Understanding the "anti-kick" in the merger of binary black holes

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The generation of a large recoil velocity from the inspiral and merger of binary black holes represents one of the most exciting results of numerical-relativity calculations. While many aspects of this process have been investigated and explained, the *"anti-kick"*, namely the sudden deceleration after the merger, has not yet found a simple explanation. We show that the anti-kick can be easily understood in terms of the radiation from a deformed black hole where the intrinsically *anisotropic* curvature distribution on the horizon determines the direction and intensity of the recoil. Our analysis is focussed on the properties of Robinson-Trautman spacetimes and allows us to measure both the energies and momenta radiated in a gauge-invariant manner. At the same time, this simpler setup provides all the qualitative but also quantitative features of inspiralling black hole binaries, thus opening the way to a deeper understanding of the nonlinear dynamics of black-hole spacetimes.

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Introduction. The merger of two black holes (BHs) is one of the most important sources of gravitational waves (GW) and this is generally accompanied by the recoil of the final BH as a result of anisotropic GW emission. While this scenario has been investigated for decades [1] and first estimates have been made using approximated and semi-analytical methods such as a particle approximation [2], post-Newtonian methods [3] and the close-limit approximation (CLA) [4], it is only thanks to the recent progress in numerical relativity that accurate values for the recoil velocity have been computed [5–12].

Besides being a genuine nonlinear effect of general relativity, the generation of a large recoil velocity during the merger of two BHs has a direct impact in astrophysics. Depending on its size and its variation with the mass ratio and spin, in fact, it can play an important role in the growth of supermassive BHs via mergers of galaxies and on the number of galaxies containing BHs [13]. Numerical-relativity simulations of BHs inspiralling on quasi-circular orbits have already revealed many of the most important features of this process showing, for instance, that asymmetries in the mass can lead to recoil velocities $v_{\rm k} \lesssim 175\,{\rm km/h}$ [5, 6], while asymmetries in the spins can lead respectively to $v_{\rm k} \lesssim 450\,{\rm km/h}$ or $v_{\rm k} \lesssim 4000\,{\rm km/h}$ if the spins are aligned [8, 9, 11] or perpendicular to the orbital angular momentum [7, 14, 15] (see [16] for a review).

At the same time, however, there are a number of aspects of the nonlinear processes leading to the recoil that are far from being clarified even though interesting work has been recently carried out to investigate such aspects [17–19]. One of these features, and possibly the most puzzling one, is the generic presence of an *"anti-kick"*, namely, of one (or more) decelerations experienced by the recoiling BH. Such anti-kicks take place after a single apparent horizon (AH) has been found and have been reported in essentially all of the mergers simulated so far (see Fig. 8 of ref. [11] for some examples).

This paper is dedicated to elucidate the stages during which the anti-kick is generated and to provide a simple and qualitative interpretation of the physics underlying this process. Our focus will be on the head-on collision of two nonspinning BHs with different mass and although this is the simplest scenario for a BH-merger, it contains all the important aspects that can be encountered in more generic conditions. Our qualitative picture will then be made quantitative and gauge-invariant by studying the logical equivalent of this process in the evolution of a Robinson-Trautman (RT) spacetime, with measurements of the recoil made at future null infinity. As we will comment below, the insight gained with the RT evolution will be precious to explain the anti-kick under generic conditions and to contribute to the understanding of nonlinear BH physics.

The basic picture. Before discussing how to use the RT spacetime to compute the anti-kick, it is useful to illustrate the basic BH physics leading to such process and for this we can consider the simple head-on collision of two Schwarzschild BHs with unequal masses. This is shown in a schematic cartoon in Fig. 1, where we have considered a coordinate system centred in the total centre of mass of the system and where the smaller black hole is initially on the positive z-axis, while the larger one is on the negative axis. As the two BHs free-fall towards each other, the smaller one will move faster and will be more efficient in "forward-beaming" its GW emission [3]. As a result, the linear momentum will be radiated mostly downwards, thus leading to an upwards recoil of the BH binary [*cf.* stage (1) in Fig. 1]. At the merger the BH velocities will be the largest and so will also be the anisotropic GW emission and the corresponding recoil of the system. However, when a single AH is formed comprising the two BHs, the curvature distribution on this 2-surface will be highly *anisotropic*, being higher in the upper hemisphere (*cf.* red-blue shading in stage (2) of Fig. 1). Because the newly formed BH will want to radiate all of its deviations away from the final Schwarzschild configuration, it will do so more effectively there where the curvature is larger, thus with a stronger emission of GWs from the northern hemisphere. As a result, after the merger the linear momentum will be emitted mostly upwards and this sudden sign change will lead to the anti-kick. The anisotropic GW emission will decay exponentially as the curvature gradients are erased and the quiescent BH will have reached its final and decelerated recoil velocity [*cf.* stage (3)].

