambda}\right]\bar{e}^{\lambda 5} \\
&- e^{-2U+V}\nabla^{\alpha}(V-U)J_{ij}\bar{e}^{ij} \\
\bar{\Theta}^{ij} =& \Theta^{ij} - \tfrac{1}{2}e^{-2U+2V}F_{\alpha\beta}J^{ij}\bar{e}^{\alpha\beta} - e^{-2U+V}\nabla_{\alpha}(V-U)J^{ij}\bar{e}^{\alpha 5} \\
&- \left[e^{-4U+2V}(J^{ij}J_{hk} + J^{i}{}_{[h}J^{j}{}_{k]}) + \nabla_{\gamma}U\nabla^{\gamma}U\delta^{i}_{[h}\delta^{j}_{k]}\right]\bar{e}^{hk} - e^{-3U+V}\nabla_{k}J^{ij}\bar{e}^{k5} \\
\bar{\Theta}^{i5} =& \left[-e^{-2U+V}\nabla_{\alpha}(V-U)J^{i}{}_{j} + \tfrac{1}{2}e^{V}F_{\alpha\gamma}\nabla^{\gamma}U\delta^{i}_{j}\right]\bar{e}^{\alpha j} + \left[e^{-4U+2V} - \nabla_{\gamma}U\nabla^{\gamma}V\right]\delta^{i}_{j}\bar{e}^{j5}
\end{aligned} \qquad \text{(B.13)}$$

Finally the Ricci tensor, $\bar{R}^{A}{}_{B} = \bar{\Theta}^{AC}{}_{BC}$, is given by

$$\begin{aligned}
\bar{R}_{\alpha\beta} &= R_{\alpha\beta} - 4\left(\nabla_{\beta}\nabla_{\alpha}U + \partial_{\alpha}U\partial_{\beta}U\right) - \left(\nabla_{\beta}\nabla_{\alpha}V + \partial_{\alpha}V\partial_{\beta}V\right) - \tfrac{1}{2}e^{2V}F_{\alpha\gamma}F_{\beta}{}^{\gamma} \\
\bar{R}_{\alpha i} &= 0 \\
\bar{R}_{\alpha 5} &= -\tfrac{1}{2}e^{-2V-4U}\nabla_{\gamma}\left(e^{3V+4U}F^{\gamma\alpha}\right) \\
\bar{R}_{ij} &= \delta_{ij}\left[6e^{-2U} - 2e^{2V-4U} - \nabla_{\gamma}\nabla^{\gamma}U - 4\partial_{\gamma}U\partial^{\gamma}U - \partial_{\gamma}U\partial^{\gamma}V\right] \\
\bar{R}_{i5} &= 0 \\
\bar{R}_{55} &= 4e^{2V-4U} - \nabla_{\gamma}\nabla^{\gamma}V - 4\partial_{\gamma}U\partial^{\gamma}V - \partial_{\gamma}V\partial^{\gamma}V + \tfrac{1}{4}e^{2V}F_{\alpha\beta}F^{\alpha\beta} \qquad \text{(B.14)}
\end{aligned}$$

The Einstein equations (A.10) now reduce to the following four equations in $D =$

5:

$$
\begin{aligned}
R_{\alpha\beta} = & \, 4\left(\nabla_\beta\nabla_\alpha U + \partial_\alpha U \partial_\beta U\right) + \left(\nabla_\beta\nabla_\alpha V + \partial_\alpha V \partial_\beta V\right) + \tfrac{1}{2}e^{2\phi}\partial_\alpha a \partial_\beta a + \tfrac{1}{2}\partial_\alpha\phi\partial_\beta\phi \\
& -e^{-4U-2V}\left(4e^{-4U+2Z} + e^{-\phi}|M_0|^2 + e^{\phi}|N_0|^2\right)\eta_{\alpha\beta} \\
& +2e^{-8U}\left(K_\alpha K_\beta - \tfrac{1}{2}\eta_{\alpha\beta}K_\lambda K^\lambda\right) + e^{-4U-2V}\left(K_{\alpha\lambda}K_\beta{}^\lambda - \tfrac{1}{4}\eta_{\alpha\beta}K_{\lambda\mu}K^{\lambda\mu}\right) \\
& +\tfrac{1}{2}e^{2V}F_{\alpha\gamma}F_\beta{}^\gamma + 4e^{-4U-2V}\left(-L_{\lambda(\alpha}L^*_{\beta)}{}^\lambda - \tfrac{1}{4}\eta_{\alpha\beta}L^*_{\lambda\mu}L^{\lambda\mu}\right) \\
& +\tfrac{1}{4}e^{-\phi}\left(H_{\alpha\lambda\mu}H_\beta{}^{\lambda\mu} - \tfrac{1}{12}\eta_{\alpha\beta}H_{\lambda\mu\nu}H^{\lambda\mu\nu}\right) + \tfrac{1}{2}e^{-2V-\phi}\left(H_{\alpha\lambda}H_\beta{}^\lambda - \tfrac{1}{8}\eta_{\alpha\beta}H_{\lambda\mu}H^{\lambda\mu}\right) \\
& +e^{-4U-\phi}\left(H_\alpha H_\beta - \tfrac{1}{4}\eta_{\alpha\beta}H_\lambda H^\lambda\right) + 4e^{-4U-\phi}\left(M_{(\alpha}M^*_{\beta)} - \tfrac{1}{4}\eta_{\alpha\beta}M^*_\lambda M^\lambda\right) \\
& +\tfrac{1}{4}e^{\phi}\left(G_{\alpha\lambda\mu}G_\beta{}^{\lambda\mu} - \tfrac{1}{12}\eta_{\alpha\beta}G_{\lambda\mu\nu}G^{\lambda\mu\nu}\right) + \tfrac{1}{2}e^{-2V+\phi}\left(G_{\alpha\lambda}G_\beta{}^\lambda - \tfrac{1}{8}\eta_{\alpha\beta}G_{\lambda\mu}G^{\lambda\mu}\right) \\
& +e^{-4U+\phi}\left(G_\alpha G_\beta - \tfrac{1}{4}\eta_{\alpha\beta}G_\lambda G^\lambda\right) + 4e^{-4U+\phi}\left(N_{(\alpha}N^*_{\beta)} - \tfrac{1}{4}\eta_{\alpha\beta}N^*_\lambda N^\lambda\right) \qquad \text{(B.15)}
\end{aligned}
$$

$$
\begin{aligned}
& d(e^{4U+3V} * F_2) + K_2\wedge K_2 + 4L_2\wedge L_2^* - 8e^{-4U+V+Z} * K_1 - e^{4U+V-\phi}H_2\wedge *H_3 \\
& \quad -e^{4U+V+\phi}G_2\wedge *G_3 - 4e^{V-\phi}(M_0^* * M_1 + M_0 * M_1^*) \\
& \quad -4e^{V+\phi}(N_0^* * N_1 + N_0 * N_1^*) = 0 \qquad \text{(B.16)}
\end{aligned}
$$

$$
\begin{aligned}
& d(e^{4U+V} * dU) + e^{-4U+V}K_1\wedge *K_1 - \tfrac{1}{8}e^{4U+V-\phi}H_3\wedge *H_3 - \tfrac{1}{8}e^{4U+V+\phi}G_3\wedge *G_3 \\
& \quad -\tfrac{1}{8}e^{4U-V-\phi}H_2\wedge *H_2 - \tfrac{1}{8}e^{4U-V+\phi}G_2\wedge *G_2 + \tfrac{1}{4}e^{V-\phi}H_1\wedge *H_1 + \tfrac{1}{4}e^{V+\phi}G_1\wedge *G_1 \\
& \quad +e^{V-\phi}M_1\wedge *M_1^* + e^{V+\phi}N_1\wedge *N_1^* \\
& \quad +\left(-6e^{2U+V} + 2e^{3V} + 4e^{-4U-V+2Z} + e^{-V-\phi}|M_0|^2 + e^{-V+\phi}|N_0|^2\right)\mathrm{vol}_5 = 0 \\
& \qquad \text{(B.17)}
\end{aligned}
$$