Although this picture refers to a head-on collision, it is sup-

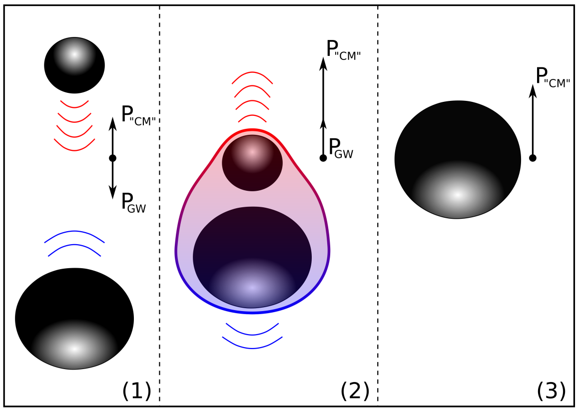

FIG. 1: Cartoon of the generation of the anti-kick in the head-on collision of two unequal-mass Schwarzschild BHs. Initially the smaller BH moves faster and linear momentum is radiated mostly downwards, thus leading to an upwards recoil of the system [stage (1)]. At the merger the curvature is higher in the upper hemisphere of the distorted BH (*cf.* red-blue shading) and linear momentum is radiated mostly upwards leading to the anti-kick [stage (2)]. The BH decelerates till a uniform curvature is restored on the horizon [stage (3)].

ported by the findings in the CLA (where the direction of the ringdown kick is approximately opposite to that of the accumulated inspiral plus plunge kick) [17] and it is straightforward to generalize it to a situation in which the BHs have different masses, different spins and are merging through an inspiral. Also in a more generic case, in fact, the newly formed AH will have a complicated but overall anisotropic distribution of the curvature, which will determine the direction (which is in general varying in time) along which the GWs will be emitted. We next discuss how to use the RT spacetime to validate, in a gauge-invariant manner, this basic picture.

The Robinson Trautman Spacetime. The RT spacetime is the class of solutions of the vacuum Einstein equations admitting a congruence of null geodesics which are hypersurface orthogonal, shear-free but with expansion. As such, it can be regarded as an isolated nonspherical BH emitting GWs, thus representing a valuable tool for studying in a fully gauge-invariant manner, physical conditions that are similar to the final stages on the dynamics of BH binaries [20].

In a coordinate system $\{u, r, \theta, \phi\}$, the RT metric is [21]

$$ds^2 = -\left(K - \frac{2M_\infty}{r} - \frac{2r\partial_u Q}{Q}\right)du^2 - 2du dr + \frac{r^2}{Q^2}d\Omega^2, \tag{1}$$

where $Q = Q(u,\Omega)$, u is the standard null coordinate, r is the affine parameter of the outgoing null geodesics, and $\Omega = \{\theta, \phi\}$ are the usual angular coordinates on the unit sphere S^2. Here M_∞ is a constant and is related to the mass of the asymptotic Schwarzschild BH, while the function $K(u,\Omega)$ is the Gaussian curvature of the surface corresponding to $r = 1$ and $u =$ constant, and is given by

$$K(u,\Omega) \equiv Q^2(1 + \nabla^2_\Omega \ln Q), \tag{2}$$

∇^2_Ω being the Laplacian operator on S^2. The Einstein equations then lead to the "RT" evolution equation

$$\partial_u Q(u,\Omega) = -Q^3\nabla^2_\Omega K(u,\Omega)/(12M_\infty). \tag{3}$$

Any regular initial data $Q = Q(0,\Omega)$ will smoothly evolve according to (3) until it achieves a stationary configuration corresponding to a Schwarzschild BH at rest or moving with a constant speed [22]. Eq. (3) implies the existence of the constant of motion $\mathcal{A} \equiv \int_{S^2} d\Omega/Q^2$, which clearly represents the area of the surface $u, r =$ constant and can be used to normalise Q so that $\mathcal{A} = 4\pi$. All the physically relevant information is contained in the function $Q(u,\Omega)$, and this includes the gravitational radiation, which can be extracted by relating $Q(u,\Omega)$ to the radiative part of the Riemann tensor [23, 24].

RT spacetimes have a past AH with a vanishing expansion of the ingoing future-directed null geodesics. Such past AH is described by the function $R(u,\Omega)$ satisfying [25, 26]

$$Q^2\nabla^2_\Omega \ln R = K - 2M_\infty/R. \tag{4}$$

The mass and momentum of the BH are computed at future null infinity using the Bondi 4-momentum [21]

$$P^\alpha(u) \equiv \frac{M_\infty}{4\pi}\int_{S^2}\frac{\eta^\alpha}{Q^3}d\Omega, \tag{5}$$

with $\{\eta^\alpha\} = \{1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\}$. Given smooth initial data, the spacetime will evolve to a stationary non-radiative solution which, in axisymmetry, has the form [21]

$$Q(\infty,\theta) = (1 \mp vx)/\sqrt{1-v^2}, \tag{6}$$

with $x \equiv \cos\theta$. The Bondi 4-momentum associated to (6) is

$$\{P(\infty)\}^\alpha = \left(M_\infty/\sqrt{1-v^2}\right)\{1, 0, 0, \pm v\}, \tag{7}$$

so that the parameter v in (6) can be interpreted as the velocity of the Schwarzschild BH in the z-direction.

One of the difficulties with RT spacetimes is the definition of physically meaningful initial data. Although we are more interested in a proof-of-principle than in describing a realistic configuration, we have adopted the prescription in [24]

$$Q(0,\theta) = Q_0\left[\frac{1}{\sqrt{1-wx}} + \frac{q}{\sqrt{1+wx}}\right]^{-2}, \tag{8}$$

and which was interpreted to represent the final stages (*i.e.* after a common AH is formed) of a head-on collision of two boosted BHs with opposite velocities w and mass ratio q [24]. In practice, to reproduce the situation shown in Fig. 1, we have set $w < 0$ and taken $q \in [0,1]$; a more general class of initial data and the corresponding phenomenology will be presented in a longer companion paper [27]. Note that Q_0 is chosen so that to $\mathcal{A} = 4\pi$ and that in general the deformed BH will not be initially at rest. As a result, given the initial velocity $v_0 \equiv P^3(0)/P^0(0)$, we perform a boost transformation $\overline{P}^\alpha = \Lambda^\alpha_\beta(v_0)P^\beta$ so that $\overline{P}^3(0) = 0$ by construction. The numerical solution of eq. (3) with initial data (8) is performed using a Galerkin decomposition as discussed in detail in [21].

Discussion. Figure 2 reports the typical evolution of a RT spacetime with the lower panel showing the evolution of the curvature of the past horizon $K_{\rm AH} \equiv 2M_\infty/R^3(x)$ at the north ($x = 1$) and south pole ($x = -1$), and with the upper

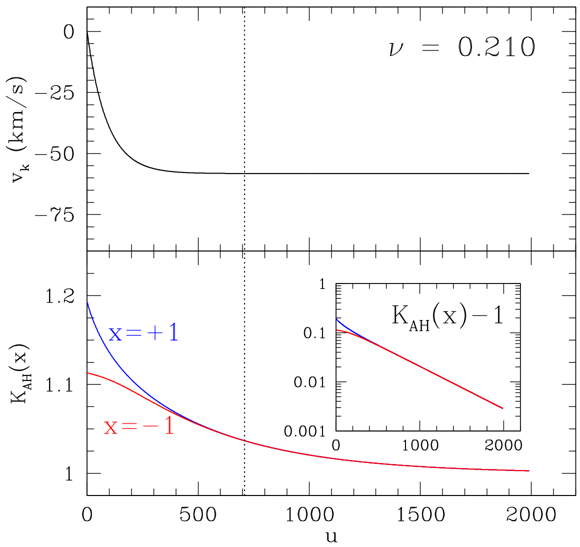

FIG. 2: Typical evolution of a RT spacetime. Shown in the lower panel is the evolution of the curvature $K_{\rm AH}$ at the north ($x = 1$) and south pole ($x = -1$). Shown in the upper panel is the evolution of the recoil, which stops decreasing when the curvature difference is erased by the emitted radiation (dotted line). Note that the curvature decays exponentially to that of a Schwarzschild BH (inset).