$$
\begin{aligned}
& d(e^{4U+V} * dV) - \tfrac{1}{8}e^{4U+V-\phi}H_3\wedge *H_3 - \tfrac{1}{8}e^{4U+V+\phi}G_3\wedge *G_3 - \tfrac{1}{2}e^{4U+3V}F_2\wedge *F_2 \\
& \quad -e^{-4U+V}K_1\wedge *K_1 + \tfrac{1}{2}e^{-V}K_2\wedge *K_2 + 2e^{-V}L_2\wedge *L_2^* + \tfrac{3}{8}e^{4U-V-\phi}H_2\wedge *H_2 \\
& \quad +\tfrac{3}{8}e^{4U-V+\phi}G_2\wedge *G_2 - \tfrac{1}{4}e^{V-\phi}H_1\wedge *H_1 - \tfrac{1}{4}e^{V+\phi}G_1\wedge *G_1 - e^{V-\phi}M_1\wedge *M_1^* \\
& \quad -e^{V+\phi}N_1\wedge *N_1^* + \left(-4e^{3V} + 4e^{-4U-V+2Z} + 3e^{-V-\phi}|M_0|^2 + 3e^{-V+\phi}|N_0|^2\right)\mathrm{vol}_5 = 0 \\
& \qquad \text{(B.18)}
\end{aligned}
$$

All the dependence on the internal SE_5 has dropped out from the type IIB equations of motion. This proves the consistency of the KK ansatz (B.1), (B.2). The Lagrangian that gives rise to the above equations of motion is given by

$$
\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{top}} \qquad \text{(B.19)}
$$

with

$$
\begin{aligned}
\mathcal{L}_{\rm kin} = {}& e^{4U+V} R \ {\rm vol}_5 + e^{4U+V} \left(12 dU\wedge *dU + 8 dU \wedge * dV\right) - \tfrac{1}{2} e^{4U+V+2\phi} da\wedge *da \\
& -\tfrac{1}{2} e^{4U+V} d\phi \wedge * d\phi - 4 e^{V-\phi} M_1 \wedge * M_1^* - 4 e^{V+\phi} N_1 \wedge * N_1^* - 2 e^{-4U+V} K_1 \wedge * K_1 \\
& - e^{V-\phi} H_1 \wedge * H_1 - e^{V+\phi} G_1 \wedge * G_1 - \tfrac{1}{2} e^{4U+3V} F_2 \wedge * F_2 \\
& - e^{-V} K_2 \wedge * K_2 - 4 e^{-V} L_2 \wedge * L_2^* - \tfrac{1}{2} e^{4U-V-\phi} H_2 \wedge * H_2 \\
& -\tfrac{1}{2} e^{4U-V+\phi} G_2 \wedge * G_2 - \tfrac{1}{2} e^{4U+V-\phi} H_3 \wedge * H_3 - \tfrac{1}{2} e^{4U+V+\phi} G_3 \wedge * G_3
\end{aligned} \tag{B.20}
$$

and

$$
\begin{aligned}
\mathcal{L}_{\rm pot} = {}& \Big[24 e^{2U+V} - 4 e^{3V} - 8 e^{-4U-V} \left(1 + \tfrac{i}{3}(M_0^* N_0 - M_0 N_0^*)\right)^2 \\
& -4 e^{-V-\phi} |M_0|^2 - 4 e^{-V+\phi} |N_0|^2 \Big] {\rm vol}_5 \\
= {}& \Big[24 e^{2U+V} - 4 e^{3V} - 8 e^{-4U-V} \left(1 + 3i(\xi^* \chi - \xi \chi^*)\right)^2 \\
& -36 e^{-V-\phi} |\xi|^2 - 36 e^{-V+\phi} |\chi - a\xi|^2 \Big] {\rm vol}_5
\end{aligned} \tag{B.21}
$$

and $\mathcal{L}_{\rm top}$ is given in (3.12). The Einstein frame Lagrangian can be obtained by the change of metric $g_{\mu\nu}^{(E)} = e^{\frac{2}{3}(4U+V)} g_{\mu\nu}$, to obtain

$$
\mathcal{L}^{(E)} = \mathcal{L}^{(E)}_{\rm kin} + \mathcal{L}^{(E)}_{\rm pot} + \mathcal{L}_{\rm top} \ , \tag{B.22}
$$

where $\mathcal{L}^{(E)}_{\rm kin}$ and $\mathcal{L}^{(E)}_{\rm pot}$ are given in (3.10) and (3.11), respectively, and $\mathcal{L}_{\rm top}$ is unchanged.

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