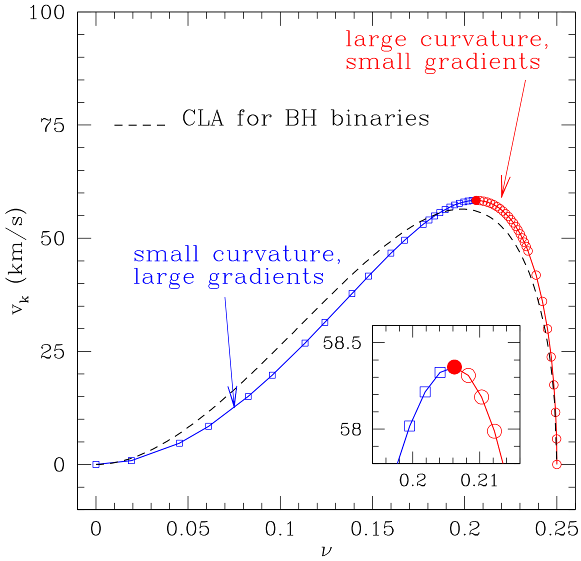

FIG. 3: Recoil velocity shown as a function of the symmetric mass ratio ν when $w = -0.425$, with the dashed line refers to the anti-kick from the inspiral of nonspinning binaries in the CLA [17]. Note that the curve can be thought as composed of two different branches.

panel showing the evolution of the recoil velocity. Note that the two local curvatures are different initially, with the one in the upper hemisphere being larger than the on in the lower hemisphere (*cf.* Fig. 1). However, as the gravitational radiation is emitted, this difference is erased. When this happens, the deceleration stops and the BH attains its asymptotic recoil velocity. The inset reports the curvature difference relative to the asymptotic Schwarzschild one, $K_{\rm AH} - 1$, whose exponentially decaying behaviour is the one expected in a ringing BH.

As mentioned before, that shown in Fig. 2 is a typical evolution of a RT spacetime and is not specific of the initial data (8) [27]. By varying the values of w, in fact, it is possible to increase or decreases the final recoil and a sign change in w simply inverts the curvature at the poles so that, for instance, initial data with $w > 0$ would yield a BH accelerating in the positive z-direction. Interestingly, it is even possible to fine-tune the parameter w so that the recoil produced for a RT spacetime mimics that produced by the quasi-circular inspiral of nonspinning binaries. This is shown in Fig. 3, which reports the recoil for $w = -0.425$ as a function of the symmetric mass ratio $\nu \equiv q/(1+q)^2$, and where the dashed line refers to the anti-kick for the inspiral of nonspinning binaries in the CLA with $r_{12} = 2\,M$ [17]. Considering that the two curves are related only logically and that the CLA one contains all the information about inspiralling BHs, including the orbital rotation, the match is surprisingly good.

It is also suggestive to think that the curve in Fig. 3 is actually composed of two different branches, one of which is characterized by large curvature gradients across the AH but small values of the curvature (this is the low-ν branch and is indicated with squares), while the other is characterized by small curvature gradients and large values of the curvature (this is the high-ν branch and is indicated with circles). Recalling that in electromagnetism the same electric current can be produced in two circuits having different potential jumps and resistances, it is then plausible that the same recoil velocity can be produced by two different values of ν, for which the effects of large curvature gradients and small local curvatures are the same as those produced by small curvature gradients but large local curvatures. When cast in these terms, the otherwise curious result that two different mass ratios should lead to the same recoil, becomes instead rather obvious.

To go from this intuition to a mathematically well-defined measure we have computed the mass multipoles of the intrinsic curvature of the initial data using the formalism developed in [28] for dynamical horizons. Namely, we compute the mass moments as (the mass-current are obviously zero)

$$M_n \equiv \oint \frac{P_n(\tilde{x})}{Q^2(\theta)R(\theta)} d\Omega, \tag{9}$$

where $P_n(\tilde{x})$ is the Legendre polynomial in terms of the coordinate $\tilde{x}(\theta)$ which obeys $\partial_\theta \tilde{x} = -\sin\theta R(\theta)^2/(R^2_{\rm AH} Q(\theta)^2)$, with $R_{\rm AH} \equiv \sqrt{\mathcal{A}_{\rm AH}/(4\pi)}$ and $\tilde{x}(0) = 1$ [27]. Using these multipoles it is possible to construct an effective curvature number $K_{\rm eff}$ that represents a measure of the global curvature properties of the initial data and from which the recoil depends in an injective way. Recognisizing even and odd multipoles in (9), respectively, as capturing geometrically the resistance and the potential jumps in the electromagnetic analogy built around Fig. 3, we have computed an effective curvature $K_{\rm eff} = M_2|\sum_{n=1} M_{2n+1}/3^{n-1}|$, which reproduces exactly what expected (note $M_1 = 0$ to machine precision). This is shown in Fig. 4, which reports the recoil velocity as a function of $K_{\rm eff}$. As predicted, and in contrast with Fig. 3, the relation between the curvature and the recoil is now injective, with the maximum recoil velocity being given by the maximum value

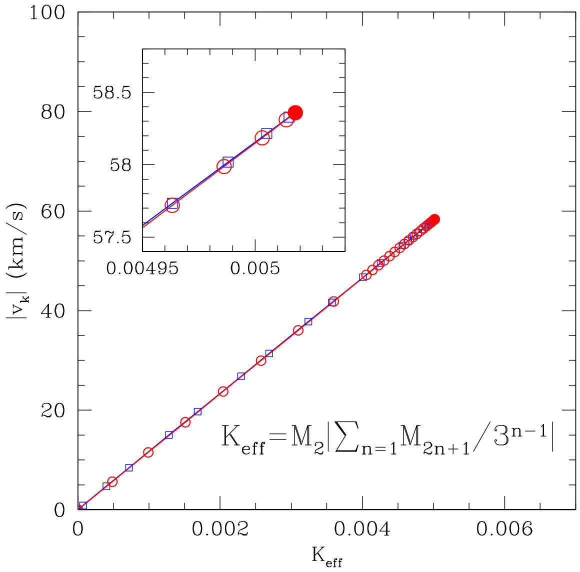

FIG. 4: Recoil velocity shown as a function of the effective curvature. In contrast with Fig. 3, which uses the same symbols employed here, the relation between the curvature and the recoil is now injective.

of $K_{\rm eff}$ (see inset), and with the two branches coinciding. We do not expect the expression found here for $K_{\rm eff}$ to be unique and indeed a more generic one will have to include also the mass-current multipoles to account for the spin contributions. However, lacking a rigorous mathematical guidance, our phenomenological $K_{\rm eff}$ is a reasonable, intuitive approximation.

Conclusions. We have outlined a simple picture to explain the deceleration observed during the merger of binary BHs in terms of the dissipation of an anisotropic distribution of curvature on the horizon of the newly formed BH. We have analyzed this picture for the head-on collision of two nonspinning BHs with unequal mass but its extension to generic systems is direct as the same features will be present also when including the spin and the orbital contributions: mass current multipoles will either add/substract in prograde/retrograde orbits. The qualitative arguments made on the head-on collision have then been made quantitative by analyzing the gauge-independent dynamics of RT spacetimes. More specifically we have shown that the deceleration is associated to the radiation of curvature differences and persists as long as the gradients are not erased. Furthermore, the directionality of the recoil is dictated by the north-south curvature gradients and a one-to-one mapping between the recoil and an effective curvature is possible. These results presented here can help in understanding the nonlinear of curved spacetimes and will be extended to include more generic initial data and a comparison with simulations [27].

We finally note that an alternative interpretation of the recoil phenomenology can be given in terms of the momentum flux coming from the Landau-Lifshitz pseudotensor [29], where the recoil is the result of the cancellation of large and opposite fluxes of momentum, part of which are "swallowed" by the BH. While this interesting route may help in exploring the dynamics of BHs, it also relies on gauge-dependent measurements which may themselves be counter-intuitive.

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